Optimization and Control Theory for Smart Grids

Michael Chertkov
Center for Nonlinear Studies & Theory Division,
Los Alamos National Laboratory

April 21, 2010, MIT/LIDS
Outline

1. Introduction

2. Switching (Phase) Transitions in Distribution Grid
   - Redundancy & Switching
   - SAT/UNSAT Transition & Message Passing

3. An Optimization Approach to Design of Transmission Grids
   - Intro (II): Power Flow & DC approximation
   - Network Optimization

4. Distance to Failure in Power Networks
   - Model of Load Shedding
   - Error Surface & Instantons

5. Conclusions and Path Forward
Optimization and Control Theory for Smart Grids

• So what? Impact.
  - savings: (a) 30b$ annually is the cost of power losses,
    10% efficiency improvement=> 3b$ savings,
    (b) cost of 2003 blackout is 7-10b$, 80b$ is the total cost of blackouts annually in US
  - further challenges (more vulnerable, cost of not doing planning, control, mitigation)

• Grid is being redesigned [stimulus]
  The research is timely.
  -2T$ in 20 years (at least)
The greatest Engineering Achievement of the 20th century will require a smart revolution in the 21st century.

US power grid

Distance to Failure in Power Networks

Conclusions and Path Forward

Michael Chertkov – chertkov@lanl.gov

http://cnls.lanl.gov/~chertkov/SmarterGrids/
Preliminary Remarks

The power grid operates according to the laws of electrodynamics

- Transmission Grid (high voltage) vs Distribution Grid (low voltage)
- Alternating Current (AC) flows ... but DC flow is often a valid approximation
- No waiting period $\Rightarrow$ power constraints should be satisfied immediately. Adiabaticity.
- Loads and Generators are players of two types (distributed renewable will change the paradigm)
- At least some generators are adjustable - to guarantee that at each moment of time the total generation meets the total load
- The grid is a graph ... but constraints are (graph-) global
Introduction

Switching (Phase) Transitions in Distribution Grid
An Optimization Approach to Design of Transmission Grids
Distance to Failure in Power Networks
Conclusions and Path Forward

Our (LANL) Road Map for Smart Grids

New Systems

- Renewables
- PHEV & Storage
- Metering

Grid Stability
- distance to failure
- dynamic stability
- outages/rare events
- cascading
- signature detection

Grid Control
- load balancing
- queuing and scheduling
- optimal power flows
- feeder lines control
- distribution and switching with redundancy

Grid Design
- placement of generations and storage
- intermittent wind as an integrated capacity
- accounting for outages and intermittency in planning

New Challenges

All of the above also requires scientific advances in

- Analysis & Control
- Stability/Reliability Metrics
- State Estimation

- Data Aggregation & Assimilation
- Middleware for the Grid
- Modeling Consumer Response

Michael Chertkov – chertkov@lanl.gov

http://cnls.lanl.gov/~chertkov/SmarterGrids/
Outline

1. Introduction
2. Switching (Phase) Transitions in Distribution Grid
   - Redundancy & Switching
   - SAT/UNSAT Transition & Message Passing
3. An Optimization Approach to Design of Transmission Grids
   - Intro (II): Power Flow & DC approximation
   - Network Optimization
4. Distance to Failure in Power Networks
   - Model of Load Shedding
   - Error Surface & Instantons
5. Conclusions and Path Forward
Methodology/Disclaimer

- Approach A: Take a realistic power grid model and run simulations. Time Consuming ... and need a new setting when details change

- Approach B (probabilistic + physicist/applied.math way): Study behavior of simple abstract models that facilitate the analysis, and look for universal features. Model choice criteria (in physics): The simpler but richer the better.
Optimization & Control of Power Grid

[L. Zdeborova, A. Decelle, MC ’09]

A: \( R = 0; 1; 2; 3 \). Graph samples. Ancillary connections to foreign generators/consumers are shown in color.

B: \( R = 1 \). Three valid (SAT) configurations (shown in black, the rest is in gray) for a sample graph shown in Fig. A.

