Electric Power to the People!

Power System Optimization in the Age of Renewable Energy

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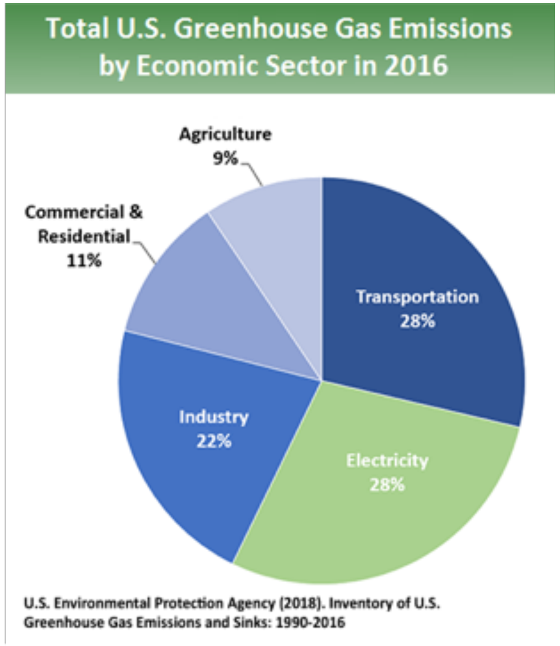
Electric Power to the People!

Sustainable, local, green energy!

Renewable Energy

Renewable Energy



Reduced greenhouse gas emissions!

Renewable Energy

Jobs and innovation.

Energy security,
globally and locally.

Will not run out!

Living off the grid?

Less pollution.

Reduced greenhouse
gas emissions!

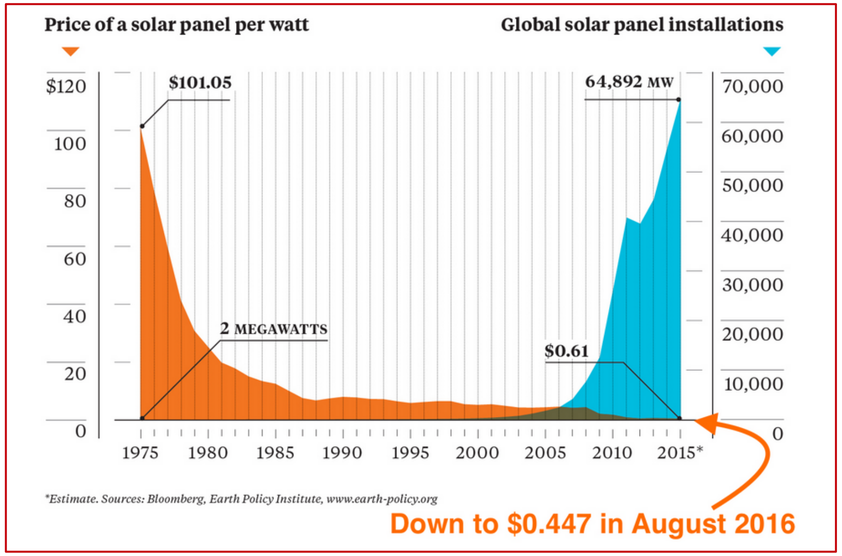
Resilience...

Renewable Energy

Jobs and innovation.

Energy security, globally and locally.

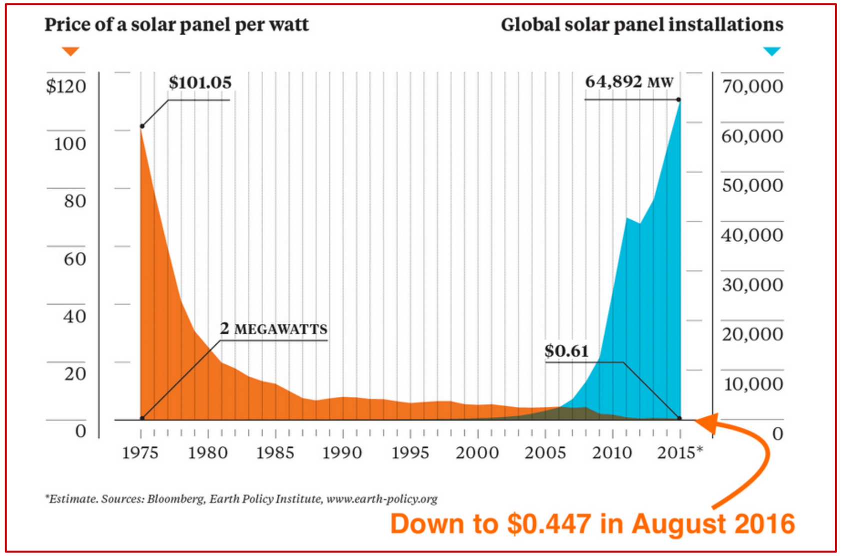
Living off the grid?



Reduced greenhouse gas emissions!

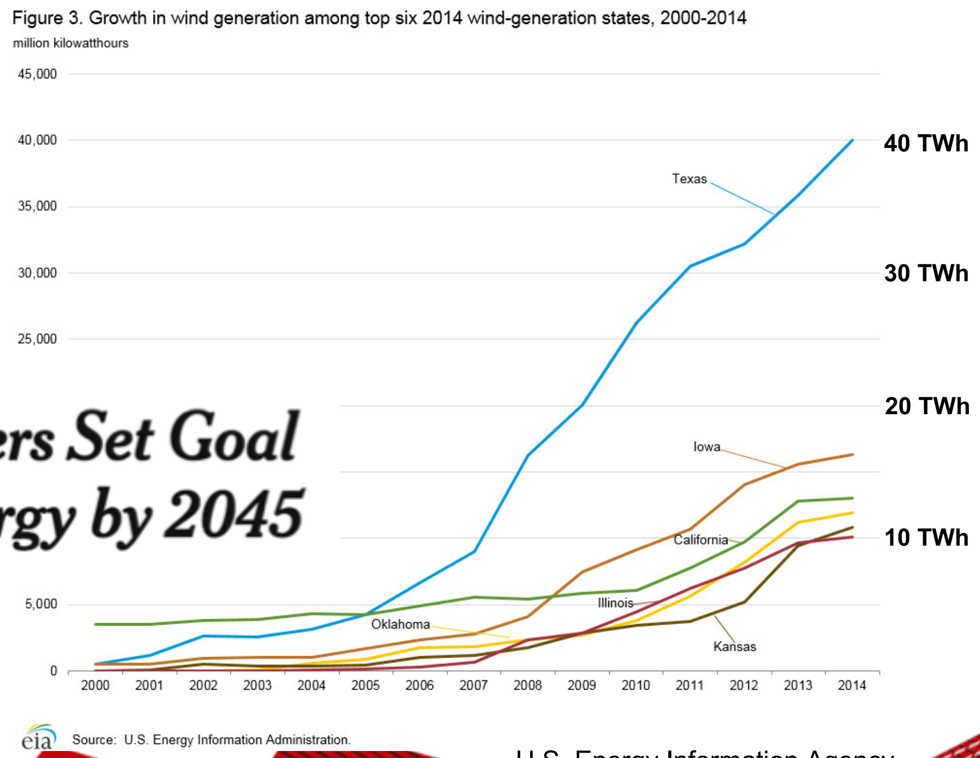
Low cost!

Renewable Energy



Low cost!

Significant energy



California Lawmakers Set Goal for Carbon-Free Energy by 2045

New York Times, Aug 28 2018

U.S. Energy Information Agency

Renewable Energy

Electric Power Grids

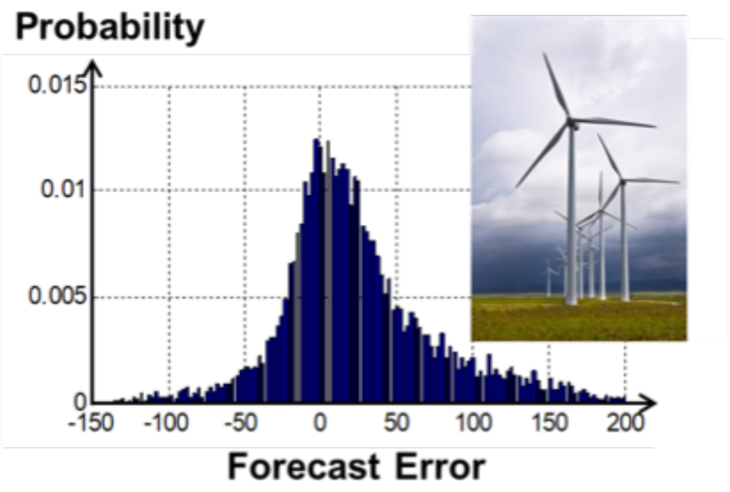
Renewable Energy

+

Electric Power Grids

=

True



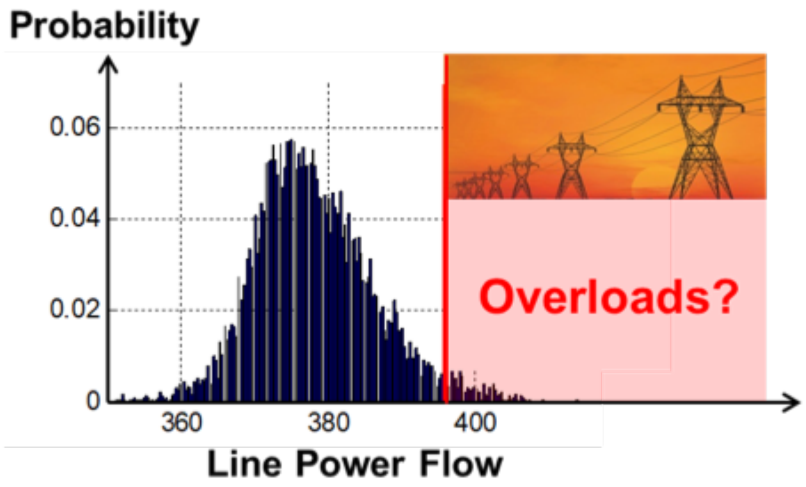
Renewable Energy

+

Electric Power Grids

=

??? True $$$



Electric Power to the People!

Affordable, reliable,
sustainable, local, green energy!

Power System Optimization

1. Optimal Power Flow

Power System Optimization

1. Optimal Power Flow

Power System Optimization

in the Age of Renewable Energy

1. **Optimal Power Flow**
2. **Modelling and Managing Uncertainty**

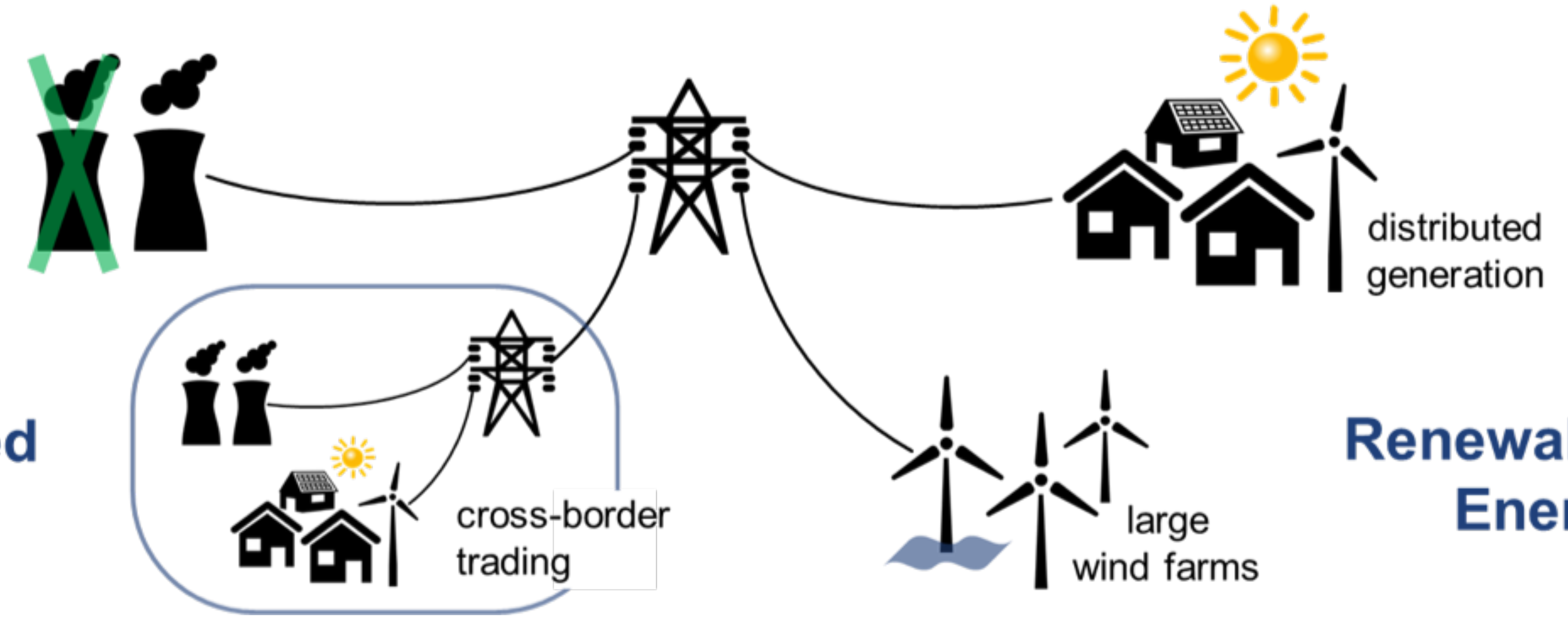
Power System Optimization

in the Age of Renewable Energy

1. **Optimal Power Flow** — First half
2. **Modelling and Managing Uncertainty** — Second half

Optimal Power Flow

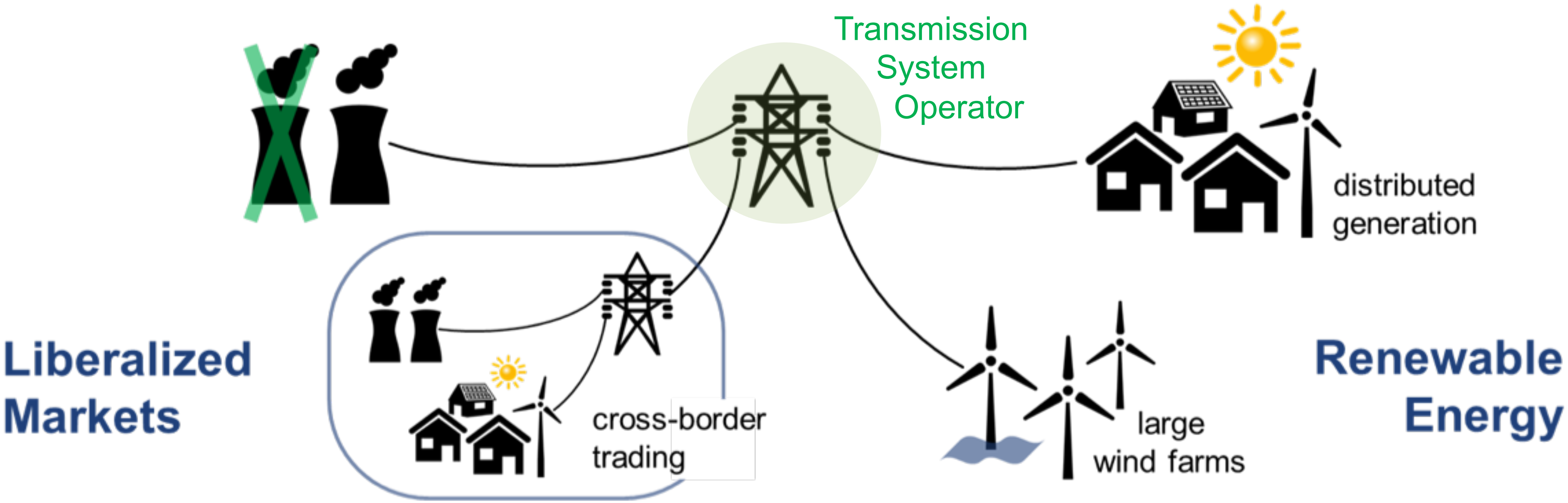
Power Systems Operation



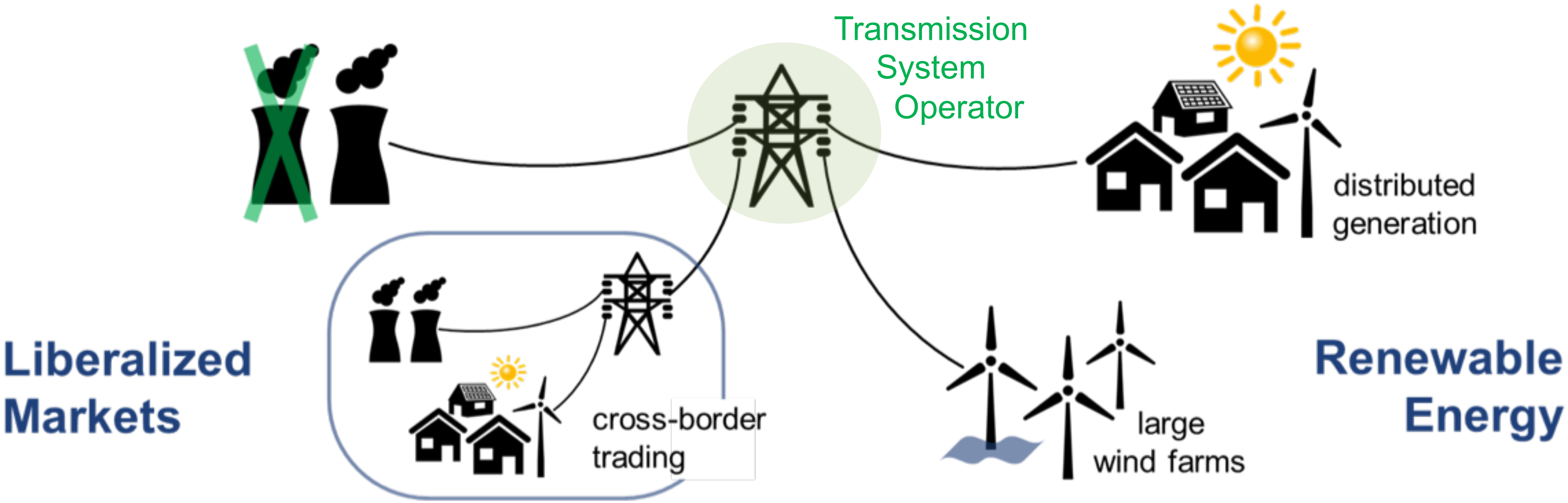
Liberalized Markets

Renewable Energy

Power Systems Operation

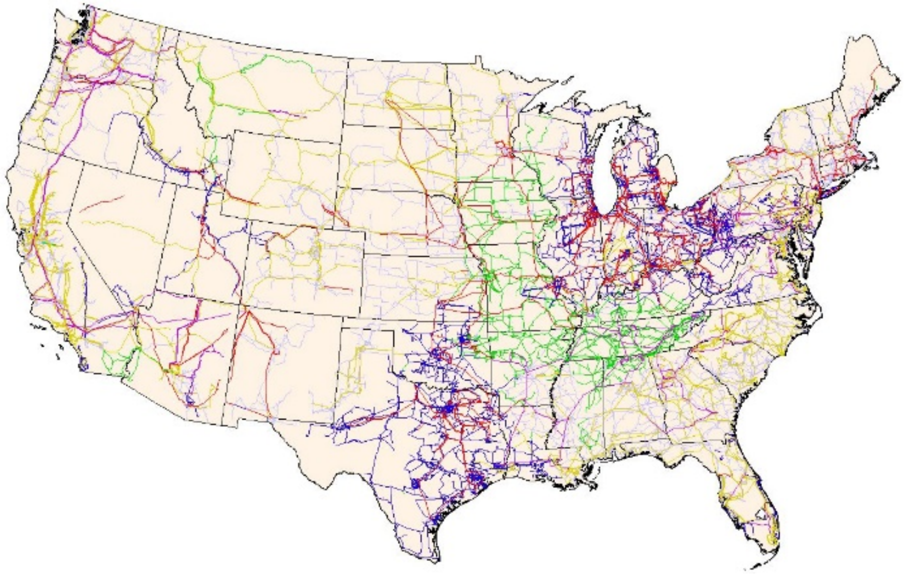


Power Systems Operation



Find the best operating strategy – use optimization!

Power Systems Optimization



Objectives

Affordable 

Reliable 

Constraints

Power balance


Grid constraints

Power Systems Optimization

Objectives

Affordable 

Reliable 

Large-scale, complex mathematical optimization

$$
\begin{aligned}
\min_{P_G(\boldsymbol{\omega})} \quad & C_G^T P_G(\boldsymbol{\omega}) \\
\text{s.t.} \quad & \textstyle\sum_{i=1}^{N_B} \left(P_{G(i)}(\boldsymbol{\omega}) + P_{W(i)} + \boldsymbol{\omega}_{(i)} - P_{D(i)}\right) = 0 \\
& P_{G(g)}(\boldsymbol{\omega}) \le P_{G(g)}^{max}, \\
& P_{G(g)}(\boldsymbol{\omega}) \ge P_{G(g)}^{min}, \\
& \quad \forall\, g = 1, \dots, N_G \\
& A_{(l,\cdot)}(P_G(\boldsymbol{\omega}) + P_W + \boldsymbol{\omega} - P_D) \le P_{L(l)}^{max}, \\
& A_{(l,\cdot)}(P_G(\boldsymbol{\omega}) + P_W + \boldsymbol{\omega} - P_D) \ge -P_{L(l)}^{max}, \\
& \quad \forall\, l = 1, \dots, N_L
\end{aligned}
$$

- Electricity market clearing
- Security assessment
- Operational support
- Planning studies
- …

Constraints

Power balance

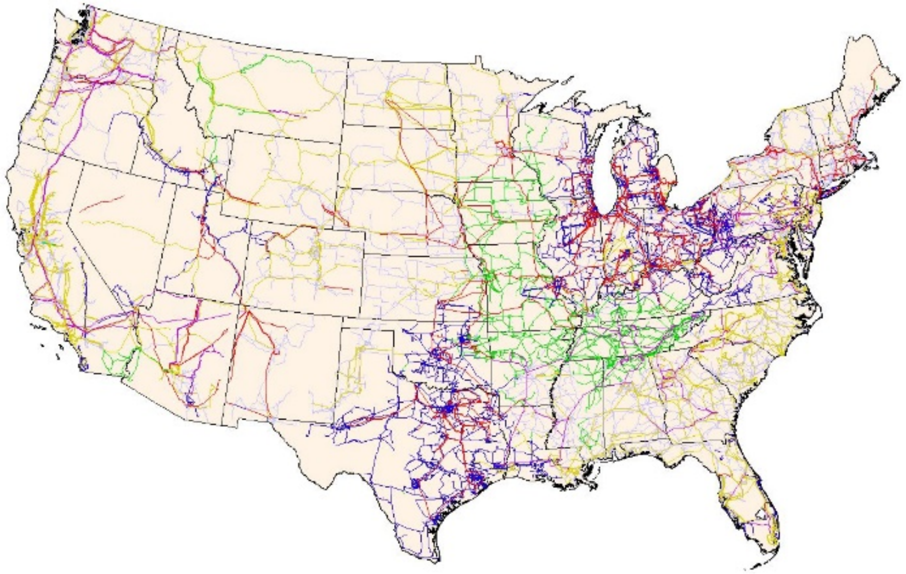
Grid constraints

Power Systems Optimization

Objectives

Affordable 

Reliable 



Constraints

Power balance


Grid constraints

First step: Understand how power flows across the network.

AC Power Flow – Some Notation

Active Power: p

Reactive Power: q

Voltage: $V = v_{re} + j \cdot v_{im} = |V|e^{j\theta_v}$

Current: $I = i_{re} + j \cdot i_{im} = |I|e^{j\theta_i}$

AC Power Flow – Some Notation

Active Power: p

Reactive Power: q

Complex Power: $S = p + j \cdot q$

Voltage: $V = v_{re} + j \cdot v_{im} = |V|e^{j\theta_v}$

Current: $I = i_{re} + j \cdot i_{im} \quad = |I|e^{j\theta_i}$

$S = VI^*$

AC Power Flow – Some Notation

Active Power: p

Reactive Power: q

Complex Power: $S = p + j \cdot q$

Apparent Power: $|S| = \sqrt{p^2 + q^2}$

Power Factor: $cos\ \varphi = \frac{p}{|S|}$

Voltage: $V = v_{re} + j \cdot v_{im} = |V|e^{j\theta_v}$

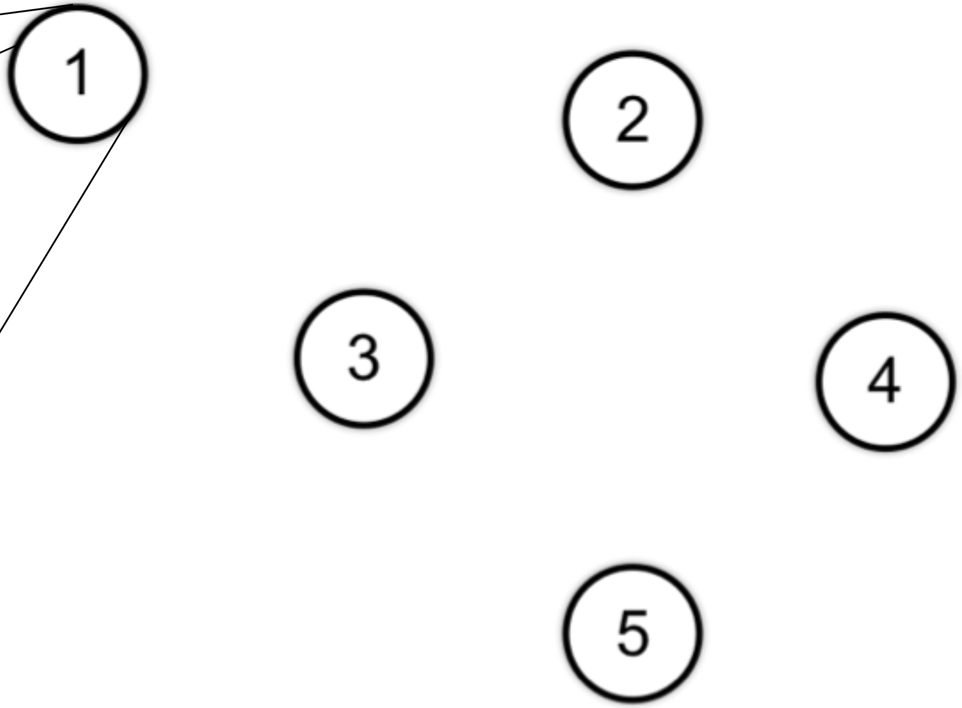
Current: $I = i_{re} + j \cdot i_{im} \ = |I|e^{j\theta_i}$

$S = VI^*$

What is an electric power grid?

