Dynamic Pricing and Stabilization of Supply and Demand in Modern Power Grids

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Dynamic Pricing

How should it be done?
Dynamic Pricing

Various forms of Dynamic Pricing:

1. Time of Use Pricing  
2. Critical Peak Pricing  
3. Real Time Pricing

Borenstein et al [1]:

“We conclude by advocating much wider use of dynamic retail pricing, under which prices faced by end-use customers can be adjusted frequently and on short notice to reflect changes in wholesale prices.”

“The goal of the RTP can be to reflect wholesale prices or to transmit even stronger retail price incentives...An RTP price might also differ between locations to reflect local congestion, reliability, or market power factors.”

“...Such price-responsive demand holds the key to mitigating price volatility in wholesale electricity spot markets.”

Dynamic Pricing

Various forms of Demand Response [2]:

- RTP DR
- Explicit Contract DR
- Imputed DR

Consumers pay the LMP for their marginal consumption.

W. Hogan [2]:

“…any consumer who is paying the RTP for energy is charged the full LMP for its consumption and avoids paying the full LMP when reducing consumption.”

“Expanding the use of dynamic pricing, particularly real-time pricing, to provide smarter prices for the smart grid would be a related priority....”

The Independent System Operator (ISO)

- Non-for-profit organization
- Operates the wholesale markets and the TX grid
- Primary function is to optimally match supply and demand -- adjusted for reserve -- subject to network constraints.
- Operation of the real-time markets involves solving a constrained optimization problem to maximize the aggregate benefits of the consumers and producers. *(The Economic Dispatch Problem (EDP))*
- In real-time, the objective usually is to minimize total cost of dispatch for a fixed demand
- Constraints are: KVL, KCL, TX line capacity, generation capacity, local and system-wide reserve, other ISO-specific constraints.
California ISO Control Room in Folsom – photo by Donald Satterlee

Courtesy of the California ISO: http://www.caiso.com

CA ISO operates 25,000 circuit miles of high-voltage, long distance power lines
Power Systems
Wholesale Markets and System Operation

Generation

Cleared for dispatch

Transmission Grid

Market Clearing

System Operation

Cleared demand

Market

bids or fixed demand

Demand

G1

G2

Gn

D1

D2

Dn

bids or fixed offers
Simplified DC Model:

\[
\begin{align*}
\text{min} & \quad \sum_{i=1}^{n} c_i S_i \\
\text{s.t.} & \quad \begin{bmatrix} K & E \\ 0 & R \end{bmatrix} \begin{bmatrix} S \\ I \end{bmatrix} = \begin{bmatrix} D \\ 0 \end{bmatrix} \\
& \quad -I_{\text{max}} \leq I \leq I_{\text{max}} \\
& \quad S_{\text{min}} \leq S \leq S_{\text{max}}
\end{align*}
\]

Generators as ideal current sources

Demand is fixed

KCL

KVL

Line Capacity limits

Generation limits

Locational Marginal Prices are the dual variables corresponding to the constraint \( K S + EI = D \).
Locational Marginal Prices

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Midwest ISO Market data is based on Eastern Standard Time (EST) while PJM Market data is based on Eastern Prevailing Time.

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Economic Dispatch
Primal and dual problems

Dual objective $\mathcal{D}(\lambda)$ depends on $d$: $g(\lambda, d)$

\[
\mathcal{D}(\lambda) = \min_{S,I} \sum_{i=1}^{n} c_i(S_i) - \lambda (K S + EI - d);
\]

s.t. $RI = 0$

$-I_{\text{max}} \leq I \leq I_{\text{max}}$

$S_{\text{min}} \leq S \leq S_{\text{max}}$

Primal objective: $f(S)$

\[
\min_{S,I} \sum_{i=1}^{n} c_i(S_i)
\]

s.t.

\[
\begin{bmatrix}
K & E \\
0 & R
\end{bmatrix}
\begin{bmatrix}
S \\
I
\end{bmatrix}
= \begin{bmatrix}
d \\
0
\end{bmatrix}
\]

$-I_{\text{max}} \leq I \leq I_{\text{max}}$

$S_{\text{min}} \leq S \leq S_{\text{max}}$

Primal feasible set: $\Omega(d)$
Passive Consumption
The System is Open Loop

The ISO – Primal and Dual Economic Dispatch Problems

\[ \lambda_{t+1} = \arg \max_{\lambda} \ g(\lambda, \hat{d}_{t+1}) \]

\[ S_{t+1} = \arg \min_{(S,I) \in \Omega(\hat{d}_{t+1})} \ f(S) \]

\[ \hat{d}_{t+1} = \hat{D}_t(d_t, \ldots, d_0) \]