- Can the ancillary lines (redundancy) help?
- Design and efficient switching algorithm for finding SAT solution.
Combinatorial Model of Switching

- $M$ producers, $N = DM$ consumers
- Of $D$ consumers $R$ has an auxiliary line
- Consumer $i$ consumes $x_i$
- In [Zdeborova, Backhaus, MC ’09] we also model renewables = allow consumer to generate $z_i$

- Capacity of producer $\alpha$ is $y_\alpha$
- Setting: switch variables $\sigma_{i\alpha} = 0, 1$ for power lines
- Constraint (1): each consumer has exactly one line - $\sum_{i \sim \alpha} \sigma_{i\alpha} = 1$
- Constraint (2): Producers are not overloaded - $\sum_{i \sim \alpha} \sigma_{i\alpha}(x_i - z_i) \leq y_\alpha$
- Assume certain statistics of consumption/production for consumers
Highlights of our Approach

- To analyze the SAT-UNSAT transition we solved Cavity Equations with Population Dynamics Algorithm. The approach allows
  (a) to compute the probability that the given switch is off/on
  (b) to estimate number of valid (not overloading) configurations.

- We also developed Belief Propagation and WalkGrid (greedy stochastic search, younger brother of WalkSum for K-SAT) algorithms which find SAT-switching efficiently.
Simple (no renewables, \( z = 0 \)) Model of Switching

Statistics of load fluctuations

\[ P(x) = \prod_i [(1 - \epsilon)p(x_i) + \epsilon \delta(x_i)] \]
\[ p(x) = \begin{cases} 
1/\Delta, & |x - \bar{x}| < \Delta/2 \\
0, & \text{otherwise}
\end{cases} \]

- \( \bar{x}, \Delta \) and \( \epsilon \) are parameters.

Some simple conditions

- \( 2\bar{x} \geq \Delta \) - individual consumption is non-negative
- \( (1 - \epsilon)\bar{x} \leq \frac{M}{N} = \frac{1}{D} \) - typical case is SAT
- \( (D - R + 1)\left(\bar{x} + \frac{\Delta}{2}\right) \leq 1 \) - SAT condition (for a generator) with redundancy
- \( \epsilon = 0 : (D + 1)(\bar{x} - \frac{\Delta}{2}) \leq 1 \) - (\( D + 1 \)) consumers connected to a generator and drawing minimum amount do not overload the generator

Michael Chertkov – chertkov@lanl.gov

http://cnls.lanl.gov/~chertkov/SmarterGrids/
Average case (cavity/population dynamics) analysis

Less trivial conditions (in color)

(1 − ε)\bar{x} and \Delta are the distribution mean and width

SAT/UNSAT = bottom-left/up-right
Walkgrid is an efficient algorithm for switching

WalkGrid
1. Assign each value of \( \sigma \) 0 or 1 randomly (but such that \( \forall i \in G : \sum_{\alpha \in \partial i} \sigma_{i\alpha} = 1 \));
2. repeat
   Pick a random power generator \( \alpha \) which shows an overload, and denote the value of the overload, \( \delta \);
3. Choose a random consumer \( i \) connected to the generator \( \alpha \), i.e. \( \sigma_{i\alpha} = 1 \);
4. Pick an arbitrary other generator which is not overloaded and consider switching connection from \((i\alpha)\) to \((i\beta)\).
5. if (in the result of this switch \( \alpha \) is relieved from being overloaded
   and \( \beta \) either remains under the allowed load or it is overloaded but by the amount less than \( \delta \))
6. Accept the move, i.e. disconnect \( i \) from \( \alpha \) and connect it to \( \beta \) thus setting \( \sigma_{i\beta} = 1, \sigma_{i\alpha} = 0 \).
7. else With probability \( p \) connect consumer \( i \) to \( \beta \) instead of \( \alpha \);
8. until Solution found or number of iterations exceeds \( MT_{\text{max}} \).
We are working on

- Application of the approach to more realistic distribution grids
- Extending the story beyond “the commodity flow” approach towards accounting for AC/DC specifics of the power flows

Bibliography

- L. Zdeborova, S. Backhaus, M. Chertkov, in proceedings of HICSS 43 (Jan 2010).
1 Introduction

2 Switching (Phase) Transitions in Distribution Grid
   - Redundancy & Switching
   - SAT/UNSAT Transition & Message Passing

3 An Optimization Approach to Design of Transmission Grids
   - Intro (II): Power Flow & DC approximation
   - Network Optimization

4 Distance to Failure in Power Networks
   - Model of Load Shedding
   - Error Surface & Instantons

5 Conclusions and Path Forward
Grid Design: Motivational Example

- Cost dispatch only (transportation, economics)
- Power flows highly approximate
- Unstable solutions
- Intermittency in Renewables not accounted

Hybrid Optimization - is current “engineering” solution developed at LANL: Toole, Fair, Berscheid, Bent 09 extending and built on NREL “20% by 2030 report for DOE