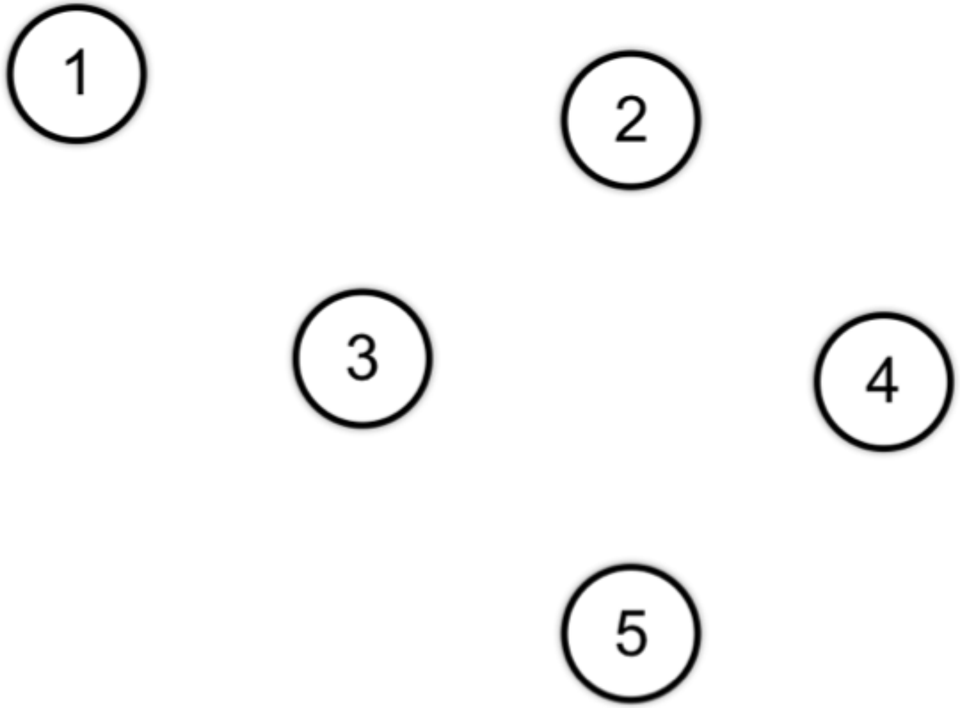
- Nodes (buses) N

substation



What is an electric power grid?

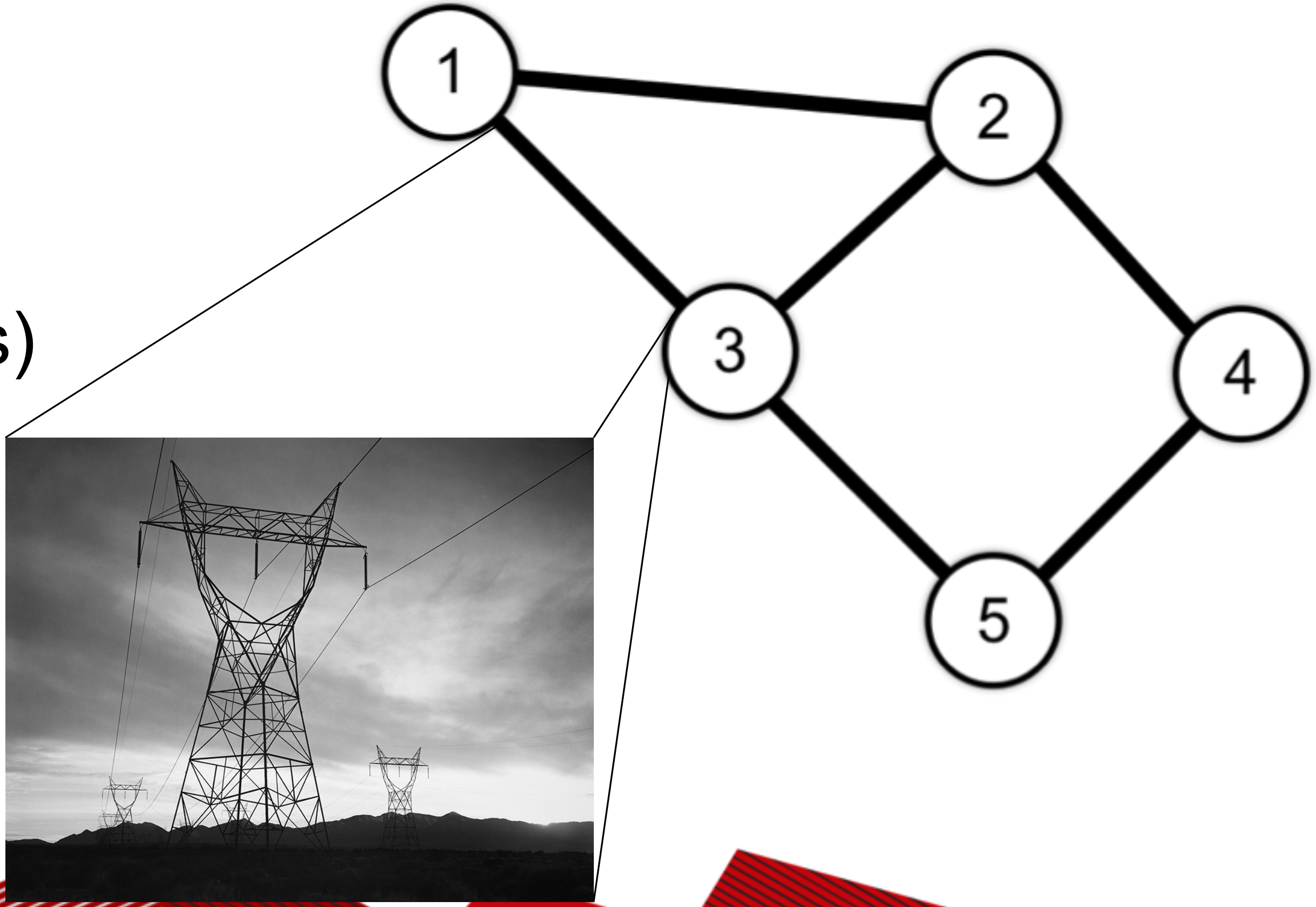
- Nodes (buses) N



What is an electric power grid?

- Nodes (buses) N

- Edges (transmission lines)



What is an electric power grid?

- Nodes (buses) N

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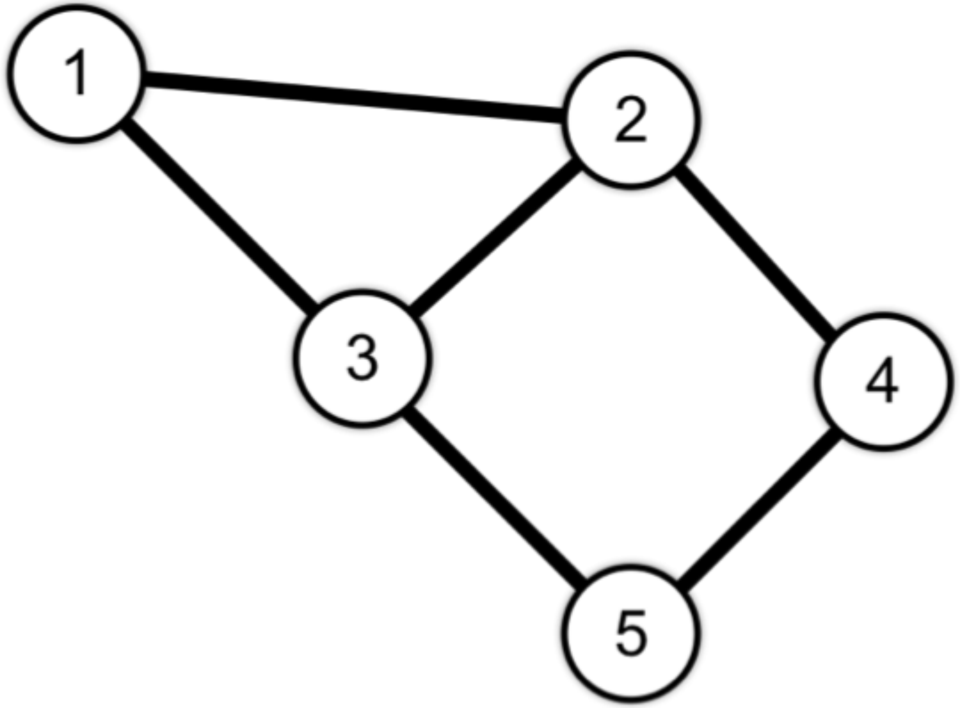
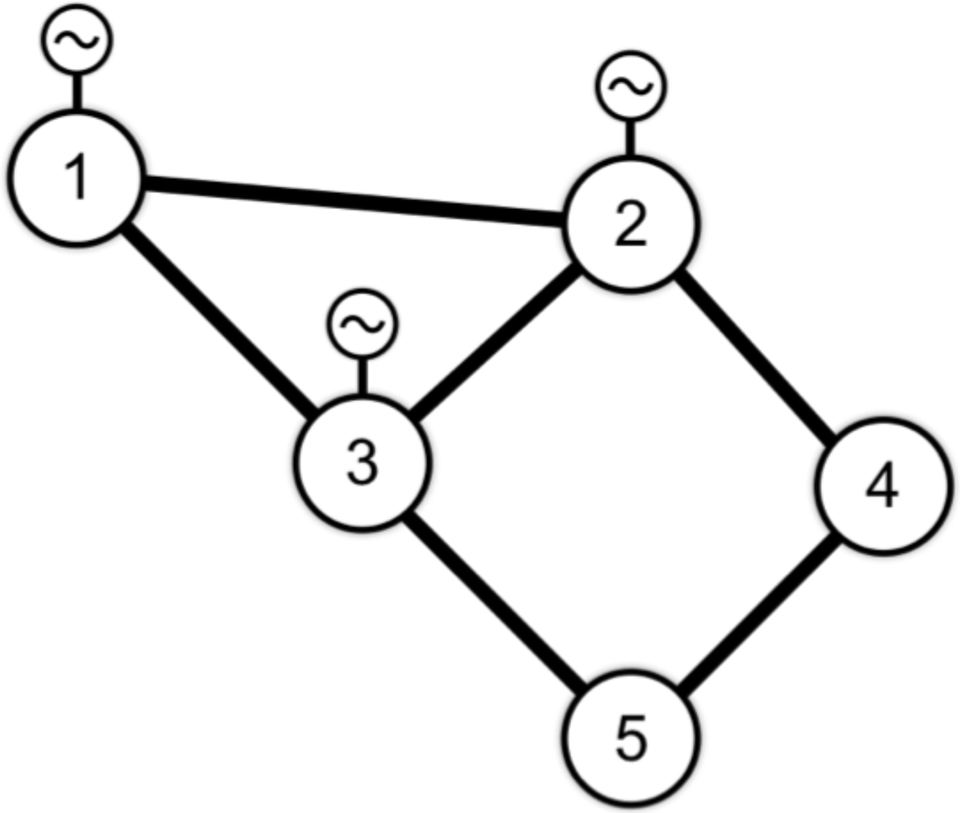


What is an electric power grid?

- Nodes (buses) N
  - Sources (generators) $S_{G,i} \quad \forall i \in N$
- Edges (transmission lines) E



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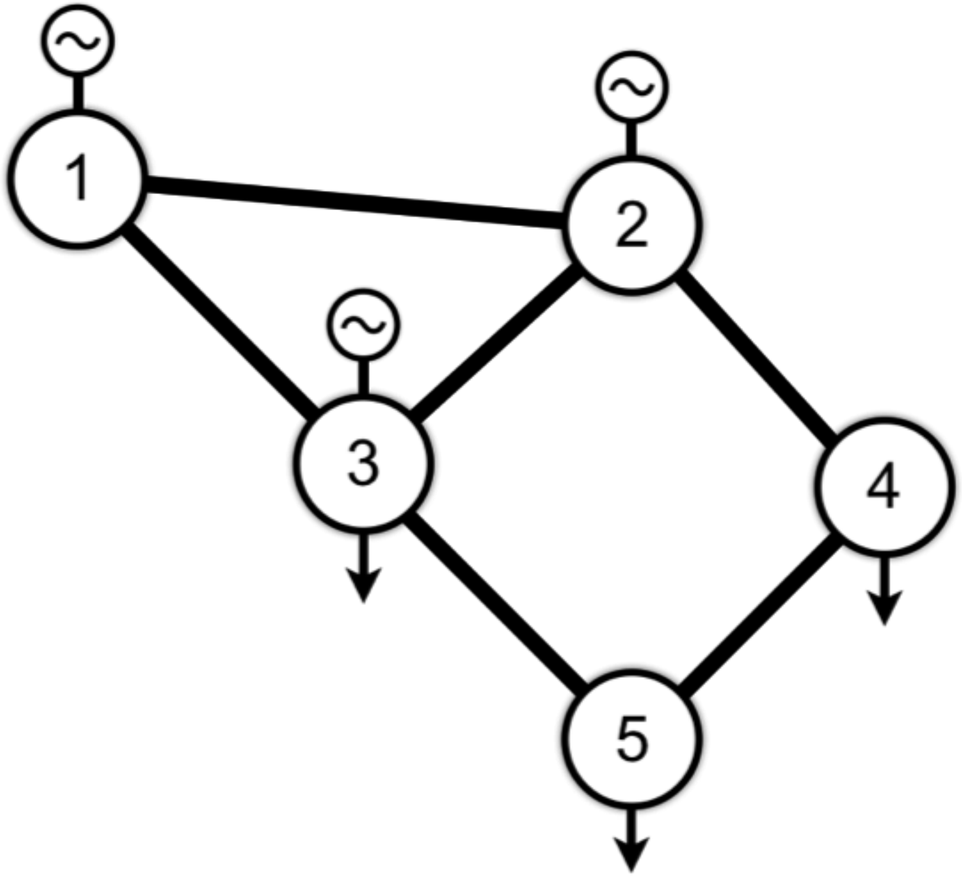

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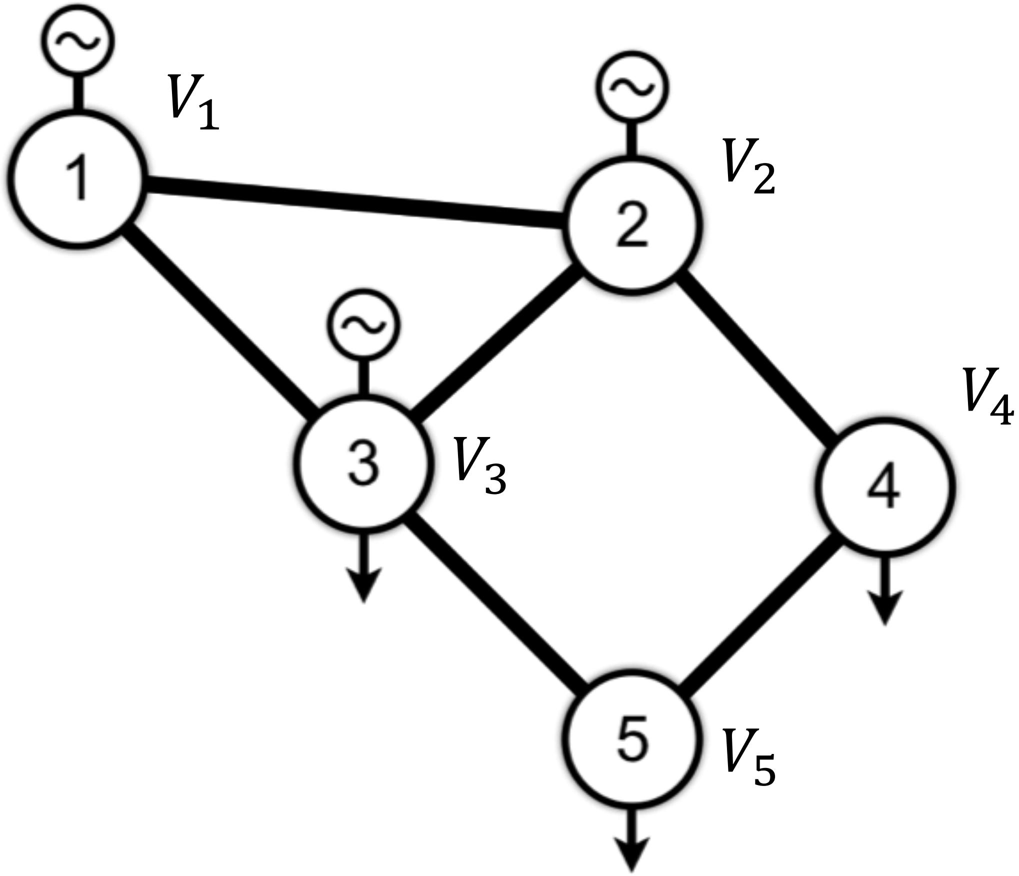

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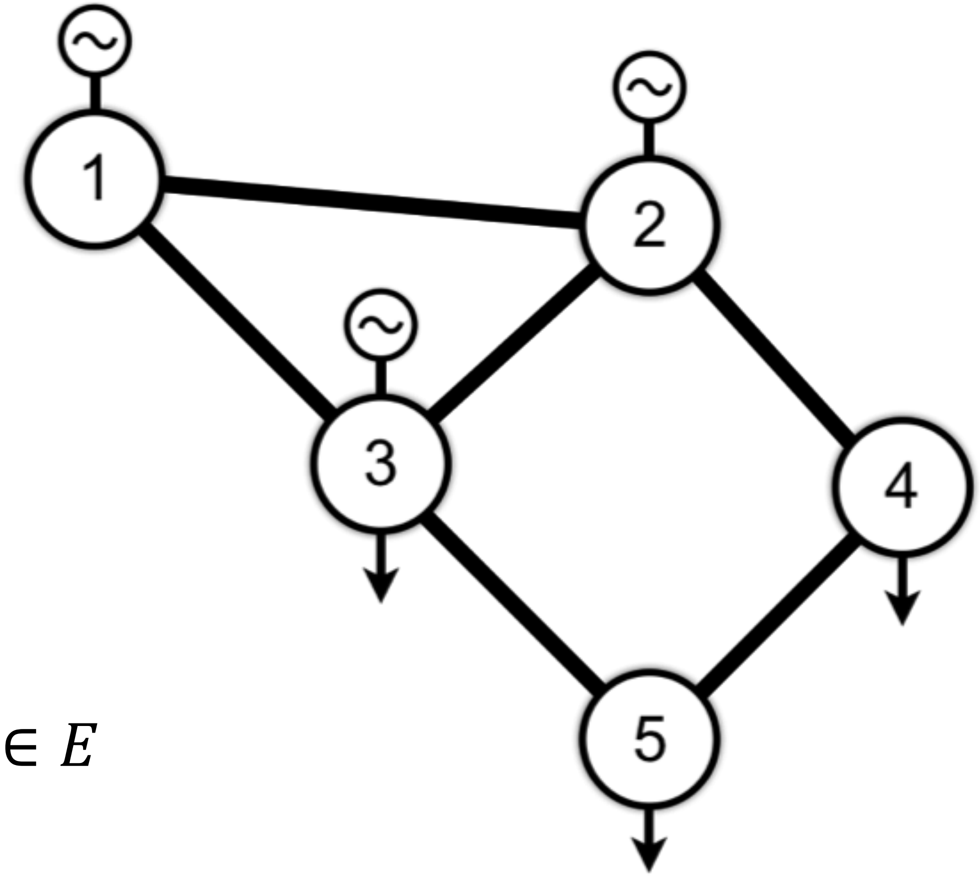
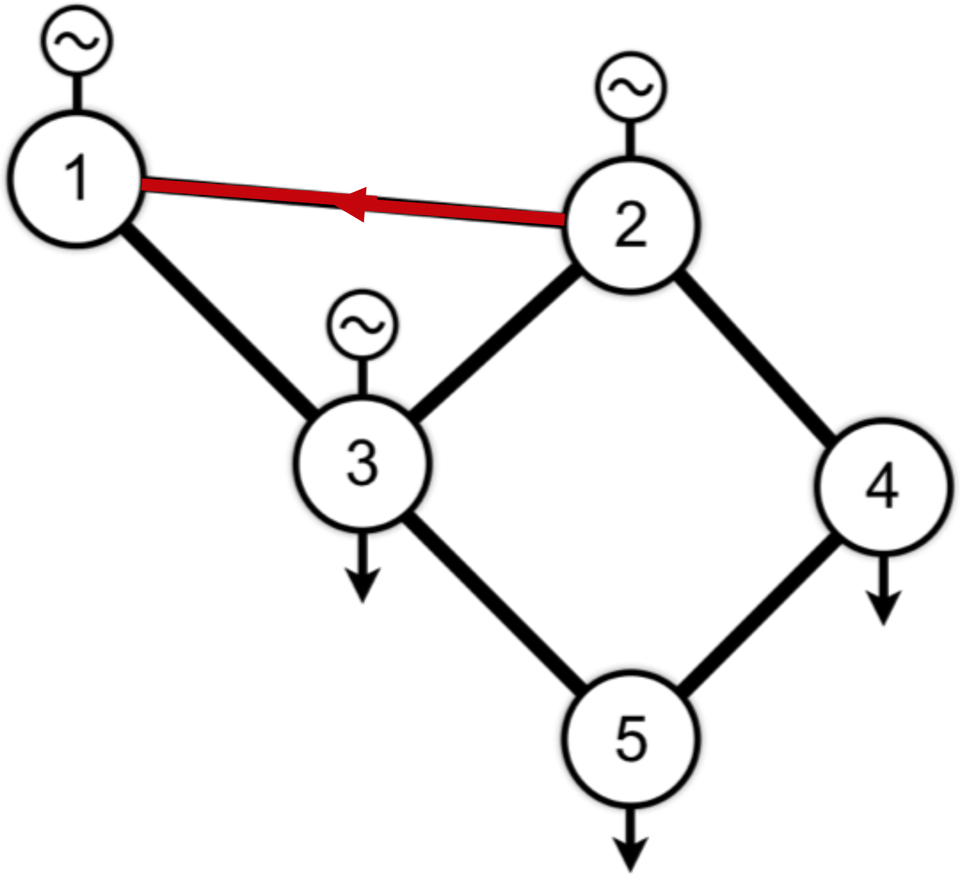
  - Voltages $V_i \quad \forall i \in N$
- Edges (transmission lines) E
  - Admittance (1/resistance) $Y_{ij} \;\forall\, (i, j) \in E$



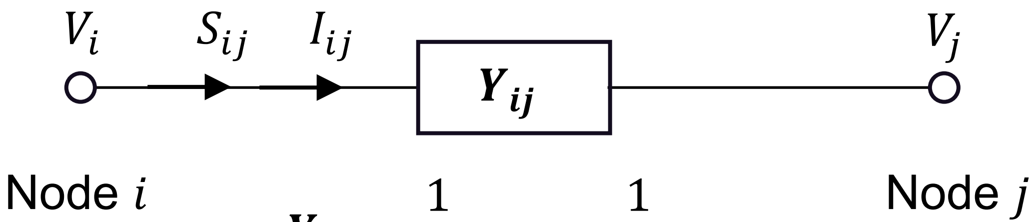
How much power flows on a transmission line?



Transmission Lines

- Connecting two nodes in the network

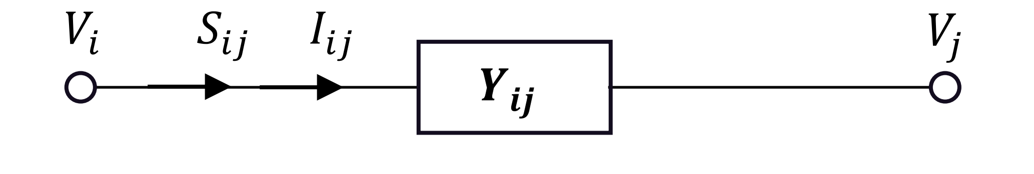
Transmission Lines



$$\boldsymbol{Y_{ij}} = \frac{1}{\boldsymbol{Z_{ij}}} = \frac{1}{r_{ij} + j \cdot x_{ij}}$$

- Connecting two nodes in the network
- Simplest model: Series impedance

Transmission Lines



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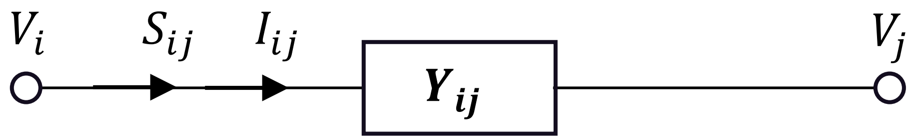
- Ohm's law $I_{ij} = \boldsymbol{Y_{ij}}(V_i - V_j)$

Transmission Lines



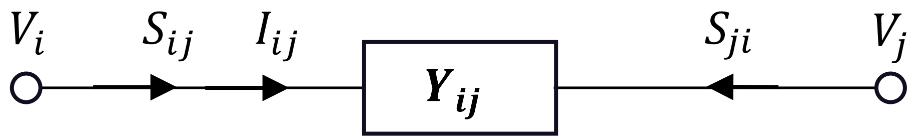
Node i

Node j

$$\boldsymbol{Y_{ij}} = \frac{1}{\boldsymbol{Z_{ij}}} = \frac{1}{r_{ij} + j \cdot x_{ij}}$$

- Connecting two nodes in the network
- Simplest model: Series impedance
- Ohm's law $\quad I_{ij} = \boldsymbol{Y_{ij}}(V_i - V_j)$
- Power Flow $\quad S_{ij} = V_i I_{ij}^* = V_i \boldsymbol{Y_{ij}^*}\left(V_i - V_j\right)^*$

Transmission Lines



Node i Node j

$$\boldsymbol{Y_{ij}} = \frac{1}{\boldsymbol{Z_{ij}}} = \frac{1}{r_{ij} + j \cdot x_{ij}}$$

- Connecting two nodes in the network
- Simplest model: Series impedance
- Ohm's law $I_{ij} = \boldsymbol{Y_{ij}}(V_i - V_j)$
- Power Flow $S_{ij} = V_i I_{ij}^* = V_i \boldsymbol{Y_{ij}^*}(V_i - V_j)^* \neq S_{ji} = V_j I_{ij}^* = V_j Y_{ij}^*(V_i - V_j)^*$

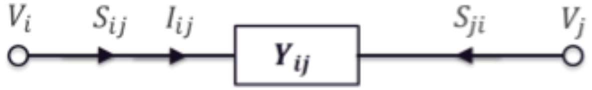
power losses!

