Opinion Dynamics: 
Rise and Fall of Political Parties

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Complex Systems (T-13)

With: Paul Krapivsky, Sidney Redner (Boston/CNLS)
Thanks: Lev Tsimring (San Diego), Michael Cross (Caltech), Harvey Rose (T-13)

papers, talk available at http://cnls.lanl.gov/~ebn
Plan

1. Motivation: modeling social dynamics
2. Noisy opinion dynamics
   -- Single party dynamics
   -- Two party dynamics
   -- Multiple party dynamics
3. Noiseless opinion dynamics

E. Ben-Naim, cond-mat/0411427
Modeling social dynamics

- Ultimate goal: predictive models of human opinions
- Relevance: politics, economics, consumer, sports

Questions

• Are “physics” concepts useful?
  Microscopic interactions $\rightarrow$ collective phenomena

• Are humans predictable?

This should help

• Large data sets available
• Large number of humans $N \sim 10^9$
• Human opinions can be quantified
Quantifying opinions

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Humans interact, opinions evolve

tendency to reach consensus?

### NCAA Football Bowl Season

#### Rankings: Week 17

<table>
<thead>
<tr>
<th>Division I-A Polls</th>
<th>USA Today/ESPN</th>
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<tr>
<td><strong>AP Top 25</strong></td>
<td><strong>USA Today/ESPN</strong></td>
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<td>8. Georgia</td>
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The Compromise Process

- Opinion measured by a single variable
  
  \[- \Delta < n < \Delta\]

- Compromise: reached via pairwise interactions
  
  \((n_1, n_2) \rightarrow \left(\frac{n_1 + n_2}{2}, \frac{n_1 + n_2}{2}\right)\)

- Conviction: restricted interaction range
  
  \[|n_1 - n_2| \leq \delta\]

- Minimal, one parameter model
- Mimics competition between compromise and conviction

Individuals may change opinion spontaneously

\[ n \xrightarrow{D} n \pm 1 \]

- Adds noise ("temperature")
- Linear process: no interaction
- Mimics unstable, varying opinion
- Influence of environment, news, editorials, events
Rate equations

simplest compromise process
total opinion, total population conserved

\[(n - 1, n + 1) \rightarrow (n, n) \quad \delta = 2\]

Probability distribution \(P_n(t)\)

**Kinetic theory: nonlinear rate equations**

\[
\frac{dP_n}{dt} = 2P_{n-1}P_{n+1} - P_n(P_{n-2} + P_{n+2}) + D(P_{n-1} + P_{n+1} - 2P_n)
\]

- Numerical integration of probability distribution
- Monte Carlo simulation of stochastic process
Single party dynamics

- Initial condition: large isolated party
  \[ P_n(0) = m(\delta_{n,0} + \delta_{n,-1}) \]

- Steady-state: compromise and diffusion balance
  \[ DP_n = P_{n-1}P_{n+1} \]

- Core of party: localized to a few opinion states
  \[ P_{-1} = P_0 = m \quad P_1 = D \quad P_2 = D^2m^{-1} \]

- Compromise negligible for n>2

Well defined core
The Tail

- Diffusion dominates outside the core
  \[ \frac{dP_n}{dt} = D(P_{n-1} + P_{n+1} - 2P_n) \quad P << D \]

- Standard problem of diffusion with source
  \[ P_n \sim m^{-1} \Phi(\sqrt{nt}) \]

- Tail mass
  \[ M_{tail} \sim m^{-1} t^{1/2} \]

- Party dissolves when
  \[ M_{tail} \sim m \quad \Rightarrow \quad \tau \sim m^4 \]

Party lifetime grows fast with its size
Core versus Tail

Party height = $m$
Party depth $\sim m^{-1}$

Self-similar shape
Gaussian tail
Qualitative features

- Exists in a quasi-steady state
- Tight core localized to a few sites
- Random opinion changes of members do not affect party position
- Party lifetime grows very fast with size
- Ultimate faith of a party: demise
- Its remnant: a diffusive cloud
- Depth inversely proportional to size, the larger the party the more stable
Two party dynamics

- **Initial condition:** two large isolated parties
  \[ P_n(0) = m_1(\delta_{n,0} + \delta_{n,-1}) + m_2(\delta_{n,l+2} + \delta_{n,l+3}) \]

- **Interaction between parties mediated by diffusion**
  \[ 0 = P_{n-1} + P_{n+1} - 2P_n \]

- **Boundary conditions set by parties depths**
  \[ P_0 = 1/m_1 \quad P_l = 1/m_2 \]