Network Optimization ⇒

- Design of the Grid as a tractable global optimization

Michael Chertkov – chertkov@lanl.gov

http://cnls.lanl.gov/~chertkov/SmarterGrids/

The Kirchhoff Laws

∀a ∈ G₀ : \( \sum_{b \sim a} J_{ab} = J_a \) for currents
∀(a, b) ∈ G₁ : \( J_{ab} z_{ab} = U_a - U_b \) for potentials

Loss Function

\[
Q(\hat{G}) = \sum \{|a,b\} \in G₁ R \left( \frac{1}{z_{ab}} \right) |U_a - U_b|^2 = J^+(\hat{G}')^{-1} J^* = U^+ \hat{G} U^*
\]

\[
U = (\hat{G}')^{-1} J, \quad \hat{G}' = \hat{G} + 11^+, \quad \hat{G} = (G_{ab}|a, b \in G₀)
\]

\[
G_{ab} = \begin{cases} 
0, & a \neq b, \ a \not\sim b \\
g_{ab}, & a \neq b, \ a \sim b \\
\sum_{c \neq a} g_{ac}, & a = b.
\end{cases}
\]

\[
(z_{ab})^{-1} = g_{ab} + i \beta_{ab}
\]

Complex Power Flow [balance of power]

∀a : \( P_a = U_a J_a^* = J_a^* \sum_b (\hat{G}')^{-1}_{ab} J_b = U_a \sum_{b \sim a} J_a^* = U_a \sum_{b \sim a} \frac{U_a^* - U_b^*}{Z_{ab}^*} \)
∀a : \( U_a = u_a \exp(i \varphi_a), \quad P_a = p_a + i q_a \)
### DC flow approximation

1. The amplitude of the complex potentials are all fixed to the same number (unity, after trivial re-scaling): $$\forall a : \ u_a = 1.$$

2. Phase variation between any two neighbors on the graph is small:
   $$\forall \{a, b\} : \ |\varphi_a - \varphi_b| \ll 1$$

3. Resistive (real) part of the impedance is much smaller than its reactive (imaginary) part. Typical values for the $$r/x$$ is in the $$1/27 \div 1/2$$ range:
   $$\forall \{a, b\} : \ r_{ab} \ll x_{ab}$$

4. The consumed and generated powers are mainly real, i.e. reactive components of the power are much smaller than their real counterparts:
   $$\forall a : \ p_a \gg q_a$$

It leads to:

- Linear relation between powers and phases (at the nodes): $$\hat{B} \varphi = p$$ and $$\sum_a p_a = 0$$

- Losses (in the leading order): $$Q = p^+ (\hat{B}')^{-1} \hat{G}(\hat{B}')^{-1} p,$$ $$\hat{B}' = \hat{B} + 11^+$$

- If all lines are of the same grade: $$\hat{B} = \hat{G}/\alpha$$ and $$Q = \alpha^2 p^+ (\hat{G}')^{-1} p,$$ i.e. the system is equivalent to “resistive network”, where $$\hat{B}/\alpha^2,$$ $$p$$ and $$\varphi$$ play the roles of the resistivity matrix, vector of currents and vector of voltages.
Network Optimization (for fixed production/consumption $p$)

$$\min_{\hat{g}} p^+ \left( \hat{G}(\hat{g}) \right)^{-1} p,$$
convex over $\hat{g}$

$$G_{ab} = \begin{cases} 0, & a \neq b, \ a \sim b \\ -g_{ab}, & a \neq b, \ a \sim b \\ \sum_{c \sim a} g_{ac}, & a = b. \end{cases}$$

Discrete Graph Laplacian of conductance

Network Optimization (averaged over $p$)

$$\min_{\hat{g}} \langle p^+ \left( \hat{G}(\hat{g}) \right)^{-1} p \rangle = \min_{\hat{g}} \text{tr} \left( \left( \hat{G}(\hat{g}) \right)^{-1} \langle pp^+ \rangle \right) =$$

$$\min \text{tr} \left( \left( \hat{G}(\hat{g}) \right)^{-1} \hat{P} \right), \ \hat{P} - \text{covariance matrix of load/generation}$$

still convex

Boyd, Ghosh, Saberi '06 in the context of resistive networks
also Boyd, Vandenberghe, El Gamal and S. Yun '01 for Integrated Circuits
Network Optimization: Losses + Costs [J. Johnson, MC '10]

Costs need to account for:

- “sizing lines” - grows with $g_{ab}$, linearly or faster (convex in $\hat{g}$)
- “breaking ground” - $l_0$-norm (non convex in $\hat{g}$) but also imposes desired sparsity