Transmission Lines – More realistic models

- Simple series admittance

$$S_{ij} = \boldsymbol{Y}^*_{\boldsymbol{ij}} V_i V_i^* - \boldsymbol{Y}^*_{\boldsymbol{ij}} V_i V_j^*$$

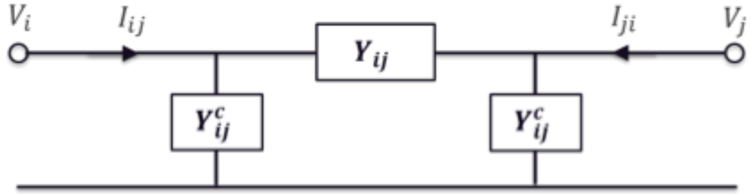
$$S_{ji} = \boldsymbol{Y}^*_{\boldsymbol{ij}} V_j V_j^* - \boldsymbol{Y}^*_{\boldsymbol{ij}} V_j V_i^*$$



- Pi-model

$$S_{ij} = \left(\boldsymbol{Y}^*_{\boldsymbol{ij}} + \boldsymbol{Y}^{\boldsymbol{c}*}_{\boldsymbol{ij}}\right) V_i V_i^* - \boldsymbol{Y}^*_{\boldsymbol{ij}} V_i V_j^*$$

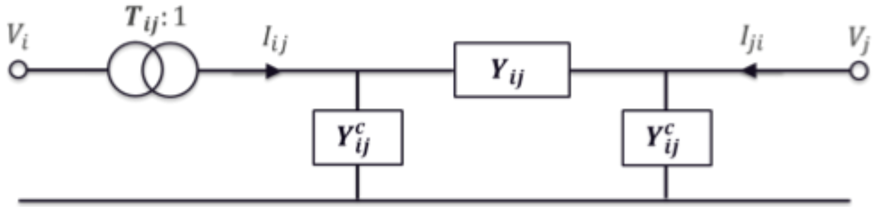
$$S_{ji} = \left(\boldsymbol{Y}^*_{\boldsymbol{ij}} + \boldsymbol{Y}^{\boldsymbol{c}*}_{\boldsymbol{ij}}\right) V_j V_j^* - \boldsymbol{Y}^*_{\boldsymbol{ij}} V_j V_i^*$$



- Pi-model with transformer

$$S_{ij} = \frac{\left(\boldsymbol{Y}^*_{\boldsymbol{ij}} + \boldsymbol{Y}^{\boldsymbol{c}*}_{\boldsymbol{ij}}\right)}{\boldsymbol{T_{ij}} \boldsymbol{T}^*_{\boldsymbol{ij}}} V_i V_i^* - \frac{\boldsymbol{Y}^*_{\boldsymbol{ij}}}{\boldsymbol{T_{ij}}} V_i V_j^*$$

$$S_{ji} = \left(\boldsymbol{Y}^*_{\boldsymbol{ij}} + \boldsymbol{Y}^{\boldsymbol{c}*}_{\boldsymbol{ij}}\right) V_j V_j^* - \frac{\boldsymbol{Y}^*_{\boldsymbol{ij}}}{\boldsymbol{T}^*_{\boldsymbol{ij}}} V_j V_i^*$$



Transmission Lines – More realistic models

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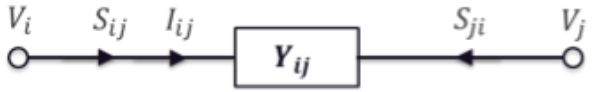


- Pi-model

$$S_{ij} = \left(\boldsymbol{Y}_{\boldsymbol{ij}}^{*} + \boldsymbol{Y}_{\boldsymbol{ij}}^{\boldsymbol{c}*}\right) V_i V_i^{*} - \boldsymbol{Y}_{\boldsymbol{ij}}^{*} V_i V_j^{*}$$

$$S_{ji} = \left(\boldsymbol{Y}_{\boldsymbol{ij}}^{*} + \boldsymbol{Y}_{\boldsymbol{ij}}^{\boldsymbol{c}*}\right) V_j V_j^{*} - \boldsymbol{Y}_{\boldsymbol{ij}}^{*} V_j V_i^{*}$$

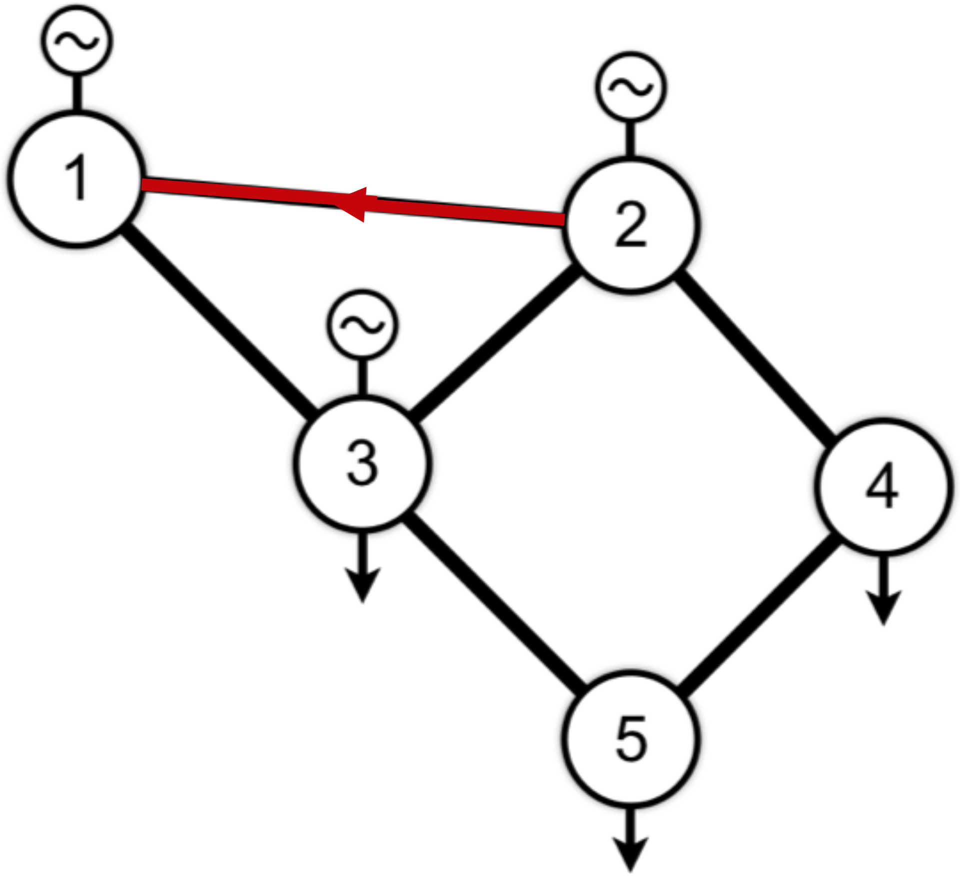
- Pi-model with transformer

$$S_{ij} = \frac{\left(\boldsymbol{Y}_{\boldsymbol{ij}}^{*} + \boldsymbol{Y}_{\boldsymbol{ij}}^{\boldsymbol{c}*}\right)}{\boldsymbol{T}_{\boldsymbol{ij}} \boldsymbol{T}_{\boldsymbol{ij}}^{*}} V_i V_i^{*} - \frac{\boldsymbol{Y}_{\boldsymbol{ij}}^{*}}{\boldsymbol{T}_{\boldsymbol{ij}}} V_i V_j^{*}$$

$$S_{ji} = \left(\boldsymbol{Y}_{\boldsymbol{ij}}^{*} + \boldsymbol{Y}_{\boldsymbol{ij}}^{\boldsymbol{c}*}\right) V_j V_j^{*} - \frac{\boldsymbol{Y}_{\boldsymbol{ij}}^{*}}{\boldsymbol{T}_{\boldsymbol{ij}}^{*}} V_j V_i^{*}$$

If $\boldsymbol{Y}_{\boldsymbol{ij}}, \boldsymbol{Y}_{\boldsymbol{ij}}^{\boldsymbol{c}}$ and $\boldsymbol{T}_{\boldsymbol{ij}}$ are constant these formulations are very similar for optimization

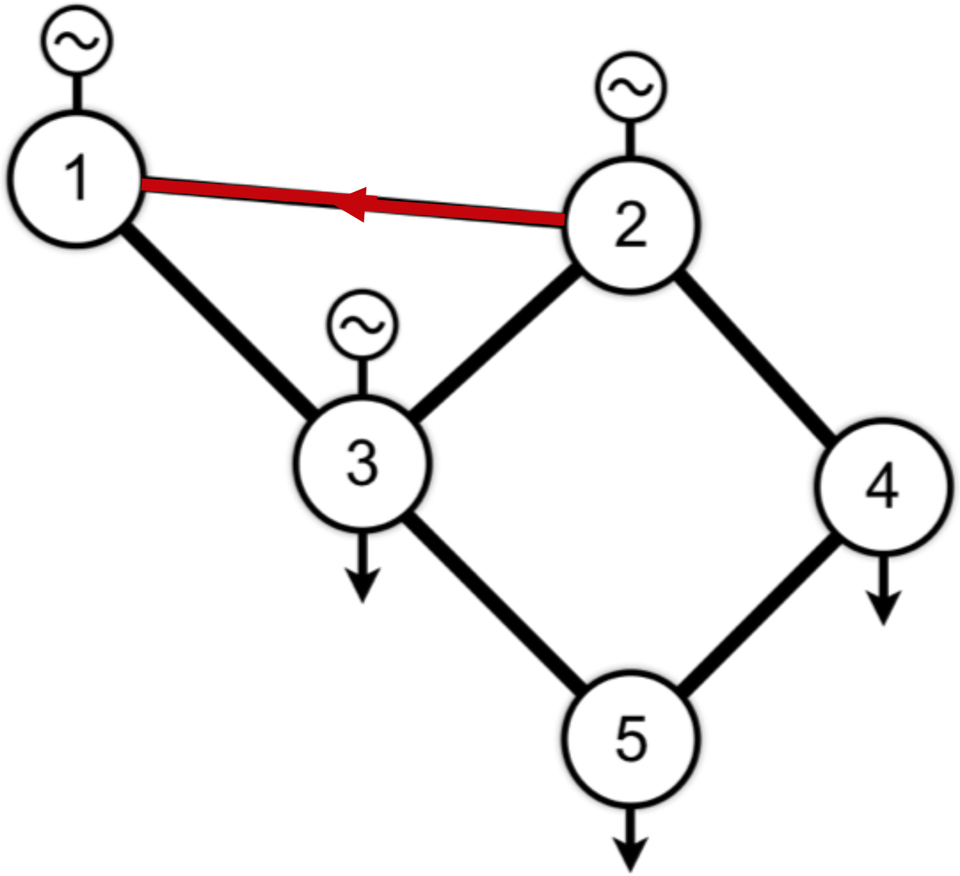
How does power flow throughout the network?



How does power flow throughout the network?

- Power Flow on Transmission Lines
  - power flow is based on voltage differences (Ohm's Law)

$$S_{ij} = \boldsymbol{Y}_{\boldsymbol{ij}}^{*} V_i V_i^{*} - \boldsymbol{Y}_{\boldsymbol{ij}}^{*} V_i V_j^{*} \quad \forall\, (i,j) \in E$$



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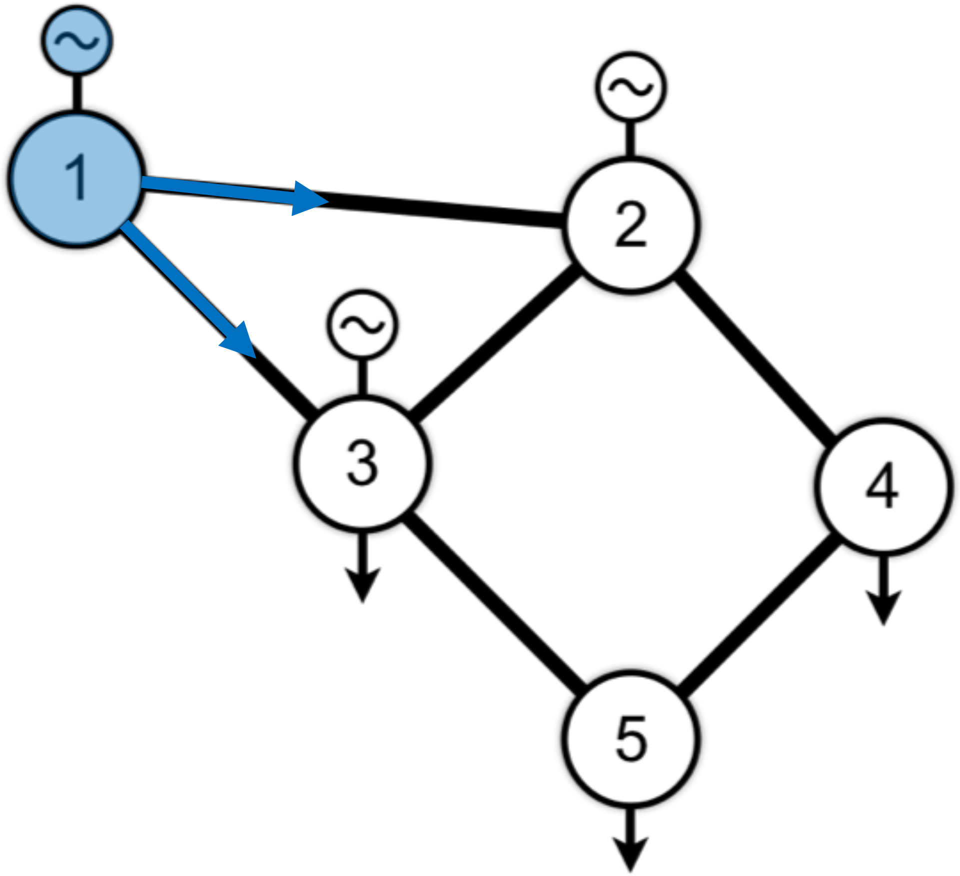
$$S_{ij} = \boldsymbol{Y}_{\boldsymbol{ij}}^{*} V_i V_i^{*} - \boldsymbol{Y}_{\boldsymbol{ij}}^{*} V_i V_j^{*} \quad \forall\, (i,j) \in E$$

- Nodal power balance (conservation of active and reactive power)
  - conservation of flow at nodes, Kirchhoff's Current Law (KCL)

$$S_{G,i} - S_{D,i} = \sum_{(i,j)\in E} S_{ij} \quad \forall\, i \in N$$



How does power flow throughout the network?

- **AC Power Flow** Equations

$$S_{ij} = \boldsymbol{Y_{ij}^*} V_i V_i^* - \boldsymbol{Y_{ij}^*} V_i V_j^* \quad \forall\ (i,j) \in E$$

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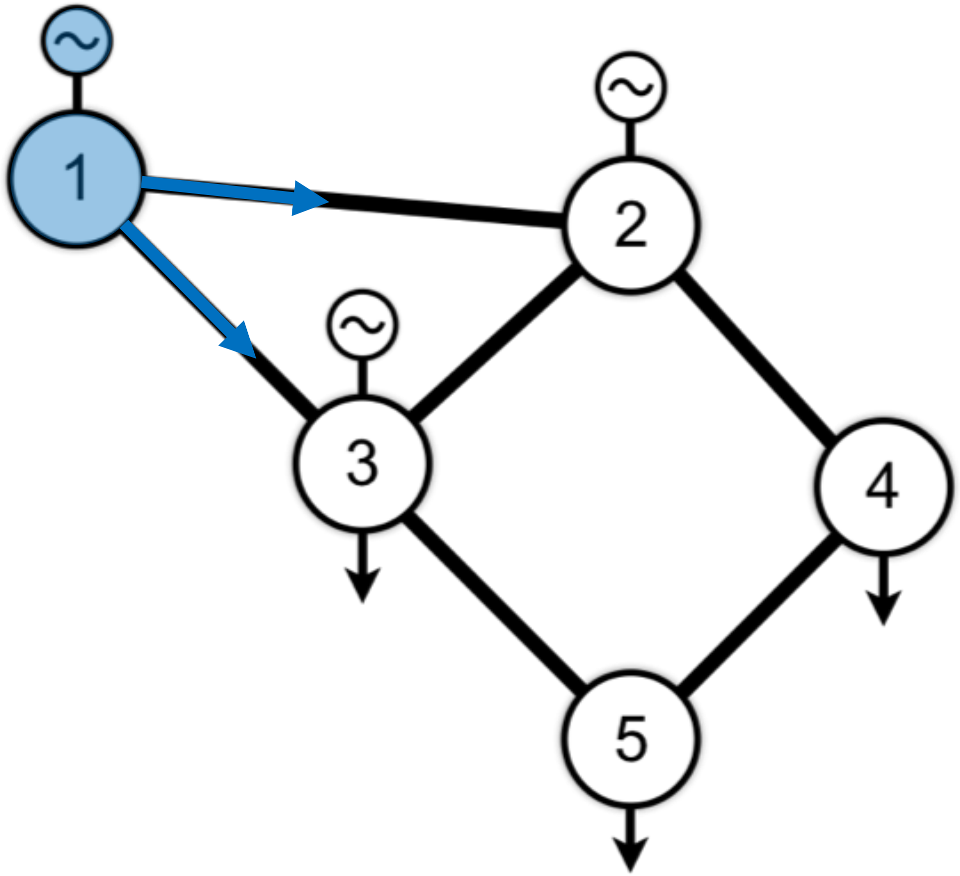

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- **AC Power Flow** Equations

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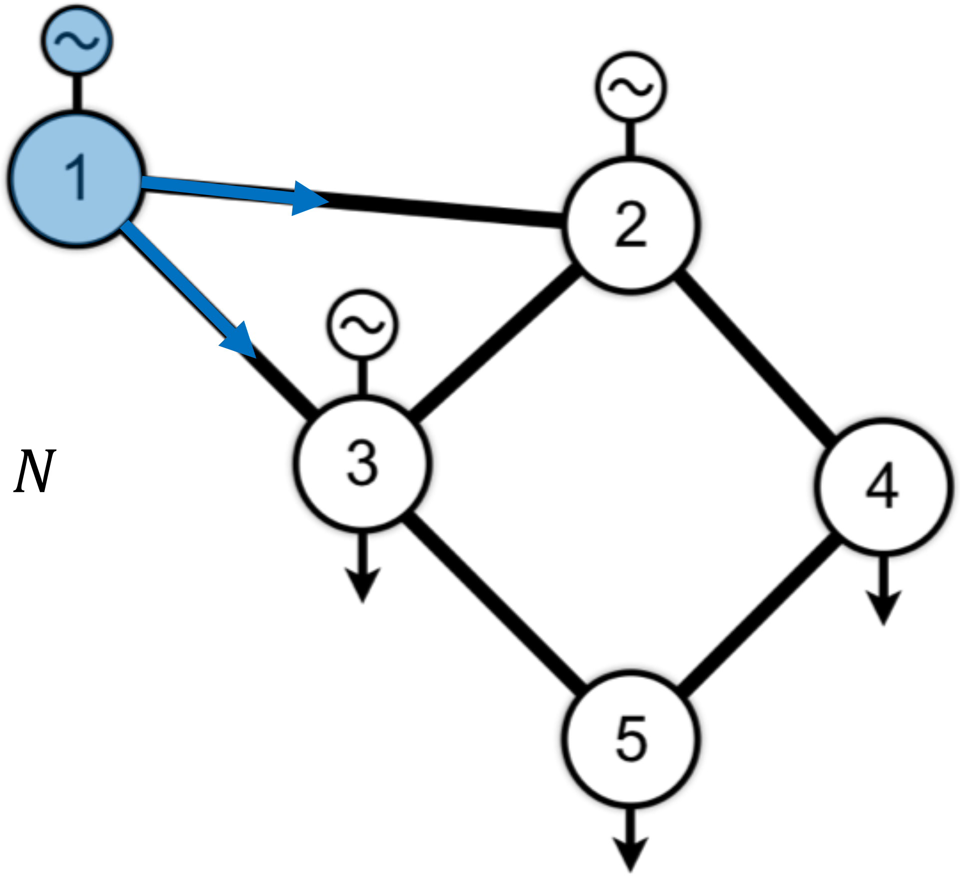

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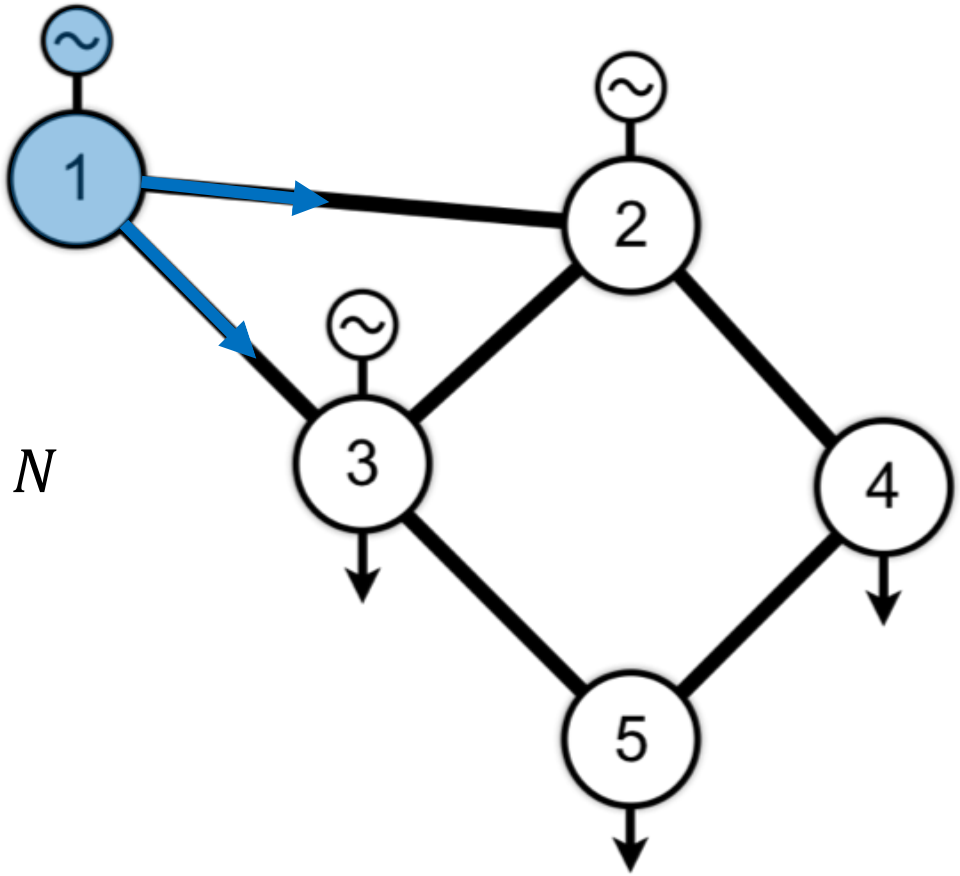
$$S = V \boldsymbol{Y}^{*} V^{*}$$



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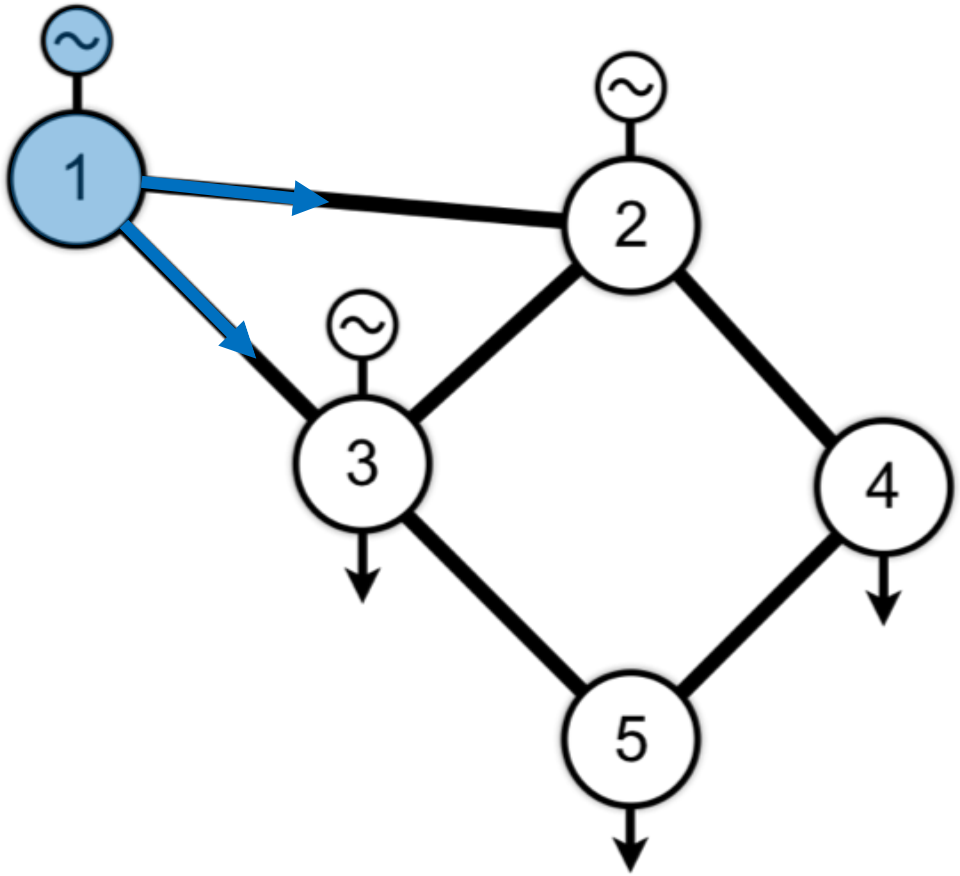
- Polar form:

Complex power: $S_i = p_i + jq_i$

Nodal voltages: $V_i = |V_i| e^{j\theta_i}$

Admittance matrix: $Y = G + jB$



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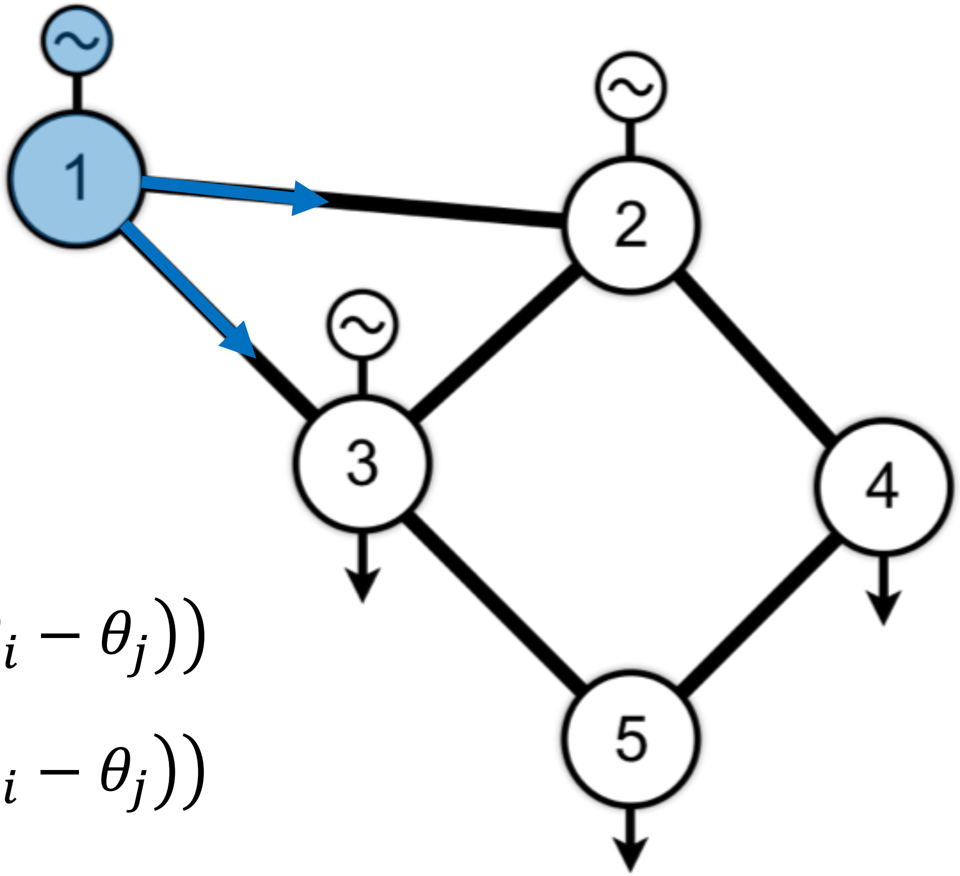
Nodal voltages: $V_i = |V_i| e^{j\theta_i}$

Admittance matrix: $Y = G + jB$

$$p_i = \sum_{j=1}^{N} |V_i||V_j| \left(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j) \right)$$

$$q_i = \sum_{j=1}^{N} |V_i||V_j| \left(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j) \right)$$



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$$S_{G,i} - S_{D,i} = \sum_{(i,j)\in E} \boldsymbol{Y_{ij}^*} V_i V_i^* - \boldsymbol{Y_{ij}^*} V_i V_j^*$$

- Polar form:

Complex power: $S_i = \boldsymbol{p_i} + j\boldsymbol{q_i}$

Nodal voltages: $V_i = |\boldsymbol{V_i}|e^{j\boldsymbol{\theta_i}}$

Admittance matrix: $Y = G + jB$

4 variables per node

$$p_i = \sum_{j=1}^{N} |V_i||V_j|\left(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right)$$

$$q_i = \sum_{j=1}^{N} |V_i||V_j|\left(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\right)$$

How does power flow throughout the network?

