LA-UR-19-20884

Tight Piecewise Convex Relaxations for Global Optimization of Optimal Power Flow

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Discussions with Prof. J. Linderoth

Jan 10, 2019

Optimal Power Flow

- **Introduced in 1962**

CONTRIBUTION A L'ÉTUDE DU DISPATCHING ÉCONOMIQUE (1)

PAR M. J. CARPENTIER,
Ingénieur à la Direction des Études et Recherches
d'Électricité de France.

- **The basis for many of the economic decisions made by modern grid operators**
  - Generator dispatch, Unit commitment, Transmission switching, etc.
  - AC OPF is NP hard (Bienstock, Verma - 2006)
- **Rising interest in solving AC OPF**
  - DOE ARPA-E Go Competition https://gocompetition.energy.gov

Optimal Power Flow

$$\min \sum_{i \in \mathcal{G}} \boldsymbol{c}_{2i}(\Re(S_i^g)^2) + \boldsymbol{c}_{1i}\Re(S_i^g) + \boldsymbol{c}_{0i}$$

Minimize cost of generation

$$s.t. \quad \sum_{k \in \mathcal{G}_i} S_k^g - \boldsymbol{S}_i^{\boldsymbol{d}} = \sum_{(i,j) \in \mathcal{E} \cup \mathcal{E}^{\mathcal{R}}} S_{ij} \qquad \forall i \in \mathcal{N}$$

Kirchhoff's law

$$S_{ij} = \boldsymbol{Y}_{ij}^* W_{ii} - \boldsymbol{Y}_{ij}^* W_{ij} \qquad \forall (i,j) \in \mathcal{E}$$

$$S_{ji} = \boldsymbol{Y}_{ij}^* W_{jj} - \boldsymbol{Y}_{ij}^* W_{ij}^* \qquad \forall (i,j) \in \mathcal{E}$$

$$W_{ii} = |V_i|^2 \quad \forall i \in \mathcal{N}$$

$$W_{ij} = V_i V_j^* \quad \forall (i,j) \in \mathcal{E}$$

Ohm's law

$$\underline{\boldsymbol{\theta}}_{ij} \leqslant \angle V_i - \angle V_j \leqslant \overline{\boldsymbol{\theta}}_{ij} \quad \forall (i,j) \in \mathcal{E}$$

$$\underline{\boldsymbol{v}}_i \leqslant |V_i| \leqslant \overline{\boldsymbol{v}}_i \quad \forall i \in \mathcal{N}$$

$$\underline{\boldsymbol{S}}_i^{\boldsymbol{g}} \leqslant S_i^g \leqslant \overline{\boldsymbol{S}}_i^{\boldsymbol{g}} \quad \forall i \in \mathcal{G}$$

$$|S_{ij}| \leqslant \overline{\boldsymbol{S}}_{ij} \quad \forall (i,j) \in \mathcal{E} \cup \mathcal{E}^{\mathcal{R}}$$

Engineering limits

Convex relaxations for OPF

$$S_i^g - \boldsymbol{S}_i^d = \sum_{(i,j)\in E\cup E^R} S_{ij} \quad \forall i \in N$$

$$S_{ij} = \boldsymbol{Y}_{ij}^* V_i V_i^* - \boldsymbol{Y}_{ij}^* V_i V_j^* \quad (i,j) \in E \cup E^R$$

SDP relaxations

- Bai, Wei, Fujisawa, Wang (2008)
- R. Madani, S. Sojoudi, J. Lavaei (2014)

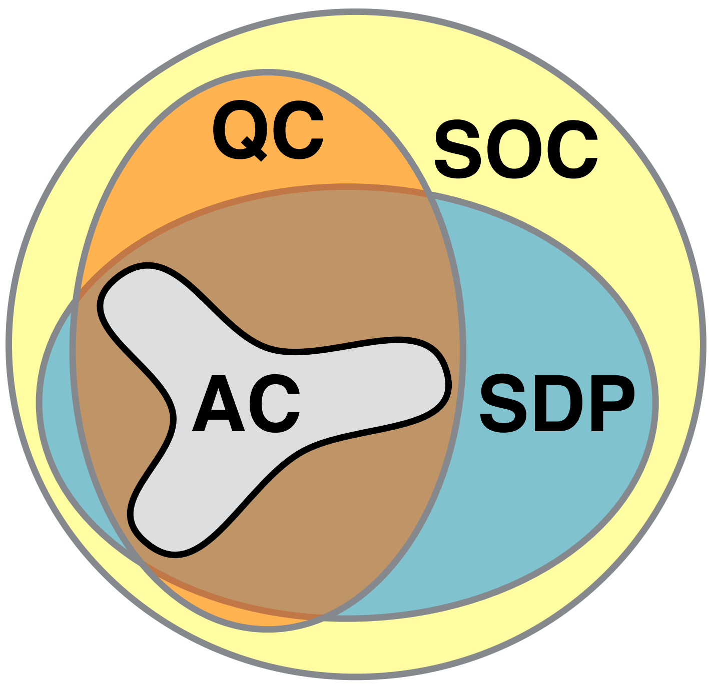
SOC-based relaxations

- R. Jabr (2006)
- B. Kocuk, S. S. Dey, and X. A. Sun (2016)

Convex quadratic (QC) relaxations

- H. Hijazi, C. Coffrin, and P. Van Hentenryck (2015)

And many others



Source: C. Coffrin, et. al. "The QC relaxation: A theoretical and computational study on optimal power flow", 2016

QC-relaxation overview

$$W_{ii} = v_i^2 \quad i \in N$$

$$\Re(W_{ij}) = v_i v_j \cos(\theta_i - \theta_j) \quad \forall (i,j) \in E$$

$$\Im(W_{ij}) = v_i v_j \sin(\theta_i - \theta_j) \quad \forall (i,j) \in E$$

- Key ideas
  - factorable-functions relaxation
  - exploit the narrow bounds in power systems
  - convexify transcendental functions (sin, cos)
- Resulting optimization model
  - quadratic and convex (computationally better)

H. Hijazi, C. Coffrin, P.V. Hentenryck, Convex Quadratic Relaxations for Mixed-Integer Nonlinear Programs in Power Systems, Math. Prog.-C, 2015

QC-relaxation overview

$$W_{ii} = v_i^2 \quad i \in N$$

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Trilinear monomials

$$v_i v_j \widehat{cs}_{ij}$$

$$v_i v_j \widehat{sn}_{ij}$$

Recursive McCormick relaxation

$$W_{ii} = \langle v_i^2 \rangle^T \quad i \in N$$

$$\Re(W_{ij}) = \langle \langle v_i v_j \rangle^M \langle \cos(\theta_i - \theta_j) \rangle^C \rangle^M \quad \forall (i,j) \in E$$

$$\Im(W_{ij}) = \langle \langle v_i v_j \rangle^M \langle \sin(\theta_i - \theta_j) \rangle^S \rangle^M \quad \forall (i,j) \in E$$

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QC-relaxation overview

$$W_{ii} = v_i^2 \quad i \in N$$
$$\Re(W_{ij}) = v_i v_j \cos(\theta_i - \theta_j) \ \ \forall (i,j) \in E$$
$$\Im(W_{ij}) = v_i v_j \sin(\theta_i - \theta_j) \ \ \forall (i,j) \in E$$

