# Shadow Price-Based Coordination of Natural Gas and Electric Power Systems

**Grid Science Winter School and Conference 2019** 



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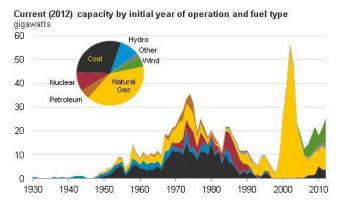
with Bining Zhao, Antonio Conejo, Ramteen Sioshansi, Alex Rudkevich

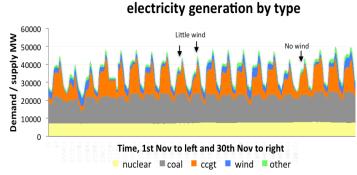
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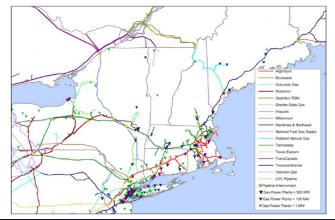
### Motivation

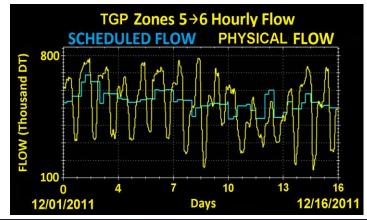
#### Expanding use of natural gas as fuel for power generation, significantly to fill the demand curve





#### Challenges: gas pipelines are fragmented, intra-day markets & operations do not use optimization





### **Motivation**

#### Gas-Electric System Issues:

- Flexible gas-fired generation lacks fuel supply flexibility
- <u>Flexibility is crucial in power systems</u>: supply must match demand continuously and instantaneously (there is no equivalent to "line pack")
- Variability and unpredictability of gas-fired generation challenges pipeline operations
- Anticipated continued growth of the gas-fired generating fleet

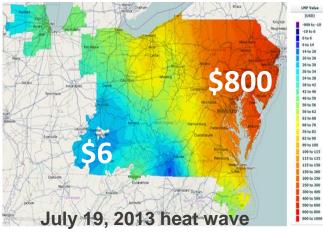
#### • Planning/Long-Term Challenges:

- Gas-fired power plants rarely procure firm gas transportation (i.e. supply guarantees)
- Under extreme conditions, there have been severe gas pipeline constraints that limited supply to gas-fired generation
- Addressing growth of gas-fired generation
  - New optimization and control technology
  - Engineering economic methods

## This Study

- Motivation
  - Pricing of natural gas using concepts that are standard in electricity markets
  - In electricity markets, shadow prices are posted as real-time prices
  - Locational Marginal Prices (LMPs) for electricity
  - Methods for coordinating gas and electricity networks with limited exchange of proprietary information
- Locational Trade Values (LTVs) for natural gas
  - Nodal pricing of natural gas delivery over a pipeline network
  - Obtained by single price two-sided auction mechanism (objective function that maximizes economic welfare of pipeline users)
- Time-dependent optimization formulation
  - What problem corresponds to Unit Commitment for gas pipelines?
  - Account for pipeline structure, physics and engineering
  - Provide operational and economic solution (flow and compressor schedule, hourly prices)

#### PJM Interconnection price per MWh



## This Study

#### Optimization model for power system

- Standard Unit Commitment (UC)
- Mixed Integer Linear Program, control variables are generator production
- Objective function is minimum production cost
- Constraints on power system and generators

#### Optimization model for gas system

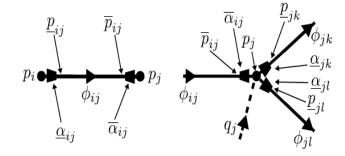
- Optimal control of flows on a network, control variables are compressors and demands
- Objective function is maximizing economic welfare for system users
- Dynamic constraints are PDEs on network edges, Kirchoff's law on nodes
- Inequality constraints on states and controls
- Iterative coordination mechanism between two models
  - Limited to exchange of generation/flow and price time-series (not network models)

### **Modeling for Gas Market Optimization**

- Network nodes
  - Physical nodes and custodial meter stations
- Network edges
  - Pipes that physically connect nodes
- Objective: a single price double auction
  - Maximize profit of gas deliveries to buyers minus cost of procuring gas from suppliers and cost of operating compressors
- Conducted subject to engineering constraints on gas pipeline network
  - Physics of pressure and flow on each pipe
  - Flow balance at nodes
  - Constraints on compressor power

