Contemporary Physics
Vol. 50, No. 3, May–June 2009, 453–461

When superfluids are a drag

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(*Received 17 December 2008; final version received 21 January 2009*)

The article considers the dramatic phenomenon of seemingly frictionless flow of slow-moving superfluids. Specifically the question of whether an object in a superfluid flow experiences any drag force is addressed. A brief account is given of the history of this problem and it is argued that recent advances in ultracold atomic physics can shed much new light on this problem. The article presents the commonly held notion that sufficiently slow-moving superfluids can flow without drag and also discusses research suggesting that scattering quantum fluctuations might cause drag in a superfluid moving at *any* speed.

Keywords: superfluids/superfluidity; drag force; quantum fluctuations; dissipationless flow; Bose–Einstein condensation

1. Introduction

Superfluidity has been in the laboratory with us since 1938 (see [1] for a history of the discovery of superfluidity), and has proven an exceptionally meaty topic of research. Its strange behaviour can be seen as the manifestation of quantum mechanics on a large scale (rather than the usual atomic and subatomic levels), representing a unique gateway into the quantum world. This prospect has been exciting enough to stir the emotions of many great physicists – Landau, Feynman, and Onsager are just a few of those who have been drawn to the subject.

Over the years and through the various avenues of research, superfluids have been found to exhibit many fascinating features (see [2] for a good overview of the subject, or [3–5] for a more detailed discussion). Among these are persistent currents (which will be discussed later), quantised vortices,[1] seemingly infinite thermal conductivity, the fountain effect, and the phenomenon of second sound.[2] One particularly curious feature of superfluidity is its ability to flow without apparent energy dissipation if its speed is sufficiently low.[3] It was this striking feature that inspired the term *super*fluid.

Picture, for instance, a submarine in a flowing river of a normal fluid, such as water. It would experience some drag from the rushing water around it and, if it weren't anchored somehow or fighting the flow with a motor, would be pushed downstream. However, if the river were made up of superfluid and the flow were sufficiently slow, experimental observations to date of superfluidity suggest that the submarine would not feel any drag even though fluid is moving past it, and would therefore not need to work against the current to stay in the same place (see Figure 1).

It remains the orthodox view that superfluid flows can be dissipationless below a certain critical velocity. Although this behaviour is counterintuitive, it rests on a theoretical foundation that is the result of careful development, and is supported by widespread experimental evidence. And yet, even in this seemingly resolved aspect of superfluidity one may still find a few surprises and questions begging answers. In this article, we will revisit the issue of whether superfluid flow indeed can be dissipationless at slow speeds, beginning with a glimpse at the history behind the orthodox view. We will then look at some new theoretical approaches made possible by recent developments in ultracold physics. We will briefly study the mean-field approach, which corroborates the orthodox view. We will also look at a recently proposed speculative theory that argues that quantum fluctuations, typically considered to have a negligible effect, could play a fundamental role in this problem. We will consider these theoretical approaches in the context of slow-flowing superfluids, i.e. superfluids in the regime where it is widely believed that dissipationless flow occurs. The question we will ask is, does an object immersed in and moving *slowly* relative to a superfluid experience a drag or not?

The next section outlines Landau's view of superfluid behaviour; Section 3 describes Bose–Einstein condensates, a new medium that will allow more exact probing of superfluid properties; Section 4 covers the

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ISSN 0010-7514 print/ISSN 1366-5812 online

DOI: 10.1080/00107510902770254
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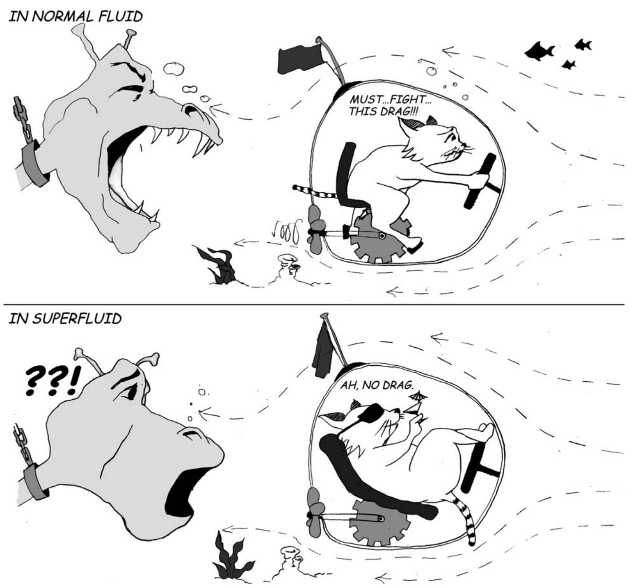

Figure 1. Illustration of the difference between normal fluid flow and superfluid flow.

mean-field approach and the superfluid behaviour it predicts; Section 5 presents the effect of quantum fluctuations in a superflow; and Section 6 discusses the issues raised in the paper.

