Turbulent Small-scale Dynamo (SSD) Action in Solar Surface Simulations

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Outline

1. Motivation: intra-network quiet-Sun B

2. Small-scale dynamo (SSD) in realistic photospheric simulations?
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1. Motivation: intra-network quiet-Sun B
   - Observed small-scale B
   - A lot of magnetic flux and energy
   - What is its source?

2. Small-scale dynamo (SSD) in realistic photospheric simulations?
   - Which realistic dynamo simulations?
   - What is a turbulent SSD and ...
   - ...how do we tell?
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Observed small-scale B
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Solar overview

http://www.ece.unm.edu/plasma/Space/solar.html
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Sunspot cycle

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

http://science.msfc.nasa.gov/sti/put/solar/images/hfdavr.gif

AVERAGE DAILY SUNSPOT AREA (% OF VISIBLE HEMISPHERE)

Solar global dynamo

- 22 year cycle
- ≈ dipolar
- many models (Babcock-Leighton, flux-transport (Dikpati et al.), surface shear (Brandenburg 2005))
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A lot of magnetic flux and energy
What is its source?

Network quiet-Sun magnetic field

Lites et al. 2008

$3\sigma < |B_z| < 500 \text{ G}$

$\langle |B_z| \rangle = 12 \text{ G}$

$\sigma = 2.4 \text{ G}$
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A lot of magnetic flux and energy
What is its source?

Intra-network quiet-Sun magnetic field

Lites et al. 2008

$3\sigma < |B_z| < 24 \text{ G}$

$\langle |B_z| \rangle = 12 \text{ G}$

$\sigma = 2.4 \text{ G}$

Magnetic carpet (Title and Schrijver 1998; Title 2000; Hagenaar et al. 2003)
Motivation: intra-network quiet-Sun B
Small-scale dynamo (SSD) in realistic photospheric simulations?

Cancellation is self-similar

Hinode observation

Portion of flux remaining after degrading resolution
\[ \propto \text{resolution}^{-\kappa} \]

\[ \kappa = 0.26 \pm 0.01 \]
Motivation: intra-network quiet-Sun B
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Cancellation is self-similar

What it tells us
- Size of magnetic structures \( \leq 20 \text{ km} \)
- Resolution-independent, true \( \langle |B_Z| \rangle \geq 40 \text{ G} \)

Portion of flux remaining after degrading resolution
\( \propto \text{resolution}^{-\kappa} \)

Hinode observation

\[ \kappa = 0.26 \pm 0.01 \]
A lot of small-scale flux

- $\langle |B_\perp| \rangle > 40 \ G$
- $\langle |B| \rangle \sim 100 \ G$ Hanle (Trujillo Bueno et al. 2004) @ 90% surface area
- More unsigned magnetic flux and energy than active regions even during solar maximum (Sánchez Almeida 2004, Trujillo Bueno et al. 2004)
- 10 times energy to heat chromosphere and corona (Trujillo Bueno et al. 2004)
Where does all this small-scale flux come from?

Turbulent small-scale dynamo (SSD)?

- More than active regions
- Petrovay & Szakaly (1993): not decay of active regions nor flux tubes → need source term: SSD?
Where does all this small-scale flux come from?

Other possibilities

- “Turbulent cascade” to small scales of large-scale $B$
- Turbulent fluid motions stretching large-scale $B_0$:
  Alfvénic turbulent induction (Schekochihin et al. 2007)
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The MURaM code (Vögler et al. 2005; Vögler 2003)

Realistic magnetoconvection
- Strong stratification
- Fully compressible
- Partial ionization
- Radiative transfer
- Open lower boundary
  (vertical upflows, \( \frac{\partial v}{\partial z} = 0 \) for downflows; \( B_{\text{hor}} \) not advected into box)
- No rotation
- Parallelized

\( B_z \) & brightness
Motivation: intra-network quiet-Sun B
Small-scale dynamo (SSD) in realistic photospheric simulations?

Which realistic dynamo simulations?
What is a turbulent SSD and ... 
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The MURaM dynamo (Vögler & Schüssler 2007)

![Graph showing magnetic energy over time with different Re_M values: Re_M = 2600, Re_M = 1300, Re_M = 300.]

![Graph showing kinetic and magnetic energy density with k_n in Mm^{-1}.

<table>
<thead>
<tr>
<th>Run</th>
<th>N_{hor}^2 \times N_Z</th>
<th>Re_M</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>648^2 \times 140</td>
<td>2600</td>
</tr>
</tbody>
</table>
Motivation: intra-network quiet-Sun B

Small-scale dynamo (SSD) in realistic photospheric simulations?

