Drift kinetics within a global MHD model: a progress report

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• What’s being added - most FLR effects
  • Particle drifts, Hall electric field, gyroviscosity

• Approach is brute force using a multifluid MHD code
  • Similar in assumptions to Rice Convection Model
  • Multiple energy channels

• Outline
  • Motivation
  • Method
  • Initial Results
Motivation

- Much of the structure is mesoscale or smaller

- What sets the scales:
  - ideal instabilities tend to have growth rates increasing with $k$
  - Increased resolution gives smaller scales without limit
  - Suspect ion FLR scales produce the scales of BBF’s, for example
Motivation II

- Coupling to Rice Convection model is hindered by incompatible physics
  - Inertial effects limit the spatial range over which the RCM can accept MHD input
  - The current LFM to RCM input is a Maxwellian which doesn’t reflect actual population in the tail
Motivation III

- Drifts are not dominant in the tail but they can be important
  - Example: 10nT field, 2 keV proton.
    - Larmor radius $1/10 R_E$
    - Drift speed = 30/L kms
    - $V_E = E(mV/m) 100$ km/s

- Geotail averages show asymmetry in direction of ion drift

- MHD calculations are approaching $d_i$
  - Does increasing resolution buy you better physics
  - Maybe put effort into something else

Guild et al., JGR, 113, 2008
Method

- Assumptions
  - At any given thermal energy the distribution function is isotropic
  - But, each energy can have a different velocity (drift) centroid

- These are essentially the same as the RCM makes
- Split distribution into discrete energy species
- No assumption is made about the parallel velocity at a given energy
- The model is essentially one in which the energy scattering rate is very slow, but pitch energy scattering is very fast
- Because the parallel velocity for each energy is unconstrained, distributions can look very different from a skewed Maxwellian
• Use the Vlasov equation and moments

\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \]

• Note that the Lorentz force contains the gyro timescale
  • usually assume \( \Omega_i >> 1/\tau_{MHD} \)
  • suggests expansion in terms of

\[ \epsilon \sim \Omega_i \tau_{MHD}, \frac{r_L}{L} \]

• but this isn’t necessary
• will rough out the perturbative approach
• Split the species velocity into

\[ \mathbf{v} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \epsilon \mathbf{v}_d + \mathbf{v}_\parallel = \mathbf{v}_E + \epsilon \mathbf{v}_d + \mathbf{v}_\parallel \]

• Then the moment equations become

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \rho \mathbf{v} \mathbf{v} + \nabla P + \epsilon \nabla \cdot \Pi_{gv} + \frac{nq}{\epsilon} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0
\]

\[
\frac{Ds}{Dt} = 0
\]

• We can write the Lorentz force as

\[ \mathbf{F}_{Lorentz} = \frac{nq}{\epsilon} \mathbf{v}_d \times \mathbf{B} \]

• This is the drift force that in lowest order gives all species the same perpendicular velocity
• Momentum and Ohm’s Law
  • Momentum contains both ExB drift and sum of the drifts
  • But, to lowest order the bulk speed is ExB/B^2
  • Thus, the drifts are specified by finding the bulk speed

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \rho \mathbf{v} \mathbf{v} + \nabla P - \mathbf{j} \times \mathbf{B} = 0
\]

\[
\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \mathbf{j}/ne + \nabla P_e/\rho ne
\]

\[
\mathbf{j} = \Sigma_{\alpha} n_{\alpha} q_{\alpha} \mathbf{v}_{d}^{\alpha} + \nabla P_e/\rho ne
\]

• Once we know the drifts to lowest order we can correct the individual velocities

\[
\mathbf{v}_{\perp}^{\alpha} = \mathbf{v}_E + \mathbf{v}_{d}^{\alpha}
\]

\[
\mathbf{v}_{\perp} = \mathbf{v}_{bulk} - \frac{1}{\rho} \Sigma_{\beta} n_{\beta} m_{\beta} \mathbf{v}_{d}^{\beta} + \mathbf{v}_{d}^{\alpha}
\]