Trilinear monomials

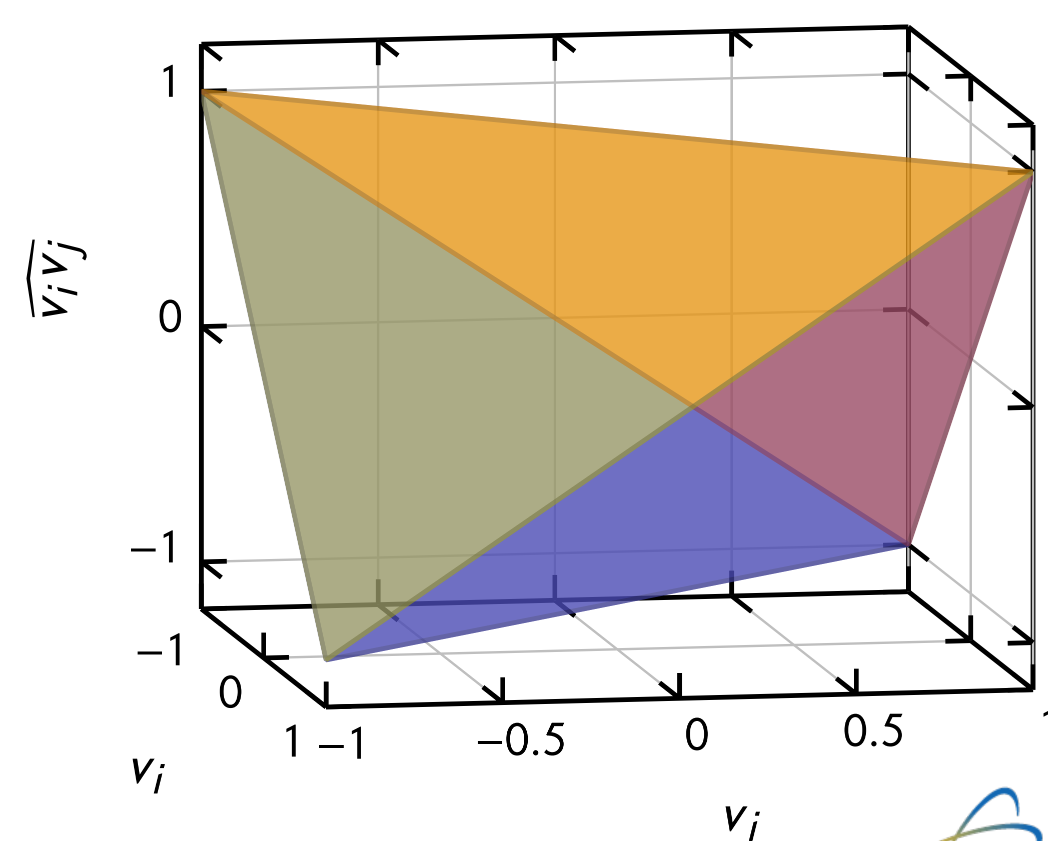
$$v_i v_j \widehat{cs}_{ij}$$
$$v_i v_j \widehat{sn}_{ij}$$

Recursive McCormick relaxation

H-representation

$$\widehat{v_i v_j} \geqslant v_i^l v_j + v_j^l v_i - v_i^l v_j^l$$
$$\widehat{v_i v_j} \geqslant v_i^u v_j + v_j^u v_i - v_i^u v_j^u$$
$$\widehat{v_i v_j} \leqslant v_i^l v_j + v_j^u v_i - v_i^l v_j^u$$
$$\widehat{v_i v_j} \leqslant v_i^u v_j + v_j^l v_i - v_i^u v_j^l$$

Convex Hull of Bilinear Function



QC-relaxation overview

$$W_{ii} = v_i^2 \quad i \in N$$
$$\Re(W_{ij}) = v_i v_j \cos(\theta_i - \theta_j) \;\; \forall (i,j) \in E$$
$$\Im(W_{ij}) = v_i v_j \sin(\theta_i - \theta_j) \;\; \forall (i,j) \in E$$

→ Trilinear monomials

$$v_i v_j \widehat{cs}_{ij}$$
$$v_i v_j \widehat{sn}_{ij}$$

Recursive McCormick relaxation

H-representation

$$\widehat{v_i v_j} \geqslant v_i^l v_j + v_j^l v_i - v_i^l v_j^l$$
$$\widehat{v_i v_j} \geqslant v_i^u v_j + v_j^u v_i - v_i^u v_j^u$$
$$\widehat{v_i v_j} \leqslant v_i^l v_j + v_j^u v_i - v_i^l v_j^u$$
$$\widehat{v_i v_j} \leqslant v_i^u v_j + v_j^l v_i - v_i^u v_j^l$$

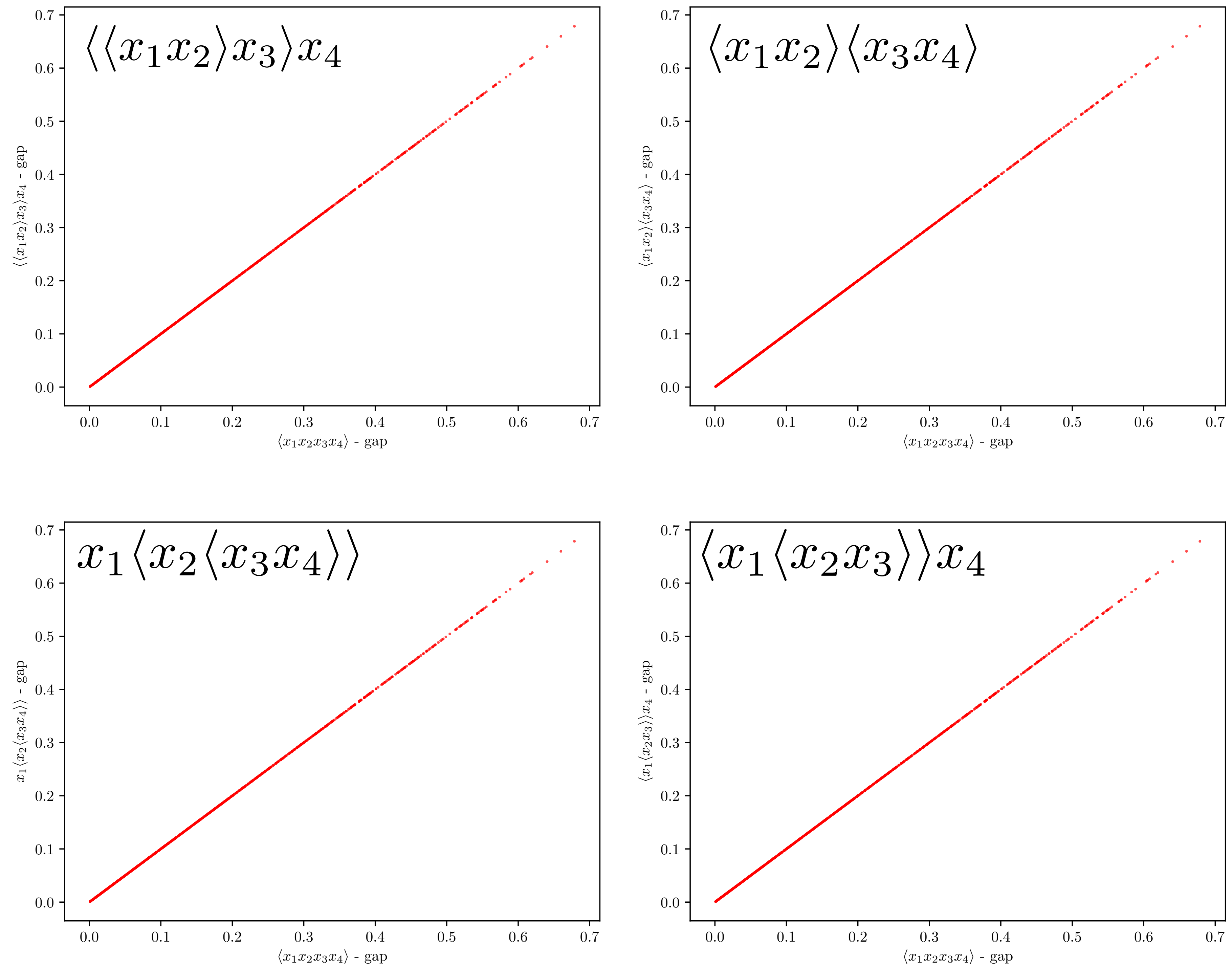
→ Apply recursively on $v_i v_j \widehat{cs}_{ij}$ $v_i v_j \widehat{sn}_{ij}$