Resulting Optimization is non-convex

$$\min_{\hat{g} > 0} \left( \text{tr} \left( \left( \hat{G}(\hat{g}) \right)^{-1} \hat{P} \right) + \sum_{\{a,b\}} \left( \alpha_{ab} g_{ab} + \beta_{ab} \phi(\gamma(g_{ab})) \right) \right), \quad \phi(\gamma) = \frac{x}{x + \gamma}$$

Tricks (for efficient solution of the non-convex problem):

- “annealing”: start from large (convex) $\gamma$ and track to $\gamma \to 0$ (combinatorial)
- Majorization-minimization (from Candes, Boyd ’05) for current $\gamma$:
  $$\hat{g}^{t+1} = \arg\min_{\hat{g} > 0} \left( \text{tr}(\mathcal{L}) + \hat{\alpha} \cdot \hat{g} + \hat{\beta} \cdot \phi'(\gamma(g_{ab}^t)) \cdot g_{ab} \right)$$
Single-Generator Examples

**GAMMA = Inf**

**GAMMA = 0.002811**
Single-Generator Examples
Single-Generator Examples
Single-Generator Examples

- **GAMMA = Inf**
- **GAMMA = 0.02811**
Single-Generator Examples
Single-Generator Examples
Single-Generator Examples
Multi-Generator Example
Adding Robustness

To impose the requirement that the network design should be robust to failures of lines or generators, we use the worst-case power dissipation:

$$L^k(\hat{g}) = \max_{\forall\{a,b\}: z_{ab} \in \{0,1\} | \sum_{\{a,b\}} z_{ab} = N - k} L(\hat{z} \ast \hat{g})$$

- It is tractable to compute only for small values of $k$.
- Note, the point-wise maximum over a collection of convex function is convex.
- So the linearized problem is again a convex optimization problem at every step continuation/MM procedure.
Single-Generator Examples

\[
\text{GAMMA = Inf}
\]

\[
\text{GAMMA = 0.02811}
\]
Single-Generator Examples

![Graphs showing single-generator examples](http://cnls.lanl.gov/~chertkov/SmarterGrids/)
Single-Generator Examples
Single-Generator Examples
Multi-Generator Example
A promising heuristic approach to design of power transmission networks. However, cannot guarantee global optimum.

Submitted to CDC10: http://arxiv.org/abs/1004.2285

Future Work:

- Bounding optimality gap?
- Use non-convex continuation approach to place generators
- Possibly useful for graph partitioning problems
- Adding further constraints (e.g. don’t overload lines)
- Extension to (exact) AC power flow?
Outline

1 Introduction

2 Switching (Phase) Transitions in Distribution Grid
   - Redundancy & Switching
   - SAT/UNSAT Transition & Message Passing

3 An Optimization Approach to Design of Transmission Grids
   - Intro (II): Power Flow & DC approximation
   - Network Optimization

4 Distance to Failure in Power Networks
   - Model of Load Shedding
   - Error Surface & Instantons

5 Conclusions and Path Forward
Normally the grid is ok (SAT) ... but sometimes failures (UNSAT) happens.

How to estimate a probability of a failure?

How to predict (anticipate and hopefully) prevent the system from going towards a failure?

Phase space of possibilities is huge (finding the needle in the haystack)
Minimize Load Shedding = Linear Programming for DC

\[ LP_{DC}(d|G; x; u; P) = \min_{f, \varphi, p, s} \left( \sum_{a \in G_d} s_a \right) \]

\[ COND = COND_{\text{flow}} \cup COND_{\text{DC}} \cup COND_{\text{edge}} \cup COND_{\text{power}} \cup COND_{\text{over}} \]

\[ COND_{\text{flow}} = \left( \forall a : \sum_{b \sim a} f_{ab} = \begin{cases} p_a, & a \in G_p \\ -d_a + s_a, & a \in G_d \\ 0, & a \in G_0 \setminus (G_p \cup G_d) \end{cases} \right) \]

\[ COND_{\text{DC}} = \left( \forall \{a, b\} : \varphi_a - \varphi_b + x_{ab} f_{ab} = 0 \right), \quad COND_{\text{edge}} = \left( \forall \{a, b\} : -u_{ab} \leq f_{ab} \leq u_{ab} \right) \]

\[ COND_{\text{power}} = \left( \forall a : 0 \leq p_a \leq P_a \right), \quad COND_{\text{over}} = \left( \forall a : 0 \leq s_a \leq d_a \right) \]

\( \varphi \) - phases; \( f \) - power flows through edges; \( x \) - inductances of edges