- **AC Power Flow** Equations

$$S_{G,i} - S_{D,i} = \sum_{(i,j)\in E} \boldsymbol{Y}_{\boldsymbol{ij}}^* V_i V_i^* - \boldsymbol{Y}_{\boldsymbol{ij}}^* V_i V_j^*$$

- Polar form:

Complex power: $S_i = \boldsymbol{p_i} + j\boldsymbol{q_i}$

Nodal voltages: $V_i = |\boldsymbol{V_i}| e^{j\boldsymbol{\theta_i}}$

Admittance matrix: $Y = G + jB$

4 variables per node

$$\left.\begin{aligned} p_i &= \sum_{j=1}^{N} |V_i||V_j| \left(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right) \\ q_i &= \sum_{j=1}^{N} |V_i||V_j| \left(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\right) \end{aligned}\right\}$$

2 equations per node

How does power flow throughout the network?

- **AC Power Flow** Equations

2 degrees of freedom per node!

$$S_{G,i} - S_{D,i} = \sum_{(i,j)\in E} \boldsymbol{Y_{ij}^*} V_i V_i^* - \boldsymbol{Y_{ij}^*} V_i V_j^*$$

- Polar form:

Complex power: $S_i = \boldsymbol{p_i} + j\boldsymbol{q_i}$

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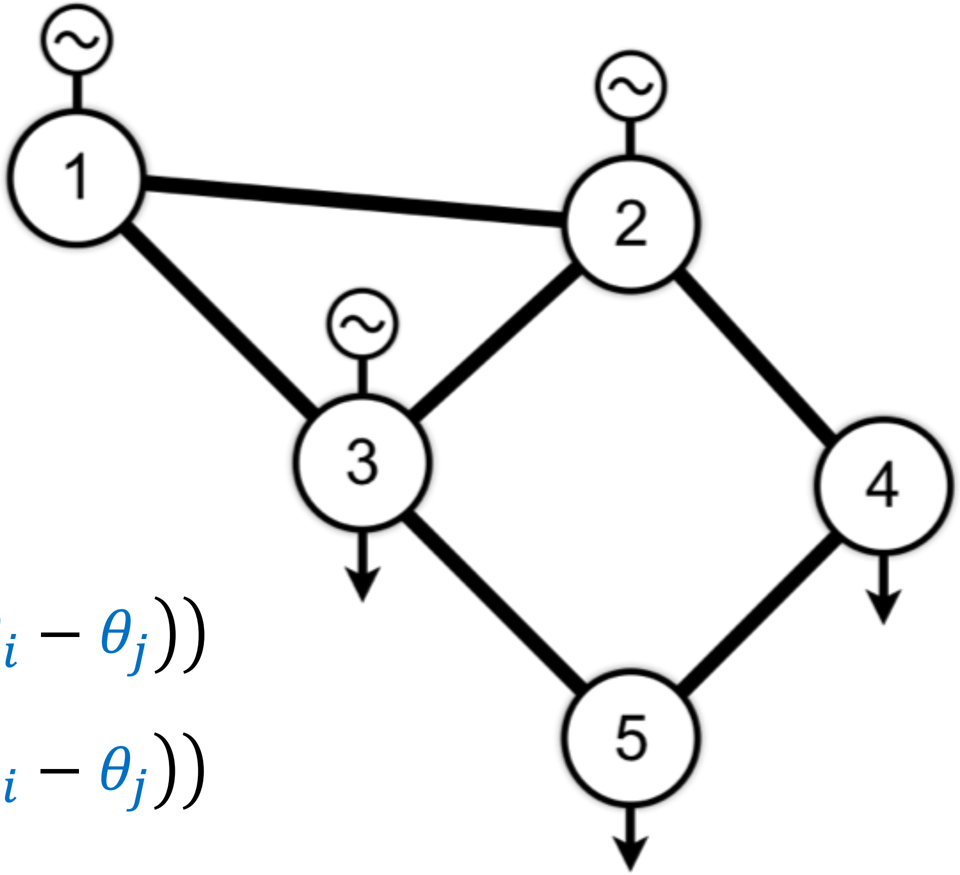
2 equations per node

How does power flow throughout the network?

- If voltages $V_i = |V_i|e^{j\theta_i}$ are given:

  Easy to solve! Always a solution



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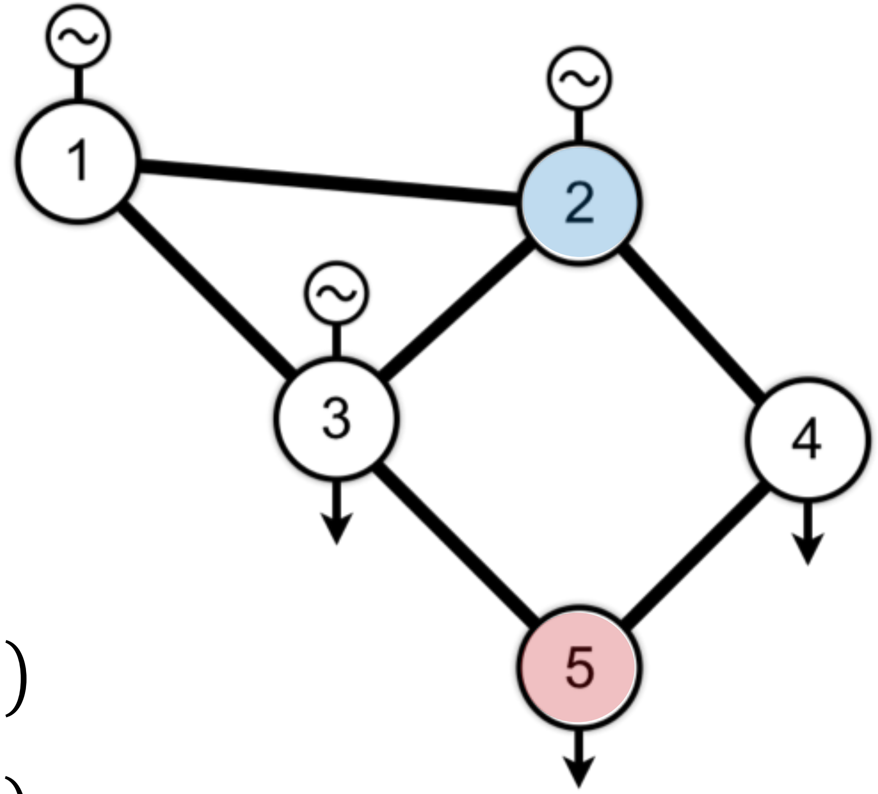
How does power flow throughout the network?

- In reality: We solve (at least partially) for the voltages

  Load buses (PQ buses): $p_{D,i}$, $q_{D,i}$ are given



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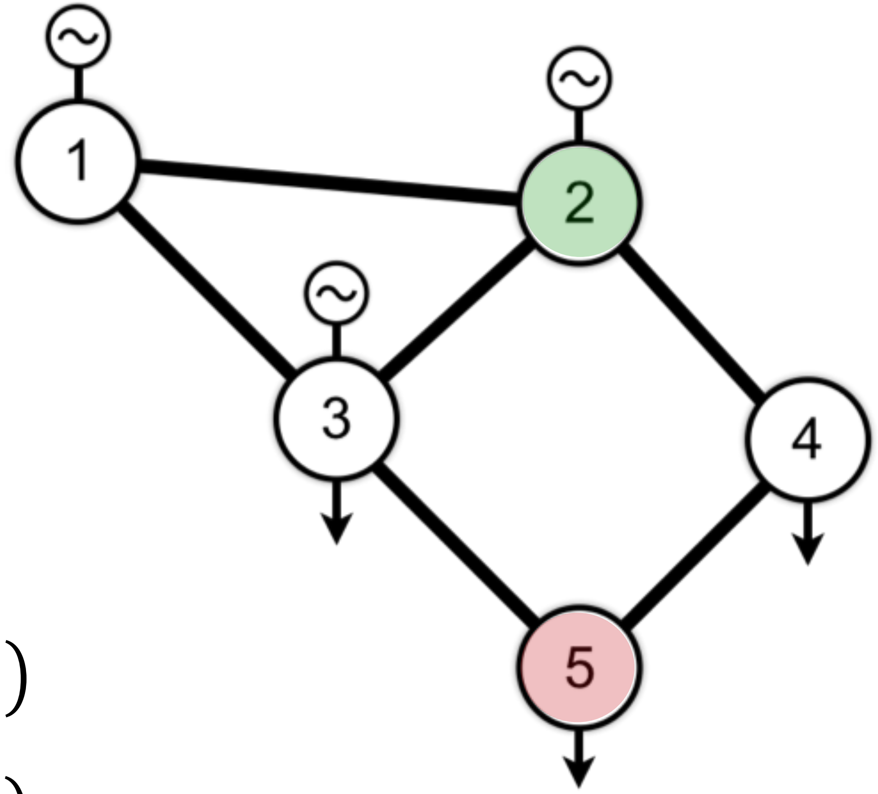
How does power flow throughout the network?

- In reality: We solve (at least partially) for the voltages

  Load buses (PQ buses): $p_{D,i}$, $q_{D,i}$ are given

  Generator buses (PV buses): $p_{G,i}$, $|V_{G,i}|$ are controllable

  ($q_{G,i}$ used to control $|V_{G,i}|$)



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How does power flow throughout the network?

- In reality: We solve (at least partially) for the voltages

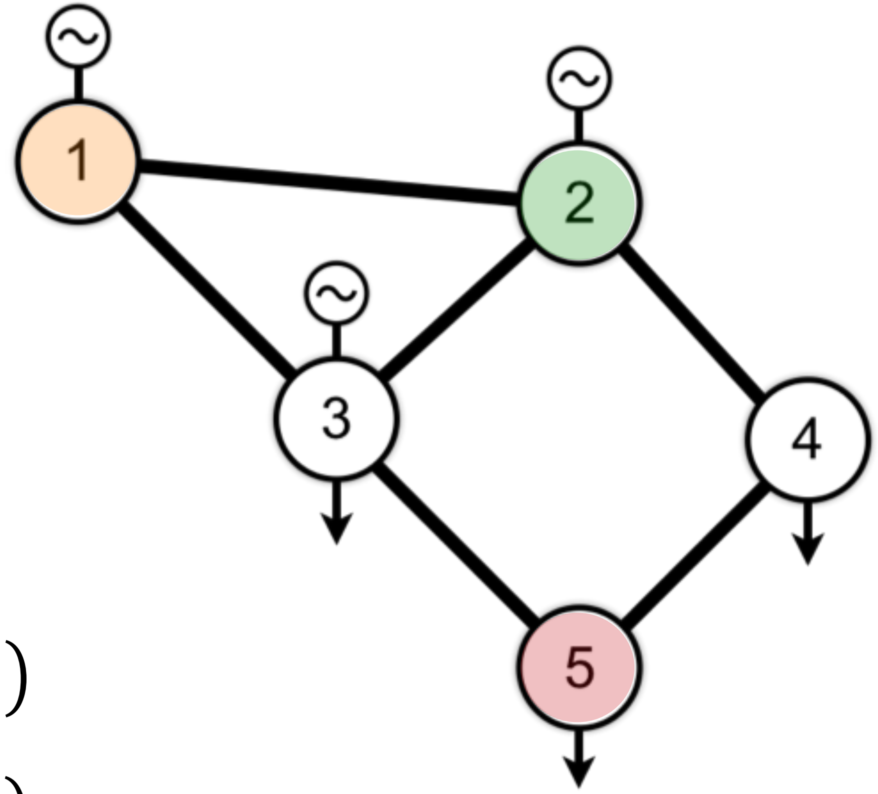
Load buses (PQ buses): $p_{D,i}$, $q_{D,i}$ are given

Generator buses (PV buses): $p_{G,i}$, $|V_{G,i}|$ are controllable

($q_{G,i}$ used to control $|V_{G,i}|$)

Reference/slack bus ($V\theta$ bus): $\theta_{ref} = 0, |V_{ref,i}|$ are defined

($q_{G,i}$ used to control $|V_{G,i}|$)



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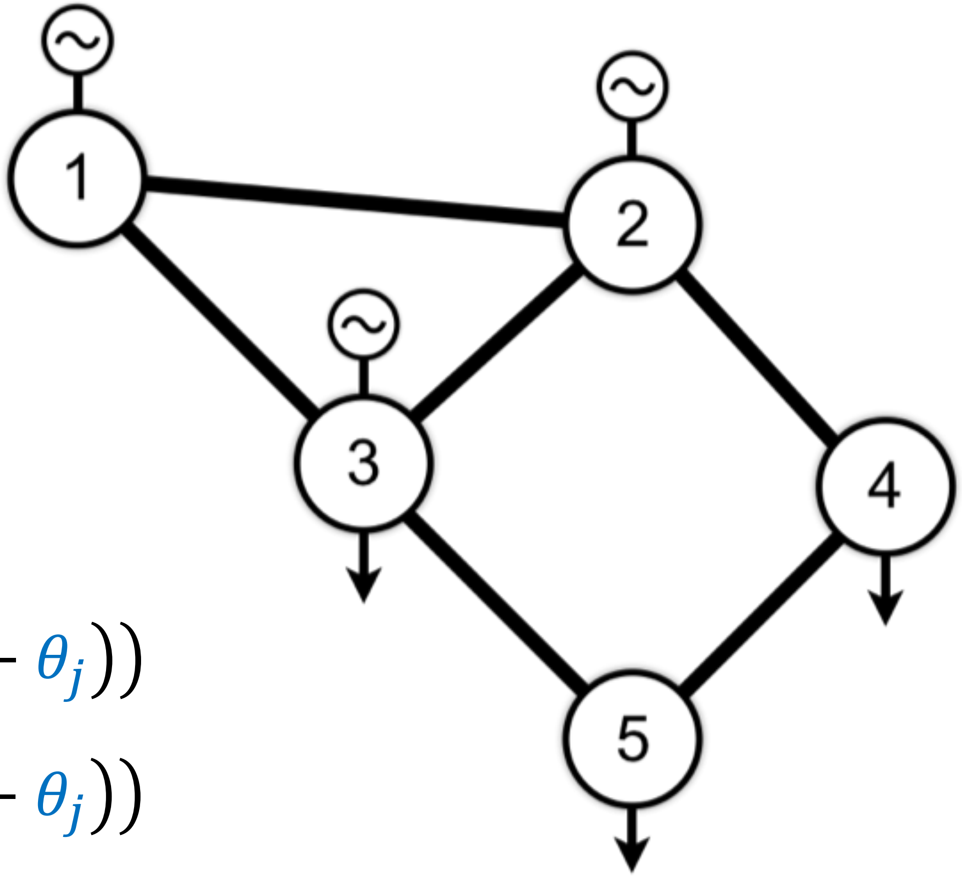
How does power flow throughout the network?

- By specifying some of the p_i, q_i

  … there may be **no solution** for V_i

  … there may be **multiple solutions** for V_i



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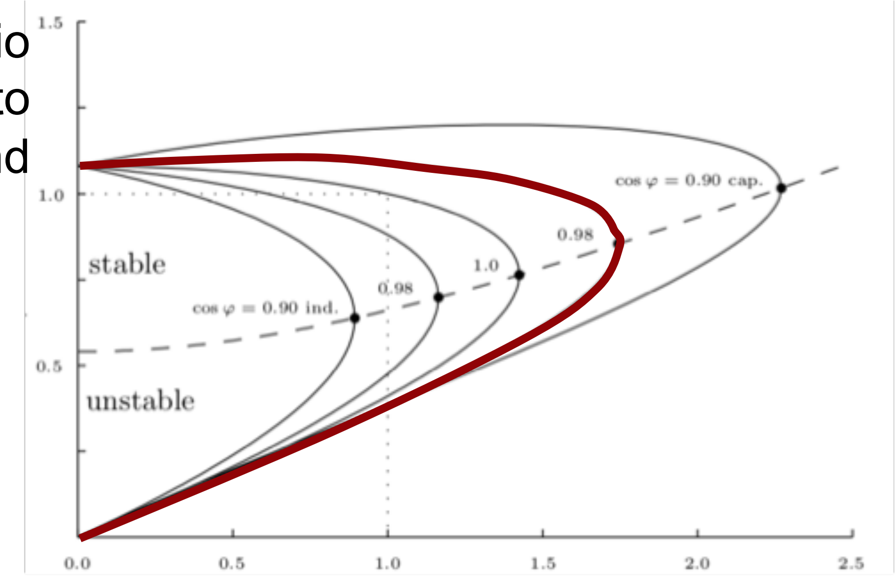
Voltage ratio sending to receiving end



Transmitted power

Franck and Andersson, Electric Power Systems, ETH Lecture Notes, 2014

Nose curve

Power flow over one transmission line

(Different curves = different power factors)

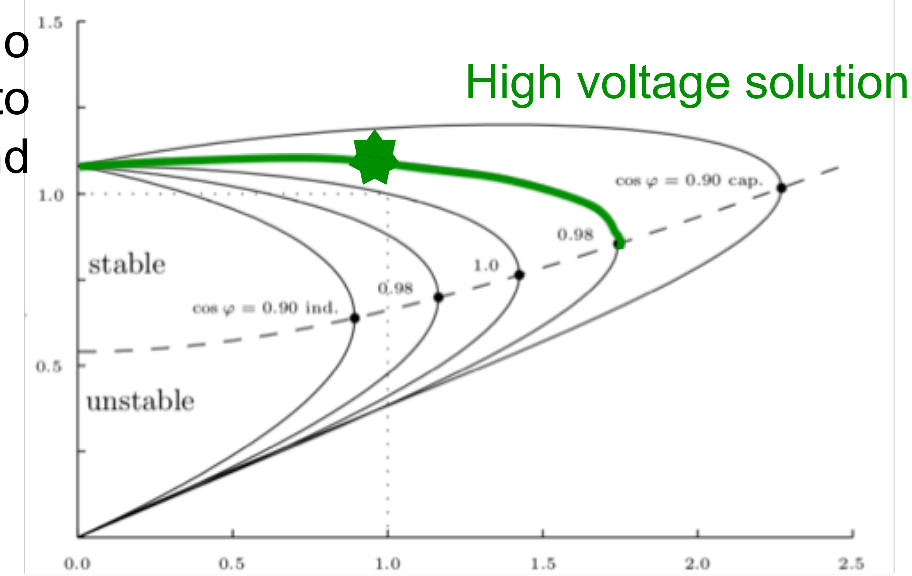
How does power flow throughout the network?

- By specifying some of the p_i, q_i

  … there may be **no solution** for V_i

  … there may be **multiple solutions** for V_i

Voltage ratio sending to receiving end



High voltage solution – stable!

Make sure to stay in stable region

Transmitted power

Franck and Andersson, Electric Power Systems, ETH Lecture Notes, 2014

How does power flow throughout the network?

- By specifying some of the p_i, q_i

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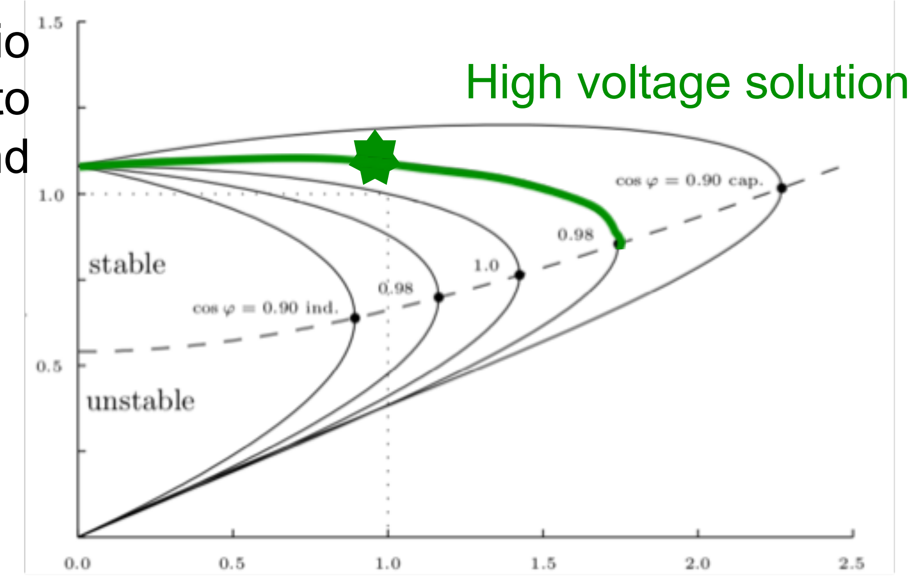

Make sure to stay in stable region

Transmitted power

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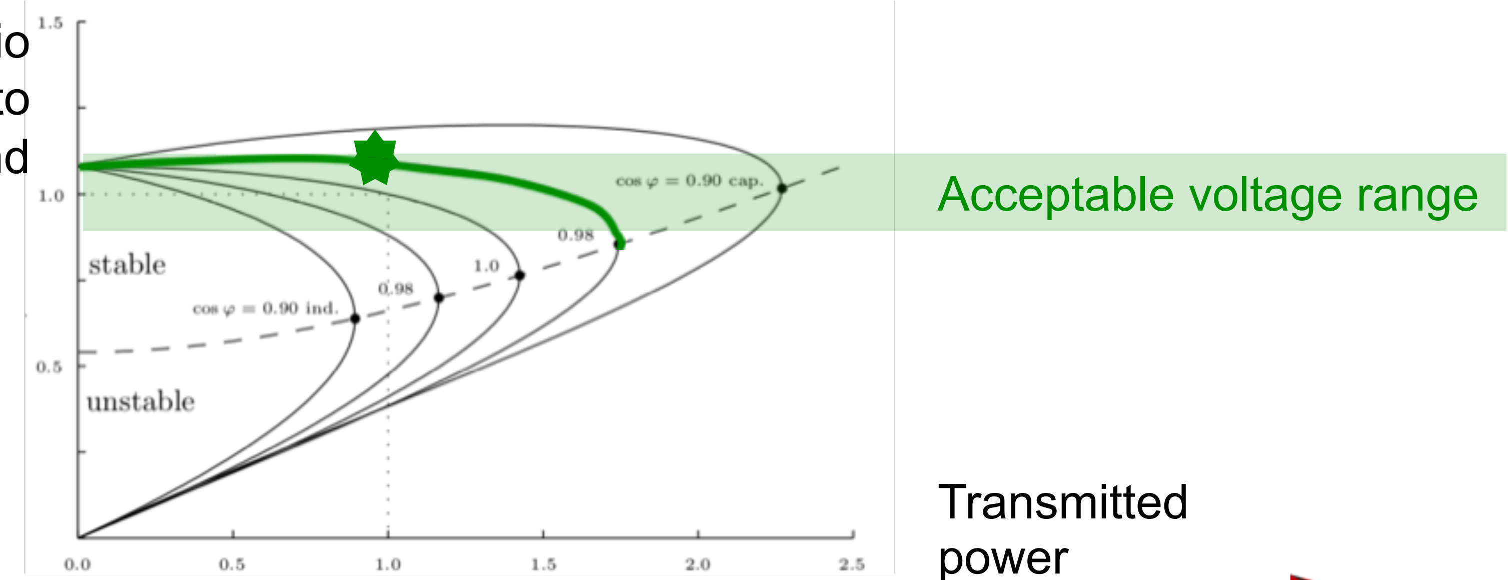
Technical constraints