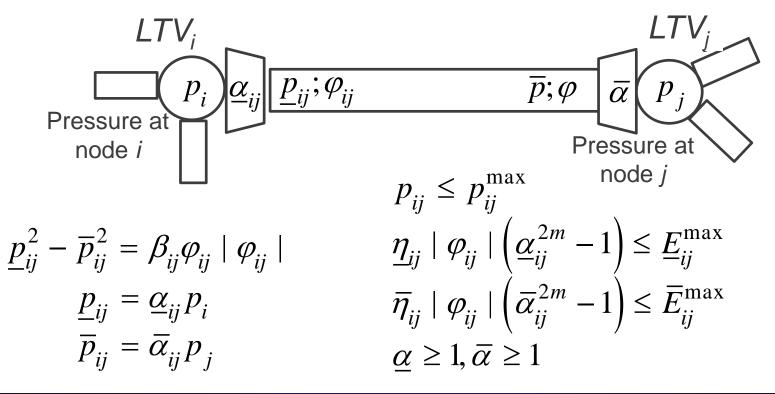
#### Participants

- Suppliers who provide node-specific Price/Quantity (P/Q) offers to sell gas
- Offtakers who provide node-specific P/Q bids to buy gas



### **Constraints on a pipe (steady-state)**

### Nodal balance equations: inflow + supply – outflow - offtake – compressor use = 0



max Social Welfare	e: $J_{MSW} \triangleq \sum_{k \in \mathcal{G}} c_k^o d_k - \sum_{k \in \mathcal{G}} c_k^s s_k$	Objective: Market surplus and Cost of compressor operation
	$-\sum_{j\in\mathcal{V}}\sum_{i\in\partial_+j}\lambda_j^e(1-\overline{\varepsilon}_{ij})\overline{\eta}_{ij}$	$\phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathscr{V}}\sum_{k\in\partial_{-}j}\lambda_{j}^{e}(1-\underline{\varepsilon}_{jk})\underline{\eta}_{jk} \phi_{jk} ((\underline{\alpha}_{jk}^{2m})-1)$
s.t. Flow balance:	$\sum_{k\in\partial_{-}j}\phi_{jk}-\sum_{i\in\partial_{+}j}\phi_{ij}=\sum_{k\in\partial_{g}j}(s_k-$	$d_k)$
	$-\sum_{i\in\partial_{+}j}\overline{\epsilon}_{ij}\overline{\eta}_{ij} \phi_{ij} \big((\overline{\alpha}_{ij}^{2m})-1\big)$	$-\sum_{k\in\partial_{-}j}\underline{\varepsilon}_{jk}\underline{\eta}_{jk} \phi_{jk} \Big((\underline{\alpha}_{jk}^{2m})-1\Big),\qquad\forall j\in\mathcal{V},\lambda_{j}$
Pressure balance:	$(\underline{\alpha}_{ij}p_i)^2 - (\overline{\alpha}_{ij}p_j)^2 = \beta_{ij}\phi_{ij} \phi_{ij} $	$\forall (i,j) \in \mathcal{E},  \mu_{ij}$
Pressure limits:	$\underline{\alpha}_{ij}p_i \leq p_{ij}^{\max},  \overline{\alpha}_{ij}p_j \leq p_{ij}^{\max},$	$\forall (i,j) \in \mathcal{E},  \underline{\xi}_{ij}^{\max}, \overline{\xi}_{ij}^{\max}$
	$p_j^{\min} \le p_j,$	$\forall j \in \mathcal{V},  \xi_j^{\min}$
Boost upper limits:	$\underline{\eta}_{ij}  \phi_{ij}  \left( (\underline{\alpha}_{ij}^{2m}) - 1 \right) \le \underline{E}_{ij}^{\max},  \overline{\eta}$	$ \phi_{ij}  ((\overline{\alpha}_{ij}^{2m}) - 1) \le \overline{E}_{ij}^{\max}  \forall (i, j) \in \mathcal{E},  \underline{\gamma}_{ij}, \overline{\gamma}_{ij}$
Boost lower limits:	$\underline{\alpha}_{ij}, \overline{\alpha}_{ij} \ge 1$	$\forall (i,j) \in \mathcal{E},  \underline{\theta}_{ij}, \overline{\theta}_{ij}$
Supply limits:	$s_k^{\min} \le s_k \le s_k^{\max}$	$\forall k \in \mathcal{G},  \sigma_k^{\min}, \sigma_k^{\max}$
Demand limits:	$d_k^{\min} \le d_k \le d_k^{\max}$	$\forall k \in \mathcal{G},  \zeta_k^{\min}, \zeta_k^{\max}$

