Security Constrained Optimal Power Flow with Distributionally Robust Chance Constraints

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**PROBLEM:** Uncertain power injections $\rightarrow$ uncertain power flows

Uncertainty from:
- Renewables and load
- Intra-day trading

Not always normally distributed!

**GOAL:** Keep system operation N-1 secure, despite uncertainty!
Chance constrained optimal power flow

- Formulation based on DC power flow
- Chance constraint reflects probability of constraint violation

Post-contingency line flow constraint:

\[ P \left( A^i_\ell \cdot P^i_{\text{inj}} + D^i_\ell \omega \leq P^{\text{max}}_{L(\ell)} \right) \geq 1 - \varepsilon \]
Analytical Reformulation of Chance Constraints

\[ \mathbb{P} \left( A^i_{(l,\cdot)} P^i_{inj} + D^i_{(l,\cdot)} \omega \leq P^{\max}_{L(l)} \right) \geq 1 - \varepsilon \]

Different (unknown) distributions of \( \omega \) lead to different expressions for \( f^{-1}(1 - \varepsilon) \):

- If multivariate normal (or elliptical): Exact reformulation
- If only partially known: Probabilistic inequalities
\[ A^i_{(l, \cdot)} P^i_{\text{inj}} \leq P^\text{max}_{L(l)} - f^{-1}(1 - \varepsilon) \left\| D^i_{(l, \cdot)} \Sigma^2 \right\|_2 - D^i_{(l, \cdot)} \mu \]

**Exact reformulation:**
- Normal distribution
- t distribution

**Distributionally robust:**
- Symmetric, unimodal with known \( \mu \) & \( \Sigma \)
- Unimodal with known \( \mu \) & \( \Sigma \)
- Chebyshev (known \( \mu \) & \( \Sigma \))
\[ A^i_{(l, \cdot)} P^i_{\text{inj}} \leq P^\text{max}_{L(l)} - f^{-1}(1 - \varepsilon) \left\| D^i_{(l, \cdot)} \Sigma^2 \right\|_2 - D^i_{(l, \cdot)} \mu \]

More information about the distribution leads to lower \( f^{-1}(1 - \varepsilon) \)!

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Case study: IEEE RTS 96 with uncertain in-feeds

- Two uncertain in-feeds (bus 8, 15)
- $\mu, \Sigma$ based on samples of historical data from APG
- Not normally distributed!

- $\varepsilon = 0.075$
- Constant $D_{(L, \cdot)}^{i}(LP)$
- Different assumptions on $\omega$
Case study: IEEE RTS 96 with uncertain in-feeds

Empirical violation probability

- Normal distribution: «good guess», no probabilistic guarantees
- Chebyshev: probabilistic guarantees, very conservative
- Unimodal: probabilistic guarantees, less conservative

Relative generation cost

- Normal
- Unimodal
- Chebyshev
Summary

- Analytic reformulation for separate chance constraints can be applied to non-normal distributions

- Assuming unimodality might be a good way to provide probabilistic guarantees, without being too conservative

- Next: German network with more uncertainty sources
Thank you!

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