May not capture it's convex hull

(asymmetric bounds on voltage and phase-angle variables)

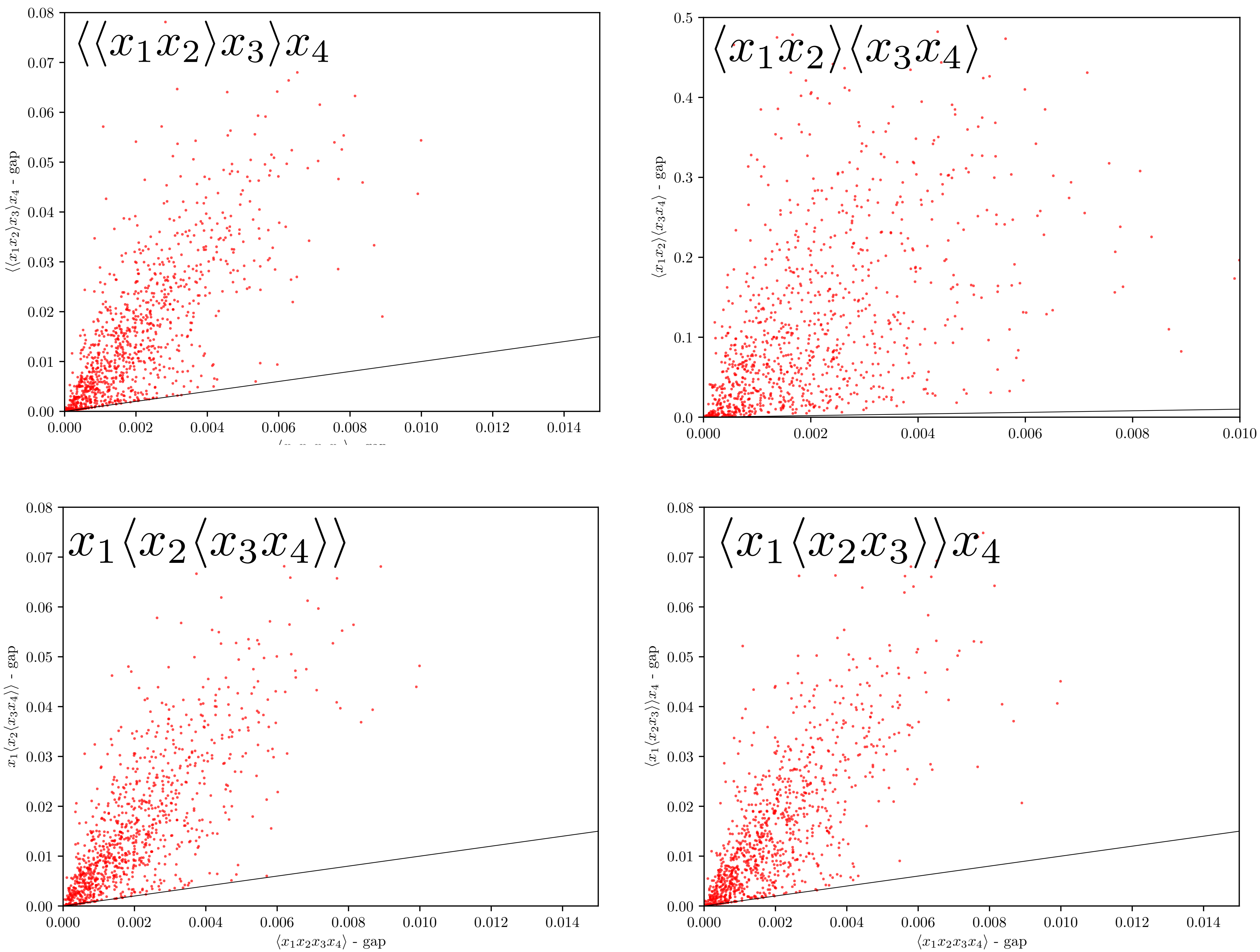
Recursive vs. Convex hull relaxations

Symmetric bounds: $x_i \in [-U, U]$



Recursive vs. Convex hull relaxations

Asymmetric bounds: $x_i \in [-U_1, U_2]$



Term-wise convex hull representation

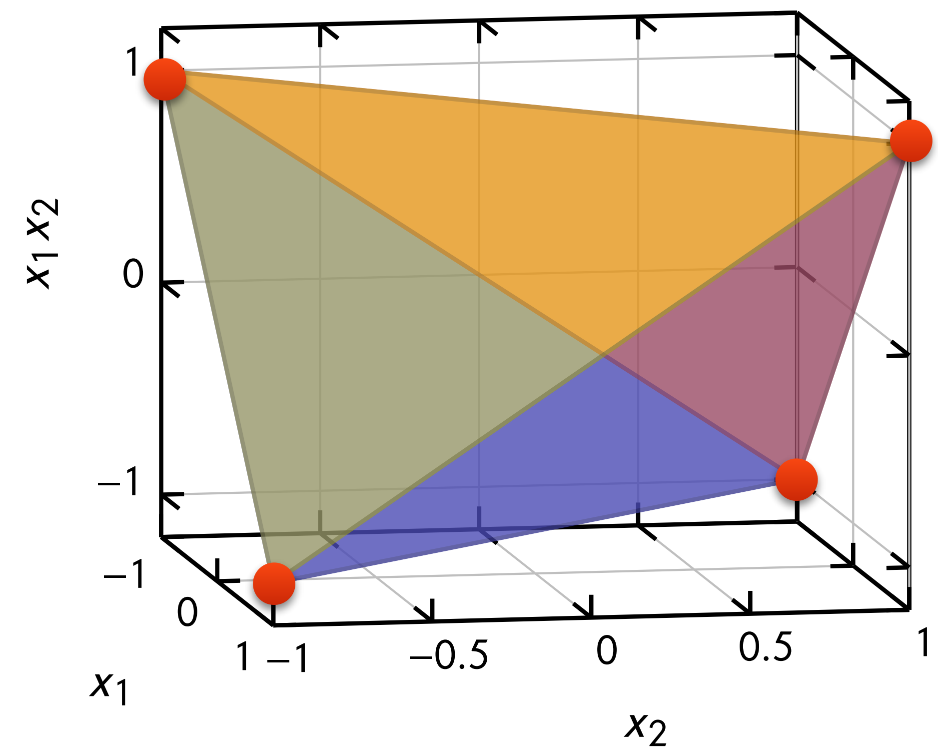
V-representation (Bilinear)

Convex combination of extreme points

$$\begin{bmatrix} x_1 \\ x_2 \\ y \end{bmatrix} = \lambda_1 \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_1\ell_2 \end{bmatrix} + \lambda_2 \begin{bmatrix} \ell_1 \\ u_2 \\ \ell_1 u_2 \end{bmatrix} + \lambda_3 \begin{bmatrix} u_1 \\ \ell_2 \\ u_1\ell_2 \end{bmatrix} + \lambda_4 \begin{bmatrix} u_1 \\ u_2 \\ u_1 u_2 \end{bmatrix}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \quad \text{and} \quad \lambda_i \geqslant 0 \quad \forall i \in \{1,2,3,4\}$$