- Voltage magnitudes: $|V_i|^{min} \leq |V_i| \leq |V_i|^{max}$

Voltage ratio sending to receiving end

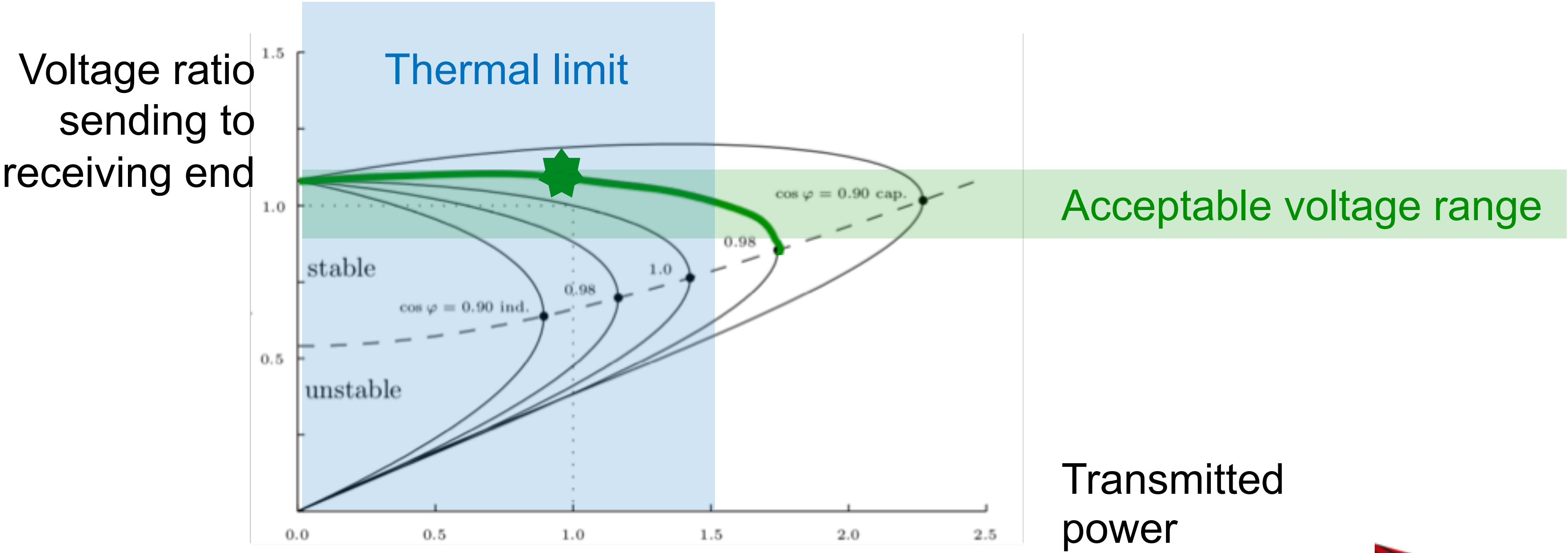


Transmitted power

Franck and Andersson, Electric Power Systems, ETH Lecture Notes, 2014

Technical constraints

- Voltage magnitudes: $|V_i|^{min} \leq |V_i| \leq |V_i|^{max}$
- Transmission lines: $|I_{ij}| \leq |I_{ij}|^{max}$ or $|S_{ij}| \leq |S_{ij}|^{max}$



Franck and Andersson, Electric Power Systems, ETH Lecture Notes, 2014

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- Transmission lines: $|I_{ij}| \leq |I_{ij}|^{max}$ or $|S_{ij}| \leq |S_{ij}|^{max}$
- Generator limits: $p_{G,i}^{min} \leq p_{G,i} \leq p_{G,i}^{max}$

  $q_{G,i}^{min} \leq q_{G,i} \leq q_{G,i}^{max}$

Optimal Power Flow

Lets gather it all together.

Optimal Power Flow

$$p_i = \sum_{j=1}^{N} |V_i||V_j|\left(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right)$$

$$q_i = \sum_{j=1}^{N} |V_i||V_j|\left(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\right)$$

Non-Linear AC Power Flow

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Non-Linear AC Power Flow

$$p_{G,g}^{min} \leq p_{G,g} \leq p_{G,g}^{max}, \quad g \in \mathcal{G}$$
$$q_{G,g}^{min} \leq q_{G,g} \leq q_{G,g}^{max}, \quad g \in \mathcal{G}$$

Generation constraints

$$|V|_i^{min} \leq |V_i| \leq |V|_i^{max}, \qquad i \in N$$

Voltage constraints

$$|S|_{ij} \leq |S|_{ij}^{max}, \quad (i,j) \in E$$

Transmission constraints

Optimal Power Flow

Objective function:

$$\min_{p_Q, q_G, |V_i|, \theta} \quad \sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i}\, p_{G,i} + c_{0,i} \right)$$

Minimize generation cost

s.t.

$$p_i = \sum_{j=1}^{N} |V_i||V_j| \left(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right)$$

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Non-Linear AC Power Flow

$$p_{G,g}^{min} \leq p_{G,g} \leq p_{G,g}^{max}, \quad g \in \mathcal{G}$$

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Generation constraints

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Voltage constraints

$$|S|_{ij} \leq |S|_{ij}^{max}, \qquad (i, j) \in E$$

Transmission constraints

Optimal Power Flow - Variations

Basic optimal power flow is a building block in many problems!

$$p_i = \sum_{j=1}^{N} |V_i||V_j|\left(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right)$$

$$q_i = \sum_{j=1}^{N} |V_i||V_j|\left(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\right)$$

Non-Linear AC Power Flow

$$p_{G,g}^{min} \le p_{G,g} \le p_{G,g}^{max}, \quad g \in \mathcal{G}$$
$$q_{G,g}^{min} \le q_{G,g} \le q_{G,g}^{max}, \quad g \in \mathcal{G}$$

Generation constraints

$$|V|_i^{min} \le |V_i| \le |V|_i^{max}, \quad i \in N$$

Voltage constraints

$$|S|_{ij} \le |S|_{ij}^{max}, \quad (i,j) \in E$$

Transmission constraints

Optimal Power Flow - Variations

$$\min_{p_Q, q_G, |V_i|, \theta} \quad \cancel{\sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i} p_{G,i} + c_{0,i} \right)}$$

s.t.

$$p_i = \sum_{j=1}^{N} |V_i||V_j| \left(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right)$$

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$$|V|_i^{min} \leq |V_i| \leq |V|_i^{max}, \quad i \in N$$

$$|S|_{ij} \leq |S|_{ij}^{max}, \quad (i,j) \in E$$

… Minimize system losses

… Optimize voltage profile

… Minimize cost of remedial actions

Non-Linear AC Power Flow

Generation constraints

Voltage constraints

Transmission constraints

Optimal Power Flow - Variations

$$\min_{p_Q, q_G, |V_i|, \theta} \quad \cancel{\sum_{i\in\mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i} p_{G,i} + c_{0,i}\right)}$$

s.t.

$$p_i = \sum_{j=1}^{N} |V_i||V_j|\left(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right)$$

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… Minimize system losses

… Optimize voltage profile

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Non-Linear AC Power Flow with transmission switching / HVDC / …

Generation constraints

Voltage constraints

Transmission constraints

Optimal Power Flow - Variations

$$\min_{p_Q, q_G, |V_i|, \theta} \quad \cancel{\sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i} p_{G,i} + c_{0,i} \right)}$$

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… Minimize system losses

… Optimize voltage profile

… Minimize cost of remedial actions

Non-Linear AC Power Flow with transmission switching / HVDC / …

Generation constraints

Voltage constraints

Transmission constraints

+ Security constraints

…

Optimal Power Flow - Approximations

$$\min_{p_Q, q_G, |V_i|, \theta} \quad \sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i}\, p_{G,i} + c_{0,i} \right)$$

s.t.

$$p_i = \sum_{j=1}^{N} |V_i| |V_j| \left(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right)$$

$$q_i = \sum_{j=1}^{N} |V_i| |V_j| \left(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right)$$

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$$|V|_i^{min} \leq |V_i| \leq |V|_i^{max}, \quad i \in N$$

$$|S|_{ij} \leq |S|_{ij}^{max}, \qquad (i,j) \in E$$

Non-Convex!

Can be hard to solve!

Simpler versions?

Optimal Power Flow - Approximations

- Linearizations
  - DC OPF
  - Linearized AC OPF

- Relaxations
  - Extend the feasible space of the problem by relaxing constraints (make constraints less restrictive)

Optimal Power Flow - Approximations

- Linearizations
  - **DC OPF**
  - Linearized AC OPF

- Relaxations
  - Extend the feasible space of the problem by relaxing constraints (make constraints less restrictive)

DC Optimal Power Flow

- DC = direct current ?

DC Optimal Power Flow

- DC = direct current ?

No, this is an approximation of the AC power flow!

$$p_{ij} = |V_i||V_j|\left(g_{ij}\cos(\theta_i - \theta_j) + b_{ij}\sin(\theta_i - \theta_j)\right)$$

$$q_{ij} = |V_i||V_j|\left(g_{ij}\sin(\theta_i - \theta_j) - b_{ij}\cos(\theta_i - \theta_j)\right)$$

DC Optimal Power Flow

- DC = direct current ?

No, this is an approximation of the AC power flow! (but considers only active power)

$$p_{ij} = |V_i||V_j|\left(g_{ij}\cos(\theta_i - \theta_j) + b_{ij}\sin(\theta_i - \theta_j)\right)$$

$$\cancel{q_{ij} = |V_i||V_j|\left(g_{ij}\sin(\theta_i - \theta_j) - b_{ij}\cos(\theta_i - \theta_j)\right)}$$

DC Optimal Power Flow

- DC = direct current ?

No, this is an approximation of the AC power flow!

1. Assume lossless system $G_{ij} = 0$

$$p_{ij} = |V_i||V_j|\left(\cancel{g_{ij}\cos(\theta_i - \theta_j)} + b_{ij}\sin(\theta_i - \theta_j)\right)$$

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- DC = direct current ?

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1. Assume lossless system $G_{ij} = 0$

$$p_{ij} = |V_i||V_j|b_{ij}\sin(\theta_i - \theta_j)$$

DC Optimal Power Flow

- DC = direct current ?

No, this is an approximation of the AC power flow!

1. Assume lossless system $G_{ij} = 0$
2. Assume voltages close to nominal values $|V_i| = 1.0$

$$p_{ij} = \underbrace{|V_i|\left|V_j\right|}_{=1.0} b_{ij} \sin\left(\theta_i - \theta_j\right)$$

DC Optimal Power Flow

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No, this is an approximation of the AC power flow!

1. Assume lossless system $G_{ij} = 0$
2. Assume voltages close to nominal values $|V_i| = 1.0$
3. Assume small angle differences $\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$

$$p_{ij} = b_{ij} \underbrace{\sin(\theta_i - \theta_j)}_{= \theta_i - \theta_j}$$

DC Optimal Power Flow

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3. Assume small angle differences $\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$

$$p_{ij} = b_{ij}(\theta_i - \theta_j)$$

Linear equations!

Variables: $\boldsymbol{p, \theta}$

DC Optimal Power Flow

- DC = direct current ?

No, this is an approximation of the AC power flow!

1. Assume lossless system $G_{ij} = 0$
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$$p_{ij} = b_{ij}(\theta_i - \theta_j)$$

Linear equations!

Variables: $\boldsymbol{p}, \boldsymbol{\theta}$

We can use the nodal equations for $\boldsymbol{p}$ to eliminate $\boldsymbol{\theta}$

DC Optimal Power Flow

$$\min_{p_G} \sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i}\, p_{G,i} + c_{0,i} \right)$$ minimize generation cost

s.t. $\sum_{i=1}^{N_B} \left(p_{G(i)} - p_{D(i)} \right) = 0$ balanced operation (instead of nodal power balance after eliminating θ)

$p_{G,g}^{min} \leq p_{G,g} \leq p_{G,g}^{max}, \quad g \in \mathcal{G}$ generator limits

$-p_{ij}^{max} \leq M_{(ij,\cdot)}(p_G - p_D) \leq p_{ij}^{max}, \quad (i,j) \in E$ transmission line limits

DC Optimal Power Flow

$$\min_{p_G} \sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i}\, p_{G,i} + c_{0,i} \right)$$

minimize generation cost

$$\text{s.t.} \quad \sum_{i=1}^{N_B} \left(p_{G(i)} - p_{D(i)} \right) = 0$$

balanced operation (instead of nodal power balance after eliminating θ)

$$p_{G,g}^{min} \leq p_{G,g} \leq p_{G,g}^{max}, \qquad g \in \mathcal{G}$$

generator limits

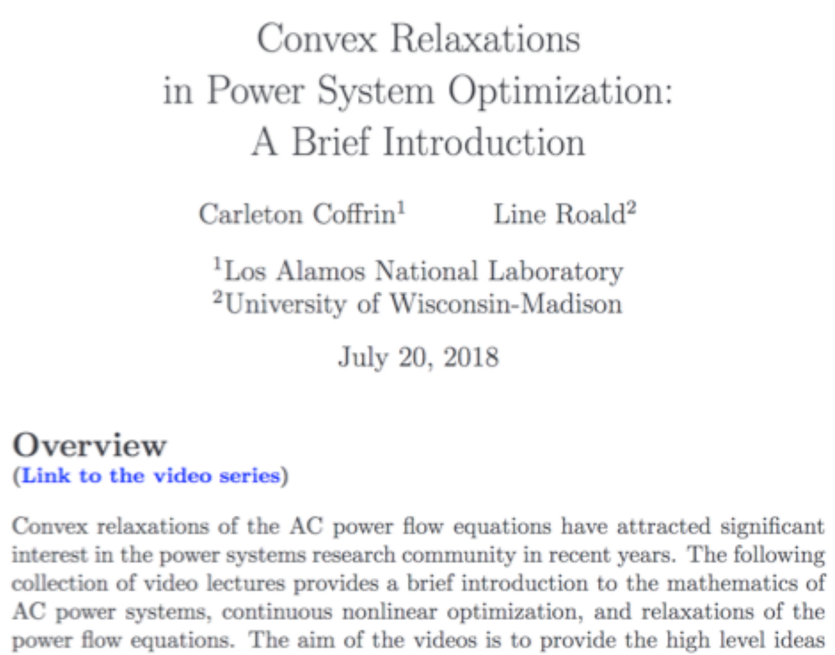
$$-p_{ij}^{max} \leq M_{(ij,\cdot)}(p_G - p_D) \leq p_{ij}^{max}, \quad (i,j) \in E$$

transmission line limits

Much simpler to handle! Used for electricity market clearing, planning, etc…

Optimal Power Flow – Approximations

D.K. Molzahn and I.A. Hiskens, "A Survey of Relaxations and Approximations of the Power Flow Equations," to appear in *Foundations and Trends in Electric Energy Systems*, 2019



Convex Relaxations
in Power System Optimization:
A Brief Introduction

Carleton Coffrin[1] Line Roald[2]

[1]Los Alamos National Laboratory
[2]University of Wisconsin-Madison

July 20, 2018

Overview
(Link to the video series)

Convex relaxations of the AC power flow equations have attracted significant interest in the power systems research community in recent years. The following collection of video lectures provides a brief introduction to the mathematics of AC power systems, continuous nonlinear optimization, and relaxations of the power flow equations. The aim of the videos is to provide the high level ideas

Carleton Coffrin and Line Roald, "Convex relaxations in Power Systems Optimization: A Brief Introduction"
A small series of videos
https://arxiv.org/abs/1807.07227

Summary

- The ***power flow*** problem describes the ***physics*** of how power flows across the network in steady-state for a given set of inputs

- The ***optimal power flow*** problem determines the optimal values of the control variables

- AC optimal power flow – accurate, but harder to solve
- DC optimal power flow – less accurate, but easier to solve
  – requires fewer inputs

Modelling and Managing Uncertainty

(Chance-Constrained Optimal Power Flow)

Collaborators and References

- Miles Lubin, Google
- Yury Dvorkin, NYU

Lubin, Dvorkin and Roald, "Chance Constraints for Improving the Security of AC Optimal Power Flow", IEEE TPWRS, in press.

- Göran Andersson, ETH Zurich

Roald and Andersson, "Chance Constrained AC Optimal Power Flow: Reformulations and Efficient Algorithms", IEEE TPWRS, 2018

- Tillmann Mühlpfort, KIT, Karlsruhe
- Sidhant Misra, Los Alamos National Lab

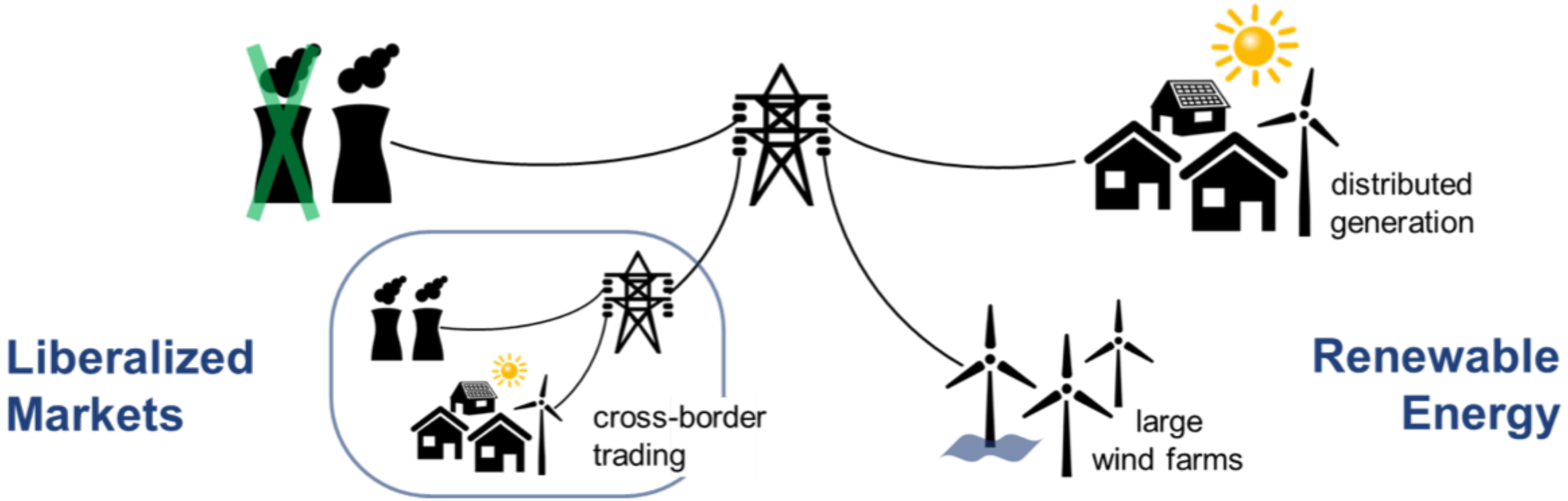
Mühlpfort, Roald, Hagenmeyer, Faulwasser and Misra, "Chance Constrained AC Optimal Power Flow – A Polynomial Chaos Approach", submitted

- Tillmann Weisser, Los Alamos National Lab

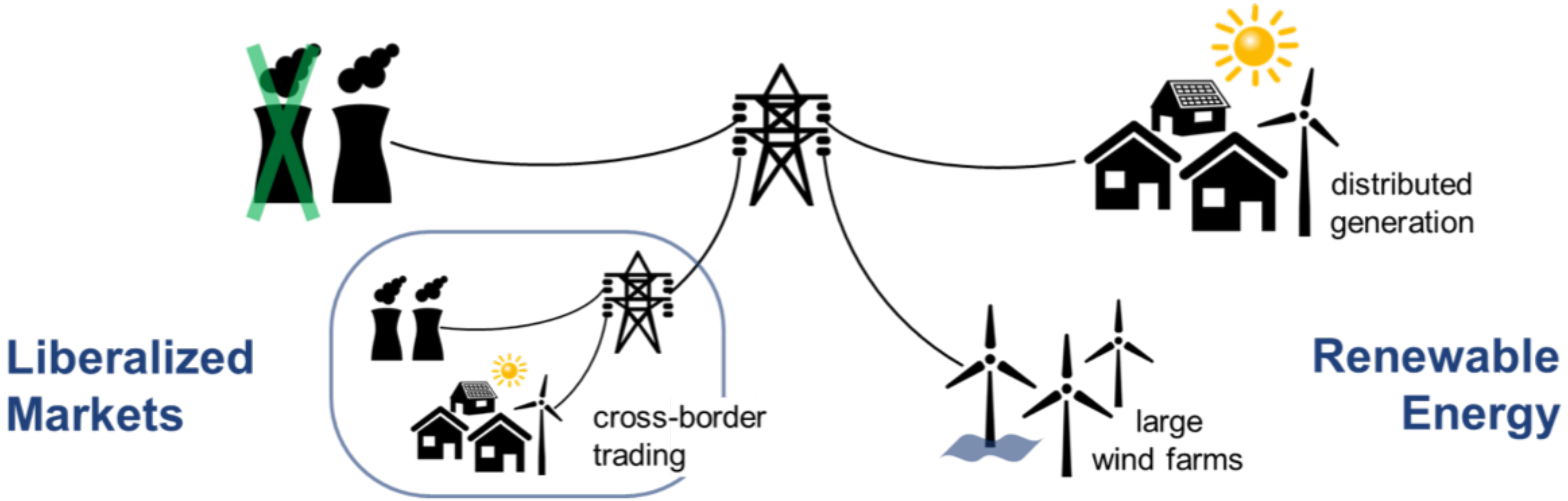
Weisser, Roald and Misra, "Chance Constrained Optimization for Non-Linear Network Flow Problems", submitted

\+ previous/ongoing work and discussions with Dan Molzahn, Dongchan Lee, Kostya Turitsyn, Andreas Waechter, Alejandra Pena Ordieres, Bart Van Parys …

Electric Power Grids



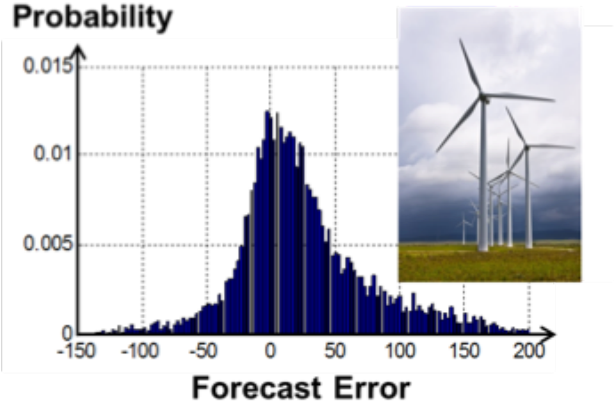
Electric Power Grids

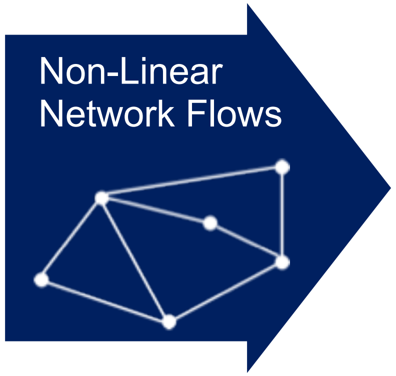


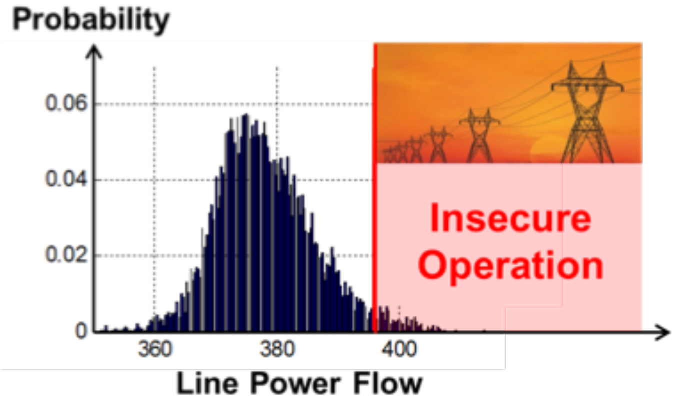
Higher and frequently changing power flows