- **Steady state:** linear profile
  \[ P_n = \frac{1}{m_1} + \left( \frac{1}{m_2} - \frac{1}{m_1} \right) \frac{n}{l} \]
Merger

- Steady flux from small party to larger one
  \[ J \sim l^{-1}(1/m_\prec - 1/m_\succ) \sim (lm_\prec)^{-1} \]
- Merger time
  \[ T \sim m_\prec / J \sim l(m_\prec)^2 \]
- Lifetime grows with separation (“niche”)
- Outcome of interaction is deterministic
- Larger party position remains fixed throughout merger process

Small party absorbed by larger one
Merger: numerical results

\[ P_n \]

\[ n \]

\[ P_n \]

\[ n \]
Multiple party dynamics

- **Initial condition**: large isolated party
  \[ P_n(0) = \text{randomly chosen number in } [1 - \varepsilon : 1 + \varepsilon] \]

- **Linear stability analysis**
  \[ P_n - 1 \sim \exp[ikn + \lambda t] \]

- **Growth rate of perturbations**
  \[ \lambda = 2(2\cos k - \cos 2k - 1) + 2D(\cos k - 1) \]

- **Long wavelength perturbations unstable**
  \[ k < k_0 \quad \cos k_0 = D / 2 \]

\[ P=1 \text{ stable only for strong diffusion } D > D_c = 2 \]
Strong noise (D>D<sub>c</sub>)

- Regardless of initial conditions
  \[ P_n \to \langle P_n(0) \rangle = 1 \]
- Relaxation time
  \[ \lambda \equiv (D_c - D)k^2 \quad \Rightarrow \quad \tau \sim (D - D_c)^{-2} \]

No parties, disorganized political system
Three scenarios

- $D=0$
- $D>D_c$
- $D>D_c$

Early | Intermediate | Late
Weak noise ($D < D_c$): Coarsening

- Smaller parties merge into large parties
- Party size grows indefinitely
- Assume a self-similar process, size scale $m$
- Conservation of populations implies separation:
  \[ l \sim m \]
- Use merger time to estimate size scale:
  \[ t \sim lm^2 \sim m^3 \implies m \sim t^{1/3} \]
- Self-similar size distribution:
  \[ P_m \sim t^{-1/3} F(mt^{-1/3}) \]

Lifshitz-Slyozov ripening
Parties are static throughout process.
A small party with a large niche may still outlast a larger neighbor!
Conclusions: noiseless dynamics

- **Isolated parties**
  - Tight, immobile core and diffusive tail
  - Lifetime grows fast with size

- **Interaction between two parties**
  - Large party grows at expense of small one
  - Deterministic outcome, steady flux

- **Multiple parties**
  - Strong noise: disorganized political system, no parties
  - Weak noise: parties form, coarsening mosaic
  - No noise: pattern formation
**Problem Setup**

- Given initial distribution (continuous opinions)
  \[ P_0(x) = \begin{cases} 
  1 & |x| < \Delta \\
  0 & |x| > \Delta 
\end{cases} \]

- Find final distribution (frozen)
  \[ P_\infty(x) = ? \]

- Multitude of final states
  \[ P_\infty(x) = \sum_{i=1}^{N} m_i \delta(x - x_i) \quad |x_i - x_j| > 1 \]

- Dynamics selects one (deterministically)

**Multiple localized clusters (parties)**
Numerical integration of probability distribution

\[ \frac{\partial}{\partial t} P(x,t) = \int \int \delta_{\leq 1} dx_1 dx_2 P(x_1,t) P(x_2,t) \left[ 2 \delta(x - (x_1 + x_2)/2) - \delta(x - x_1) - \delta(x - x_2) \right] \]

Direct simulation of stochastic process
Rise and fall of central party

0 < \Delta < 1.871

1.871 < \Delta < 2.724

Central party may or may not exist!
Reemergence of central party

$2.724 < \Delta < 4.079 \quad \quad 4.079 < \Delta < 4.956$
Emergence of extremists

Tiny parties (mass $<10^{-3}$)
Bifurcations and Patterns

![Graph showing bifurcations and patterns](image)

- Major
- Central
- Minor
Self-similar structure, universality

- **Periodic sequence of bifurcations**
  1. Nucleation of minor cluster branch
  2. Nucleation of major cluster branch
  3. Nucleation of central cluster

- **Alternating major-minor pattern**

- **Clusters are equally spaced**

- **Period gives major cluster mass, separation**

\[ x(\Delta) = x(\Delta + L) \quad L = 2.155 \]
How many political parties?

