Structure Learning and Statistical Estimation in Radial Distribution Networks

DEEPLYOTI DEKA, MICHAEL CHERTKOV, AND SCOTT BACKHAUS
deeprajotideka@utexas.edu, chertkov@lanl.gov, and backhaus@lanl.gov

ALGORITHM:
• Pick undiscovered node \( b \) with largest \( \epsilon b e^{j\theta a} - \epsilon a e^{j\theta b} \) OR
• For every discovered node \( a \), join \( b \) with \( a \) if
\[
P_a + iQ_a = \sum_{(a,b) \text{ is edge}} \epsilon_{ab} e^{j\theta a} (R_{ab} - iX_{ab})
\]

** LC lossless power flow **
\[
V_a = \epsilon_a - 1 \approx 0, \ \theta_a = 0
\]
\[
P = (M^T BM)\theta + (M^T GM)V = H^T \theta + H^T V
\]
\[
Q = (M^T GM)\theta + (M^T BM)V = -H^T \theta + H^T V
\]
\[
\theta = H^{-1} P - H^{-1} Q
\]
\[
V = H^{-1} P + H^{-1} Q
\]

** Additive & Separable (Good) **
\[
\sum_V = E[VV^T]
\]
\[
\Omega_V = E[(V - \mu_V)(V - \mu_V)^T]
\]

** Covariance **

** STATISTICAL TRENDS IN TREES **

If node \( b \) is parent of node \( a \),
\[
\Sigma_V(a,a) > \Sigma_V(b,b)
\]
\[
\Omega_V(a,a) > \Omega_V(b,b)
\]

** Mean sq. difference contributed only by descendant nodes **
\[
E[(V_a - V_b)^2] = \sum_{k_1,k_2 \in D_i} R_{ab} \sum_{P(k_1,k_2)} P(k_1,k_2) + 2R_{ab} \sum_{Q(k_1,k_2)} Q(k_1,k_2) + \sum_{Q(k_1,k_2)} Q(k_1,k_2)
\]

SUMMARY OF RESULTS

Considered Learning Problems:

- **Structure Learning**: current operational state
- **Estimation of Load Statistics**: (mean/correlations)
- **Learning with Missing Data**: (limited observations)

Potential Uses

- Failure detection
- Flow optimization
- Non-intrusive control
- Privacy quantification

ASSUMPTIONS considered

\[
\Sigma_P(a,a) > 0, \Sigma_Q(a,b) > 0, \Sigma_PQ(a,b) > 0
\]
\[
\Omega_Q(a,a) > 0, \Omega_P(a,b), \Omega_Q(a,b), \Omega_PQ(a,b) > 0
\]

** All load nodes are energy consumers **

** Fluctuations at different nodes are uncorrelated, at same node are aligned **

ALGORITHM:

- Pick undiscovered node \( b \) with largest \( \Sigma_V \) OR \( \Omega_V \)
- For every discovered node \( a \), join \( b \) with \( a \) if
\[
E[(V_a - V_b)^2] = \sum_{k_1,k_2 \in D_i} R_{ab} \sum_{P(k_1,k_2)} P(k_1,k_2)
\]

OR
\[
b = \arg \min_k \Omega_{P(k_1,k_2)} \Omega_{Q(k_1,k_2)} \Omega_{PQ(k_1,k_2)}
\]

- Estimate means, covariances of power injection using LC power flow equations.

REFERENCES

