Market Model and Algorithmic Design for Demand Response in Power Networks

Lijun Chen, Na Li, and Steven Low

Computing and Mathematical Sciences
California Institute of Technology

April 18, 2011
Demand response

Use incentive mechanisms such as real-time pricing to induce customers/appliances to shift usage or reduce (even increase) consumption

Demand response techniques

- Smart appliances responding to price/event signals
- Load shifting technologies such as storage
- Peak-eliminating techniques such as distributed generation or simply turning off appliances
- ...

...
Enabler

Smart grid

- Timely two-way communications between customers and utility companies
- Individual customers and appliances are empowered with certain computing capability
- High speed WAN allows real-time and global monitoring at control centers
- High performance computing allows faster control decisions
Outline

- Motivation for demand response
- Main issues in demand response design
- Demand response: Match the supply
- Demand response: Shape the demand
Electricity demand is highly time-varying

Provision for peak load

- Low load factor
  - US national load factor is about 55%

- Underutilized
  - 10% of generation and 25% of distribution facilities used less than 5% of the time

Source: DoE, Smart Grid Intro, 2008
Shape the demand
- Reduce peak load
- Flatten load profile

Benefits
- Lower generation cost
- Larger safety margin
- Reduce or slow down the need for new generation and distribution infrastructure
Uncertainty of renewables

change at timescale of minutes

Tehachapi Wind Generation in April – 2005

Could you predict the energy production for this wind park either day-ahead or 5 hours in advance?

Each Day is a different color.

Source: Rosa Yang
Uncertainty of renewables

change at timescale of seconds

Source: Rosa Yang
Dealing with uncertainty

- Reduce uncertainty by
  - Aggregating supply types
  - Aggregating over space
  - Aggregating over time (but large-scale storage is currently not available)

- Accommodate uncertainty
  - Reliability as resource to trade off
    - Optimize risk tolerance
  - **Match** time-varying supply (demand response)
Outline

- Motivation for demand response
- **Main issues in demand response design**
- Demand response: Match the supply
- Demand response: Shape the demand
Main challenge

Matching supply and demand

- Market challenge
  - achieve efficient and economic generation, delivery, and consumption

- Engineering challenge

  electricity must be consumed at the moments it is generated
Overall structure

generation

wholesale market

utility company

retail market

customers

- Bilateral contracts
- Auction market
  - day-ahead
  - real-time
  - ancillary service
Main issues

The role of utility as an intermediary

- Play in multiple wholesale markets to provision aggregate power to meet demands
- Resell, with appropriate pricing, to end users
- Provide two important values
  - Aggregate demand at the wholesale level so that overall system is more efficient
  - Absorb large uncertainty/complexity in wholesale markets and translate them into a smoother environment (both in prices and supply) for end users.

How to quantify these values and price them in the form of appropriate contracts/pricing schemes?
Main issues

Utility/end users interaction

- Design objective
  - Welfare-maximizing, profit-maximizing, ...
- Distributed implementation
- Real-time demand response

The impact of distribution network

- i.e., put in physical network (Kirkoff Law, and other constraints)
- How does it change the algorithm and optimality
- Can we exploit radial structure of distribution network
Retail market

Retail (utility-user) essentially uses fixed prices

- Tiered, some time-of-day

Demand response will (likely) use real-time pricing to better manage load

How should utility company design real-time retail prices to optimize demand response?
The basics of supply and demand

- **Supply function**: quantity supplied at given price
  \[ q = S(p) \]

- **Demand function**: quantity demanded at given price
  \[ q = D(p) \]

- **Market equilibrium**: \((q^*, p^*)\) such that \[ q^* = S(p^*) = D(p^*) \]

  - No surplus, no shortage, price clears the market

![Diagram of supply and demand](image-url)
Competitive vs oligopolistic markets

- Competitive market: no market participant is large enough to have market power to set the price
  - Price-taking behavior
  - e.g., individual residential customers

- Oligopolistic market: (a few) market players can influence and be influenced by the actions of others
  - Price-anticipating behavior
  - e.g., large commercial customers
Utility function

- Given the set $X$ of possible alternatives, a function
  \[ U : X \rightarrow R \]
  is a utility function representing preference relation among alternatives, if for all $x, y \in X$,
  \[ "x \text{ is at least as good as } y" \iff U(x) \geq U(y) \]

- To use utility function to characterize preferences is a fundamental assumption in economics.
Outline

- Motivation for demand response
- Main issues in demand response design
- Demand response: Match the supply
- Demand response: Shape the demand
Problem setting

- Supply deficit (or surplus) on electricity: $d$
  - weather change, unexpected events, ...