2. Landau's approach – constructing a theory of superfluidity

Following the initial observation of superfluid helium, Landau reasoned that there must be a critical value of the bulk fluid flow velocity below which a superfluid would flow without dissipating energy [11]. In the early days of superfluidity, superfluid helium was 'the' example of a (Bose) superfluid and, because of the strongly interacting nature of superfluid helium, it was not possible to calculate much from microscopic considerations (such as atomic interactions) using analytical techniques. Landau's argument was therefore necessarily restricted to a phenomenological approach.

Landau made the assumption that for the flow's energy to dissipate, excitations had to be present to carry away the energy. However, by definition, at $T = 0$ the superfluid in itself does not contain any excitations. Therefore, excitations must be created in order for there to be dissipation. Furthermore, the flow would have to have sufficient momentum to be able to produce excitations. Because of the nature of the dispersion relation, the implication is that in order for the flow to dissipate energy, the flow would have to be moving above a certain critical velocity. Based on this argument, Landau was able to determine from the dispersion relation an estimate of the value of that critical velocity for superfluid helium. In fact, this notion of dissipationless flow and a non-zero critical velocity has become so closely associated with superfluids that this has become a popular definition of superfluidity and the one most often repeated in standard statistical mechanics textbooks.

It is important to note that Landau's predicted critical velocity does not agree with all experimental observations; in some superfluid helium flow experiments the apparent critical velocity is around an order of magnitude lower than the predicted value. A number of explanations, such as the formation of a vortex ring [12] and the expansion of remnant vortex lines [13], have been proposed to explain this discrepancy, but questions persist (see for example footnote 2 in [14]). I mention this supercritical behaviour only in passing since, in this article, we will be focusing on superfluids travelling at speeds low enough as to be safely below any predicted or experimentally observed critical velocity.

3. Gaseous Bose–Einstein condensates – an ideal test bed for superfluidity

For decades physicists studying superfluidity were obliged to tangle with strongly interacting liquid helium for answers to fundamental questions. No longer. Relatively recent advances in ultracold physics have delivered to us dilute Bose–Einstein condensates, or BECs (see Box 1), in trapped atomic-gas form, which are a much more convenient medium of study than superfluid helium.

What does Bose–Einstein condensation have to do with superfluidity? The relationship, while intimate, remains to be clearly articulated. Initially, Landau did not believe that quantum statistics, and hence Bose–Einstein condensation, had anything to do with superfluidity. However, Bogoliubov showed two things that together at least made BECs consistent with superfluidity [17]. First, he demonstrated from first principles that a dilute, weakly interacting gas could undergo Bose–Einstein condensation, contradicting the common view at the time that only a noninteracting gas could Bose condense. Second, he showed that the dispersion relation of a gas with such interactions, unlike that of a noninteracting gas, had a non-zero critical velocity consistent with superfluidity. In a nutshell, the relationship is that BECs typically seem to exhibit superfluid behaviour (except in the pathological case of a noninteracting gas). Having said that, the superfluid properties in the trapped atomic gases are not as apparent as they are in superfluid helium.[4] Over the last decade, however, many experiments have succeeded in clearly showing the superfluid characteristics of the trapped atomic gases (the existence of

BOX 1 **What is Bose–Einstein condensation?** Briefly, let's recall that all known particles have an internal angular momentum, or spin, that can only take on certain values. Those that take on values of even integer times $\hbar/2$ are referred to as bosons and those that take on odd integer times $\hbar/2$ values are referred to as fermions. Photons and helium-4 atoms are examples of bosons (although made up of fermions, the internal structure is irrelevant for low temperature fluids); electrons, protons and neutrons are examples of fermions.

In the 1920s, Bose defined certain rules for determining the statistical distribution of photons given that photons are indistinguishable in the quantum sense. We now call these rules 'Bose statistics'. Einstein guessed that these same rules might apply to atoms. He worked out a theory for how atoms would behave in a gas assuming that Bose statistics applied. (Note that Einstein's theory holds only for bosons; fermions, which do not obey Bose statistics, had not yet been identified at the time.)