Increased uncertainty

Impact of Renewable Energy



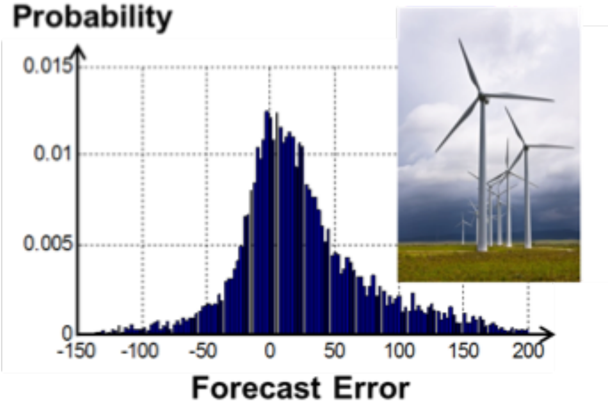


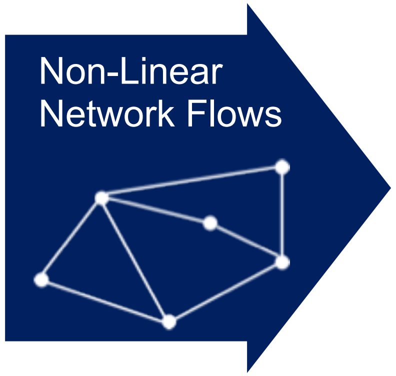


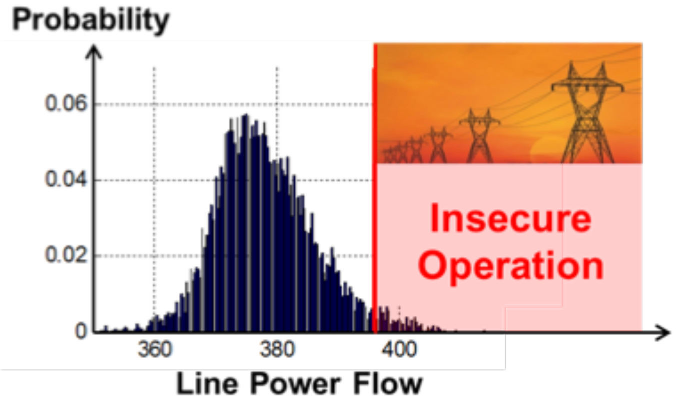
Forecast uncertainty

Increased risk

Impact of Renewable Energy







Forecast uncertainty

Increased risk

Need to take **costly** remedial actions

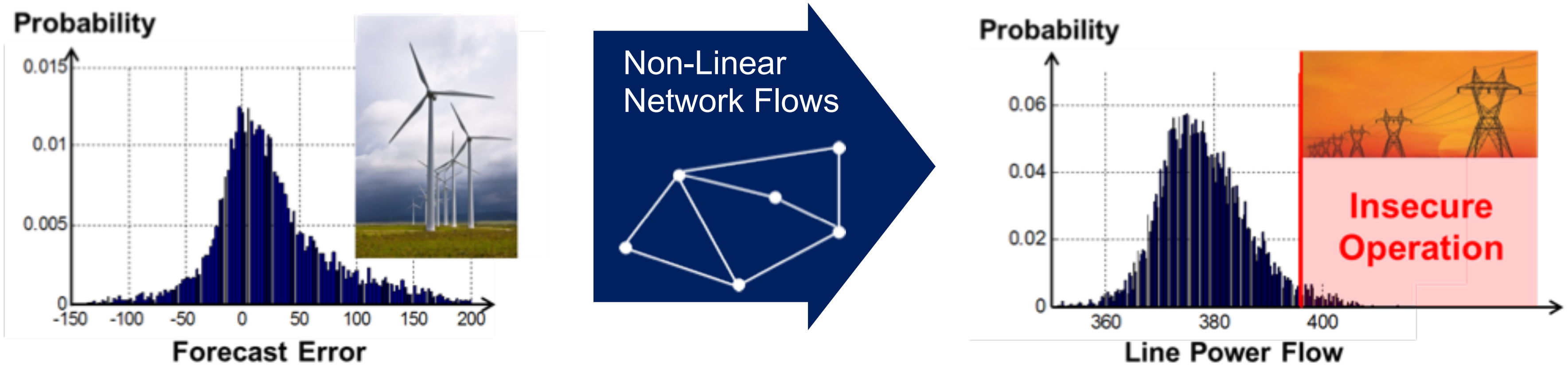
Impact of Renewable Energy Uncertainty

Redispatch of generation in Germany [GWh]

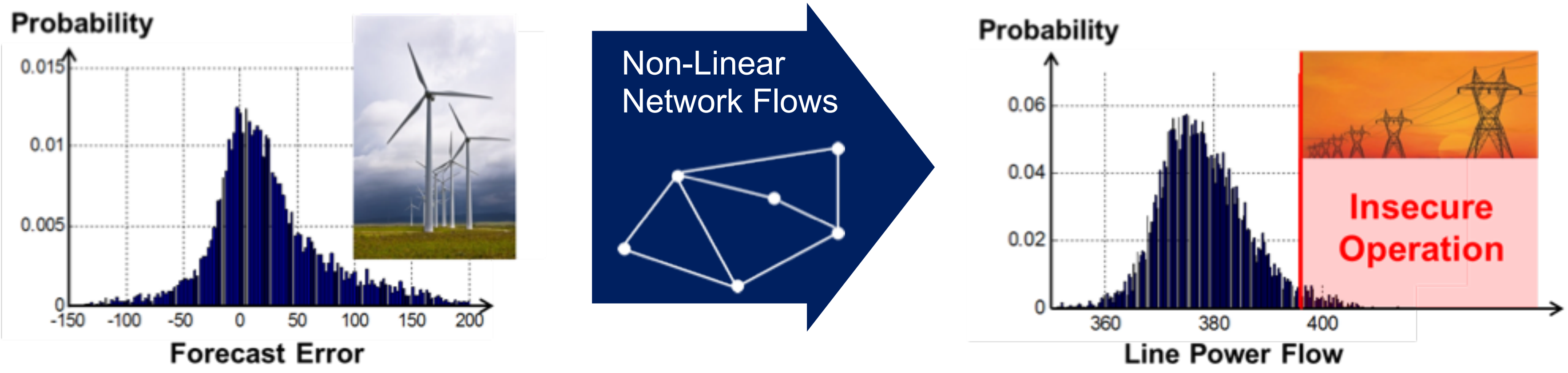
Increasing Cost!

How can we obtain a **good trade-off** between **security** and **cost**?

Why are these problems hard?



Why are these problems hard?

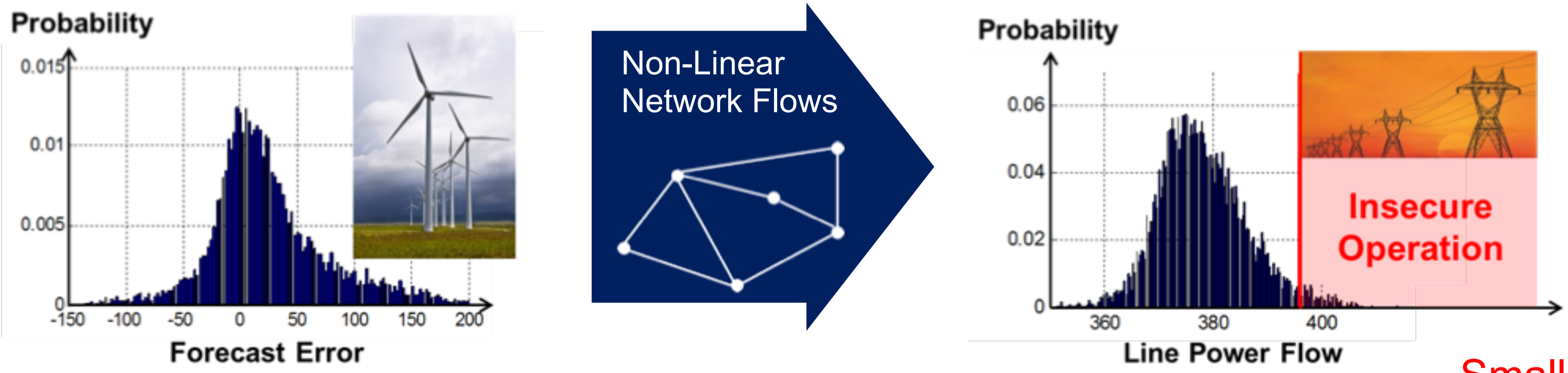


Forecast uncertainty propagates through a non-linear system.

High-Dimensional, Non-Linear

Uncertainty Quantification

Why are these problems hard?



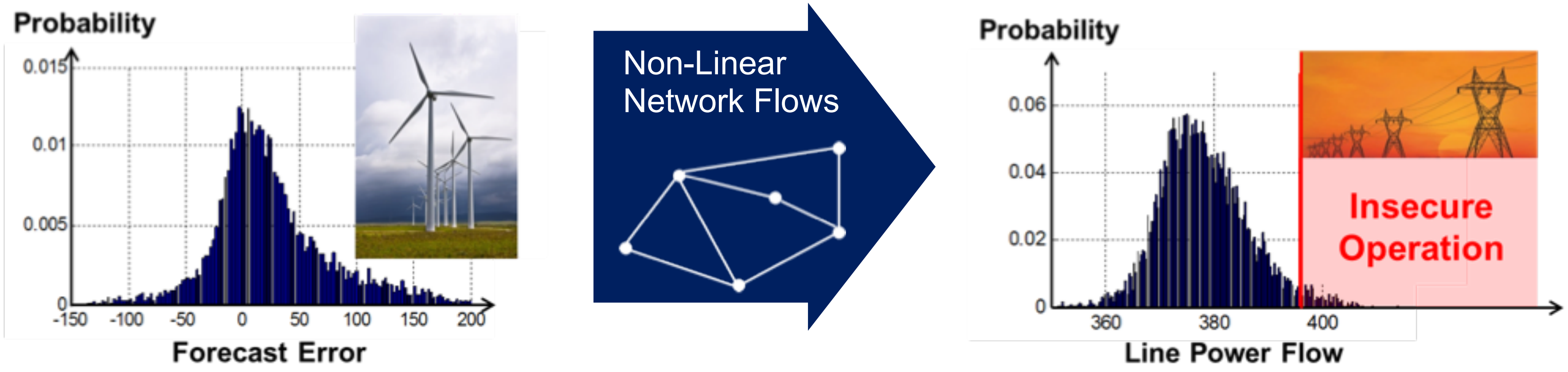
Small probability?
Small magnitude?

How do we define security?

High-Dimensional, Non-Linear Uncertainty Quantification

Risk modelling

Why are these problems hard?



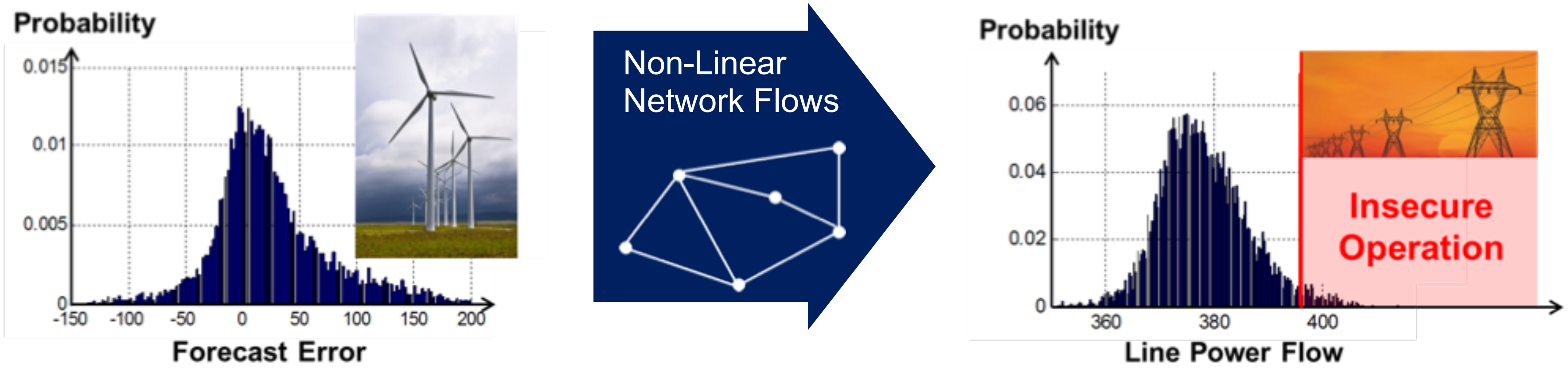
We want the best possible (lowest cost) solution!

High-Dimensional, Non-Linear Uncertainty Quantification

Risk modelling

Large-Scale, Non-Convex Optimization

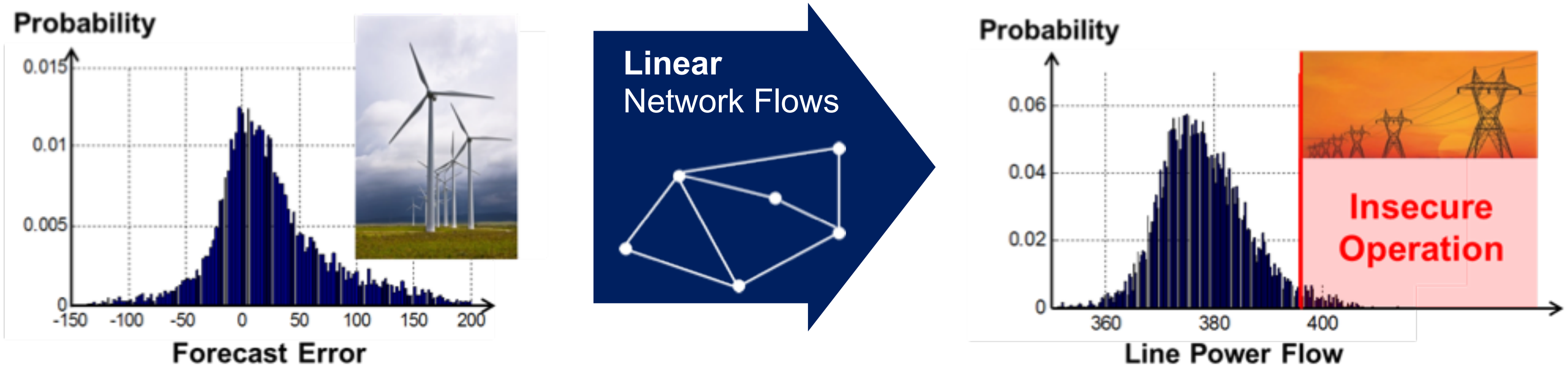
Why are these problems hard?



Integrated approach!

High-Dimensional, Non-Linear Uncertainty Quantification + **Risk modelling** + **Large-Scale, Non-Convex Optimization**

Why are these problems hard?



So let's start from a simpler problem: Use the DC power flow model!

Chance-Constrained DC Optimal Power Flow

Chance-Constrained Optimal Power Flow

$$\mathbb{P}(p(x,\omega) \le p^{max}) \ge 1 - \epsilon$$

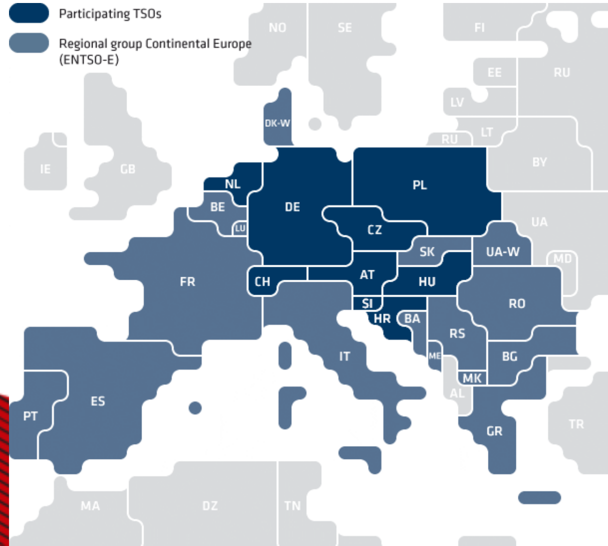
acceptable violation probability

Umbrella Project

www.e-umbrella.eu

- **Why chance constraints?**

  Transmission system operators like them!



Chance-Constrained Optimal Power Flow

$$\mathbb{P}(p(x,\omega) \leq p^{max}) \geq 1 - \epsilon$$

acceptable violation probability

- **Why NOT chance constraints?**

Chance-Constrained Optimal Power Flow

$$\mathbb{P}(p(x,\omega) \leq p^{max}) \geq 1 - \epsilon$$

acceptable violation probability

Umbrella Project

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- **Why NOT chance constraints?**

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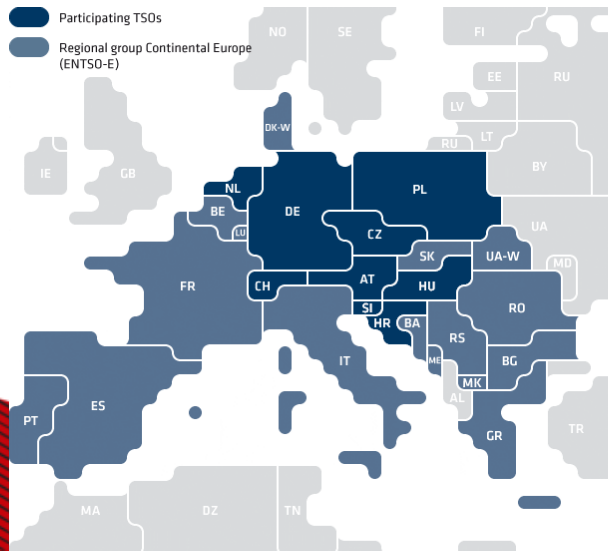


DC Optimal Power Flow with Wind Power

- Goal: Low cost operation, while enforcing technical limits

$\min_{p_G} \quad c_G^T p_G$	minimize generation cost
s.t. $\sum_{i=1}^{N_B} \left(p_{G(i)} + \boldsymbol{p}_{\boldsymbol{W(i)}} - p_{D(i)} \right) = 0$	balanced operation
$p_G \le p_{G(g)}^{max}$, $P_{G(g)} \ge p_{G(g)}^{min}$, $\forall\, g \in \mathcal{G}$	generator limits
$M_{(ij,\cdot)}(p_G + \boldsymbol{p}_{\boldsymbol{W}} - p_D) \le p_{ij}^{max}$, $M_{(ij,\cdot)}(p_G + \boldsymbol{p}_{\boldsymbol{W}} - p_D) \ge -p_{ij}^{max}$, $\forall\, ij \in \mathcal{L}$	transmission line limits

Modelling Uncertain Fluctuations

- Uncertain generation:

$$\bar{p}_{W(i)} = \underbrace{p_{W(i)}}_{\text{Forecasted power}} + \underbrace{\omega_{(i)}}_{\text{Fluctuation}}$$

- Conventional generators:

$$\bar{p}_{G(g)} = \underbrace{p_{G(g)}}_{\text{Scheduled generation}} - \underbrace{\alpha_{(i)}\,\Omega}_{\text{Balancing fluctuations}}$$

where $\Omega = \sum \omega$ total power fluctuation

$\sum \alpha_{(g)} = 1$ balanced system (AGC)

Formulation of chance constraints

- Deterministic line constraint: $M_{(ij,\cdot)}(p_G + p_W - p_D) \leq p_{ij}^{max}$,
- Changes in the wind in-feed influences the line flow: $M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \leq p_{ij}^{max}$,

Depends on fluctuations ω !

Formulation of chance constraints

- Deterministic line constraint: $M_{(ij,\cdot)}(p_G + p_W - p_D) \leq p_{ij}^{max},$
- Changes in the wind in-feed influences the line flow: $M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \leq p_{ij}^{max},$

  Depends on fluctuations ω !

Acceptable violation probability

- **Chance Constraint:** $\mathbb{P}\left(M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \leq p_{ij}^{max}\right) \geq 1 - \boldsymbol{\varepsilon}$

Chance-Constrained DC Optimal Power Flow

$$\min_{p_G,\alpha} \quad c_G^T p_G$$

minimize generation cost

s.t. $\sum_{i=1}^{N_B} \left(p_{G(i)} + p_{W(i)} - p_{D(i)}\right) = 0$

$\sum \alpha_{(g)} = 1$

power balance
with fluctuations

$$\mathbb{P}\left(p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \leq p_{G(g)}^{max}\right) \geq 1 - \varepsilon_G,$$
$$\mathbb{P}\left(p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \geq p_{G(g)}^{min}\right) \geq 1 - \varepsilon_G,$$
$$\forall \ g \in \mathcal{G}$$

generation constraints

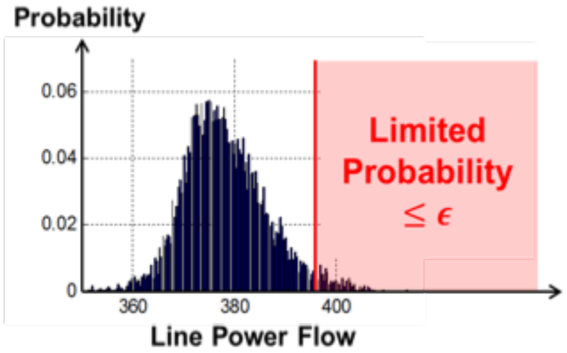
constraints with uncertainty
→ chance constraints!

$$\mathbb{P}\left(M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \leq p_{ij}^{max}\right) \geq 1 - \varepsilon,$$
$$\mathbb{P}\left(M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \leq p_{ij}^{max}\right) \geq 1 - \varepsilon,$$
$$\forall \ l \in \mathcal{L}$$

line constraints

Chance Constraint Reformulation

$$\mathbb{P}\left(p_{ij}(x,\omega) \leq p^{max}\right) \geq 1-\epsilon$$



- Needs reformulation:
  - Scenario approach, based on **randomly drawn samples**
  - **Representative samples** with assigned probability
  - **Adaptive approach**, learning distribution online
  - **Robust optimization** with suitable uncertainty set

➢ **Analytical reformulation** based on probabilistic inequalities

Chance Constraint Reformulation

scheduled power flow

change due to forecast error ω

- **DC power flow:** $\mathbb{P}\left(\boldsymbol{p_{ij}}(\boldsymbol{x},\boldsymbol{0}) + \boldsymbol{h}(\boldsymbol{x}) \cdot \boldsymbol{\omega} \le p_{ij}^{max}\right) \ge 1 - \epsilon$

Chance Constraint Reformulation

scheduled power flow

change due to forecast error ω

- **DC power flow:** $\mathbb{P}\left(\boldsymbol{p_{ij}}(\boldsymbol{x},\boldsymbol{0}) + \boldsymbol{h}(\boldsymbol{x}) \cdot \boldsymbol{\omega} \leq p_{ij}^{max}\right) \geq 1 - \epsilon$

$$\Big\downarrow$$

Rescale to zero mean, unit variance

Apply probabilistic inequality

Rearrange terms

$$\boldsymbol{p_{ij}}(\boldsymbol{x},\boldsymbol{0}) \leq p_{ij}^{max} - \boldsymbol{f^{-1}}(\boldsymbol{1}-\boldsymbol{\epsilon})\sqrt{\boldsymbol{h}(\boldsymbol{x})\,\Sigma_{\mathrm{cov}}\,\boldsymbol{h}(\boldsymbol{x})^{T}}$$

Assumption about distribution

Chance Constraint Reformulation

- **DC power flow:** $\mathbb{P}\Big(\underbrace{\boldsymbol{p_{ij}}(\boldsymbol{x},\boldsymbol{0})}_{\text{scheduled power flow}} + \underbrace{\boldsymbol{h}(\boldsymbol{x}) \cdot \boldsymbol{\omega}}_{\text{change due to forecast error } \omega} \leq p_{ij}^{max}\Big) \geq 1-\epsilon$

$$\xrightarrow{\text{Rescale to zero mean, unit variance; Apply probabilistic inequality; Rearrange terms}}$$

$$\underbrace{\boldsymbol{p_{ij}}(\boldsymbol{x},\boldsymbol{0}) \leq p_{ij}^{max}}_{\textbf{Deterministic constraint}} - \underbrace{\boldsymbol{f^{-1}}(\boldsymbol{1-\epsilon})\sqrt{\boldsymbol{h}(\boldsymbol{x})\, \Sigma_{\text{cov}}\, \boldsymbol{h}(\boldsymbol{x})^{T}}}_{\textbf{“Uncertainty margin”}}$$

Chance Constraint Reformulation

scheduled power flow — change due to forecast error ω

- **DC power flow:** $\mathbb{P}\left(\boldsymbol{p_{ij}}(\boldsymbol{x},\boldsymbol{0}) + \boldsymbol{h}(\boldsymbol{x}) \cdot \boldsymbol{\omega} \leq p_{ij}^{max}\right) \geq 1 - \epsilon$

Rescale to zero mean, unit variance

Apply probabilistic inequality

Rearrange terms

$$\boldsymbol{p_{ij}}(\boldsymbol{x},\boldsymbol{0}) \leq p_{ij}^{max} - \boldsymbol{f^{-1}}(\boldsymbol{1}-\boldsymbol{\epsilon})\sqrt{\boldsymbol{h}(\boldsymbol{x})\,\Sigma_{\text{cov}}\,\boldsymbol{h}(\boldsymbol{x})^T}$$

+ Applicable to known or partially known distributions

+ Scalable to large systems with many uncertain injections

+ Transparent and easy to interpret

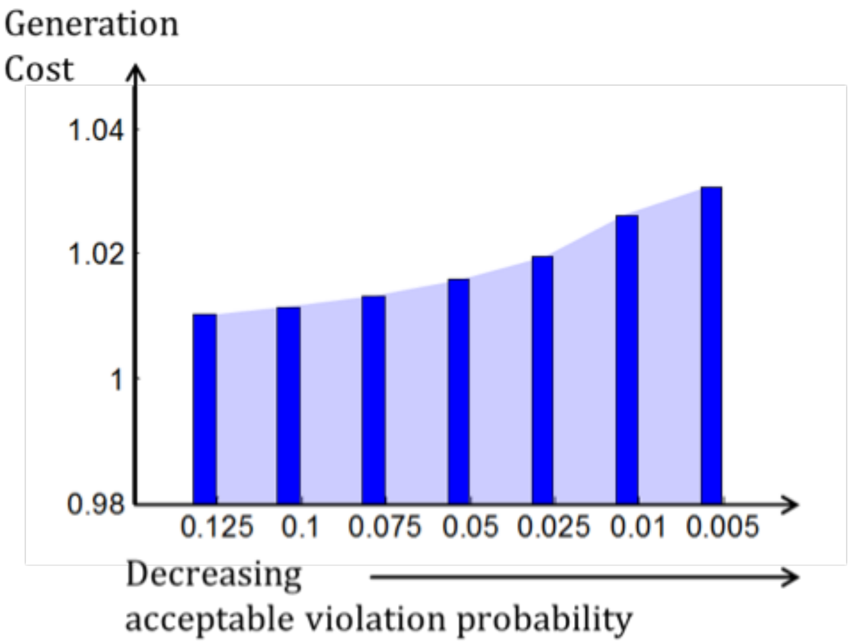
But wait…

Forecast errors

are not normally distributed???