Convex Hull of Bilinear Function



V-representation (Trilinear)

$$\phi(x_1, x_2, x_3) = x_1 x_2 x_3, \quad \underline{x}_i \leqslant x_i \leqslant \overline{x}_i \ \forall i = 1,2,3$$

$$\sum_{k=1..8} \lambda_k = 1,$$

$$\lambda_k \geqslant 0, \quad \forall k = 1, \ldots, 8,$$

$$\widehat{x} = \sum_{k=1..8} \lambda_k \phi(\xi_k),$$

$$x_i = \sum_{k=1..8} \lambda_k \xi_k^i$$

→ H-representation (Trilinear)

Meyer, C.A. and Floudas, C.A. "Trilinear monomials with mixed sign domains: Facets of the convex and concave envelopes"
Journal of Global Optimization - 2004

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Improved QC-relaxation gaps

Without Bound Tightening

Instances	QC^{rmc} (%)	QC^{conv} (%)
case3_lmbd	1.21	0.96
case30_ieee	15.64	15.20
case3_lmbd_api	1.79	1.59
case24_ieee_rts_api	11.88	8.78
case73_ieee_rts_api	10.97	9.64
case3_lmbd_sad	1.42	1.37
case4_gs_sad	1.53	0.96
case5_pjm_sad	0.99	0.77
case24_ieee_rts_sad	2.93	2.77
case73_ieee_rts_sad	2.53	2.38
case118_ieee_sad	4.61	4.14

Instances: C. Coffrin et. al, "NESTA, the NICTA energy system test archive," 2014

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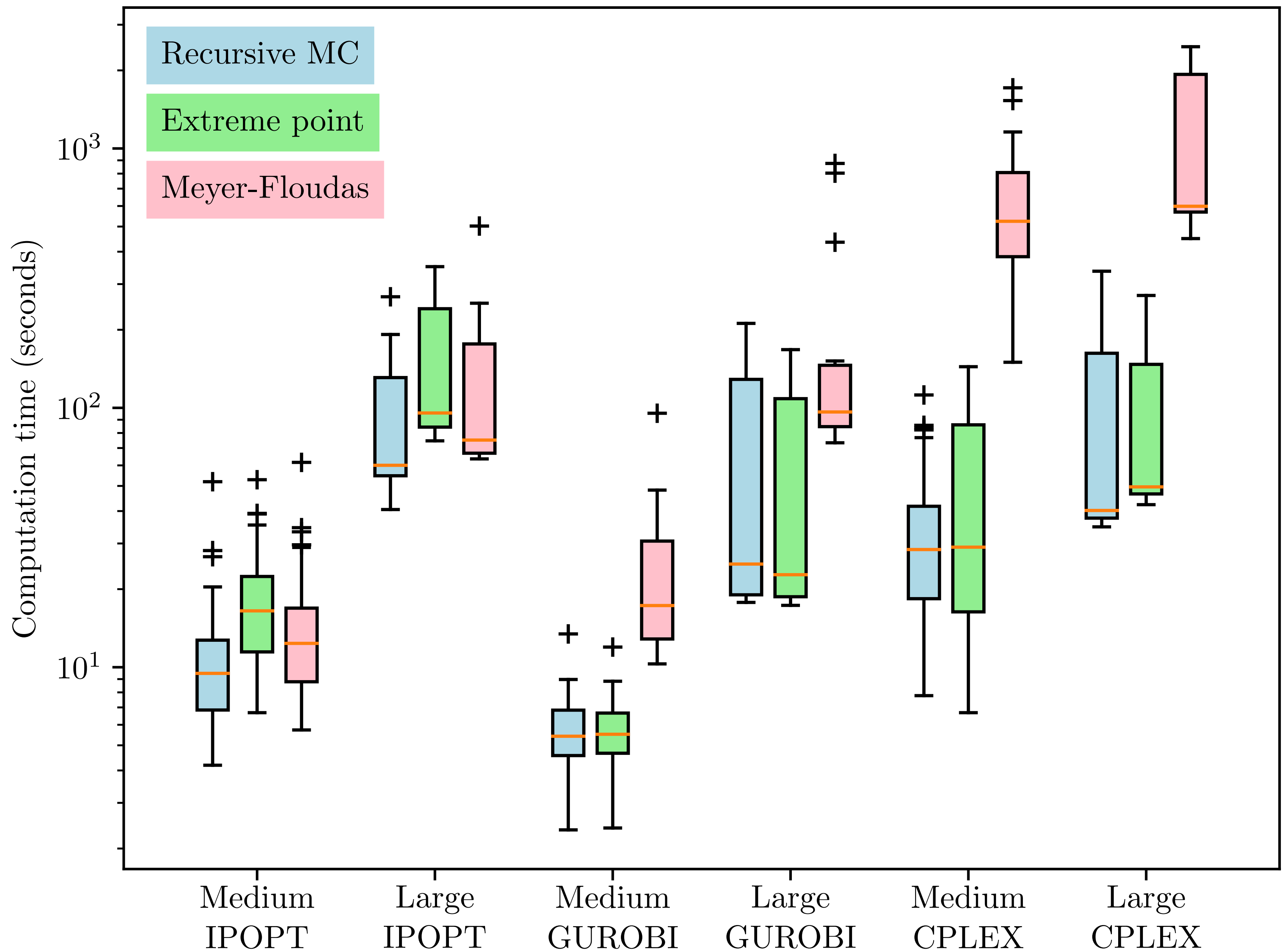
Comparison of trilinear envelopes on OPF relaxations