- Supply is inelastic
  - because of technical reasons such as supply friction

**Problem**: How to allocate the deficit/surplus among demand-responsive customers?

- load (demand) as a resource to trade
Supply function bidding

- Customer $i \in N$ load to shed: $q_i$
- Customer $i$ supply function (SF):
  \[ q_i(b_i, p) = b_ip \]
  - parameterized by $b_i \geq 0$; $b \triangleq (b_i)_{i \in N}$
  - the amount of load that the customer is committed to shed given price $p$
- Market-clearing pricing:
  \[ \sum_i q_i(b_i, p) = d \]

\[ p = p(b) \triangleq d / \sum_i b_i \]

utility company: deficit $d$

customer 1: $q_1 = b_1p$

......

customer n: $q_n = b_np$

......
Parameterized supply function

- Adapts better to changing market conditions than does a simple commitment to a fixed price or quantity (Klemper & Meyer ’89)
  - widely used in the analysis of the wholesale electricity markets
  - Green & Newbery ‘92, Rudkevich et al ‘98, Baldick et al ‘02, ‘04, ...

- Parameterized SF
  - easy to implement
  - control information revelation
  - ...


Optimal demand response

- Customer $i$ cost (or disutility) function: $C_i(q_i)$
  - continuous, increasing, and strictly convex

- Competitive market and price-taking customers

- Given price $p$, each customer $i$ solves
  
  $$\max_{b_i} pq_i(b_i, p) - C_i(q_i(b_i, p))$$

utility company: deficit $d$

customer $i$:

$$\max_{b_i} pq_i(b_i, p) - C_i(q_i(b_i, p))$$
Competitive equilibrium

- **Definition**: A competitive equilibrium (CE) is defined as a tuple \( \{(b^*_i)_{i \in N}, p^*\} \) such that
  \[
  b^*_i = \arg \max_{b_i \geq 0} p^* q_i(b_i, p^*) - C_i(q_i(b_i, p^*)), \quad \forall i
  \]
  \[
  \sum_i q_i(b^*_i, p^*) = d
  \]

- **Theorem**: There exist a unique CE. Moreover, the equilibrium is efficient, i.e., maximizes social welfare
  \[
  \max_{q_i} -C_i(q_i) \quad \text{s.t.} \quad \sum_i q_i = d
  \]
Proof

Show the equilibrium condition is the optimality condition (KKT) of the optimization problem.

Equilibrium

\[ b_i^* = \arg\max_{b_i \geq 0} p^* q_i(b_i, p^*) - C_i(q_i(b_i, p^*)), \forall i \]
\[ \sum_i q_i(b_i^*, p^*) = d \]

Social Welfare optimization

\[ \max_{q_i} -C_i(q_i) \text{ s.t. } \sum_i q_i = d \]

\[ (p^* - C_i'(q_i(b_i^*, p^*)))(b_i^* - b_i) p^* \leq 0, \forall b_i \geq 0 \]
\[ \sum_i q_i(b_i^*, p^*) = d \]

\[ (p^* - C_i'(q_i^*)) (q_i - q_i^*) \leq 0, \forall q_i \geq 0 \]
\[ \sum_i q_i^* = d \]
Iterative supply function bidding

- Upon receiving the price information, each customer \( i \) updates its supply function
  \[ b_i(k) = \left[ \frac{(C_i')^{-1}(p(k))}{p(k)} \right]^+ \]

- Upon gathering bids from the customers, the utility company updates price
  \[ p(k+1) = [p(k) - \gamma(\sum_i b_i(k)p(k) - d)]^+ \]

- Requires
  - timely two-way communication
  - certain computing capability of the customers
Strategic demand response

- Oligopoly market and price-anticipating customer

\[ p = p(b) \triangleq d / \sum_i b_i \]

- Given others’ supply functions \( b_{-i} \), each customer \( i \) solves

\[
\max_{b_i} u_i(b_i, b_{-i})
\]

with

\[ u_i(b_i, b_{-i}) = p(b)q_i(b_i, p(b)) - C_i(q_i(b_i, p(b))) \]