Performance – Cost and Security

Full knowledge:
Normal distribution

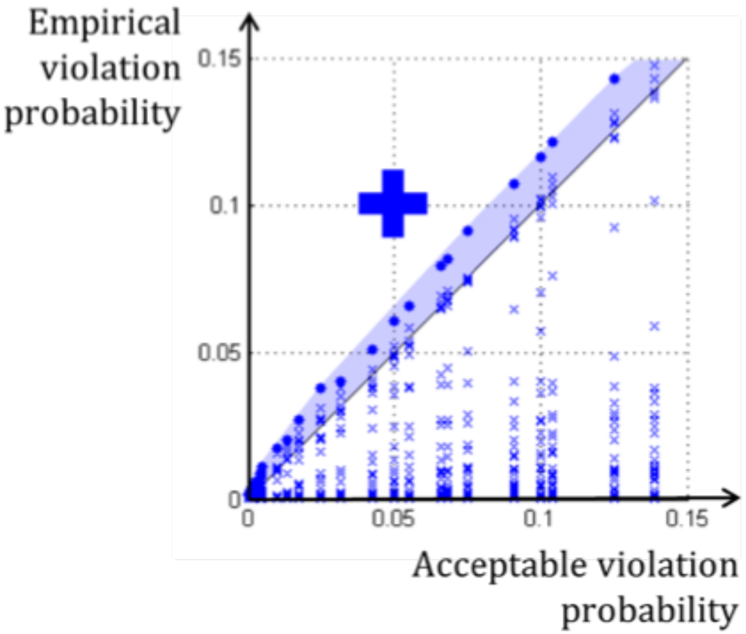


Higher security, higher cost

Performance – Cost and Security

Evaluation with historical data

Full knowledge:
Normal distribution



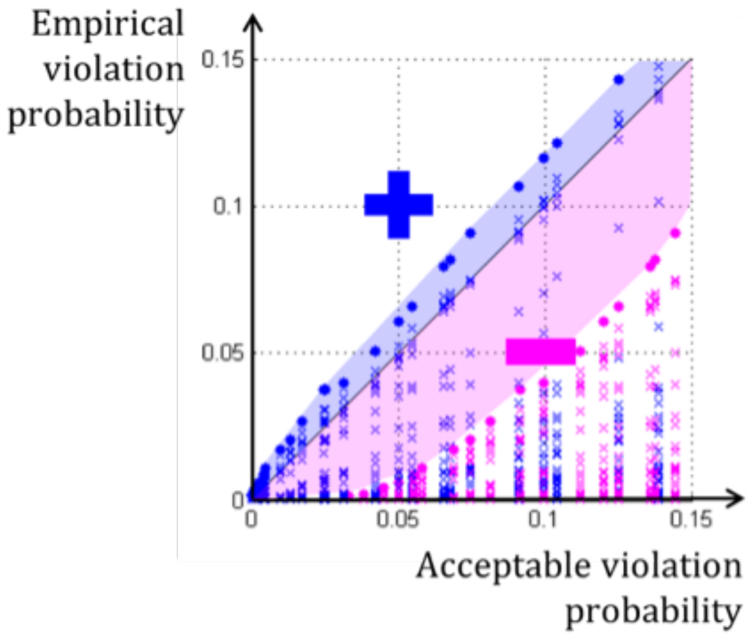
Violation probability:
Higher than prescribed

Performance – Cost and Security

Evaluation with historical data

Full knowledge:
Normal distribution

Partial knowledge:
Unimodal distribution



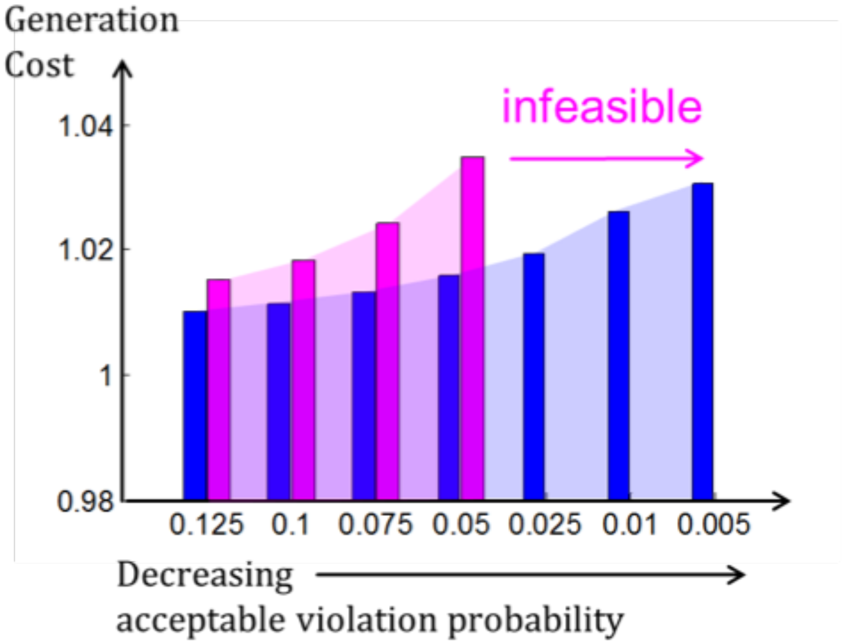
Violation probability:
Higher than prescribed

Lower than prescribed

Performance – Cost and Security

Full knowledge:
Normal distribution

Partial knowledge:
Unimodal distribution



Violation probability:
Higher than prescribed

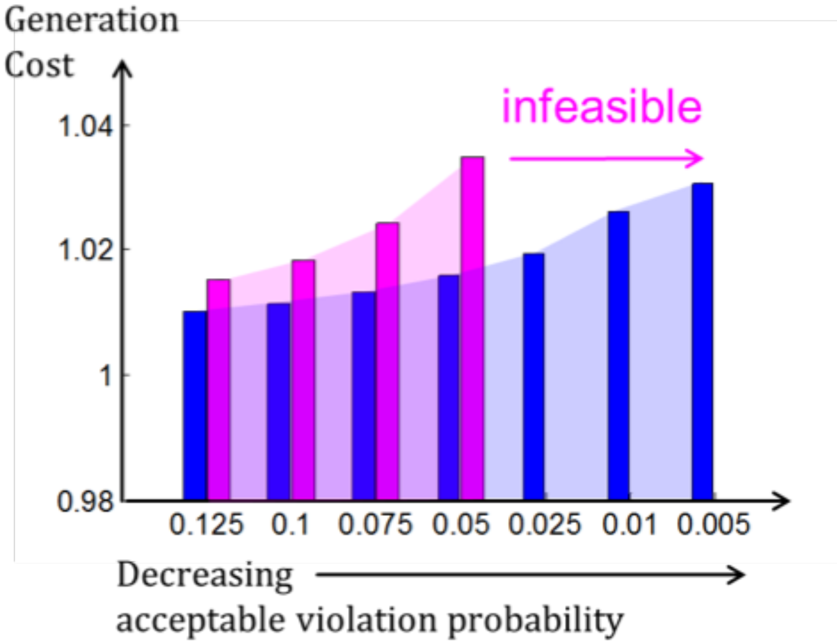
Lower than prescribed

High cost or infeasibility!

Performance – Cost and Security

Full knowledge:
Normal distribution

Partial knowledge:
Unimodal distribution



Normal distribution good assumption…

«Central limit theorem»!
(Concentration)

Joint or separate chance constraints?

Separate

$$\mathbb{P}\left(p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \le p_{G(g)}^{max}\right) \ge 1 - \varepsilon_G,$$
$$\mathbb{P}\left(p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \ge p_{G(g)}^{min}\right) \ge 1 - \varepsilon_G,$$

$$\mathbb{P}\left(M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \le p_{ij}^{max}\right) \ge 1 - \varepsilon,$$
$$\mathbb{P}\left(M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \le p_{ij}^{max}\right) \ge 1 - \varepsilon,$$

Joint

$$\begin{aligned}\mathbb{P}(\quad & p_{G(g)}^{min} \le p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \le p_{G(g)}^{max}, \\ & -p_{ij}^{max} \le M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \le p_{ij}^{max}) \\ & \ge 1 - \varepsilon,\end{aligned}$$

Joint or separate chance constraints?

Separate

$$\mathbb{P}\left(p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \le p_{G(g)}^{max}\right) \ge 1 - \varepsilon_G,$$
$$\mathbb{P}\left(p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \ge p_{G(g)}^{min}\right) \ge 1 - \varepsilon_G,$$

$$\mathbb{P}\left(M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \le p_{ij}^{max}\right) \ge 1 - \varepsilon,$$
$$\mathbb{P}\left(M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \le p_{ij}^{max}\right) \ge 1 - \varepsilon,$$

easier to assign risk to certain components

Joint

$$\begin{aligned}\mathbb{P}\big(\quad & p_{G(g)}^{min} \le p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \le p_{G(g)}^{max}, \\ & -p_{ij}^{max} \le M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \le p_{ij}^{max}\big) \\ & \ge 1 - \varepsilon,\end{aligned}$$

probability of having a peaceful afternoon at work

Joint or separate chance constraints?

Separate

$$\mathbb{P}\left(p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \leq p_{G(g)}^{max}\right) \geq 1 - \varepsilon_G,$$
$$\mathbb{P}\left(p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \geq p_{G(g)}^{min}\right) \geq 1 - \varepsilon_G,$$

$$\mathbb{P}\left(M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \leq p_{ij}^{max}\right) \geq 1 - \varepsilon,$$
$$\mathbb{P}\left(M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \leq p_{ij}^{max}\right) \geq 1 - \varepsilon,$$

easier to assign risk to certain components

easier computionally

Joint

$$\begin{aligned}\mathbb{P}\big(\quad p_{G(g)}^{min} &\leq p_{G(g)} - \alpha_{(g)}\boldsymbol{\Omega} \leq p_{G(g)}^{max}, \\ -p_{ij}^{max} &\leq M_{(ij,\cdot)}(p_G - \alpha\boldsymbol{\Omega} + p_W + \boldsymbol{\omega} - p_D) \leq p_{ij}^{max}\big) \\ &\geq 1 - \varepsilon,\end{aligned}$$

probability of having a peaceful afternoon at work

computationally expensive/more conservative

Joint or separate chance constraints?

Example: IEEE RTS96 system

	$\epsilon = 0.01$	$\epsilon = 0.05$	$\epsilon = 0.1$
Max. ϵ_{emp}	0.013	0.044	0.092
Joint ϵ_J	0.065	0.137	0.219

[Roald and Andersson, TPWRS 2018]

- Observation: Very few constraints are active!

Control joint violation probability using separate constraints

Chance-Constrained **AC** Optimal Power Flow

Modelling Renewable Energy Uncertainty

- Variations in active power injections $\boldsymbol{p_{inj}}$ due to renewable forecast errors $\boldsymbol{\omega}$:

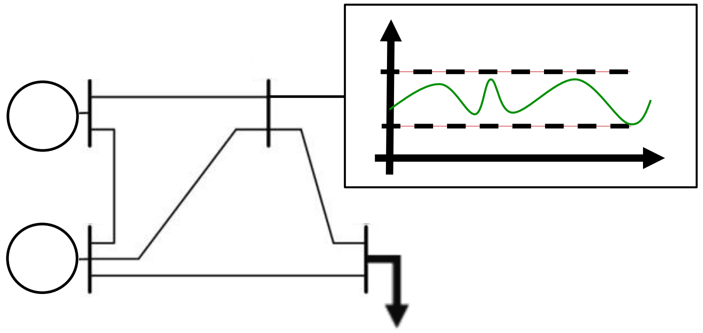
$$\boldsymbol{p_{inj}}(\boldsymbol{\omega}) = \boldsymbol{\hat{p}_{inj}} + \boldsymbol{\omega}$$

- Constant power factor:

$$\boldsymbol{q_{inj}}(\boldsymbol{\omega}) = \boldsymbol{\hat{q}_{inj}} + \boldsymbol{\gamma}\boldsymbol{\omega}$$

- Box uncertainty set:

$$\mathcal{W} = \{\omega \in [\omega_{min}, \omega_{max}]\}$$



Many generalizations possible!

Modelling Generator Response

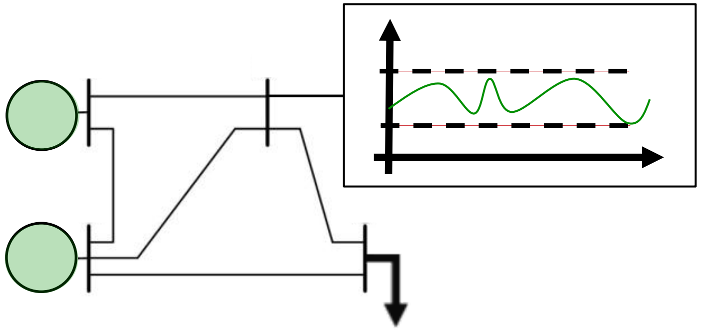
- Active Power Balance - Automatic generation control (AGC)

Scheduled generation

$$p_{G,i}(\omega) = p_{G0,i} - \alpha_i \underbrace{\left(\sum_{i\in N}\omega_i - \delta p(\omega)\right)}_{\text{Power mismatch due to uncertainty + change in losses}}$$

Actual generation

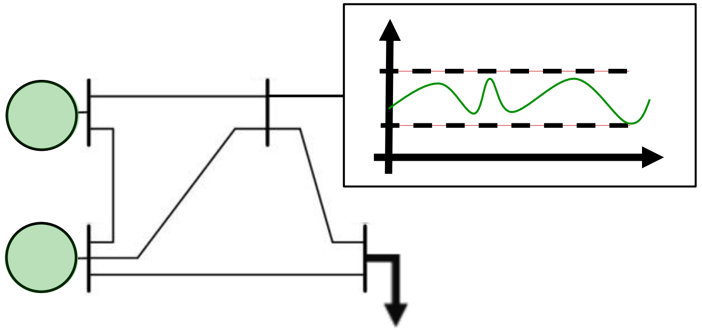
Participation factors



Modelling Generator Response

- Active Power Balance - Automatic generation control (AGC)

$$p_{G,i}(\omega) = p_{G0,i} - \alpha_i \left(\sum_{i \in N} \omega_i - \delta p(\omega) \right)$$



- Reactive Power Balance – Local Voltage Control

$$|V_{G,i}|(\omega) = |V_{G0,i}| \qquad q_{G,i}^{min} \leq q_{G,i}(\omega) \leq q_{G,i}^{max}$$

Scheduled voltage set-point

Varying reactive power

Optimal Power Flow with Uncertainty

$$\min_{p_Q, q_G, |V_i|, \theta} \quad \sum_{i\in\mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i}\, p_{G,i} + c_{0,i} \right)$$

Minimize generation cost

s.t.

$$p_i(\boldsymbol{\omega}) = \sum_{j=1}^{N} |V_i|(\boldsymbol{\omega}) |V_j|(\boldsymbol{\omega}) \left(G_{ij} \cos\left(\theta_i(\boldsymbol{\omega}) - \theta_j(\boldsymbol{\omega})\right) + B_{ij} \sin\left(\theta_i(\boldsymbol{\omega}) - \theta_j(\boldsymbol{\omega})\right)\right)$$

$$q_i(\boldsymbol{\omega}) = \sum_{j=1}^{N} |V_i|(\boldsymbol{\omega}) |V_j|(\boldsymbol{\omega}) \left(G_{ij} \sin\left(\theta_i(\boldsymbol{\omega}) - \theta_j(\boldsymbol{\omega})\right) - B_{ij} \cos\left(\theta_i(\boldsymbol{\omega}) - \theta_j(\boldsymbol{\omega})\right)\right)$$

Non-Linear AC Power Flow

$$p_{G,g}^{min} \le p_{G,g}(\boldsymbol{\omega}) \le p_{G,g}^{max}, \ g \in \mathcal{G}$$
$$q_{G,g}^{min} \le q_{G,g}(\boldsymbol{\omega}) \le q_{G,g}^{max}, \ g \in \mathcal{G}$$

Generation constraints

$$|V|_i^{min} \le |V_i|(\boldsymbol{\omega}) \le |V|_i^{max}, \quad i \in N$$

Voltage constraints

$$|S|_{ij}(\boldsymbol{\omega}) \le |S|_{ij}^{max}, \qquad (i,j) \in E$$

Transmission constraints

Chance-Constrained AC Optimal Power Flow

$$\min_{P_G} \quad \sum_{i\in\mathcal{G}}\left(c_{2,i}p_{G,i}^2 + c_{1,i}\,p_{G,i} + c_{0,i}\right)$$

Minimize generation cost

s.t.

$$\mathbb{P}\left(f\big(\theta(\omega), v(\omega), p(\omega), q(\omega)\big) = 0\right) \geq 1-\epsilon$$

Non-Linear AC Power Flow

joint chance-constraints
limit violation probability ϵ_1

$$\mathbb{P}\left(p_{G,g}(\omega) \leq p_{G,g}^{max}\right) \geq 1-\epsilon_2, \qquad g \in \mathcal{G}$$
$$\mathbb{P}\left(p_{G,g}(\omega) \geq p_{G,g}^{min}\right) \geq 1-\epsilon_2, \qquad g \in \mathcal{G}$$

Generation constraints

$$\mathbb{P}\left(|V_i|(\omega) \leq v_i^{max}\right) \geq 1-\epsilon_2, \qquad i \in N$$
$$\mathbb{P}\left(|V_i|(\omega) \geq v_i^{min}\right) \geq 1-\epsilon_2, \qquad i \in N$$

Voltage constraints

$$\mathbb{P}\left(|S_{ij}|(\omega) \leq \left|S_{ij}\right|^{max}\right) \geq 1-\epsilon_2, \qquad j \in E$$

Transmission constraints

individual chance-constraints
limit violation probability ϵ_2

Chance-constrained AC Optimal Power Flow

- Rich literature on chance-constrained for DC optimal power flow

  [Vrakopoulou et al, 2012], [Margellos et al, 2012], [Roald et al, 2013], [Bienstock, Chertkov and Harnett, SIAM Review, 2014], [Summers et al, 2014], Roald at al, 2015], [Lubin, Dvorkin and Backhaus, ‘16], [Li et al, 2018] …

- AC Power flow with uncertainty is much less mature
  - Linearization of AC power flow equations — [Dall’Anese, Baker & Summers ‘16], [Roald & Andersson ‘17] [Lubin, Dvorkin & Roald ‘18]
    - Accurate only close to linearization point
  - Convex relaxation + linearization of voltage products — [Vrakopoulou at al, ‘13], [Venzke et al ’17]
    - Are not exact (?)
  - Convex relaxation + two/multi-stage robust program — [Phan & Ghosh ‘14] , [Lorca & Sun ‘17] [Nasri, Kazempour, Conejo, & Ghandhari ‘16]
  - Convex inner approximation — [Louca & Bitar ‘17] [Misra et al, 2017]
    - Does not handle equality constraints = requires controllable injections at every bus

Challenges

of Chance-Constrained AC Optimal Power Flow

Challenges of chance-constrained AC OPF

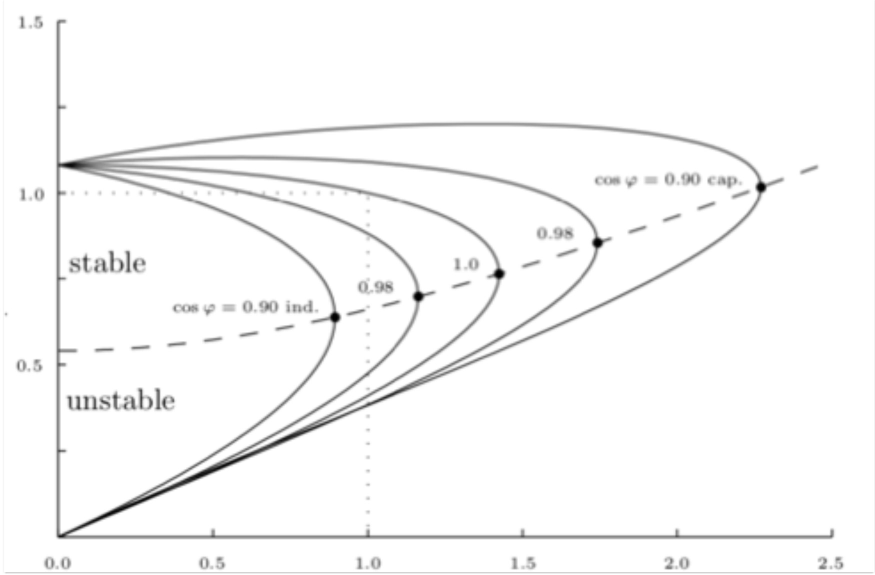
$\min_{P_G} \; \sum_{i\in\mathcal{G}}(c_{2,i}p_{G,i}^2 + c_{1,i}\,p_{G,i} + c_{0,i})$	**Minimize generation cost**
s.t.	
$\mathbb{P}\left(f\big(\theta(\omega), v(\omega), p(\omega), q(\omega)\big) = 0\right) \geq 1-\epsilon$	**Non-Linear AC Power Flow**
$\mathbb{P}\left(p_{G,g}(\omega) \leq p_{G,g}^{max}\right) \geq 1-\epsilon, \quad g \in \mathcal{G}$ $\mathbb{P}\left(p_{G,g}(\omega) \geq p_{G,g}^{min}\right) \geq 1-\epsilon, \quad g \in \mathcal{G}$	**Generation constraints**
$\mathbb{P}(\lvert V_i\rvert(\omega) \leq \lvert V\rvert_i^{max}) \geq 1-\epsilon, \quad i \in N$ $\mathbb{P}(\lvert V_i\rvert(\omega) \geq \lvert V\rvert_i^{min}) \geq 1-\epsilon, \quad i \in N$	**Voltage constraints**
$\mathbb{P}\left(i_{L,j}(\omega) \leq i_{L,j}^{max}\right) \geq 1-\epsilon, \quad j \in E$	**Transmission constraints**

- Guaranteeing **power flow solvability**

  If there exist no power flow solution…

  - all other constraints are **meaningless**.

  - we are in **big** trouble.



Franck and Andersson, Electric Power Systems, ETH Lecture Notes, 2014

Challenges of chance-constrained AC OPF

$$\min_{p_G} \sum_{i\in\mathcal{G}}\left(c_{2,i}p_{G,i}^2 + c_{1,i}\,p_{G,i} + c_{0,i}\right)$$

Minimize generation cost

s.t.

$$f\left(\theta(\omega), v(\omega), p(\omega), q(\omega)\right) = 0, \quad \forall\, \omega \in U$$

Non-Linear AC Power Flow
for all uncertainty realizations

$$\mathbb{P}\left(p_{G,g}(\omega) \le p_{G,g}^{max}\right) \ge 1-\epsilon, \quad g \in \mathcal{G}$$
$$\mathbb{P}\left(p_{G,g}(\omega) \ge p_{G,g}^{min}\right) \ge 1-\epsilon, \quad g \in \mathcal{G}$$

Generation constraints

$$\mathbb{P}(|V_i|(\omega) \le |V|_i^{max}) \ge 1-\epsilon, \quad i \in N$$
$$\mathbb{P}\left(|V_i|(\omega) \ge |V|_i^{min}\right) \ge 1-\epsilon, \quad i \in N$$

Voltage constraints

$$\mathbb{P}\left(i_{L,j}(\omega) \le i_{L,j}^{max}\right) \ge 1-\epsilon, \quad j \in \mathcal{L}$$

Transmission constraints

- Guaranteeing **power flow solvability**

If there exist no power flow solution…

- all other constraints are **meaningless**.

- we are in **big** trouble.

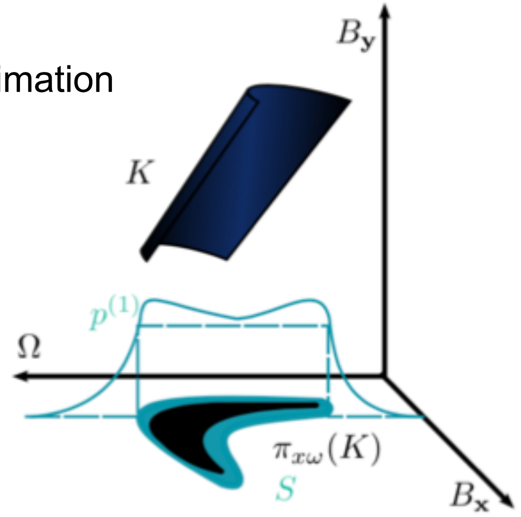
Enforce power flow solvability either robustly or with a (very) high probability.