max Social Welfare	Telfare: $J_{MSW} \triangleq \sum_{k \in \mathcal{G}} c_k^o d_k - \sum_{k \in \mathcal{G}} c_k^s s_k$ Lagrange multipliers			
	$\begin{split} &-\sum_{j\in\mathcal{V}}\sum_{i\in\partial_{+}j}\lambda_{j}^{e}(1-\overline{\epsilon}_{ij})\overline{\eta}_{ij} \phi_{ij} \left((\overline{\alpha}_{ij}^{2m})-1\right)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{-}j}\lambda_{j}^{e}(1-\overline{\epsilon}_{ij})\overline{\eta}_{ij} \phi_{ij} \left((\overline{\alpha}_{ij}^{2m})-1\right)-\sum_{k\in\partial_{-}j}\overline{\epsilon}_{jk}\underline{\eta}_{jk} \phi_{jk} \left((\underline{\alpha}_{jk}^{2m})-1\right),\\ &-\sum_{i\in\partial_{+}j}\overline{\epsilon}_{ij}\overline{\eta}_{ij} \phi_{ij} \left((\overline{\alpha}_{ij}^{2m})-1\right)-\sum_{k\in\partial_{-}j}\underline{\epsilon}_{jk}\underline{\eta}_{jk} \phi_{jk} \left((\underline{\alpha}_{jk}^{2m})-1\right),\\ &(\underline{\alpha}_{ij}p_{i})^{2}-(\overline{\alpha}_{ij}p_{j})^{2}=\beta_{ij}\phi_{ij} \phi_{ij} ,\\ &\underline{\alpha}_{ij}p_{i}\leq p_{ij}^{\max}, \overline{\alpha}_{ij}p_{j}\leq p_{ij}^{\max},\\ &p_{j}^{\min}\leq p_{j},\\ &\underline{\eta}_{ij} \phi_{ij} \left((\underline{\alpha}_{ij}^{2m})-1\right)\leq \underline{E}_{ij}^{\max}, \overline{\eta}_{ij} \phi_{ij} \left((\overline{\alpha}_{ij}^{2m})-1\right)\leq \overline{E}_{ij}^{\max}\\ &\underline{\alpha}_{ij},\overline{\alpha}_{ij}\geq 1\\ &s_{k}^{\min}\leq s_{k}\leq s_{k}^{\max}\\ &d_{k}^{\min}\leq d_{k}\leq d_{k}^{\max} \end{split}$	$-\underline{\varepsilon}_{jk})\underline{\eta}_{jk} \phi_{jk} \Big(($	$(\underline{\alpha}_{jk}^{2m}) - 1$	
s.t. Flow balance:	$\sum_{k \in \partial_{-j}} \phi_{jk} - \sum_{i \in \partial_{+j}} \phi_{ij} = \sum_{k \in \partial_g j} (s_k - d_k)$			
	$-\sum_{i\in\partial_{+}j}\overline{\varepsilon}_{ij}\overline{\eta}_{ij} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{k\in\partial_{-}j}\underline{\varepsilon}_{jk}\underline{\eta}_{jk} \phi_{jk} ((\underline{\alpha}_{jk}^{2m})-1),$	$\forall j \in \mathcal{V},$	$\lambda_j$	
Pressure balance:	$(\underline{\alpha}_{ij}p_i)^2 - (\overline{\alpha}_{ij}p_j)^2 = \beta_{ij}\phi_{ij} \phi_{ij} ,$	$\forall (i,j) \in \mathcal{E},$	$\mu_{ij}$	
Pressure limits:	$\underline{\alpha}_{ij}p_i \le p_{ij}^{\max},  \overline{\alpha}_{ij}p_j \le p_{ij}^{\max},$	$\forall (i,j) \in \mathcal{E},$	$\underline{\xi}_{ij}^{\max}, \overline{\xi}_{ij}^{\max}$	
	$p_j^{\min} \le p_j,$	$\forall j \in \mathcal{V},$	$\xi_j^{\min}$	
Boost upper limits:	$\underline{\eta}_{ij} \phi_{ij} ((\underline{\alpha}_{ij}^{2m})-1) \leq \underline{E}_{ij}^{\max},  \overline{\eta}_{ij} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1) \leq \overline{E}_{ij}^{\max}$	$\forall (i,j) \in \mathcal{E},$	$\underline{\gamma}_{ij}, \overline{\gamma}_{ij}$	
Boost lower limits:	$\underline{\alpha}_{ij}, \overline{\alpha}_{ij} \ge 1$	$\forall (i,j) \in \mathcal{E},$	$\underline{\Theta}_{ij}, \overline{\Theta}_{ij}$	
Supply limits:	$s_k^{\min} \le s_k \le s_k^{\max}$	$\forall k \in \mathcal{G},$	$\sigma_k^{\min}, \sigma_k^{\max}$	
Demand limits:	$d_k^{\min} \le d_k \le d_k^{\max}$	$\forall k \in \mathcal{G},$	$\zeta_k^{\min}, \zeta_k^{\max}$	