Locational marginal price
The producers
Demand prediction
Dual objective function
Total cost of production in EDP

\[ \hat{d}_{t+1} = \hat{D}_t(d_t, \ldots, d_0) \]
\[ d_t \]
Real-Time Pricing

Closing the Loop

\[ \lambda_{t+1} = \arg \max_{\lambda} \ g(\lambda, \hat{d}_{t+1}) \]

\[ S_{t+1} = \arg \min_{(S,I) \in \Omega(\hat{d}_{t+1})} f(S) \]

\[ \hat{d}_{t+1} = \hat{D}_t(d_t, \ldots, d_0) \]

\[ d_t = h(\lambda_t) \]

Locational marginal price

The producers

Demand prediction

The consumers

Dual objective function

Total cost of production in EDP

\[ z^{-1} \]
Consumers and Producers
Cost functions and value function

**Consumers**

Convex cost function

\[ s_t = \arg \max_x \lambda_t x - c(x) \]

\[ = \dot{c}^{-1}(\lambda_t) \]

**Producers**

Concave value function

\[ d_t = \arg \max_x v(x) - \lambda_t x \]

\[ = \dot{v}^{-1}(\lambda_t) \]
Real-Time Pricing
Closing the Loop

\[
\begin{align*}
\lambda_{t+1} &= \arg \max_{\lambda} g(\lambda, \hat{d}_{t+1}) \\
S_{t+1} &= \arg \min_{(S,I) \in \Omega(\hat{d}_{t+1})} f(S) \\
\hat{d}_{t+1} &= \hat{D}_t(d_t, \ldots, d_0) \\
d_t &= \arg \max_x u(x) - \lambda_t x \\
    &= \dot{v}^{-1}(\lambda_t)
\end{align*}
\]
Message:

Real time pricing creates a closed loop feedback system

Need good engineering to create a well-behaved closed loop system

There are tradeoffs in stability/volatility and efficiency
Closed Loop System Dynamics
Simplified Model

Assumptions:
1. Line capacities are high enough, i.e., no congestion
2. Generator capacities are high enough, i.e., no capacity constraint

Then
1. All the generators can be lumped into one representative generator
2. All the consumer can be lumped into one representative consumer

Assume the Rep. agents’ cost (value) functions are smooth convex (concave).
Theorem: The system $x_{k+1} = \psi(x_k)$, where $\psi : \mathbb{R}_+ \to \mathbb{R}_+$, is stable if there exist functions $f$ and $g$ mapping $\mathbb{R}_+$ to $\mathbb{R}_+$, and $\theta \in (-1, 1)$ satisfying:

\[ g(x_{k+1}) = f(x_k) \]  \hspace{1cm} (1)

and

\[ |\dot{f}(x)| \leq \theta |\dot{g}(x)| \]  \hspace{1cm} (2)

Note: $\psi = g^{-1} \circ f$

In our context: $\psi = \dot{c} \circ \dot{v}^{-1}$, hence, a sufficient stability criterion is:

\[ \left| \frac{d}{dx} \dot{v}^{-1}(x) \right| \leq \theta \frac{d}{dx} \dot{c}^{-1}(x) \]
Stability Theorem: The system $\lambda_{t+1} = \dot{c} \left( \dot{v}^{-1} (\lambda_t) \right)$ is stable if there exists a function $\rho : \mathbb{R}_+ \mapsto \mathbb{R}_+$, and a constant $\theta \in (-1, 1)$, s.t.

$$\left| \frac{\dot{\rho} \left( \dot{v}^{-1} (\lambda) \right)}{\ddot{v} \left( \dot{v}^{-1} (\lambda) \right)} \right| \leq \theta \frac{\dot{\rho} \left( \dot{c}^{-1} (\lambda) \right)}{\ddot{c} \left( \dot{c}^{-1} (\lambda) \right)} , \quad \forall \lambda \in \mathbb{R}_+$$

(1)

In particular,

$$|\ddot{c}| \leq \theta \ddot{v}$$

is sufficient. (obtained with $\rho = \dot{c}$)
Real-Time Pricing with a Static Pricing Function

Efficiency Loss

The function $\phi : \mathbb{R}_+ \mapsto \mathbb{R}_+$ stabilizes the system $\lambda_{t+1} = \dot{c}\left(\dot{v}^{-1}(\phi(\lambda_t))\right)$ if

$$|\dot{\phi}(\lambda)| \left| \frac{\dot{\rho}(\dot{v}^{-1}(\phi(\lambda)))}{\ddot{v}(\dot{v}^{-1}(\phi(\lambda)))} \right| \leq \theta \left| \frac{\dot{\rho}(\dot{c}^{-1}(\phi(\lambda)))}{\ddot{c}(\dot{c}^{-1}(\phi(\lambda)))} \right|, \quad \forall \lambda \in \mathbb{R}_+ \tag{2}$$

S(x) = v(x) + c(x)