Challenges of chance-constrained AC OPF

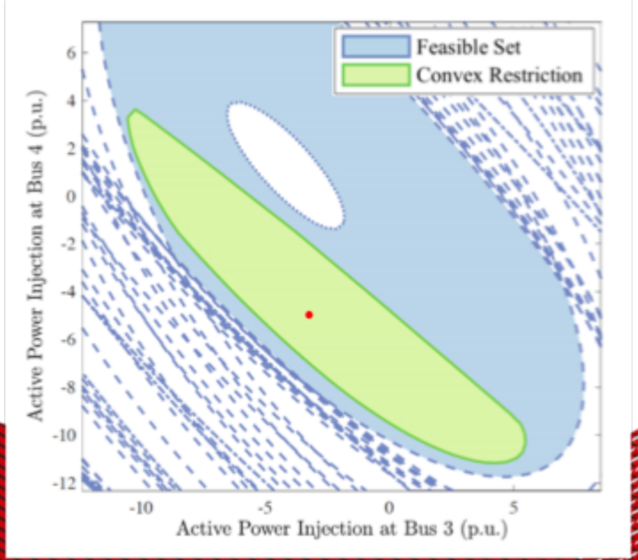
- Guaranteeing power flow solvability for all uncertainty realizations is HARD!
  - We cannot use a convex relaxation
  - We don't want to assume controllability of power injections at every node

- To do this properly → need a projection step!
  - In DC OPF: Get rid of voltage angles θ
  - In AC OPF: Get rid of dependent variables! (θ, v) at PQ buses, (θ, q) at PV buses

Polynomial outer approximation [Weisser et al, 2018]



Convex restriction [Lee et al, 2018]



Challenges of chance-constrained AC OPF

- Guaranteeing power flow solvability for all uncertainty realizations is HARD!
  - We cannot use a convex relaxation
  - We don't want to assume controllability of power injections at every node
- To do this properly → need a projection step!
  - In DC OPF: Get rid of voltage angles θ
  - In AC OPF: Get rid of dependent variables! (θ, v) at PQ buses, (θ, q) at PV buses

Work in progress…

For discussions here:
1. Assume that there exist a power flow solution
2. Aim to satisfy with high numerical accuracy

Challenges of chance-constrained AC OPF

$$\min_{p_G} \quad \sum_{i\in\mathcal{G}}\left(c_{2,i}p_{G,i}^2 + c_{1,i}\,p_{G,i} + c_{0,i}\right)$$

Minimize generation cost

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0, \quad \forall\, \omega \in U$$

Non-Linear AC Power Flow
for all uncertainty realizations

$$\mathbb{P}\left(p_{G,g}(\omega) \le p_{G,g}^{max}\right) \ge 1-\epsilon, \qquad g \in \mathcal{G}$$
$$\mathbb{P}\left(p_{G,g}(\omega) \ge p_{G,g}^{min}\right) \ge 1-\epsilon, \qquad g \in \mathcal{G}$$

Generation constraints

$$\mathbb{P}(|V_i|(\omega) \le |V|_i^{max}) \ge 1-\epsilon, \qquad i \in N$$
$$\mathbb{P}\left(|V_i|(\omega) \ge |V|_i^{min}\right) \ge 1-\epsilon, \qquad i \in N$$

Voltage constraints

$$\mathbb{P}\left(i_{L,j}(\omega) \le i_{L,j}^{max}\right) \ge 1-\epsilon, \qquad j \in E$$

Transmission constraints

- ~~Guaranteeing **power flow solvability**~~
- Find solutions $\theta(\omega), v(\omega), p(\omega), q(\omega)$ that **satisfy the power flow equations**

Tractable Reformulations

of Chance-Constrained AC Optimal Power Flow

Moment-Based Tractable Reformulations

of Chance-Constrained AC Optimal Power Flow

Moment-based chance constraint reformulation

Voltage magnitude
x decision variables
ω random variables

$$\mathbb{P}(v(x,\omega) \leq v^{max}) \geq 1-\epsilon$$

Moment-based chance constraint reformulation

Voltage magnitude
x decision variables
ω random variables

! No assumptions on ω except known (and finite) mean and variance !

$$\mathbb{P}(v(x,\omega) \leq v^{max}) \geq 1 - \epsilon$$

Moment-based chance constraint reformulation

Voltage magnitude
x decision variables
ω random variables

! No assumptions on ω except known (and finite) mean and variance !

$$\mathbb{P}(v(x,\omega) \leq v^{max}) \geq 1 - \epsilon$$

Reformulation

$$\mu_v(x,\omega) + f^{-1}(1-\epsilon)\sigma_v(x,\omega) \leq p^{max}$$

Moment-based chance constraint reformulation

$$\mathbb{P}(v(x,\omega) \leq v^{max}) \geq 1 - \epsilon$$

$$\Big\downarrow \text{ Reformulation}$$

$$\mu_v(x,\omega) + f^{-1}(1-\epsilon)\sigma_v(x,\omega) \leq p^{max}$$

$\mu_v(x,\omega)$ Mean voltage magnitude

Moment-based chance constraint reformulation

$$\mathbb{P}(v(x,\omega) \leq v^{max}) \geq 1 - \epsilon$$

$\Big\downarrow$ Reformulation

$$\mu_v(x,\omega) + f^{-1}(1-\epsilon)\sigma_v(x,\omega) \leq p^{max}$$

$\mu_v(x,\omega)$ Mean voltage magnitude

$\sigma_v(x,\omega)$ Voltage magnitude variance

Moment-based chance constraint reformulation

$$\mathbb{P}(v(x,\omega) \leq v^{max}) \geq 1 - \epsilon$$

$\Big\downarrow$ Reformulation

$$\mu_v(x,\omega) + f^{-1}(1-\epsilon)\sigma_v(x,\omega) \leq p^{max}$$

$\mu_v(x,\omega)$ Mean voltage magnitude

A constant depending on ϵ and assumption about **distribution of v**

- inverse normal CDF
- Chebyshev bound
- …

$\sigma_v(x,\omega)$ Voltage magnitude variance

Moment-based chance constraint reformulation

$$\mathbb{P}(v(x,\omega) \leq v^{max}) \geq 1-\epsilon$$

$\downarrow$ Reformulation

$$\mu_v(x,\omega) + f^{-1}(1-\epsilon)\sigma_v(x,\omega) \leq p^{max}$$

$\mu_v(x,\omega)$ Mean voltage magnitude

How do we compute the voltage magnitude and variance?

$\sigma_v(x,\omega)$ Voltage magnitude variance

1. Linearize the AC power flow

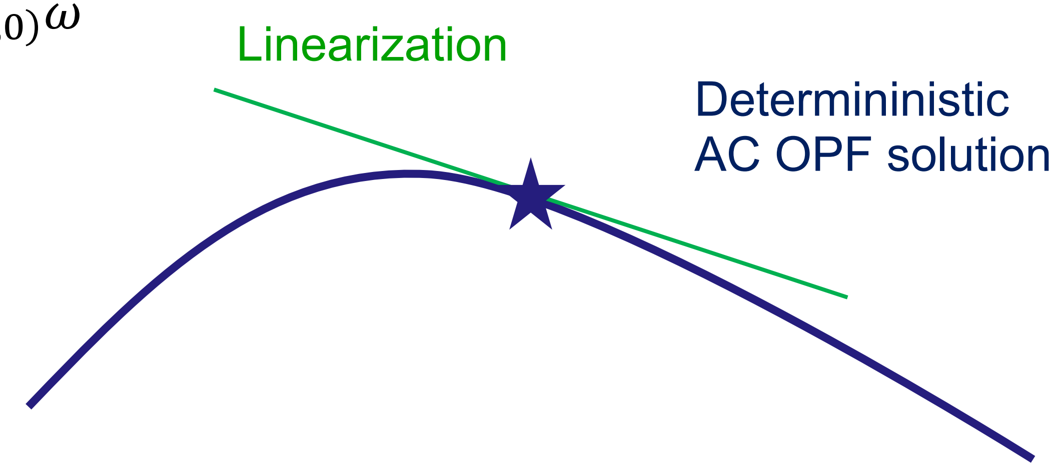
[Dall'Anese, Baker & Summers '16], [Lubin, Dvorkin & Roald '19], …

Taylor expansion for x and ω

$$v(x,\omega) \approx v(x_0,0) + \frac{dv}{dx}\Big|_{(x_o,0)}(x - x_0) + \frac{dv}{d\omega}\Big|_{(x_o,0)}\omega$$

$$\mu_v(x,\omega) \approx v(x_0,0) + \frac{dv}{dx}\Big|_{(x_o,0)}(x - x_0)$$

$$\sigma_v(x,\omega) \approx \sqrt{\left(\frac{dv}{d\omega}\Big|_{(x_o,0)}\right)\Sigma_\omega\left(\frac{dv}{d\omega}\Big|_{(x_o,0)}^T\right)}$$



2. Partial linearization of the AC power flow

[Schmidli, Roald, Chatzivasileiadis and Andersson '16]
[Roald and Andersson '18]

Taylor expansion for ~~x and~~ ω

$$v(x,\omega) \approx v(x,0) + \frac{dv}{d\omega}|_{(x_o,0)}\omega$$

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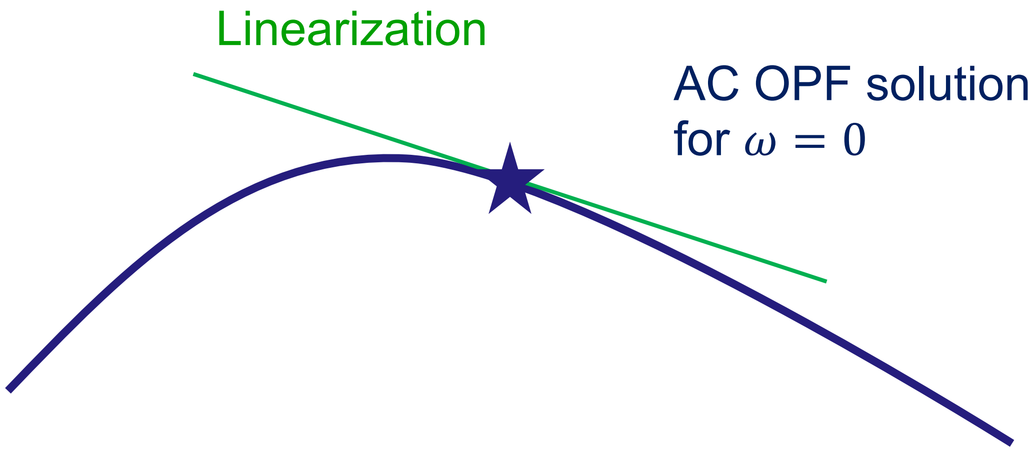
$$\sigma_v(x,\omega) \approx \sqrt{\left(\frac{dv}{d\omega}|_{(x_o,0)}\right)\Sigma_\omega\left(\frac{dv}{d\omega}|_{(x_o,0)}^T\right)}$$



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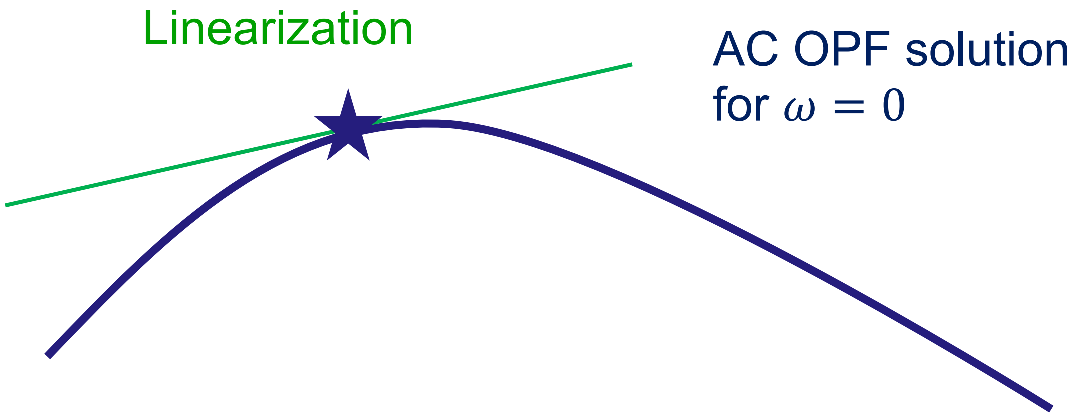

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$$v(x,\omega) \approx v(x,0) + \frac{dv}{d\omega}\Big|_{(x_o,0)}\omega$$

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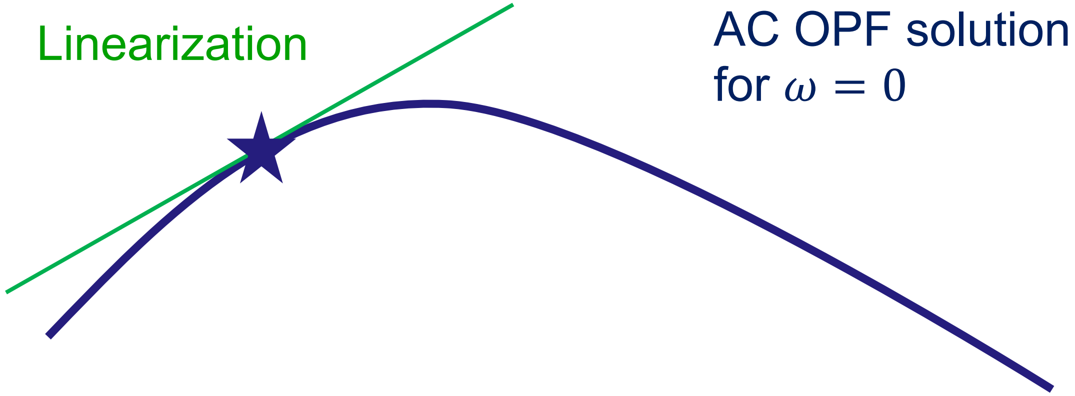

3. Polynomial Chaos Expansion

[Mühlpfort, Roald, Hagenmeyer, Faulwasser and Misra, submitted ‘18]

1. Build a polynomial basis based on orthogonal polynomials from random variables

2. Express power flow and decision variables in terms of basis polynomials with unknown coefficients

3. Truncate at finite dimension

4. Solve optimal power flow with polynomials as constraints

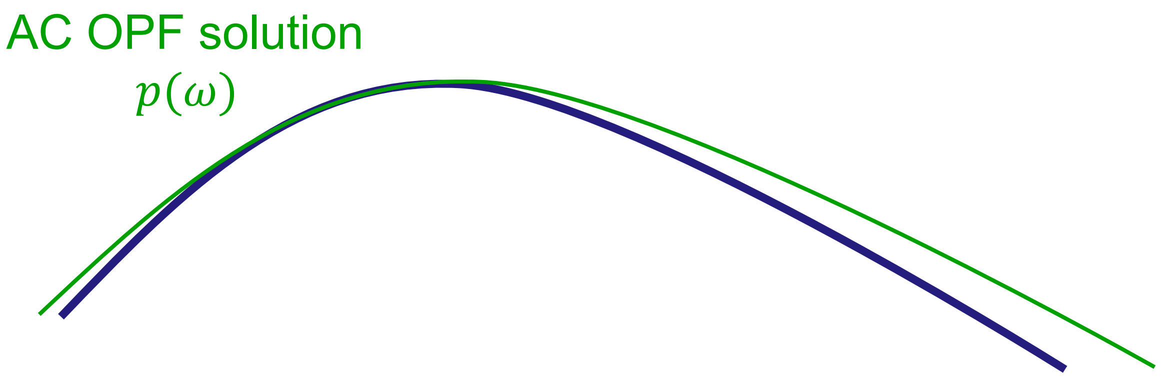

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Table I. REFORMULATIONS OF POWER FLOW EQUATIONS AND MOMENTS IN TERMS OF PCE COEFFICIENTS.

Rectangular power flow in terms of PCE coefficients with $i \in \mathcal{N}$, $k \in \mathcal{K}$
$\langle\psi_k,\psi_k\rangle(p^{\mathrm{g}}_{i,k} - p^{\mathrm{d}}_{i,k}) = \sum_{j\in\mathcal{N}} \sum_{k_1,k_2\in\mathcal{K}} \langle\psi_{k_1}\psi_{k_2},\psi_k\rangle(G_{ij}(v^{\mathrm{re}}_{i,k_1} v^{\mathrm{re}}_{j,k_2} + v^{\mathrm{im}}_{i,k_1} v^{\mathrm{im}}_{j,k_2}) + B_{ij}(v^{\mathrm{im}}_{i,k_1} v^{\mathrm{re}}_{j,k_2} - v^{\mathrm{re}}_{i,k_1} v^{\mathrm{im}}_{j,k_2}))$
$\langle\psi_k,\psi_k\rangle(q^{\mathrm{g}}_{i,k} - q^{\mathrm{d}}_{i,k}) = \sum_{j\in\mathcal{N}} \sum_{k_1,k_2\in\mathcal{K}} \langle\psi_{k_1}\psi_{k_2},\psi_k\rangle(G_{ij}(v^{\mathrm{im}}_{i,k_1} v^{\mathrm{re}}_{j,k_2} - v^{\mathrm{re}}_{i,k_1} v^{\mathrm{im}}_{j,k_2}) - B_{ij}(v^{\mathrm{re}}_{i,k_1} v^{\mathrm{re}}_{j,k_2} + v^{\mathrm{im}}_{i,k_1} v^{\mathrm{im}}_{j,k_2}))$
Moments of squared line current magnitudes with $ij \in \mathcal{L}$, $v^{\mathrm{re}}_{ij,k} = v^{\mathrm{re}}_{i,k} - v^{\mathrm{re}}_{j,k}$, $v^{\mathrm{im}}_{ij,k} = v^{\mathrm{im}}_{i,k} - v^{\mathrm{im}}_{j,k}$
$\mathbb{E}[\mathrm{i}^2_{i\text{-}j}] = \lvert y^{\mathrm{br}}_{ij}\rvert^2 \sum_{k\in\mathcal{K}} \langle\psi_k,\psi_k\rangle((v^{\mathrm{re}}_{ij,k})^2 + (v^{\mathrm{im}}_{ij,k})^2)$
$\sigma[\mathrm{i}^2_{i\text{-}j}]^2 = \lvert y^{\mathrm{br}}_{ij}\rvert^4 \sum_{k_1,k_2,k_3,k_4\in\mathcal{K}} \langle\psi_{k_1}\psi_{k_2}\psi_{k_3},\psi_{k_4}\rangle(v^{\mathrm{re}}_{i,k_1} v^{\mathrm{re}}_{ij,k_2} v^{\mathrm{re}}_{i,k_3} v^{\mathrm{re}}_{ij,k_4} + 2v^{\mathrm{re}}_{ij,k_1} v^{\mathrm{re}}_{ij,k_2} v^{\mathrm{im}}_{ij,k_3} v^{\mathrm{im}}_{ij,k_4} + v^{\mathrm{im}}_{ij,k_1} v^{\mathrm{im}}_{ij,k_2} v^{\mathrm{im}}_{ij,k_3} v^{\mathrm{im}}_{ij,k_4}) - \mathbb{E}[\mathrm{i}^2_{i\text{-}j}]^2$
Moments of squared voltage magnitudes with $i \in \mathcal{N}$
$\mathbb{E}[\mathrm{v}^2_i] = \sum_{k\in\mathcal{K}} \langle\psi_k,\psi_k\rangle((v^{\mathrm{re}}_{i,k})^2 + (v^{\mathrm{im}}_{i,k})^2)$
$\sigma[\mathrm{v}^2_i]^2 = \sum_{k_1,k_2,k_3,k_4\in\mathcal{K}} \langle\psi_{k_1}\psi_{k_2}\psi_{k_3},\psi_{k_4}\rangle(v^{\mathrm{re}}_{i,k_1} v^{\mathrm{re}}_{i,k_2} v^{\mathrm{re}}_{i,k_3} v^{\mathrm{re}}_{i,k_4} + 2v^{\mathrm{re}}_{i,k_1} v^{\mathrm{re}}_{i,k_2} v^{\mathrm{im}}_{i,k_3} v^{\mathrm{im}}_{i,k_4} + v^{\mathrm{im}}_{i,k_1} v^{\mathrm{im}}_{i,k_2} v^{\mathrm{im}}_{i,k_3} v^{\mathrm{im}}_{i,k_4}) - \mathbb{E}[\mathrm{v}^2_i]^2$

Similar structure as power flow equations…

JUST MANY MORE!

When can we truncate?

3. Polynomial Chaos Expansion

[Mühlpfort, Roald, Hagenmeyer, Faulwasser and Misra, submitted ‘18]

1. Build a polynomial basis based on orthogonal polynomials from random variables

2. Express power flow and decision variables in terms of basis polynomials with unknown coefficients

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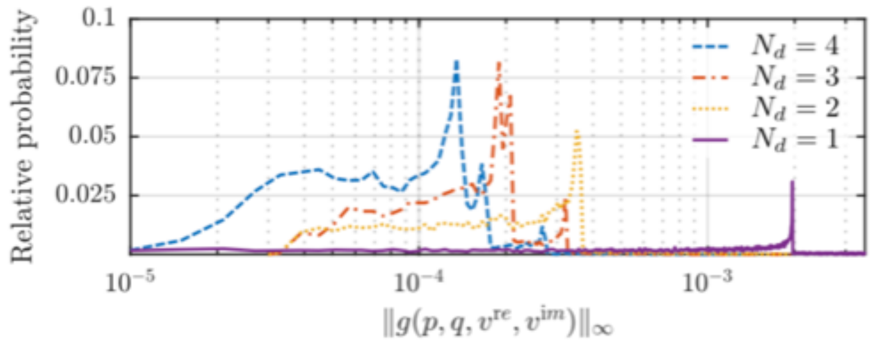

PCE bases of degree 2

(quadratic polynomials) already provide good results.

Different approaches

- Linearize the AC power flow
  - ++ Computational speed
  - - - Inaccuracy

- Partial linearization of the AC power flow
  - \+ Easy to compute moments,
  - \+ Computational speed
  - \- (less) inaccuracy

Provide good approximations.

Linearization error ≈ Distribution error

In-sample testing (normal distribution)

	$\epsilon = 0.01$	$\epsilon = 0.05$	$\epsilon = 0.1$
Max. ϵ_{emp}	0.013	0.044	0.092
Joint ϵ_J	0.065	0.137	0.219

- Polynomial Chaos Expansion
  - \+ Efficient computation of moments
  - \+ Accuracy
  - \- Computational tractability

Out-of-sample testing (non-normal)

	$\epsilon = 0.01$	$\epsilon = 0.05$	$\epsilon = 0.1$
Max. ϵ_{emp}	0.014	0.054	0.093
Joint ϵ_J	0.074	0.145	0.233

Different approaches

- Linearize the AC power flow
  - ++ Computational speed
  - - - Inaccuracy
- Partial linearization of the AC power flow
  - + Easy to compute moments,
  - + Computational speed
  - - (less) inaccuracy

Provide good approximations.

- Polynomial Chaos Expansion
  - + Efficient computation of moments
  - + Accuracy
  - - Computational tractability

How much better is

Polynomial Chaos Expansion?

Errors in Polynomial Chaos and Linearized AC

Polynomial Chaos expansion order(s) of magnitude more accurate than linearized AC.

Table IV. Error in the computed moments for the PCE method (PCE) and the linearization method (LIN) for 30-bus system.

AC vs.	s	p^g $\|\Delta\mu\|_\infty$	p^g $\|\Delta\sigma\|_\infty$	q^g $\|\Delta\mu\|_\infty$	q^g $\|\Delta\sigma\|_\infty$	v $\|\Delta\mu\|_\infty$	v $\|\Delta\sigma\|_\infty$	i_{i-j} $\|\Delta\mu\|_\infty$	i_{i-j} $\|\Delta\sigma\|_\infty$
PCE	0.05	1.8 E-5	0.6 E-5	1.7 E-5	0.7 E-5	0.3 E-5	0.4 E-5	5.1 E-5	3.9 E-5
	0.10	10.1 E-5	0.4 E-5	2.0 E-5	2.0 E-5	2.2 E-5	1.0 E-5	33.4 E-5	5.7 E-5
	0.15	2.9 E-5	19.8 E-5	10.7 E-5	6.4 E-5	3.8 E-5	1.1 E-5	19.3 E-5	12.1 E-5
lin. AC	0.05	431.2 E-5	4.0 E-5	0.131	81.1 E-5	108.4 E-5	2.3 E-5	4901.5 E-5	104.0 E-5
	0.10	411.0 E-5	12.9 E-5	0.136	294.3 E-5	105.8 E-5	7.5 E-5	4812.1 E-5	196.3 E-5
	0.15	387.6 E-5	7.0 E-5	0.146	700.3 E-5	101.8 E-5	17.1 E-5	4715.7 E-5	348.1 E-5
	Reference	$\|\mathbb{E}[p^g]_{AC}\|_\infty$ 0.5800	$\|\sigma[p^g]_{AC}\|_\infty$ 0.1132	$\|\mathbb{E}[q^g]_{AC}\|_\infty$ 0.3829	$\|\sigma[q^g]_{AC}\|_\infty$ 0.0038	$\|\mathbb{E}[v]_{AC}\|_\infty$ 1.0792	$\|\sigma[v]_{AC}\|_\infty$ 0.0012	$\|\mathbb{E}[i_{i-j}]_{AC}\|_\infty$ 0.3951	$\|\sigma[i_{i-j}]_{AC}\|_\infty$ 0.0100

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Linearized AC introduces errors in estimating the mean!

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Polynomial Chaos expansion order(s) of magnitude more accurate than linearized AC.

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AC vs.	s	p^g $\|\Delta\mu\|_\infty$	p^g $\|\Delta\sigma\|_\infty$	q^g $\|\Delta\mu\|_\infty$	q^g $\|\Delta\sigma\|_\infty$	v $\|\Delta\mu\|_\infty$	v $\|\Delta\sigma\|_\infty$	i_{i-j} $\|\Delta\mu\|_\infty$	i_{i-j} $\|\Delta\sigma\|_\infty$
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lin. AC	0.05	431.2 E-5	4.0 E-5	0.131	81.1 E-5	108.4 E-5	2.3 E-5	4901.5 E-5	104.0 E-5
	0.10	411.0 E-5	12.9 E-5	0.136	294.3 E-5	105.8 E-5	7.5 E-5	4812.1 E-5	196.3 E-5
	0.15	387.6 E-5	7.0 E-5	0.146	700.3 E-5	101.8 E-5	17.1 E-5	4715.7 E-5	348.1 E-5
	Reference	$\|\mathbb{E}[p^g]_{AC}\|_\infty$	$\|\sigma[p^g]_{AC}\|_\infty$	$\|\mathbb{E}[q^g]_{AC}\|_\infty$	$\|\sigma[q^g]_{AC}\|_\infty$	$\|\mathbb{E}[v]_{AC}\|_\infty$	$\|\sigma[v]_{AC}\|_\infty$	$\|\mathbb{E}[i_{i-j}]_{AC}\|_\infty$	$\|\sigma[i_{i-j}]_{AC}\|_\infty$
		0.5800	0.1132	0.3829	0.0038	1.0792	0.0012	0.3951	0.0100

particularly for reactive power!

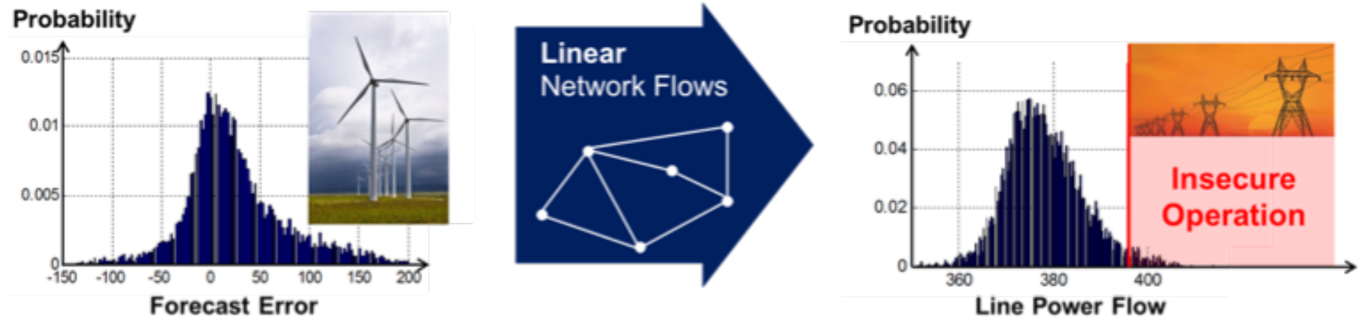
Errors in Polynomial Chaos and Linearized AC

Polynomial Chaos expansion order(s) of magnitude more accurate than linearized AC.

Linearized AC introduces errors in estimating the mean!

Polynomial Chaos provides better (but not perfect) approximation of chance constraints.

Problem: Which **distribution** are we dealing with?



Key observations

- Chance-Constrained AC Optimal Power Flow is not easy…
  - Guaranteeing power flow solvability
  - Accurately satisfying AC power flow constraints
  - Propagating uncertainty from input to output

- Moment-based chance constraint reformulations
  - Tractable, but approximate reformulations
  - Separate chance-constraints are effective (also to limit joint violation prob!)
  - Use limited information about distribution
  - Normal distribution often “accurate enough”

Thanks!

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Ansel Adams,
*Transmission Lines
in Mojave Desert*, 1947