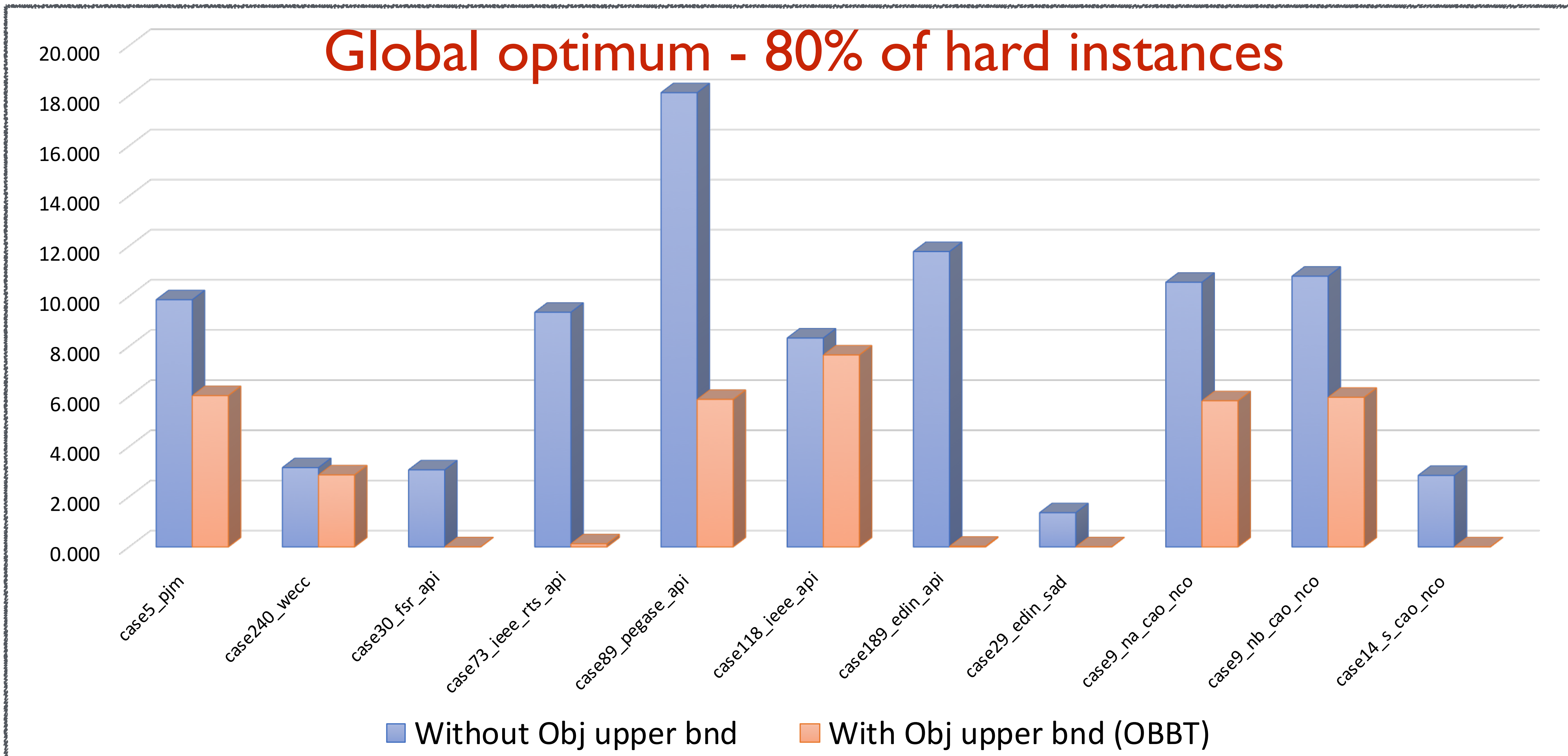


Narimani, M.R, Molzahn, D.K, Nagarajan, H, Crow, M.L. "Comparison of Various Trilinear Monomial Envelopes for Convex Relaxations of Optimal Power Flow Problems". *IEEE Global Conference on Signal and Information Processing (GlobalSIP). IEEE, 2018.*

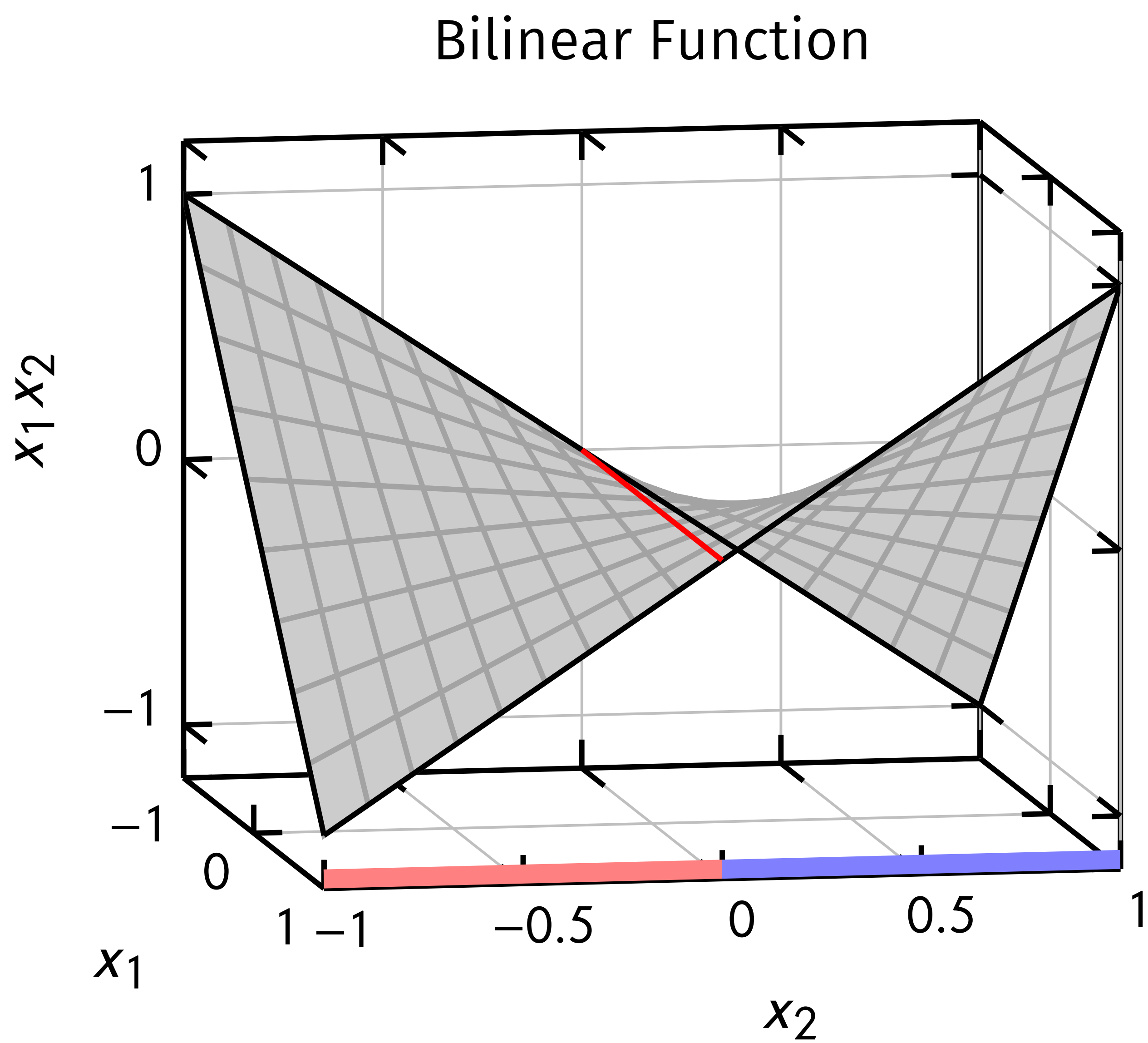
Improved QC-relaxation gaps

With Optimization-based Bound Tightening (OBBT)