It turns out that the behaviour for bosons and fermions do not differ significantly, except at very low temperatures where quantum effects become important. At sufficiently high temperatures, atoms would populate many different levels regardless of whether they are bosons or fermions. However, all things being equal, bosons prefer to squeeze in together in the same quantum state and, at low enough temperatures, nothing prevents them from doing so. Thus, if a substance composed of bosons becomes cold enough, a significant fraction of the bosons would suddenly fall into the very lowest energy level; the substance undergoes a phase transition into a Bose–Einstein condensate (BEC). By contrast, fermions are – thankfully – antisocial. In fact, the Pauli exclusion principle dictates that there can be only one fermion per state, and this is what keeps ordinary matter stable.

In one of the great breakthroughs in physics in the last few decades, this form of matter was unambiguously and independently witnessed in its (almost) pure form by Cornell, Wieman, and Ketterle in 1995, 70 years after Einstein predicted it (see Figure 2). They succeeded in cooling trapped atomic gases to temperatures in the region of 10^{-9} degrees above absolute zero, which is colder than anything in the known universe, and thereby achieved Bose–Einstein condensation. The remarkable achievement of this novel form of matter, which had been thought of as the holy grail of atomic physics since the late 1970s, attracted a Nobel prize shortly after its discovery and has launched a new and rapidly expanding discipline devoted to the study of its properties and exploration of its potential applications (see for example [3,15,16].

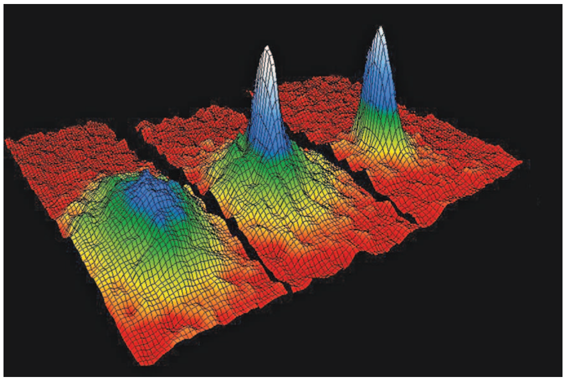

Figure 2. 3D density plot of Bose–Einstein condensate formation in ultracold trapped Rb atoms at different temperatures (400, 200, 50 nK from left to right). The peak of the density distribution, which consists of almost stationary atoms acting in lockstep, is the Bose–Einstein condensate; the flatter part of the density distribution represents the thermal atoms. (Figure courtesy of the Physics Department, University of Colorado.)

scissor modes [18], quantised vortices [19], and reduced drag on impurities [8–10], etc.).

For the purposes of the present discussion, however, the most important difference between the dilute BECs in trapped atomic gases and superfluid helium, the original bosonic superfluid discovered in 1938, is that of density and therefore, the importance of the interaction between particles. Superfluid helium is more dense, so the interactions between the atoms are a significant factor in determining the fluid's behaviour and cannot be ignored. The new BECs to be found in the trapped atomic gases, on the other hand, are dilute (100,000 times more dilute than air); the average distance between atoms is much greater and so the interaction is far less important. In this dilute limit, Bogoliubov (as well as Lee, Huang and Yang [20] and others) showed that one can derive the system's macroscopic behaviour from its microscopic properties.

In many ways, experimentalists are also able to do much more with dilute, weakly interacting BECs than with the strongly interacting superfluid helium. Not only can scientists change the geometry of the trap for the dilute BEC almost at will, but they can do amazing things like tune the atomic interactions (even change their sign!) which, at present, is only a dream in the dense world of superfluid helium.

The theoretical and experimental advantages of trapped atomic gases over liquid helium make them an excellent medium through which to probe superfluidity, giving new hope of resolving some of the lingering paradoxes of superfluidity, including the unintuitive behaviour of frictionless flow in superfluids. The remainder of this article will focus on these trapped atomic gas systems; helium systems will be revisited briefly in the discussion.

4. Mean field calculation

From a theoretical perspective, the moving superfluid is completely characterised by the full bosonic quantum field operator $\hat{\psi}(\boldsymbol{r})$. ($\hat{\psi}(\boldsymbol{r})$ is quantum in the sense that it and $\hat{\psi}^{\dagger}(\boldsymbol{r})$ are noncommuting operators.) With that, and since we know that force is determined by the gradient of the external potential, we can immediately write the exact expression for the drag on an object immersed in the flow in second quantised notation as

$$F = -\langle \hat{\psi}^{\dagger}(\boldsymbol{r})[\nabla \Phi(\boldsymbol{r})]\hat{\psi}(\boldsymbol{r})\rangle_{T=0}, \quad (1)$$

where for convenience we have represented the object by an external field $\Phi(\boldsymbol{r})$ and we assume that the superfluid is at $T = 0$. It is here that we encounter the stumbling block – generally speaking it is not feasible to solve $\hat{\psi}(\boldsymbol{r})$ exactly. This is where the diluteness of the new BECs comes in. It means that the interactions are far less important than they are in a dense medium, allowing us to make a justifiable, controlled approximation of $\hat{\psi}(\boldsymbol{r})$. In this section we will look at one approximation, known as the mean-field approximation, where one assumes that each individual atom experiences the average of the interactions of all the other atoms.