Our simulations

<table>
<thead>
<tr>
<th>Run</th>
<th>Grid (km)</th>
<th>$P_{M,\text{eff}}$</th>
<th>$\eta$</th>
<th>$Re_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$9^2 \times 10$</td>
<td>$\sim 2.0$</td>
<td>$1.6 \cdot 10^{10}$</td>
<td>$\approx 2100$</td>
</tr>
<tr>
<td>2=C</td>
<td>$7.5^2 \times 10$</td>
<td>$\sim 1.3$</td>
<td>$1.25 \cdot 10^{10}$</td>
<td>$\approx 2600$</td>
</tr>
<tr>
<td>3</td>
<td>$5^2 \times 7$</td>
<td>$\sim 1.1$</td>
<td>$6.25 \cdot 10^{9}$</td>
<td>$\approx 5300$</td>
</tr>
<tr>
<td>4</td>
<td>$4^2 \times 4$</td>
<td>-</td>
<td>$4 \cdot 10^{9}$</td>
<td>$\approx 8300$</td>
</tr>
</tbody>
</table>
Motivation: intra-network quiet-Sun B
Small-scale dynamo (SSD) in realistic photospheric simulations?
Which realistic dynamo simulations?
What is a turbulent SSD and ...
...how do we tell?

Structure of MURaM dynamo

Run 1
Appearance of MURaM dynamo (Danilovic et al. 2010)

Run 2
Is it a SSD?

- Magnetic
- Kinetic

Run 3

Which realistic dynamo simulations?
What is a turbulent SSD and ...
...how do we tell?
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Small-scale field $\Rightarrow$ SSD (Ponty et al. 2007)
Alternate source: time-averaged mean flow

Kinetic Spectra

\[ E_v(k)/E_v \]

Magnetic Spectra

\[ E_M(k)/E_M \]
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MURaM mean flow

Instantaneous flow

Time-averaged mean flow
Our goal: distinguish between possible sources
Alexakis et al. 2007

3 Possible sources

- Turbulent energy cascade
- Stretching of large-scale $B_0$ – Alfvénic response to small-scale velocity fluctuations
- Stretching of small-scale field – SSD
**What is a turbulent dynamo?**

<table>
<thead>
<tr>
<th>Turbulent dynamo</th>
<th>Stretching $\gg \eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretching of B-field lines by turbulence <em>(Batchelor 1950, Moffat 1978, Parker 1979)</em></td>
<td>$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$</td>
</tr>
<tr>
<td>“Fast” dynamo for chaotic &amp; sufficiently complex flows <em>(Childress &amp; Gilbert 1995)</em></td>
<td>$Re_M = \frac{v_0 l_0}{\eta} &gt; Re^C_M \rightarrow$ dynamo</td>
</tr>
</tbody>
</table>
Motivation: intra-network quiet-Sun $B$
Small-scale dynamo (SSD) in realistic photospheric simulations?

What **does** SSD look like?

$$P_M \equiv \frac{\nu}{\eta} \geq 1$$
Schekochihin et al. 2004

**Growth rate, $\gamma$**

$$\gamma^{-1} \sim \tau I \sim \frac{l}{\nu}$$

K41: $\nu \sim l^{1/3}$, $l_\nu \sim Re^{-3/4}$

$$\Rightarrow \gamma \sim Re^{1/2}$$

Never yet seen (Schekochihin et al. 2004)

**Fig. 1.**—Sketch of scale ranges and energy spectra in a large-$Pr_m$ medium.

**eddy**s $l_\nu > l_\eta$ **stretch** $B$
Motivation: intra-network quiet-Sun B
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What is a turbulent SSD and ...
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MURaM growth rate

\[
\gamma = Re^{1/2} - C
\]
Kazantsev 1968 model of SSD

Kazantsev 1968 model

- Analytical, isotropic, kinematic model
- $M(k) \equiv E_M(k) \propto K_0(k)e^{\lambda t}k^{3/2}$
- All modes grow at same rate
- $k^{3/2}$

Brandenburg 2004
Motivation: intra-network quiet-Sun B
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Which realistic dynamo simulations?
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MURaM looks similar

<table>
<thead>
<tr>
<th>$Re_M$</th>
<th>exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2100</td>
<td>0.5</td>
</tr>
<tr>
<td>2600</td>
<td>0.6</td>
</tr>
<tr>
<td>5300</td>
<td>0.8</td>
</tr>
</tbody>
</table>
How do we know at what scales?
Incomparable transfer functions (Alexakis et al. 2005, Mininni et al. 2005)

What does a transfer function do?