- Data: CIA world factbook 2002
- 120 countries with multi-party parliaments
- Average=5.8 standard deviation=2.9
- Masses are periodic
  \[ m(\Delta) = m(\Delta + L) \]
- Major mass
  \[ M \rightarrow L = 2.155 \]
- Minor mass
  \[ m \rightarrow 3 \times 10^{-4} \]
Scaling near bifurcation points

- Minor mass vanishes

\[ m \sim (\Delta - \Delta_c)^\alpha \]

- Universal exponents

\[ \alpha = \begin{cases} 
3 & \text{type 1} \\
4 & \text{type 3} 
\end{cases} \]

L-2 is the small parameter explains small saturation mass
Heuristic derivation of exponents

- Perturbation theory \( \Delta = 1 + \varepsilon \)
- Central cluster \( x(\infty) = 0 \)
- Extremist minor cluster \( x(\infty) = 1 + \varepsilon / 2 \)

- Rate of transfer from minor cluster to major cluster
  \[
  \frac{dm}{dt} = -mM \quad \rightarrow \quad m(t) \sim \varepsilon e^{-t}
  \]

- Process stops when
  \[
  x \sim e^{-t_f/2} \sim \varepsilon
  \]

- Final minor cluster mass
  \[
  m(\infty) \sim m(t_f) \sim \varepsilon^3
  \]
Consensus

- **Integrable for** $\Delta < 1/2$
  \[
  \langle x^2(t) \rangle = \langle x^2(0) \rangle e^{-\Delta t}
  \]

- **Final state: localized**
  \[
  P_\infty(x) = 2\Delta \delta(x)
  \]

- **Rate equations in Fourier space**
  \[
  p_t(k) + P(k) = P^2(k/2)
  \]

- **Self-similar collapse dynamics**
  \[
  \Phi(z) \propto (1 + z^2)^{-2} \quad z = \frac{x}{\langle x^2(t) \rangle}
  \]

Pattern selection

- **Linear stability analysis**
  
  \[ P - 1 \propto e^{i(kx + \omega t)} \quad \Rightarrow \quad \omega(k) = \frac{8}{k} \sin \left( \frac{k}{2} - \frac{2}{k} \sin k - 2 \right) \]

- **Fastest growing mode**
  
  \[ d\omega / dk = 0 \quad \Rightarrow \quad L = \frac{2\pi}{k} = 2.2515 \]

- **Traveling wave (FKPP extremal selection)**
  
  \[ d\omega / dk = \text{Im}(\omega) / \text{Im}(k) \quad \Rightarrow \quad L = \frac{2\pi}{k} = 2.0375 \]

Patterns induced by wave propagating from boundary. However, emerging period is different L=2.155!

Pattern selection intrinsically nonlinear
Traveling waves

\[ p - 1 \propto \exp[-\lambda(x - vt) + i(kx + wt)] \]

**Discrete opinions**

\[
\begin{align*}
L_{\text{max}} &= 6 \\
L &= 5.67 \\
L_{\text{trav wave}} &= 5.31
\end{align*}
\]
Exponential initial conditions

- Bifurcations induced at the boundary
- Periodic structure, nontrivial period
- Two types of bifurcations
  1. Nucleation of major branch
  2. Nucleation of minor branch

Central cluster is stable
Two kinds of opinions

symmetry breaking, packing
Conclusions: noiseless dynamics

- Clusters form via bifurcations
- Periodic structure
- Alternating minor-major pattern
- Central party not always exists
- Power-law behavior near transitions
Outlook

- Pattern selection criteria
  - Gaps
  - Role of initial conditions, classification
  - Role of spatial dimension, correlations
  - Disorder, inhomogeneities
  - Tiling/Packing in 2D
  - Discord dynamics (seceder model, Halpin-Heally 03)

Many open questions
General features

- Dissipative system, volume contracts
- Energy (Lyapunov) function exists: $<x^2>$
- No cycles or strange attractors
- Uniform state is unstable (Cahn-Hilliard)

\[ P_i = 1 + \phi_i \quad \phi_t + \left( \phi + a \phi_{xx} + b \phi^2 \right)_{xx} = 0 \]

Discrete case yields useful insights
Discrete opinions

- Compromise process
  \[(i-1, i+1) \rightarrow (i, i)\]

- Master equation
  \[
  \frac{dP_i}{dt} = 2P_{i-1}P_{i+1} - P_i(P_{i-2} + P_{i+2})
  \]

- Example: 6 states

- Symmetry + normalization:
  two-dimensional problem

Initial conditions determine final state

Isolated fixed points, lines of fixed points