To leading order in a zero-temperature weakly interacting Bose gas, we can approximate $\hat{\psi}(\boldsymbol{r})$ as a classical field $\Psi^{(0)}(\boldsymbol{r})$ representing the condensate wave function. This is roughly equivalent to saying that adding or subtracting a few atoms from this state will not change the properties of the system, akin to treating light classically when many photons are present. This is a good approximation in our case as one can self-consistently show that a macroscopic number of atoms reside in the condensate field.

Being a classical field, $\Psi^{(0)}(\boldsymbol{r})$ obeys the well-known nonlinear Schrödinger equation, better known as the Gross–Pitaevskii equation (GPE) in the superfluid context. Assuming a steady state, for a stationary object in a moving superfluid flow, the GPE can be written as

$$(\hat{T} + \Phi(\boldsymbol{r}) - \mu)\Psi^{(0)}(\boldsymbol{r}) + g|\Psi^{(0)}(\boldsymbol{r})|^2\Psi^{(0)}(\boldsymbol{r}) = 0, \quad (2)$$

where

$$\hat{T} \equiv -\frac{\hbar^2\nabla^2}{2m} + \mathrm{i}\hbar v_{\mathrm{s}}\frac{\partial}{\partial x} + \frac{1}{2}mv_{\mathrm{s}}^2$$

and $g = 4\,\pi\,\hbar^2 a/2m$, m being the atoms' mass, v_{s} being the superfluid velocity relative to the object, and a being the scattering length.

The force on the object can be expressed as

$$\boldsymbol{F}_{\mathrm{GPE}} = -\int \mathrm{d}^3\boldsymbol{r}|\hat{\Psi}^{(0)}(\boldsymbol{r})|^2\nabla\Phi(\boldsymbol{r}), \quad (3)$$

which means that for an object with a potential symmetric about $x = 0$, a density asymmetry in $|\Psi^{(0)}|^2$ would lead to a drag.

Theorists working in this mean-field approximation have shown that a superfluid flowing slowly enough past an object with just such a symmetric potential would not develop a density asymmetry, so there would be no drag on the object [21–23]. It is interesting to note that the GPE can be mapped into a classical ideal fluid (differing only by an additional quantum pressure term), and that lack of drag at subcritical speeds has been a known feature of these classical fluids since the eighteenth century, i.e. d'Alembert's paradox.

So, analysed in the mean-field approximation, it appears that there *can* be flow without dissipation if the fluid travels below a certain velocity, much as Landau had argued. However, from the mean-field studies done in various geometries it would appear that for the flow to be dissipationless, it is the maximum local fluid velocity that must be less than the speed of sound rather than the bulk fluid velocity of Landau's argument. Above the geometry dependent critical velocity, nonlinear effects such as vortex shedding and soliton shedding cause the superfluid flow to break down. The bottom line from the mean-field approach is that there is a nontrivial critical (bulk fluid) velocity below which the object would not feel any drag, i.e. the flow would not dissipate energy.

5. Effect of quantum fluctuations

In the previous section we showed how $\hat{\psi}(\boldsymbol{r})$ could be tamed by a leading order approximation. But what if we let $\hat{\psi}(\boldsymbol{r})$ retain a little bit more of its character? Around the classical solution discussed in the last section there exist fluctuations, even at zero temperature, because of the quantum zero point motion. Therefore, in this section, we will keep not only the leading order term (the classical-field picture introduced in the previous section) but also the next-order term in the expansion of $\hat{\psi}(\boldsymbol{r})$. The approximation becomes

$$\hat{\psi}(\boldsymbol{r}) \approx \Psi^{(1)}(\boldsymbol{r}) + \hat{\phi}(\boldsymbol{r}), \tag{4}$$

where $\hat{\phi}(\boldsymbol{r})$ is the quantum fluctuating operator and $\Psi^{(1)}(\boldsymbol{r})$ is the condensate field modified by the quantum fluctuations. Since the condensate is dilute and weakly interacting, we will assume $\hat{\phi}(\boldsymbol{r})$ to be small. One can show self-consistently that corrections due to the quantum fluctuations would be on the order of the diluteness parameter, which can be written as $(n_0\, a^3)^{1/2}$, where n_0 is the number density [24].