Michael Chertkov – chertkov@lanl.gov

http://cnls.lanl.gov/~chertkov/SmarterGrids/
Introduction
Switching (Phase) Transitions in Distribution Grid
An Optimization Approach to Design of Transmission Grids
Distance to Failure in Power Networks
Conclusions and Path Forward

SAT/UNSAT & Error Surface

Statistics of Loads

\[ P(d|D; c) \propto \exp \left( -\frac{1}{2c} \sum_i \frac{(d_i-D_i)^2}{D_i^2} \right) \]

D is the normal operational position in the space of demands

Instantons (special instances of demands from the error surface)

- Points on the error-surface maximizing \( P(d|D; c) \) - locally!
- \( \arg \max_d P(d)|_{LP_{DC}(d)>0} \) - most probable instanton
- The maximization is not concave (multiple instantons)

No Shedding (SAT) - Boundary - Shedding (UNSAT) = Error Surface

The task: to find the most probable failure modes [instantons]
Borrowed (with modifications) from Error-Correction studies: analysis of error-floor [MC, M. Stepanov, et al '04-'10]

- Construct \( Q(d) = \begin{cases} \mathcal{P}(d), & LP_{DC}(d) > 0 \\ 0, & LP_{DC}(d) = 0 \end{cases} \)
- Generate a simplex (N+1 points) of UNSAT points
- Use Amoeba-Simplex [Numerical Recepies] to maximize \( Q(d) \)
- Repeat multiple times (sampling the space of instantons)

Point at the Error Surface closest to normal operational point

Error Surface

load shedding

no load shedding

normal operational point

\( \frac{\partial}{\partial d} \frac{\partial}{\partial d} \frac{\partial}{\partial d} \)

\frac{\partial}{\partial \text{demand}_1}

\frac{\partial}{\partial \text{demand}_2}

\frac{\partial}{\partial \text{demand}_3}

point at the Error Surface closest to normal operational point
The instantons are sparse (localized on troubled nodes)

The troubled nodes are repetitive in multiple-instantons

Instanton structure is not sensitive to small changes in $D$ and statistics of demands
The instantons are well localized (but still not sparse)

The troubled nodes and structures are repetitive in multiple-instantons

Instanton structure is not sensitive to small changes in $D$ and statistics of demands
Conclusions and Path Forward (for the distance to failure)

Conclusions

- Formulated Load Shedding (SAT/UNSAT condition) as an $LP_{DC}$
- Posed the problem of the Error-Surface and Instantons description in the power-grid setting
- Instanton-amoeba algorithm was suggested and tested on examples

Path Forward

- The instanton-amoeba allows upgrade to other (than $LP_{DC}$) network stability testers, e.g. for AC flows and transients
- Instanton-search can be accelerated, utilizing LP-structure of the tester (in the spirit of [MC,MS '08])
- This is an important first step towards exploration of “next level” problems in power grid, e.g. on interdiction [Bienstock et. al ’09], optimal switching [Oren et al ’08], cascading outages [Dobson et al ’06], and control of the extreme [outages] [Illic et al ’05]

Michael Chertkov – chertkov@lanl.gov

http://cnls.lanl.gov/~chertkov/SmarterGrids/
A lot of interesting power grid settings for CS/IT, Physics analysis

The research is timely (blackouts, renewables, stimulus)

Stay tuned ... the field is growing explosively

Other Problems under investigation by the team


Efficient PHEV charging via queuing/scheduling with and without communications and delays [S. Backhaus, MC, K. Turitsyn, N. Sinitsyn]

For more info - check:
http://cnls.lanl.gov/~chertkov/SmarterGrids/
Thank You!

Michael Chertkov – chertkov@lanl.gov

http://cnls.lanl.gov/~chertkov/SmarterGrids/
The Switching Story with Renewables

Switching/control with renewables (I)

Example n. 1

Fraction 1/3 of consumers produce amount $z$

Every consumer consumes random number in (0.9,1.1)

# producers

$M \to \infty$

# consumers

$N = 3M$

generation > consumption

$y/3 = 1 - z/3$

somebody must serve $D-R+1$

fully demanding consumers

Main Switching Story

Michael Chertkov – chertkov@lanl.gov

http://cnls.lanl.gov/~chertkov/SmarterGrids/
Every consumer produced a random number between (0, z)
Example n. 3

Produced > consumed

\[ \frac{y}{3} > 1 - fz \]

Amount \( z \) is produced by fraction \( f \) of consumers

D=3, R=3, x=1, \( \Delta=0.2 \)

firm generation per customer, \( y/D \)
coverage of renewables, \( f \)