max Social Welfare	e: $J_{MSW} \triangleq \sum_{k \in \mathcal{G}} c_k^o d_k - \sum_{k \in \mathcal{G}} c_k^s s_k$ Mass flow balance gas consumed by	/ compress	
	$-\sum_{j\in\mathcal{V}}\sum_{i\in\partial_{+}j}\lambda_{j}^{e}(1-\overline{\varepsilon}_{ij})\overline{\eta}_{ij} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{-}j}\lambda_{j}^{e}(1-\overline{\varepsilon}_{ij})\overline{\eta}_{ij} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{+}j}\lambda_{j}^{e}(1-\overline{\varepsilon}_{ij})\overline{\eta}_{ij} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{+}j}\lambda_{j} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{+}j}\lambda_{j} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{+}j}\lambda_{j} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{+}j}\lambda_{j} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{+}j}\lambda_{j} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{+}j}\lambda_{j} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{+}j}\lambda_{j} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{j\in\mathcal{V}}\sum_{k\in\partial_{+}j}\lambda_{j} (($	$\cdot \underline{\mathbf{e}}_{jk})\underline{\mathbf{\eta}}_{jk} \mathbf{\phi}_{jk} \Big((\underline{\mathbf{e}}_{jk})\Big)\Big $	$\underline{\alpha}_{jk}^{2m}$ ) – 1)
s.t. Flow balance:	$\begin{split} \sum_{k\in\partial_{-}j} \phi_{jk} &- \sum_{i\in\partial_{+}j} \phi_{ij} = \sum_{k\in\partial_{g}j} (s_k - d_k) \\ &- \sum_{i\in\partial_{+}j} \overline{\epsilon}_{ij} \overline{\eta}_{ij}  \phi_{ij}  \left( (\overline{\alpha}_{ij}^{2m}) - 1 \right) - \sum_{k\in\partial_{-}j} \underline{\epsilon}_{jk} \underline{\eta}_{jk}  \phi_{jk}  \left( (\underline{\alpha}_{jk}^{2m}) - 1 \right), \end{split}$		
	$-\sum_{i\in\partial_+j}\overline{\varepsilon}_{ij}\overline{\eta}_{ij} \phi_{ij} ((\overline{\alpha}_{ij}^{2m})-1)-\sum_{k\in\partialj}\underline{\varepsilon}_{jk}\underline{\eta}_{jk} \phi_{jk} \Big((\underline{\alpha}_{jk}^{2m})-1\Big),$	$\forall j \in \mathcal{V},$	$\lambda_j$
Pressure balance:	$(\underline{\alpha}_{ij}p_i)^2 - (\overline{\alpha}_{ij}p_j)^2 = \beta_{ij}\phi_{ij} \phi_{ij} ,$	$\forall (i,j) \in \mathcal{E},$	$\mu_{ij}$
Pressure limits:	$\underline{\alpha}_{ij}p_i \le p_{ij}^{\max},  \overline{\alpha}_{ij}p_j \le p_{ij}^{\max},$	$\forall (i,j) \in \mathcal{E},$	$\underline{\xi}_{ij}^{\max}, \overline{\xi}_{ij}^{\max}$
	$p_j^{\min} \le p_j,$	$\forall j \in \mathcal{V},$	$\xi_j^{\min}$
Boost upper limits:	$\underline{\eta}_{ij}  \phi_{ij}  ((\underline{\alpha}_{ij}^{2m}) - 1) \leq \underline{E}_{ij}^{\max},  \overline{\eta}_{ij}  \phi_{ij}  ((\overline{\alpha}_{ij}^{2m}) - 1) \leq \overline{E}_{ij}^{\max}$	$\forall (i,j) \in \mathcal{E},$	$\underline{\gamma}_{ij}, \overline{\gamma}_{ij}$
Boost lower limits:	$\underline{\alpha}_{ij}, \overline{\alpha}_{ij} \ge 1$	$\forall (i,j) \in \mathcal{E},$	$\underline{\Theta}_{ij}, \overline{\Theta}_{ij}$
Supply limits:	$s_k^{\min} \le s_k \le s_k^{\max}$	$\forall k \in \mathcal{G},$	$\sigma_k^{\min}, \sigma_k^{\max}$
Demand limits:	$d_k^{\min} \le d_k \le d_k^{\max}$	$\forall k \in \mathcal{G},$	$\zeta_k^{\min}, \zeta_k^{\max}$

max Social Welfare: 
$$J_{MSW} \triangleq \sum_{k \in G_{i}} c_{k}^{\alpha} d_{k} - \sum_{k \in G_{i}} c_{k}^{\alpha} s_{k}$$

$$= \sum_{j \in \mathcal{V}' i \in \partial_{+j}} \lambda_{j}^{\alpha} (1 - \overline{\epsilon}_{ij}) \overline{\eta}_{ij} |\phi_{ij}| ((\overline{\alpha}_{ij}^{2m}) - 1) - \sum_{j \in \mathcal{V}' k \in \partial_{-j}} \lambda_{j}^{\alpha} (1 - \underline{\epsilon}_{jk}) \underline{\eta}_{jk} |\phi_{jk}| ((\underline{\alpha}_{jk}^{2m}) - 1)$$
s.t. Flow balance: 
$$\sum_{k \in \partial_{-j}} \phi_{jk} - \sum_{i \in \partial_{+j}} \phi_{ij} = \sum_{k \in \partial_{k}j} (s_{k} - d_{k})$$