• The new velocities can be used to correct the convective terms in the equations
Hall term and the Biermann Battery

- Coupled ring current models by and large ignore the Hall effect - this is inconsistent
- The velocity coming out of the momentum equation contains both the ExB and diamagnetic drifts, but Faraday’s Law usually uses the total
- The drifts can be much larger than ExB, so we need to include jxB/ne
- To a good approximation in the inner magnetosphere \( \nabla P = -\vec{j} \times \vec{B} \)
- Substituting we get

\[
\frac{\partial \vec{B}}{\partial t} = -\frac{1}{n^2e} \nabla n \times \nabla P
\]

- If the flow is smooth probably no effect, but if, for example, we have fingers:

No effect

Some effect
Pros and Cons

• Advantages:
  • Handles inertial terms automatically
  • Keeps field-aligned behavior
  • Effectively provides a physically reasonable closure for the integrated equations
  • *e.g.*, sum over species implies a heat flux for the ensemble

• Disadvantages
  • Computationally intensive
  • As approach \(d_i\) scales it’s, not clear that expansion works
  • Some hope for the plasma sheet (note Usadi *et al.*, JGR, 101, 1996)
  • More related to the assumptions about the distribution function than the gyro scales
  • Electrons aren’t handled well.
• Gyroviscosity is usually ignored in magnetospheric fluid models
  • finite Larmor radius effects other than the Hall term are usually dropped
  • Except for very simple cases the GV tensor is very messy, requires tensor velocity moments up to fourth order

Huba (GRL, 23, 1996) did KH calculation with Hall term and gyroviscosity

Huba and Winske (Phys. Plasma, 5, 1998) compared hybrid and FLR MHD with mixed results
• For a single energy, isotropic distribution the gyroviscous tensor reduces to something manageable.

• When sum over energy channels the bulk properties recover higher order moment corrections.

\[
\Pi_{gv} = \frac{P}{4\Omega_i} \left[ (b \times \mathcal{W}) \cdot (\mathcal{I} + 3bb) + (\mathcal{I} + 3bb) \cdot (\mathcal{W} \times b) \right] \\
\mathcal{W} = \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]
Drifts

• Song *et al.* (JGR, 113, 2008), a cautionary tale
  • Compared the effort of Heinemann and Wolf (JGR, 106, 2001) to provide a single fluid model for inner magnetosphere drifts to The RCM
  • Results were not encouraging
  • Test problem of drifts in a dipole field
    • drifting blob (Gaussian in space) - fluid model breaks into two separate blobs doesn’t spread out like RCM

Fluid model density, pressure

RCM density, temperature
MHD drifts

- Basically same as Song et al problem.
- Blob at $5.5 \, R_E$, 64 species, $0.1 \, R_E$ resolution
- Higher energy particles drift almost entire way around Earth
Comparisons

Low res MHD, 0.2 $R_E$, 15 species

Movie frame

Song et al. 200 species left frame - density, right - temperature
Ring current simulations

- Preliminary - doing one-way coupling
  - use LFM fields to push drifting distribution, e.g. $\nabla P$ rather than $\nabla B$
  - 16 energy channels
  - base run, $V_{sw} = 400$ km/s, $n = 5$ cm$^{-3}$, $B_z = -15$ nT

- Show development of ring current
  - Too close, too weak
  - Probably resolution related and lack of self-consistency
To Do List

• Full implementation in the Multi-Fluid LFM
• Anisotropic formalism
• Modify RCM coupling to take more energy info
  • Plan is to use both codes for what they do best
• 2 and 3D reconnection studies
  • Have been working on Hall only version
• Particle tracing in simulations
  • validation / improvements
  • energization
• Series of 2D equilibria that may allow collisionless tearing (Schindler and Sitnov)

• Some similarities between fully kinetic and ideal MHD
  • how do FLR effects change the MHD picture
Summary

• Developed a formalism for self-consistent application of drifts to MHD in a magnetospheric context

• Diamagnetic drifts in code are consistent with RCM results

• Initial application to the magnetosphere is promising