This extra term *does* make calculations a little more complicated and, yes, one of the great benefits of trapped gaseous condensates was the idea that one could ignore these higher order effects and still be able to describe experiments with a high degree of precision – theorists could argue that their effects were insignificant relative to those of the mean field because of the diluteness of the gas. Furthermore, even though the effects of quantum fluctuations have measurable effects in various experimental systems (see for example [25–27]), these experimental consequences have typically been a small correction to the dominant mean-field effects. So why bother taking into account quantum fluctuations in this case? Since the mean field makes no contribution to drag in the object/superfluid system considered in this article (as discussed in the previous section), any effect from quantum fluctuations would dominate and therefore merits a closer look.

As a brief aside, let us look at how quantum fluctuations can give rise to a force in a superfluid system. Casimir, in 1948, argued that quantum fluctuations in an electromagnetic (EM) vacuum would give rise to a force (see Box 2 for details). One can make an analogous argument about fluctuations in a zero-temperature superfluid (for similar vacuum/superfluid analogies see [30]). A condensate at zero temperature can be thought of as a vacuum in that it is devoid of excitations and, like Casimir's EM vacuum, one must still take into account quantum fluctuations, as represented by $\hat{\phi}(\boldsymbol{r})$, that exist in a zero temperature superfluid. To adapt Casimir's thought experiment to our superfluid situation, instead of introducing conducting plates into our vacuum as Casimir did, let us imagine immersing two parallel, thin, hard planar walls. In the mean-field approximation there would be no force because the pressure of the fluid would be equal between and outside of the parallel walls, just as there would not be a Casimir force in an EM vacuum according to classical argument. However, if one includes quantum fluctuations, an attractive force does arise between these two walls, a force that is directly analogous to the Casimir force in an EM vacuum [31,32]. (Note that this Casimir force from the excitation vacuum in condensates is very much a *quantum*, not a thermal, effect due to the quantum

BOX 2 **The Casimir force**

In a classical mechanics analysis, two uncharged conducting plates placed parallel to each other in a perfect electromagnetic (EM) vacuum would not result in any force. However, Casimir (with a conceptual push from Bohr) showed that in a quantum mechanical conception a small attractive force would arise between these plates as a result of the vacuum, i.e. from nothing. Although in a vacuum no real photons are present, virtual photons still exist due to the quantum nature of the EM field. The resulting field is a superposition of many modes, and the parallel plates in the vacuum effectively place boundary conditions on which of these modes are allowed between the plates (see Figure 3). Waves with nodes that coincide with the surface of both plates form standing waves (at least for perfect conductors); all other waves are suppressed. This effectively filters out many modes from the space between the plates. One can show that the energy of the system depends on the separation of the plates and thus a force (which is attractive) exists between the plates. This force has recently been conclusively observed and its study has since developed into a booming area of research [28,29].

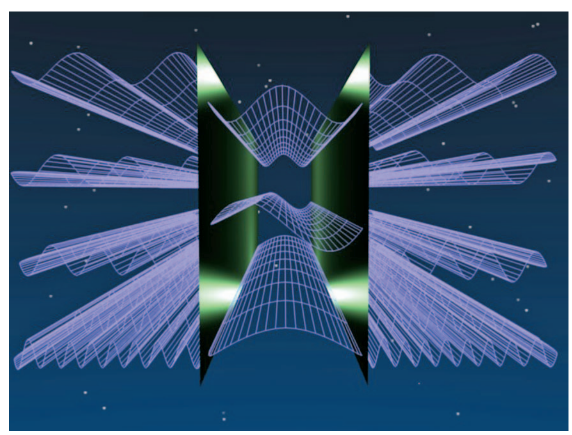

Figure 3. Schematic showing the boundary effect of conducting plates on the vacuum modes, which is responsible for the Casimir force.

fluctuation operator.) To leading order, the force expression in the EM case is identical to that in the condensate-vacuum case, except for the speed of sound replacing the speed of light and a factor of two arising from polarisation of the EM waves. This is because the dispersion relations for photons and phonons are identical (at least for low momenta, which dominate the force).

How does this apply to the problem of an object in a moving superfluid? First, it is important to note that the direct Casimir analogy can only go so far. We do not expect there to be a drag on a single object moving at constant velocity in an electromagnetic vacuum. This is because such a force would imply a preferred reference frame and an ether, violating the fundamental principle that all non-accelerating frames in an EM vacuum must be equivalent. By contrast, a preferred non-accelerating reference frame *can* be identified in the object/superfluid system – the lab frame – without breaking any fundamental principles of physics. This makes it possible to have drag on an object in a moving superfluid although, of course, this does not prove that one exists.