$$= \sum_{i \in \partial_{+j}} \overline{\epsilon}_{ij} \overline{\eta}_{ij} |\phi_{ij}| ((\overline{\alpha}_{ij}^{2m}) - 1) - \sum_{k \in \partial_{-j}} \underline{\epsilon}_{jk} \underline{\eta}_{jk} |\phi_{jk}| ((\underline{\alpha}_{jk}^{2m}) - 1), \quad \forall j \in \mathcal{V}, \quad \lambda_{j}$$
Pressure balance: 
$$(\underline{\alpha}_{ij}p_{i})^{2} - (\overline{\alpha}_{ij}p_{j})^{2} = \beta_{ij}\phi_{ij} |\phi_{ij}|, \quad \forall (i, j) \in \mathcal{E}, \quad \mu_{ij}$$
Pressure limits: 
$$\underline{\alpha}_{ij}p_{i} \leq p_{ij}, \quad \overline{\alpha}_{ij}p_{j} \leq p_{ij}^{\max}, \quad \overline{\eta}_{ij}p_{j} \leq p_{ij}^{\max}, \quad \overline{\eta}_{ij} |\phi_{ij}| ((\overline{\alpha}_{ij}^{2m}) - 1) \leq \overline{E}_{ij}^{\max}, \quad \forall (i, j) \in \mathcal{E}, \quad \underline{\gamma}_{ij}, \overline{\gamma}_{ij}$$
Boost upper limits: 
$$\underline{\alpha}_{ij}, \overline{\alpha}_{ij} \geq 1$$
Supply limits: 
$$s_{k}^{\min} \leq s_{k} \leq s_{k}^{\max}$$
Demand limits: 
$$d_{k}^{\min} \leq d_{k} \leq d_{k}^{\max}$$

$$\sum_{j \in \mathcal{V}, i \in \mathcal$$

### Locational Trade Values (LTVs) of gas

- A binding constraint may not lead to price separation in the network
- To cause price separation, the pipe must be constrained at both ends
  - Minimum pressure constraint must bind at the receiving node
  - At the sending end of the pipe, either pressure constraint must bind at maximum or compressor must bind at maximum power

 $\lambda_j - \lambda_i = Compression_{ij} + Congestion_{ij}^c + Congestion_{ij}^p$ 

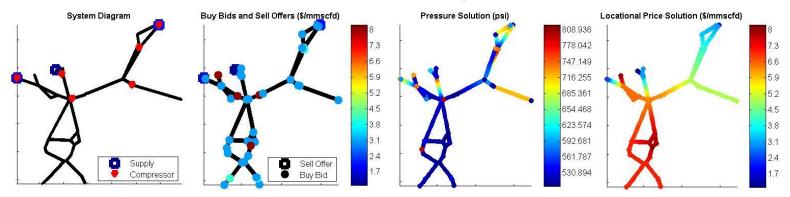
- Proof that prices cannot decrease in the direction of the flow
  - The compression and congestion components are non-negative in the direction of flow
  - Optimal LTVs assure revenue adequacy: offtakers' payments are greater or equal suppliers' receipts
  - The auctioneer's net position equals  $R = \sum_{ij \in \mathcal{E}} \left[ \varphi_{ij} (\lambda_j \lambda_i) C_{ij}^{Comp} \right]$

- Price difference over each pipe is sufficient to recover the cost of compression

### **Usage and model outputs**

#### Obtaining system properties

- Pressure bounded between 500 and 800 psi, respectively.
- Supply at nodes 1, 2, and 3 at \$1, \$1.5, and \$2 per mmscfd.
- Bids at \$3 per mmscfd with higher bids of \$4 at nodes 22, 24, 25, and 34, and bids of \$8 at nodes 16, 18, 20, and 31.
- Maximum offtakes are 800 mmscfd, and this constraint was binding at nodes 16, 18, 22, and 39; nodes 6, 20, 26, 31, 34, and 36 had lesser nonzero offtakes.
- Maximum power for the compressors on edges 43 and 44 were limited to 3000 and 2000 horsepower, respectively, and were binding.