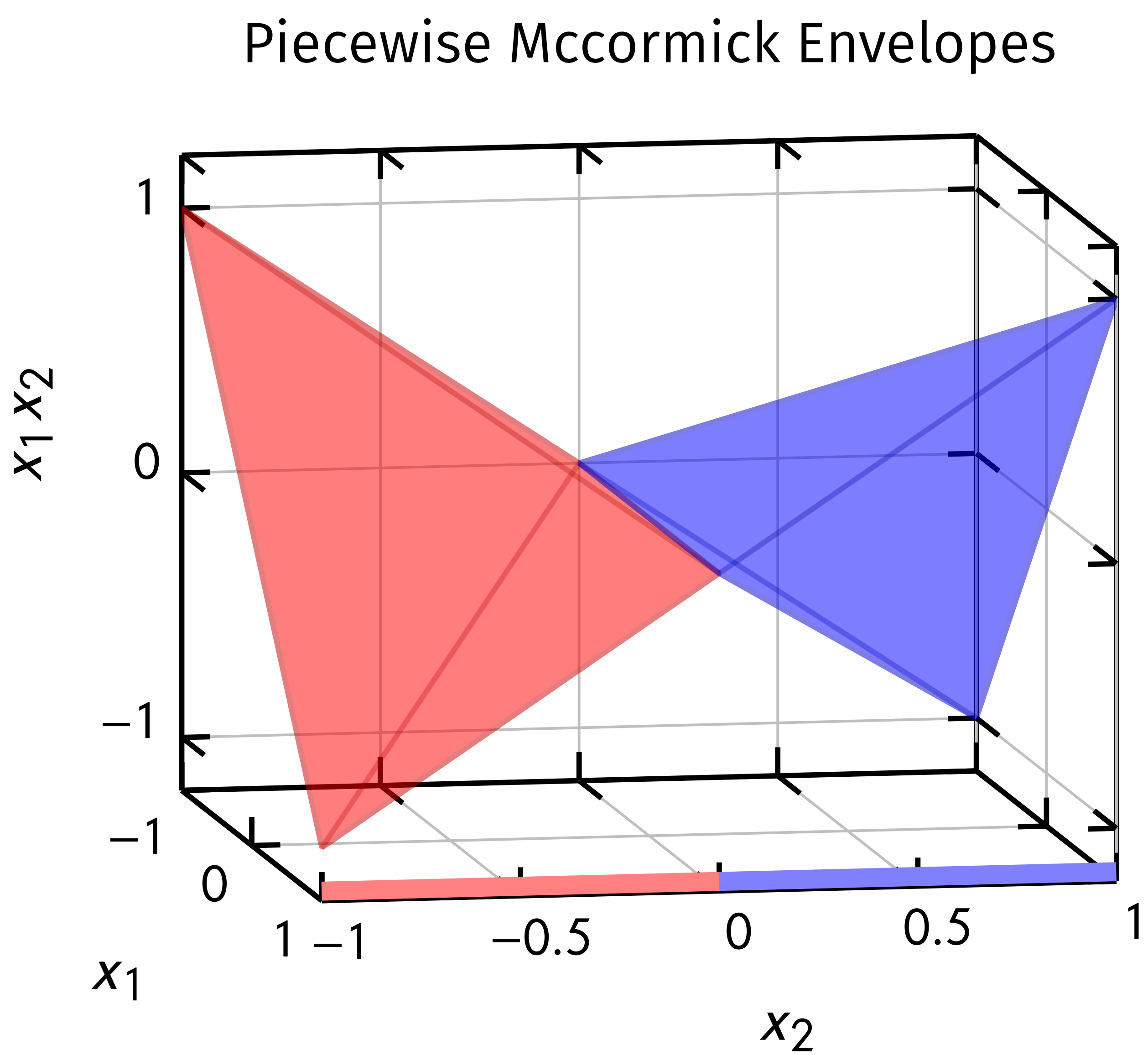
$$\sum_{i \in \mathcal{G}} \boldsymbol{c}_{2i} (\Re(S_i^g)^2) + \boldsymbol{c}_{1i} \Re(S_i^g) + \boldsymbol{c}_{0i} \leqslant \boxed{UB^{feas}}$$ Local-solver (Ipopt)



Global optimization: Tight piecewise relaxations



Tight piecewise formulations



$$z_1 = 1 \Rightarrow -1 \leqslant x_2 \leqslant 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ y \end{bmatrix} = \lambda_1 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + \lambda_4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \quad \text{and} \quad \lambda_5 + \lambda_6 = 0$$

$$z_2 = 1 \Rightarrow 0 \leqslant x_2 \leqslant 1$$

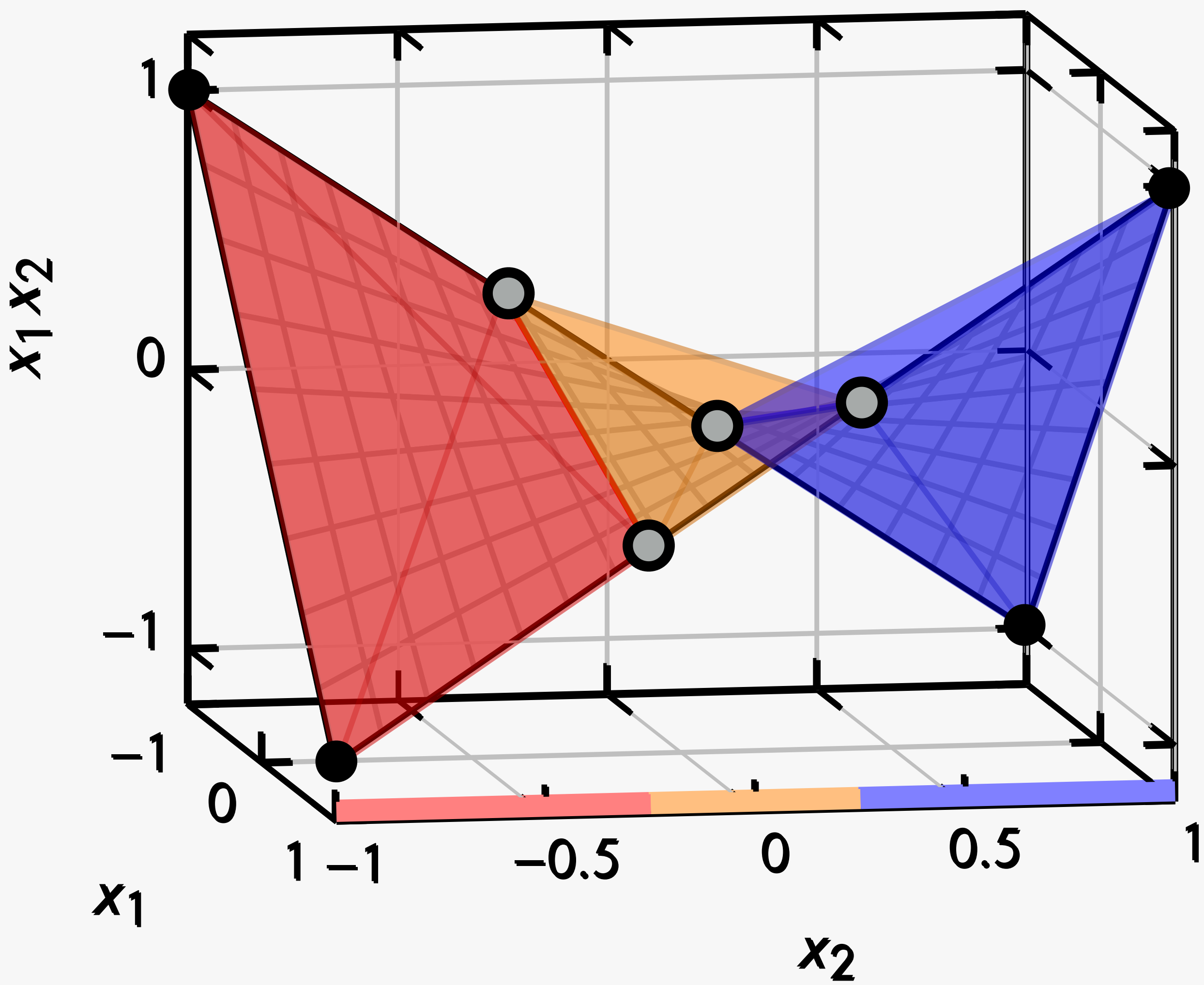
$$\begin{bmatrix} x_1 \\ x_2 \\ y \end{bmatrix} = \lambda_2 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \lambda_5 \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + \lambda_4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \lambda_6 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 = 1 \quad \text{and} \quad \lambda_1 + \lambda_3 = 0$$