- \( E_M = \frac{1}{8\pi} |B|^2 \)
- \( \partial_t 4\pi E_M = B \cdot [\partial_t B] = B \cdot [\nabla \times (v \times B) + \eta \nabla^2 B] \)

\[
E_M(k) = \frac{1}{8\pi} \hat{B}^*(k) \cdot \hat{B}(k) \\
\partial_t 4\pi E_M(k) = \hat{B}^*(k) \cdot \mathcal{F}[-v \cdot \nabla B] + B \cdot \nabla v + \eta \nabla^2 B(k) \\
\partial_t E_M(k) = T_{BB}(k) + T_{VB}(k) + D(k)
\]

- Convolution theorem: \( \mathcal{F} [v \cdot B] (k) = \int_{-\infty}^{\infty} \hat{v}(k - p) \cdot \hat{B}(p) dp \)

\( q \equiv k - p \)
Motivation: intra-network quiet-Sun B
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Which realistic dynamo simulations?
What is a turbulent SSD and ...
...how do we tell?

Work against/by magnetic tension force

\[
\hat{v}^* \cdot \mathcal{S} \left[ \partial_t \hat{v} \right] + \nabla \cdot \mathbf{v} + \nabla (P + \frac{1}{2} |\mathbf{B}|^2) = \begin{pmatrix} T_{VV}(k) \\ T_{BV}(k) \end{pmatrix} + D_{\nu}(k)
\]

\[
\hat{B}^* \cdot \mathcal{S} \left[ \partial_t \hat{B} \right] + \mathbf{v} \cdot \nabla \mathbf{B} = \begin{pmatrix} T_{BB}(k) \\ T_{VB}(k) \end{pmatrix} + D_{\eta}(k)
\]
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Which realistic dynamo simulations?
What is a turbulent SSD and ...
...how do we tell?

How do we know from where?
Incompressible transfer functions (Alexakis et al. 2005, Mininni et al. 2005)

\[
T_{BV}(k) = \frac{1}{4\pi} B \cdot (B \cdot \nabla) v
\]

\[
T_{BV}(q) = \frac{1}{4\pi} v \cdot (B \times (\nabla \times B))
\]

\[
T_{BB}(k) = -\frac{1}{4\pi} B \cdot (v \cdot \nabla) B
\]
Isotropic, incompressible dynamo example

Kinetic energy loss/gain

$T_{av}(t)$
Motivation: intra-network quiet-Sun B
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Which realistic dynamo simulations?
What is a turbulent SSD and ...
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Isotropic, incompressible dynamo example

Magnetic energy

\[ T_X(k) \]

- **Stretching gain**
- **Cascade loss**
- **Cascade gain**
Isotropic, incompressible dynamo example

Kinetic energy **loss/gain**

Magnetic energy

- stretching gain
- cascade loss
- cascade gain
Summary of incompressible MHD transfer

Magnetic tension

Larger scales

Smaller scales

KINETIC ENERGY

ENERGY

Smaller scales

ENERGY

=0% Net

Dynamo

Larger scales

Smaller scales

MAGNETIC ENERGY

Stretching
Extend transfers functions to compressible MHD
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MURaM dynamo transfer analysis
Pietarila Graham et al. 2010, arXiv:

Kinetic energy loss

![Graph showing kinetic energy loss with axes and logarithmic scales.]
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**Kinetic energy loss**

**Magnetic energy gain**
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What is a turbulent SSD and ... how do we tell?

Rule out: cascade

Analysis

1. Cascade below $\sim 20$ km
2. $B$ generated at $\sim 65$ km

Run 3

$E_M(k)$ gain

$E_M(k)$ loss

Net transfer rate (erg cm$^{-2}$ s$^{-1}$)

$magnetic$

$kinetic$

$E_M(k)$ gain

$E_M(k)$ loss
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Which realistic dynamo simulations?
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Rule out: Alfvénic turbulent induction

**Analysis**

1. B generated at $\sim 65$ km
2. Dynamo driven by $\sim 140$ km motions
3. Triad $\Rightarrow$ stretched $B$ at $\sim 80$ km

<table>
<thead>
<tr>
<th>$Re_M$</th>
<th>$\nu_{gen}$</th>
<th>$\nu_v$</th>
<th>$\nu_{stretch}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2100</td>
<td>110 km</td>
<td>200 km</td>
<td>160 km</td>
</tr>
<tr>
<td>2600</td>
<td>100 km</td>
<td>180 km</td>
<td>140 km</td>
</tr>
<tr>
<td>5300</td>
<td>65 km</td>
<td>140 km</td>
<td>80 km</td>
</tr>
</tbody>
</table>

**Small-scale dynamo**

- $k_{gen} B \sim \frac{1}{65 \text{ km}}$
- $k_{stretching} v \sim \frac{1}{140 \text{ km}}$
- $k_{stretched} B \sim \frac{1}{80 \text{ km}}$

**Alfvénic**

- $k_{Alfven} B \sim \frac{1}{\text{small}}$
- $k_{Alfven} v \sim \frac{1}{\text{small}}$
- $k_{B_0} \sim \frac{1}{\text{large}}$
Summary

MURaM small-scale dynamo

- Small-scale field produced by SSD
  - Not “compressive cascade”
  - Not Alfvénic turbulent induction

- SSD should play a role in the Sun as well