Given $\hat{\psi}(\boldsymbol{r}) \approx \Psi^{(1)}(\boldsymbol{r}) + \hat{\phi}(\boldsymbol{r})$, the drag on an object immersed in a moving superfluid becomes

$$\begin{aligned}\boldsymbol{F} &= \boldsymbol{F}_{\mathrm{GGPE}} + \boldsymbol{F}_{\mathrm{fluc}} \\ &= -\int \mathrm{d}^3\boldsymbol{r}(|\Psi^{(1)}(\boldsymbol{r})|^2 + \langle\hat{\phi}^\dagger(\boldsymbol{r})\hat{\phi}(\boldsymbol{r})\rangle_{T=0})\nabla\Phi(\boldsymbol{r}), \qquad (5)\end{aligned}$$

where the first term on the right-hand side, $\boldsymbol{F}_{\mathrm{GGPE}}$, represents the force contribution from the generalised GPE [33,34], which describes the condensate field modified by quantum fluctuations; the second term represents the force contribution from quantum fluctuations, $\boldsymbol{F}_{\mathrm{fluc}}$. (Note that $\boldsymbol{F}_{\mathrm{GPE}}$ does not feature here since it is zero at subcritical velocities, as described in the previous section.) To calculate $\boldsymbol{F}$, we must first determine $\hat{\phi}(\boldsymbol{r})$ and then $\Psi^{(1)}(\boldsymbol{r})$, using $\hat{\phi}(\boldsymbol{r})$ in the generalised GPE (GGPE).

To make a long story short, $\hat{\phi}(\boldsymbol{r})$ can be calculated by performing a canonical transformation from a system that describes weakly interacting particles to a system describing noninteracting quasiparticles. This effectively diagonalises the system, making it easier to compute the system's properties (see [24] for details). We can write $\hat{\phi}(\boldsymbol{r})$ in terms of the normal modes of the system, i.e. the quasiparticle operators $\hat{\alpha}_k$ and $\hat{\alpha}_k^\dagger$ [5] such that

$$\hat{\phi}(\boldsymbol{r}) = \sum_{k'}\left(u_k(\boldsymbol{r})\hat{\alpha}_k - v_k^*(\boldsymbol{r})\hat{\alpha}_k^\dagger\right), \qquad (6)$$

where, in order for the transformation mentioned above to hold, $u_k(\boldsymbol{r})$ and $v_k(\boldsymbol{r})$ must satisfy the Bogoliubov–de Gennes (BdG) equations, which govern the behaviour of quantum fluctuations in the dilute limit. The BdG equations, which determine the behaviour of these normal modes, are

$$\hat{\mathcal{L}}u_k(\boldsymbol{r}) - g(\Psi^{(0)})^2 v_k(\boldsymbol{r}) = E_k u_k(\boldsymbol{r}), \qquad (7)$$

$$\hat{\mathcal{L}}v_k(\boldsymbol{r}) - g(\Psi^{(0)*})^2 u_k(\boldsymbol{r}) = -E_k v_k(\boldsymbol{r}), \qquad (8)$$

where $\hat{\mathcal{L}} = \hat{T} + \Phi(\boldsymbol{r}) - \mu + 2g|\Psi^{(0)}|^2$, E_k is the eigenvalue for momentum state $\boldsymbol{k}$, and * indicates the complex conjugate. This describes an effective scattering problem which must be solved in order to determine $u_k(\boldsymbol{r})$ and $v_k(\boldsymbol{r})$ and thus the force in a superfluid at subcritical speeds.

With these, we can return to Equation (5) and find that it yields a non-zero drag at all velocities [34–36]. [36] goes beyond the perturbative calculations of [32,35] in a quasi-ld geometry; it also discusses the regimes where the drag force is dominated by quantum fluctuations and where it is dominated by thermal fluctuations. Unsurprisingly given its origination from the quantum fluctuations term, the calculated force is proportional to the small diluteness parameter, $(n_0 a^3)^{1/2}$.[6]

This does not square with either the orthodox or the mean-field view, both of which determine that there is dissipationless flow below a certain (non-zero) critical velocity. In the approximation that includes the fluctuation term, there is drag at *any* velocity, even at very low velocities where there is no mean-field contribution to the force. If quantum fluctuations in a superflow do indeed give rise to a force on an immersed object then, in the regime below the critical velocity, this effect would be the dominant one.