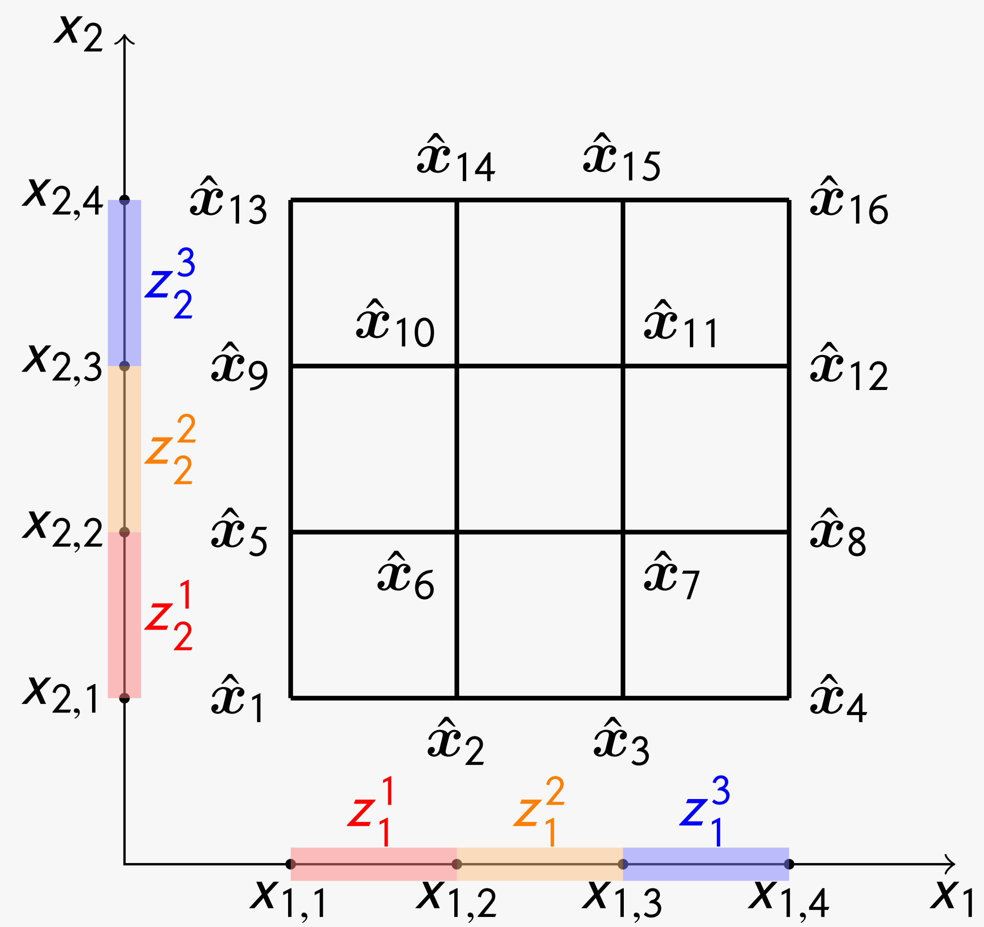
$$z_1 + z_2 = 1,\ z_1, z_2 \in \{0, 1\}, \text{ and } \lambda_i \geqslant 0 \quad i \in \{1, \ldots, 6\}$$

Tight piecewise formulations

Piecewise Mccormick Envelopes



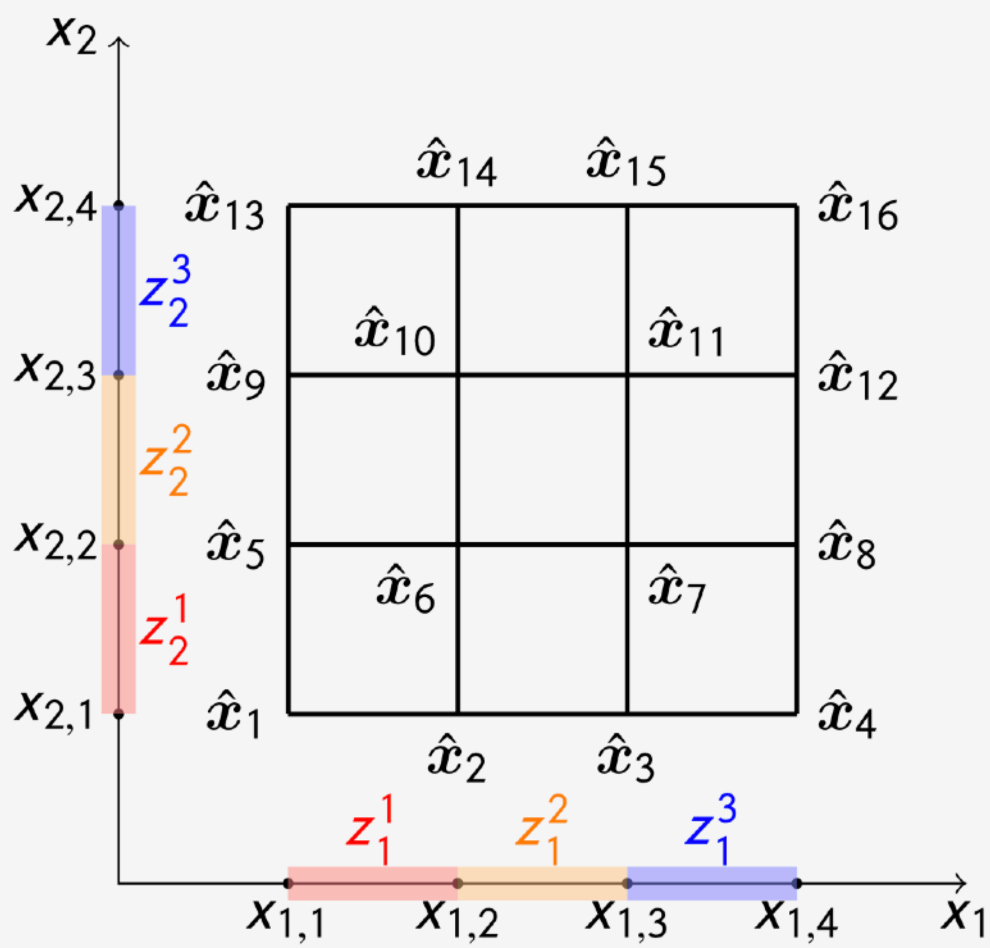
$\phi(\hat{x}_s) = x_{1,i} \cdot x_{2,j}$



Bivariate partitioning

Tight piecewise formulations

$$\phi(\hat{x}_s) = x_{1,i} \cdot x_{2,j}$$



$$\begin{bmatrix} x_1 \\ x_2 \\ y \end{bmatrix} = \sum_{s=1}^{16} \lambda_s \begin{bmatrix} \hat{x}_s^1 \\ \hat{x}_s^2 \\ \phi(\hat{x}_s) \end{bmatrix}$$