$S_{\phi}(\lambda) = c(\dot{c}^{-1}(\lambda)) + v(\dot{v}^{-1}(\phi(\lambda)))$

The farther the wholesale and retail prices at the equilibrium, the more is the efficiency loss.
Stable for sufficiently small $\gamma$:

$$\lambda_{t+1} = \lambda_t + \gamma (\dot{c} (\dot{v}^{-1} (\lambda_t)) - \lambda_t)$$

Retrieve the original dynamics when $\gamma = 1$

The idea can be used to construct a stabilizing sub-gradient algorithm for the full model of EDP with DC OPF constraints.
Real-Time Pricing

Subgradient-based Stabilizing Pricing Mechanism

\[
\lambda_{t+1} = \arg\max_{\lambda} g(\lambda, \hat{d}_{t+1})
\]

\[
S_{t+1} = \arg\min_{(S,I) \in \Omega(\hat{d}_{t+1})} f(S)
\]

\[
\hat{d}_{t+1} = \hat{D}_t(d_t, \ldots, d_0)
\]

\[
\pi_t = \Pi(\lambda_t, \pi_{t-1})
\]

\[
d_t = \arg\max_x v(x) - \pi_t x
\]

\[
= \dot{v}^{-1}(\pi_t)
\]

\[
G(\pi_t) = -Ks_t - EI_t + d_t
\]

\[
= -Ks_t - EI_t + \dot{v}^{-1}(\pi_t)
\]
Real-Time Pricing
Subgradient-based Stabilizing Pricing Mechanism

\[ \mathcal{D}(\lambda) = \min_{s,I} \sum_{i=1}^{n} c_i(S_i) - \lambda (K S + E I - d) \]

\[
\text{s.t. } R I = 0 \\
-I_{\text{max}} \leq I \leq I_{\text{max}} \\
S_{\text{min}} \leq S \leq S_{\text{max}}
\]

\[
\text{primal} \\
\min \sum_{i=1}^{n} c_i(S_i) \\
\text{s.t. } \begin{bmatrix} K & E \\ 0 & R \end{bmatrix} \begin{bmatrix} S \\ I \end{bmatrix} = \begin{bmatrix} d \\ 0 \end{bmatrix} \\
-I_{\text{max}} \leq I \leq I_{\text{max}} \\
S_{\text{min}} \leq S \leq S_{\text{max}}
\]

The dual is concave and non-differentiable
\(-Ks_t - EI_t + d_t\) is a subgradient direction
Real-Time Pricing

Subgradient-based Stabilizing Pricing Mechanism

\[ \mathcal{D}(\lambda) = \min_{s,I} \sum_{i=1}^{n} c_i(S_i) - \lambda(KS + EI - d) \]

s.t. \( RI = 0 \)

\(-I_{\max} \leq I \leq I_{\max}\)

\( S_{\min} \leq S \leq S_{\max} \)

**Theorem:** The pricing mechanism

\[ \pi_{t+1} = \pi_t + \gamma G(\pi_t) = \pi_t - \gamma(KS_t + EI_t - d_t) \]

stabilizes the system: For sufficiently small \( \gamma \), \((\pi_t, S_t, d_t)\) converge to a small neighborhood of \((\pi^*, S^*, d^*)\) where \( \pi^* \) is the dual optimal solution and \( S^* \) and \( d^* \) are the corresponding optimal supply and demand.
consumer value functions are logarithmic:

\[ v_l(d) = \log(d) \]
\[ d_l = \alpha_l / \pi_l \]

producer cost functions are quadratic:

\[ c_l(s) = \beta_l s^2 \]
\[ s_l = \lambda_l / (2 \beta_l) \]

To make the simulations more realistic, we approximated the quadratic costs with piecewise linear functions to get an LP for EDP. Also added noise to \( \alpha, \beta \)
Both demand and price are very volatile

\[ \pi_t = \lambda_t \]

3 Bus system

LMP

demand
Real-Time Pricing
Subgradient-based Stabilizing Pricing Mechanism

disturbance was introduced at node 3 at time t=100

\[ \pi_{t+1} = \pi_t + \gamma G(\pi_t) \]

3 Bus system

\[ d_{l,t} = \alpha_l / \pi_{l,t} \quad s_{l,t} = \lambda_{l,t}^{cp} / 2\beta_l \]

LMP demand
1. More sophisticated models (consumer behavior, power flow, market clearing)
2. Price anticipating consumers / consumers with rational expectations
3. Incorporate reserve capacity markets in the model
4. Stochastic model of supply / demand
5. Dynamic model of the Economic Dispatch over a rolling time horizon
6. Partial knowledge of demand value function -- demand prediction
7. Tradeoffs between wholesale market volatility and retail price volatility
8. Control always has a cost, in this case real money. There will be discrepancies between retail revenue and wholesale cost. Who pays for it? Consumers? How does that change consumer behavior?
9. Fairness: If only a portion of population is participating in RTP, those with fixed-price contract can drive the prices very high for the RTP consumers at a time of shortage, exposing them to undue risk and inconvenience…
Thank you!