6. Bringing theory and experiment together

The mean-field analysis finds that superfluids moving slowly enough can flow without dissipation while findings using a closer approximation of superfluids that takes into account quantum fluctuations point to there being dissipation at *all* velocities. One would expect that greater precision in the approximation would lead to more correct conclusions. However, experimental verification, the great arbiter, has yet to come down definitively on either side.

Experiments on superfluid helium [6,7] as well as on dilute BECs [8,9] have observed a sharp drop in drag below a certain relative velocity of a superfluid to the object immersed in it. This is consistent with the idea that there is dissipationless flow below a certain velocity. However, these experimental observations cannot be said to be inconsistent with the notion of drag from quantum fluctuations. Demonstrating no quantum fluctuation-induced drag at low velocities would require not only that a drop in dissipation is observed, but that there is no drag as large as that predicted to be due to quantum fluctuations. For dilute BECs, the predicted force from quantum fluctuations is very small, and dilute BEC experiments have yet to achieve the precision needed to rule out drag on this order of magnitude.

Newly developed techniques in ultracold atomic gas experiments are making it possible to improve precision. For example, containing the superfluid in a toroidal form rather than in a harmonic trap may reduce inhomogeneities in the superfluid density, improving sensitivity to drag effects from an immersed object moving relative to the superfluid. The toroidal set-up is, conveniently, already an experimental reality. It is a feature of persistent current experiments, which have been conducted on superfluid helium decades ago [37] but have only recently been achieved using dilute BECs [38,39]. In these experiments the superfluid is set flowing around the principal axis of the toroidal, rough-walled container or trap and, without further external intervention, the superfluid flows essentially forever without noticeable dissipation. Certainly, the fact that the flow maintains its speed indefinitely is an indisputable example of drag-free flow, i.e. no drag from quantum fluctuations?

Well, perhaps it is possible to reconcile drag at all velocities with the observation of persistent currents. One can speculate with the phenomenological argument below, the corollary of which should be verifiable through simulations (currently in progress) as well as through new, more sensitive experimental techniques.

The calculation outlined in Section 5 treats the system as if it were infinitely extended – quantum fluctuations that deflect off the object in such a system would travel away and not return. This theoretical infinitely extended situation can be likened to that of wave drag on a boat moving on an ocean, where waves that hit the boat are reflected once and travel away unhindered. Persistent current experiments, on the other hand, are conducted in a finite geometry and are therefore subject to finite geometry effects. For simplicity, let us now treat all the friction due to the roughness of the container walls as if it were embodied in one single object immersed in the toroidal flow. (Surface roughness will be dealt with more directly below.) In this way, we can see that the experimental situation is more akin to a boat moving in a circular channel where the scattered waves are eventually reflected back towards the boat, fundamentally changing the nature of the problem from that treated theoretically.

In finite-geometry experiments, therefore, the fluctuations scattered once off the object in the superfluid eventually re-encounter the object. This should happen on some geometry dependent timescale, which would be on the order of the characteristic length of the system over the speed of sound. (Note that we are only concerned with scattering fluctuations whose interaction with the object has a component along the flow axis; the force from fluctuations that scatter perpendicular to the direction of bulk fluid motion should average out to zero.) One can surmise that below this timescale, i.e. before this system recognises it is a finite system, the object would act as if it were in an infinitely extended medium and there would be drag due to the quantum fluctuations; above this timescale the drag would die away as the rescattered quantum fluctuations become important. This predicted observable provides one way to test for drag from quantum fluctuations in superfluids moving at subcritical speeds.[7] If the argument above is borne out, then the phenomenon of persistent currents can be understood as a finite-size effect, i.e. the bigger the toroidal system, the longer it would take for persistent currents to settle in.

In the approximation discussed in Section 5 of this article, Equation (4), quasiparticles (which diagonalise the Hamiltonian) do not interact,[8] meaning that the system including quantum fluctuations are unable to relax to any thermal equilibrium. However, as the time increases, quasiparticle interactions play a more prominent role in the system and higher order terms (the Beliaev and Landau terms [40] truncated in the approximation described in Section 5, i.e. Equation (4)) become significant. These interactions permit the scattered waves to reach a local thermal equilibrium, generating some normal fluid (fluid with the viscous properties we are familiar with in our regular non-zero temperature world). The presence of superfluid and normal fluid means that the system behaves as a two-fluid system as discussed by Landau [11]. The scattered fluctuations should in principle be detectable as normal gas, providing a second way to test for drag due to quantum fluctuations in the subcritical regime.

