Data assimilation for the inner ring current using RAM-SCB

Humberto C Godinez, Earl Lawrence, Dave Moulton, Michael Henderson, and Yiqun Yu

4/6/2016
Data Assimilation

- Models have approximate physics.
- Data is typically sparse.
- Data Assimilation combines the two to improve our estimate of the true state.

We will combine fluxes from RAM-SCB with flux data from the Van Allen Probes.
Ensemble Kalman Filter

- EnKF is Monte Carlo approx. to Kalman filter: uses ensemble of simulation to estimate uncertainty about the state (fluxes)
- Data and model are combined using covariances (Model and data are approximated by Gaussian)
- Successfully used in weather forecasting, hurricane prediction, etc.
- EnKF is non-intrusive (doesn’t require model modification or derivatives), easy to implement.
Ensemble Kalman Filter

\( \mathbf{x} \) contains the state, fluxes (size ~ \(10^7\))

\( \mathbf{M} \) is the forward model, RAM-SCB

\( \mathbf{y} \) contains the observations, Van Allen probes (size ~ 100s)

\( \mathbf{H} \) interpolates the model to the observation

\( \mathbf{K} \) calibrates the adjustment based on the covariance of the ensembles (\( \mathbf{P} \)) and the observation error (\( \mathbf{R} \)).

\( \mathbf{P} \) is the empirical covariance of the ensemble of states.

\( \mathbf{N} \) is size of ensembles (~30)

\[
\mathbf{x}^f(t_i) = \mathbf{M}\left[\mathbf{x}^f(t_{i-1})\right]
\]

\[
\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}\left(\mathbf{y}^o - \mathbf{H}\mathbf{x}^f\right)
\]

\[
\mathbf{K} = \mathbf{PH}^T\left(\mathbf{PH}^T + \mathbf{R}\right)^{-1}
\]

\[
\mathbf{P} = \frac{1}{\mathbf{N} - 1} \sum_{i=1}^{\mathbf{N}} \left(\mathbf{x}_i^f - \bar{\mathbf{x}}\right)\left(\mathbf{x}_i^f - \bar{\mathbf{x}}\right)^T
\]

The size of ensembles is much less than the size of the state, so the naïve estimate of \( \mathbf{P} \) is not full rank. Modeling \( \mathbf{P} \) is the challenge.
Local EnKF

- Update only parts of the state that are “close” to observed data.
- Localization operates on a small part of the state space for which a full rank covariance can be computed.
- Successfully developed and used for atmospheric models

Example: 7/18/2013 Event
Local EnKF results (cont.)
Local EnKF results (cont.)

- Spin-averaged proton flux along RBSP-B for E=100 KeV
- E=150 KeV
- RMS of flux along RBSP-A
- MLT vs. time (UTC) for July 18, 2013
New Developed Assimilation Method

Decompose the state into a combination of a small number of basis vectors. The state is replaced by the basis weights for which a full rank covariance can be computed. **Project onto basis and perform EnKF.**
Reduced Basis EnKGF Method

Define ensemble matrix $\mathbf{X} = [\mathbf{x}_1 \ \mathbf{x}_2 \ \ldots \ \mathbf{x}_N]$ and compute the singular value decomposition (PCA)

$$\mathbf{X} = \mathbf{U} \Sigma \mathbf{V}^T$$

define a basis

$$\mathbf{B} = \frac{1}{\sqrt{N-1}} \mathbf{U}_k \Sigma_k$$

project (weights)

$$\mathbf{w}_i = \left( \mathbf{B}^T \mathbf{B} \right)^{-1} \mathbf{B}^T \mathbf{x}_i$$

Perform EnKGF on the weights.
Reduced Basis EnKF Results

Model

Assimilation

UNCLASSIFIED

2013-07-18 14:30

2013-07-18 14:30
Reduced Basis EnKF Results
RMS error

---

Spin-averaged proton flux along RBSP-B for $E=100$ KeV

- Flux ($cm^{-2} s^{-1} sr^{-1} KeV^{-1}$)

- $E=150$ KeV

- RMS of flux along RBSP-A

- RMS error

- MLT

- Time (UTC) for July 18, 2013

- RBSP-A flux obs
- non-assim RAM flux
- assim RAM flux
Summary

- LEnKF reduces error, but does not provide injection (not noticeable)
- Developed new assimilation algorithm using Singular Value Decomposition (SVD) to define new basis; captures main model signals
- Update weights of basis using EnKF
- Results look very promising, reduces error and provides an injection behaviour
- Outer boundary is important for injection, investigating empirical injection models on boundary
Future Work

- Use $K_p$ and empirical model boundary model to build more physics intuition into the covariance modeling.
- Joint parameter (e.g. $K_p$) and state estimation.
- Couple assimilated RAM-SCB with BATS-R-US, provide simulations with more realistic outer boundary conditions.