### Intra-day Gas Balancing Market

max Social Welfare:	$\sum_{m \in \mathcal{G}} \int_0^T c_m^d(t) \hat{d}_m(t) \mathrm{d}t - \sum_{m \in \mathcal{G}} \int_0^T c_p^d$	$\hat{s}_m(t)\hat{s}_m(t)\mathrm{d}t$	$\frac{p}{i^{j}}$	$\overline{p}_{ij}$ $\overline{p}_{ij} \setminus \overline{p}_{ij} \setminus p_j /$
s.t. Mass conservation:	$\begin{split} \partial_t p_{ij} &+ \frac{a^2}{A_{ij}} \partial_x \phi_{ij} = 0, \\ \partial_x \left( \frac{1}{2} p_{ij}^2 \right) &+ a^2 r_{ij} \phi_{ij}  \phi_{ij}  = 0, \end{split}$	$\forall  (i,j) \in \mathcal{E},$	$\mu_{ij}(t,x) \qquad \qquad$	$p_j \qquad p_j \qquad \phi_{ij}$
Momentum conservation:	$\partial_x \left(\frac{1}{2} p_{ij}^2\right) + a^2 r_{ij} \phi_{ij}  \phi_{ij}  = 0,$	$\forall  (i,j) \in \mathcal{E},$	$\eta_{ij}(t,x) \qquad \frac{\dot{\alpha}_{ij}}{2}$	$\overline{lpha}_{ij}$ $q_j$
Nodal flow balance:	$\sum_{k\in\partial_{-j}} \underline{\phi}_{-jk}(t) - \sum_{i\in\partial_{+j}} \overline{\phi}_{ij}(t) - \overline{q}_j(t)$			
	$-\sum_{m\in\partial_g j} (\hat{s}_m(t) - \hat{d}_m(t)) = 0,$	$\forall j \in \mathcal{V},$	$\lambda_j(t)$	
Pressure compatibility:	$\underline{p}_{ij}(t) = \underline{\alpha}_{ij}(t)p_i(t),$	$\forall  (i,j) \in \mathcal{E},$	$\underline{\Delta}_{ij}(t),$	
Pressure limits:	$\begin{split} \overline{p}_{ij}(t) &= \overline{\alpha}_{ij}(t)p_j(t), \\ p_{ij}^{\min} &\leq p_{ij}(t,0) \leq p_{ij}^{\max}, \\ p_{ij}^{\min} &\leq p_{ij}(t,L_{ij}) \leq p_{ij}^{\max}, \end{split}$		$\underline{\beta}_{ij}^{\min}(t), \underline{\beta}_{ij}^{\max}(t)$	
Boost upper limits:	$\frac{\underline{\varepsilon}_{ij} \underline{\phi}_{ij}(t) \left((\underline{\alpha}_{ij}(t))^{h}-1\right) \leq \underline{E}_{ij}^{\max},\\ \overline{\varepsilon}_{ij} \overline{\phi}_{ij}(t) \left((\overline{\alpha}_{ij}(t))^{h}-1\right) \leq \overline{E}_{ij}^{\max},$			• A two-sid
Boost lower limits:	$\underline{\alpha}_{ij}(t) \ge 1,  \overline{\alpha}_{ij}(t) \ge 1$	$\forall  (i,j) \in \mathcal{E},$	$\underline{\theta}_{ij}(t), \ \overline{\theta}_{ij}(t)$	Shadow
			$\sigma_m^{\min}(t),  \sigma_m^{\max}(t)$	– On ma
			$\zeta_m^{\min}(t), \ \zeta_m^{\max}(t)$	(conge
Time periodicity:	$p_{ij}(0, x) = p_{ij}(T, x),$ $\phi_{ij}(0, x) = \phi_{ij}(T, x),$	$\forall (i, j) \in \mathcal{E},$ $\forall (i, j) \in \mathcal{E},$	•	– On pre (capad

Parameter		Set	Doma	in	Units (SI)				
	$c_m^s(t), c_m^d(t)$		$m \in G$	[0, T	]	\$·kg <sup>−1</sup>			
jk .	$s_m^{\min}(t), s_m^{\max}(t), d_m^{\min}(t), d_m^{\max}(t)$			$m \in G$	[0, T	[0,T] kg·s <sup>-1</sup>			
$\mathbf{I}_{\phi_{jk}}$	$\bar{q}_j(t)$	$j \in \mathcal{V}_F$	[0, T	1	kg·s <sup>−1</sup>				
	$p_j(t)$			$j \in \mathcal{V}_P$	[0, T	1	kg· m <sup>−1</sup> · s <sup>−2</sup>	(Pa)	
0	- <u>Ajk</u> Table 1: Market Parameters								
$\underline{\underline{n}}_{jl}^{\underline{\alpha}_{jl}}$	Variables		Set	Doma	uin		Units (SI	)	
$\sum_{jl}$	$\hat{s}_m(t), \hat{d}_m(t)$	1	$n \in G$	[0, T]	]		kg∙ s <sup>−1</sup>		
$\phi_{jl}$	$p_j(t)$	j	$\in \mathcal{V}_F$	[0, T]	]	k	$sg \cdot m^{-1} \cdot s^{-2}$	(Pa)	
$\varphi_{JI}$	$p_{ij}(t,x)$	(i,	$(j) \in \mathcal{E}$	$[0,T] \times [0,T]$	0, <i>L</i> <sub>ij</sub> ]	k	$sg \cdot m^{-1} \cdot s^{-2}$	(Pa)	
	$\phi_{ij}(t,x)$	(i,	$(j) \in \mathcal{E}$	$[0,T] \times [0,T]$	0, <i>L</i> <sub>ij</sub> ]		kg∙ s <sup>−1</sup>		
	$\underline{\alpha}_{ij}(t), \overline{\alpha}_{ij}(t)$	(i,	$(j) \in \mathcal{E}$	[0, T]	]		-		
			Table 2:	Primal V	ariab	les			
Variable			Set	Domai	n	Units (SI)			
$\lambda_j(t)$			$j \in \mathcal{V}$	[0, T]					
	$\chi_n^{\max}(t),  \zeta_m^{\min}(t),  \zeta_m^{\max}(t)$	(t)	$m \in G$	L / J			\$ kg <sup>-1</sup>		
$\mu_{ij}(t,x)$			$(i, j) \in \mathcal{E}$				2 1.		
$\eta_{ij}(t,x)$			$(i, j) \in \mathcal{E}$	$[0,T] \times [0]$					
$\underline{\Delta}_{ij}(t), \overline{\Delta}_{ij}(t)$		$(i, j) \in \mathcal{E}$	[0, T]		\$.s.m.kg <sup>-1</sup> (\$.Pa <sup>-1</sup> .		$\cdot s^{-1}$ )		
$\underline{\beta}_{ij}^{\min}(t), \underline{\beta}_{ij}^{\max}(t), \overline{\beta}_{ij}^{\min}(t), \overline{\beta}_{ij}^{\max}(t)$		$(i,j)\in \mathcal{E}$	[0,T]	\$.s.m.kg <sup>-1</sup> (\$.Pa <sup>-1</sup> .		$\cdot s^{-1}$ )			
$\underline{\gamma}_{ii}(t), \overline{\gamma}_{ij}(t)$		$(i, j) \in \mathcal{E}$	[0, T]		$\cdot W^{-1} \cdot s^{-1}$				
$\underline{\theta}_{ij}(t), \overline{\theta}_{ij}(t)$		$(i,j)\in \mathcal{E}$	[0, T]		-				
$\tau^{\hat{p}}_{ij}(x), \tau^{\phi}_{ij}(x)$	r)		$(i,j)\in \mathcal{E}$	[0, T]			\$- s-kg <sup>−1</sup>		
Table 3: Dual Variables									