In real superfluid flow experiments, whether using helium or dilute BECs, the primary source of surface roughness is typically the container walls rather than an immersed object. So let us now return to thinking of the persistent current experimental set-up with rough container walls and no immersed object. As one can imagine, the problem of quantum fluctuations scattering off rough container walls is far more complex than that of scattering off one localised object. Nevertheless, with the 'reversible' conversion between superfluid and normal fluid just described, coupled with a healthy dose of speculation (in keeping with the spirit of this article), one can theorise a hydrodynamic boundary condition for superfluid flowing along a rough wall.

If one ventures that the equilibrium temperature is proportional to the kinetic energy of the superfluid component, v_s^2, weighted by the roughness of the walls,

Figure 4.

and that v_s is more or less constant, one can hypothesise that the boundary condition is of the form

$$\boldsymbol{n} \cdot (\boldsymbol{j}_n - \boldsymbol{j}_s) = \alpha(T_b - T) + \beta v_s^2, \tag{9}$$

where $\boldsymbol{n}$ is the average unit vector normal to the rough surface, $\boldsymbol{j}_n$ is the mass flux for the normal fluid component and $\boldsymbol{j}_s$ that for the superfluid component, T is the local fluid temperature, T_b is the boundary temperature, and α and β are parameters dependent on the roughness of the surface (for more details see [14]). According to this boundary condition, a superfluid moving along a rough boundary will necessarily be at a higher temperature than the boundary due to the scattering of quantum fluctuations. This should, in principle, be detectable with current experimental technology. For instance, in immersed torsional oscillator experiments [41], more normal fluid will be carried by the oscillating discs with increasing angular velocity, creating a unique nonlinear signal to detect the presence of normal fluid.

7. Conclusion

I hope to have given you a little taste of what is super about superfluids. I also hope that you come away persuaded that, although it has been around for a long time, certain aspects of superfluidity – such as low-speed flow behaviour – might not yet have given up all their secrets. It is not yet clear that the slow-moving superfluid does not push gently on an object in its flow (see Figure 4).

However, recent experimental breakthroughs in dilute BECs give us new hope of resolving some of these longstanding issues, making now a particularly exciting time to work on superfluids.

Acknowledgements

The author gratefully acknowledges Keith Burnett, Alexander Fetter, Yves Pomeau, and Felipe da Rosa for constructive comments on the manuscript. The author is also grateful to Martin Goldman, Elaine Kuok, and Andrew Sykes for contributing the figures.

Notes

1. Superfluids don't spin as stirred-up water would in a cylindrical container. Instead, they form regularly spaced vortices in the superfluid, with axes parallel to what would have been the axis of rotation if we were dealing with a normal fluid. These vortices are quantised in terms of $\hbar$ which shows the direct connection between the superfluid's macroscopic behaviour and the underlying subatomic quantum mechanics.
2. In addition to normal sound waves, which are pressure waves, superfluids can support temperature waves, known as second sound.
3. The absence of friction on objects moving in superfluids has been the subject of a number of experiments in both liquid helium [6,7] and dilute condensates [8–10].
4. By contrast, the superfluid properties of liquid helium below the lambda temperature were very prominent but the presence of Bose–Einstein condensation in superfluid helium was not observed until much later. In fact, it was only in the 1980s that scientists were finally able to confirm – through neutron scattering experiments and surface absorption that – about 10% of the atoms in zero-temperature liquid helium were in a Bose–Einstein condensed state.
5. As $\hat{\alpha}_k$ and $\hat{\alpha}_k^\dagger$ are, respectively, the annihilation and creation quasiparticle operators, the quasiparticle vacuum at $T = 0$ means that the term $\langle\hat{\phi}^\dagger(\boldsymbol{r})\hat{\phi}(\boldsymbol{r})\rangle$ from Equation (5) reduces to $\sum_k |v_k(\boldsymbol{r})|^2$.
6. In superfluid helium, $(n_0\, a^3)^{1/2} \sim 1$, much larger than in dilute BECs. Were the above force calculation to hold, one would expect the drag to be greater in the superfluid helium system. However, the Bogoliubov theory used to derive the force calculation is only valid in the regime of small $(n_0\, a^3)^{1/2}$ (the regime that is the focus of this article). Therefore, the size of the drag, if any, in a superfluid helium system remains unresolved.
7. It is unsurprising that the above-mentioned timescale of initial dissipation has not yet been observed since the speed of sound of the liquid helium used in previous persistent current experiments is very large compared to the spatial scales of the system. At 50 m s^{-1}, the system realises its finite boundaries very quickly. However, in ultracold gas experiments the speed of sound is much slower ($\sim$0.01 m s^{-1}), which should make such a timescale easier to observe.
8. Lee, Huang, and Yang's celebrated correction [20] to the ground state energy degenerate Bose gases occurs at this order.

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