It is a game
**Game-theoretic equilibrium**

- **Definition**: A supply function profile $b^*$ is a Nash equilibrium (NE) if, for all customers $i$ and $b_i \geq 0$

$$u_i(b_i^*, b_{-i}^*) \geq u_i(b_i, b_{-i}^*).$$

- **Theorem**: There exists a unique NE when the number of customers is larger than 2. Moreover, the equilibrium solves

\[
\max_{0 \leq q_i \leq d/2} -D_i(q_i) \quad \text{s.t.} \quad \sum_i q_i = d \\
D_i(q_i) = (1 + \frac{q_i}{d-2q_i})C_i(q_i) - \int_0^{q_i} \frac{d}{(d-2x_i)^2}C_i(x_i)dx_i
\]
Proof

Show the equilibrium condition is the optimality condition (KKT) of the optimization problem.

**Nash Equilibrium**

\[
\max_{b_i} \quad p q_i(p(b), p) - C_i(q_i(p(b), p))
\]

\[
= d^2 b_i / (\Sigma_j b_j)^2 - C_i(db_i / \Sigma_j b_j)
\]

\[
\sum_i q_i(b_i, q) = d
\]

\[
\left( p^* - \left(1 + \frac{q_i^*}{d-2q_i^*}\right) C_i'(q_i^*) \right) (b_i p^* - q_i^*) \leq 0
\]

\[
\forall b_i \geq 0
\]

\[
\sum_i b_i p^* = d
\]

\[
p^* = \sum_i b_i^*; \quad q_i^* = b_i^* p^*
\]

**Optimization**

\[
\max_{q_i} \quad -D_i(q_i) \quad \text{s.t.} \quad \sum_i q_i = d
\]

\[
D_i(q_i) = (1 + q_i / d - 2q_i) C_i(q_i)
\]

\[
-\int_0^{q_i} d / (d - 2x_i)^2 C_i(x_i)dx_i
\]
Iterative supply function bidding

- Each customer $i$ updates its supply function
  
  \[ b_i(k) = \left[ \frac{(D'_i)^{-1}(p(k))}{p(k)} \right]^+ \]

- The utility company updates price
  
  \[ p(k+1) = [p(k) - \gamma(\sum_i b_i(k)p(k) - d)]^+ \]
Numerical example

Optimal supply function bidding (upper panels) v.s. strategic bidding (lower panels)
Outline

- Motivation for demand response
- Main issues in demand response design
- Demand response: Match the supply
- Demand response: Shape the demand
Problem setting

- Load is deferrable and reducible
- Subject to various constraints, depending on the types of appliances
  - minimal/maximal load over certain period of time
  - minimal/maximal load at each time
  - battery has finite capacity and usage-dependent cost
  - ...

**Problem**: How to shape deferrable load over certain period of time, so as to reduce peak, flatten load profile and even conserve energy?
Customer-side model (abstract)

- Each customer $i$, each of the appliances $a \in A_i$:
  - Load at time $t$: $q_{i,a}(t)$; define: $q_{i,a} \triangleq (q_{i,a}(t))_{t \in T}$
  - Load constraint: $q_{i,a} \in C_{i,a}$
  - Total load at time $t$: $Q_i(t) = \sum_a q_{i,a}(t) + r_i(t)$
  - Utility: $U_{i,a}(q_{i,a})$
  - The appliances divided into 4 categories

- Energy Storage: one battery for each customer $i$
  - Load at time $t$: $r_i(t)$; define $r_i \triangleq (r_i(t))_{t \in T}$
    - positive means charging
    - negative means discharging
  - Load constraints: $r_i \in R_i$
  - Cost function: $D_i(r_i)$
Utility-side model

- The utility company incurs cost $C(Q)$ when the supply is $Q$
- convex, with a positive, increasing marginal cost

- Piecewise quadratic cost functions

$$C(Q) = \begin{cases} 
  c_1Q^2 + b_1Q + a_1; & 0 \leq Q \leq Q_1 \\
  c_2Q^2 + b_2Q + a_2; & Q_1 \leq Q \leq Q_2 \\
  \vdots \\
  c_mQ^2 + b_mQ + a_m; & Q_{m-1} \leq Q
\end{cases}$$

with $c_m > c_{m-1} > \cdots > c_1 > 0$
Utility-side model

Objective: induce customers’ consumption to maximize social welfare

\[
\max_{q,r} \sum_i \left( \sum_{a \in A_i} U_{i,a}(q_{i,a}) - D_i(r_i) \right) - \sum_t C \left( \sum_i Q_i(t) \right)
\]

s.t. \( q_{i,a} \in C_{i,a} \)
\( r_i \in R_i \)
\( 0 \leq Q_i(t) \leq Q_i^{\text{max}} \)

proof of conception, to see how effective real-time pricing can be
Utility-customer interaction

- Utility sets prices \( p \triangleq (p(t))_{t \in T} \) to induce customer behaviors.