A two-sided auction over pipeline network

#### Shadow prices (dual variables)

- On mass flow withdrawal at nodes (congestion price)
- On pressure and compressor limits (capacity price)

### **Intra-Day Gas Balancing Market**

#### Single price double auction market

- For Shippers and other Buyers and Sellers
- Trade deviations from steady-state flows purchased in existing markets

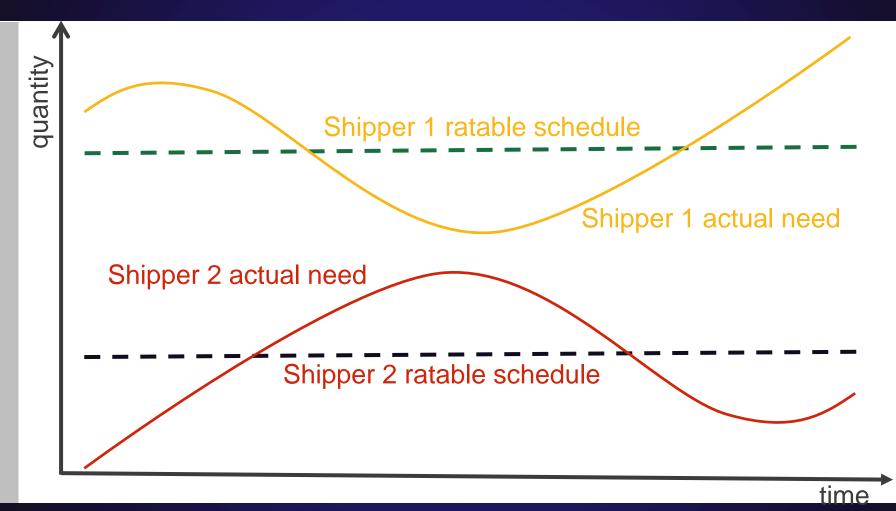
#### Opportunistic buyers and sellers

- may have no reserved capacity rights but are allowed to participate to increase liquidity
- No capacity rights = no congestion hedging

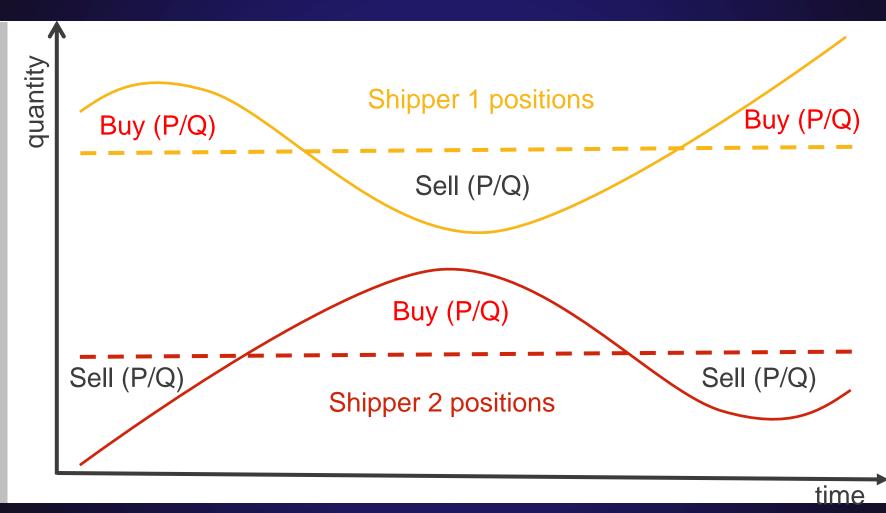
#### Offers and bids are node-specific

- submitted with hourly time step for the optimization horizon (e.g., 36 hours)
- Auctioneer's objective function is to maximize market surplus
  - over the optimization horizon
  - accounting for accepted bids & offers less pipeline operating costs

#### **Ratable schedules vs. non-ratable needs**



### Need more - schedule buy; Need less - schedule sell

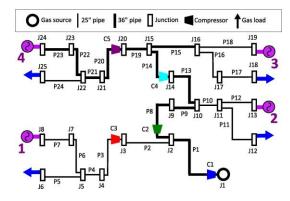


### **Coordination Mechanism**

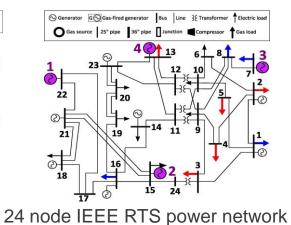
• At each iteration:

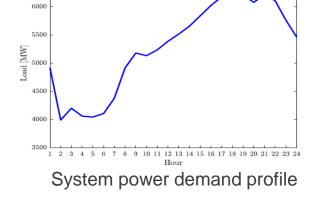
	Optimal Production Schedule $p_i(t)$	$d_i^{\max} = h_1(p_i)$	$d_i^{\max}(t)$ : Maximum gas demand of generators	
Power System (Unit	Locational Marginal Prices $\lambda_i^p(t)$	$c_i^g = h_2(\lambda_i^p)$ Generator (Heat Rate	Bid (buy) price $c_i^g(t)$ for gas	Gas System (Gas
Commitment)	$p_i^{\max}(t)$ : Maximum Production Schedule $c_i^p(t)$ : Marginal price of generation (of fuel)	Curve) $p_i = h_1^{-1}(d_i^{\max})$ $c_i^p = h_2^{-1}(\lambda_i^g)$	Optimal gas delivery to power generators $d_i(t) \le d_i^{\max}(t)$ Locational Trade Values of gas $\lambda_i^g(t)$	Balancing Market)

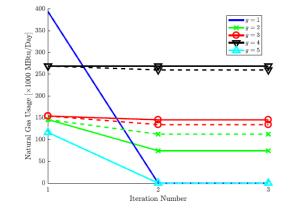
### **Computational Example**



24 pipe gas test network



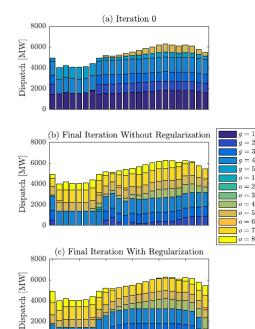




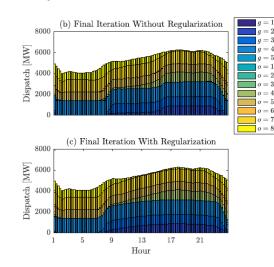
# Procedure converges after 1 iteration!

6500

### **Computational Example**



(a) Iteration 0 8000 (b) Iteration 0 (c) Iteration 0 



Generation Schedule: 1 hour increments

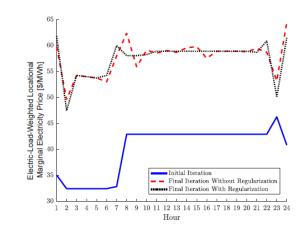
Hour

9 13

21

17

Generation Schedule: 15 minute increments



Hourly electricity price Initial Iteration Final iteration

0

### Conclusion

#### Gas-electric coordination using optimization-based markets

- Time-dependent locational marginal pricing (electricity LMPs and natural gas LTVs)
- Requires only limited exchange of information to produce price/quantity (P/Q) bids and production/demand constraints

#### Properties

- Revenue adequacy for the administrators of both markets
- Operation of systems is not altered if all demands can be met
- Convergence after only one iteration of the procedure (by ~linearity of UC)

### Acknowledgement

#### Los Alamos National Laboratory

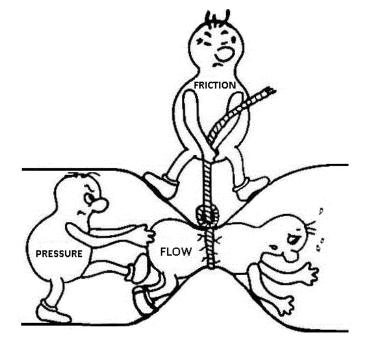
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### **Questions?**