- Customer \( i \) maximizes his own \textbf{net benefit}

\[
\max_{q_{i,r_i}} \sum_{a} U_{i,a}(q_{i,a}) - D_i(r_i) - \sum_{t} Q_i(t)p(t)
\]

s.t.
\[
q_{i,a} \in C_{i,a} \\
q_{i,a} \in C_{i,a} \\
r_i \in R_i \\
0 \leq Q_i(t) \leq Q_i^{\max}
\]

\textbf{price-taking}
Market equilibrium

- **Definition**: The prices and customer demands \((p^*, q_{i,a}^*, r_i^*)\) is in equilibrium if \((q_{i,a}^*, r_i^*)\) maximizes the social-welfare, and also maximizes customer \(i\) net benefit for given price \(p^*\).

- **Theorem**: There exists an equilibrium \((p^*, q_{i,a}^*, r_i^*)\). Moreover, the equilibrium price \(p^*(t) = C' \left( \sum_i Q_i^*(t) \right)\).
  - follow from the welfare theorem and imply that setting the price to be the marginal cost of power is optimal
  - similar proof
Customer-side model (appliances)

Utility function:
\[ U_{i,a}(q_{i,a}) = \sum_t U_{i,a}(T_{i,a}(t), T_{i,a}^{\text{comf}}) \]

Constraints:
\[ T_{i,a}^{\text{min}} \leq T_{i,a}(t) \leq T_{i,a}^{\text{max}} \]
\[ T_{i,a}(t) = g(T_{i,a}(t-1), q_{i,a}(t)) \]
\[ 0 \leq q_{i,a}(t) \leq q_{i,a}^{\text{max}}(t) \]

Utility function:
\[ U_{i,a}(q_{i,a}) = U_{i,a}\left(\sum_t q_{i,a}(t)\right) \]

Constraints:
\[ 0 \leq q_{i,a}(t) \leq q_{i,a}^{\text{max}}(t) \]
\[ Q_{i,a}^{\text{min}} \leq \sum_t q_{i,a}(t) \leq Q_{i,a}^{\text{max}} \]
Customer-side model (appliances)

Utility function:
\[ U_{i,a}(q_{i,a}) = \sum_t U_{i,a}(q_{i,a}(t), t) \]

Constraints:
\[ 0 \leq q_{i,a}(t) \leq q_{i,a}^{\text{max}}(t) \]

Utility function:
\[ U_{i,a}(q_{i,a}) = \sum_t U_{i,a}(q_{i,a}(t), t) \]

Constraints:
\[ 0 \leq q_{i,a}(t) \leq q_{i,a}^{\text{max}}(t) \]
\[ Q_{i,a}^{\text{min}} \leq \sum_t q_{i,a}(t) \leq Q_{i,a}^{\text{max}} \]

a crude model
Customer-side model (Battery)

Cost function:

\[ D_i(r_i) = \eta_1 \sum_t r_i^2(t) - \eta_2 \sum_t r_i(t)r_i(t+1) + \eta_3 \sum_t (\min\{B_i(t) - \delta B_i, 0\})^2 \]

- charging & discharging
- charging - dis cycles
- deep discharging

Constraints:

\[ 0 \leq B_i(t) \leq B_i \]
\[ B_i(T) \geq \gamma_i B_i \]
\[ r_i^{\min} \leq r_i(t) \leq r_i^{\max} \]
Numerical example: no battery

4 households with people at home all the day;
4 with no person at home during day time
Numerical example: with battery
Numerical experiments

Load Factor

\[ \text{load factor} = \frac{\text{average load}}{\text{peak load}} \]

Peak Demand Per Household, kwh

Total Demand Per Household, kwh
Concluding remarks

- Demand response: Match the supply
  - iterative supply function bidding (competitive vs oligopolistic)

- Demand response: Shape the demand
  - Real-time pricing based on marginal cost is “ideally” very effective

- Future work: extend the models to study the aforementioned issues in demand response design
  - Current focus: real-time demand response; coordinated control with Volt/Var
References

Thanks