$$z_1^1 + z_1^2 + z_1^3 = 1$$
$$z_2^1 + z_2^2 + z_2^3 = 1$$

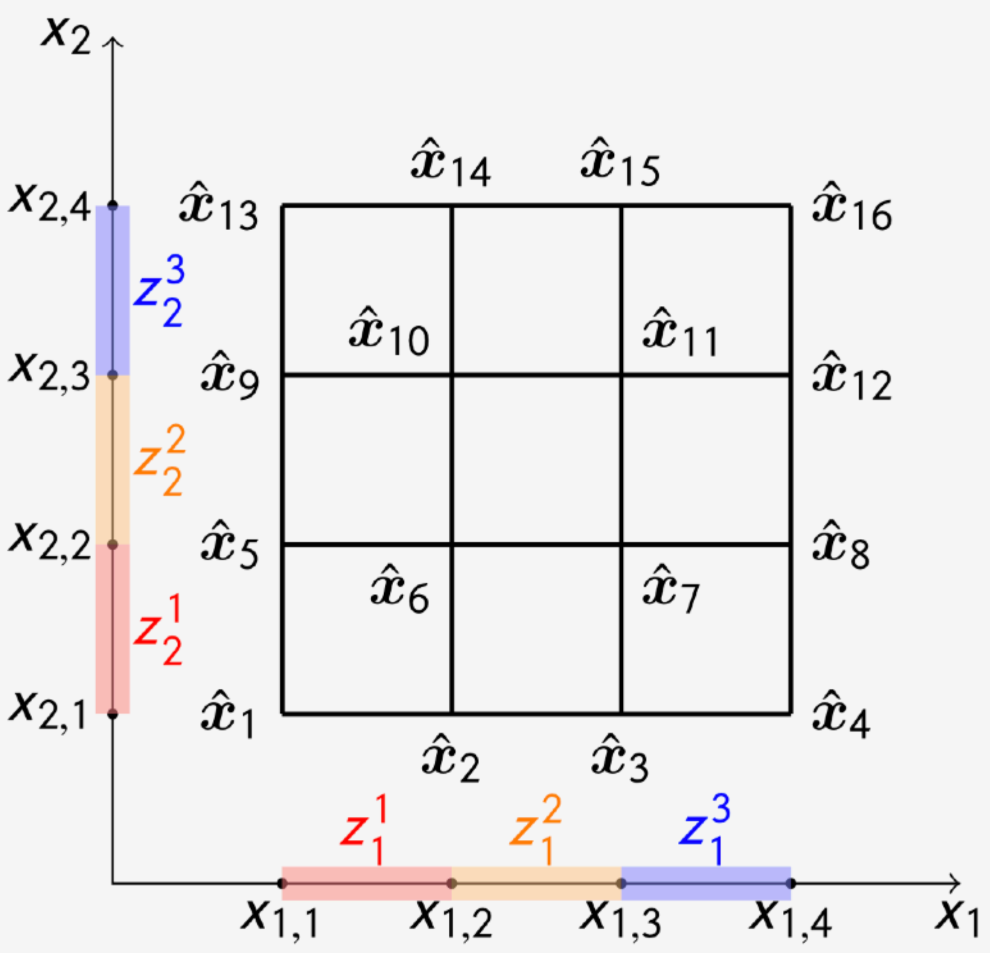
$$\sum_{s=1}^{16} \lambda_s = 1 \quad \text{and} \quad \lambda_s \geqslant 0 \quad \forall s \in \{1, \ldots, 16\}$$

$$\lambda_1 + \lambda_5 + \lambda_9 + \lambda_{13} \leqslant z_1^1$$
$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \leqslant z_2^1$$
$$\lambda_2 + \lambda_6 + \lambda_{10} + \lambda_{14} \leqslant z_1^1 + z_1^2$$
$$\lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 \leqslant z_2^1 + z_2^2$$
$$\lambda_3 + \lambda_7 + \lambda_{11} + \lambda_{15} \leqslant z_1^2 + z_1^3$$
$$\lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} \leqslant z_2^2 + z_2^3$$
$$\lambda_4 + \lambda_8 + \lambda_{12} + \lambda_{16} \leqslant z_1^3$$
$$\lambda_{13} + \lambda_{14} + \lambda_{15} + \lambda_{16} \leqslant z_2^3$$

SOS-2 type constraints: Extreme points of the lifted-variable polytope are integral

Tight piecewise formulations

$$\phi(\hat{x}_s) = x_{1,i} \cdot x_{2,j}$$



$$\begin{bmatrix} x_1 \\ x_2 \\ y \end{bmatrix} = \sum_{s=1}^{16} \lambda_s \begin{bmatrix} \hat{x}_s^1 \\ \hat{x}_s^2 \\ \phi(\hat{x}_s) \end{bmatrix}$$

$$z_1^1 + z_1^2 + z_1^3 = 1$$
$$z_2^1 + z_2^2 + z_2^3 = 1$$

$$\sum_{s=1}^{16} \lambda_s = 1 \quad \text{and} \quad \lambda_s \geqslant 0 \quad \forall s \in \{1, \ldots, 16\}$$

$$\lambda_1 + \lambda_5 + \lambda_9 + \lambda_{13} \leqslant z_1^1$$
$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \leqslant z_2^1$$
$$\lambda_2 + \lambda_6 + \lambda_{10} + \lambda_{14} \leqslant z_1^1 + z_1^2$$
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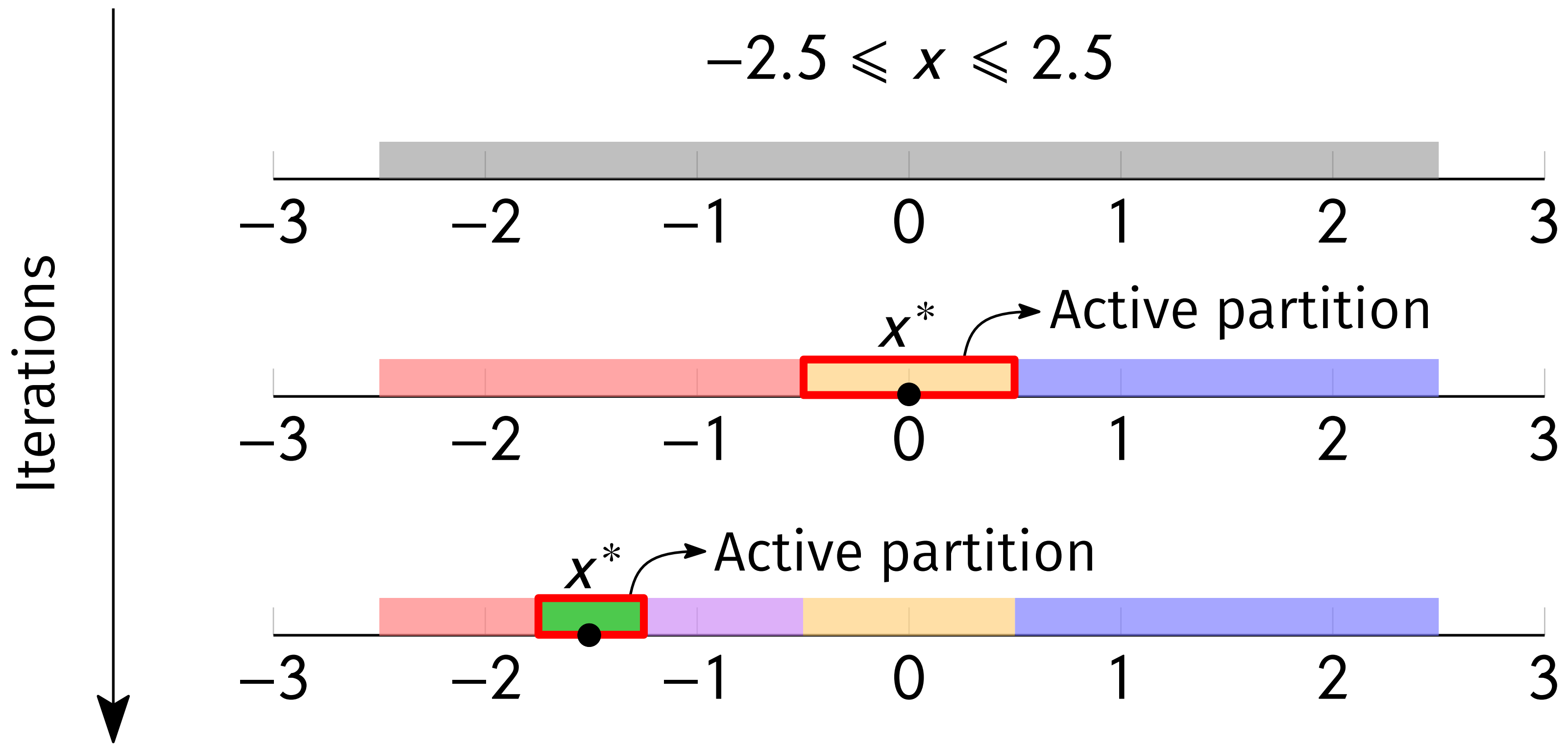
$$\lambda_4 + \lambda_8 + \lambda_{12} + \lambda_{16} \leqslant z_1^3$$
$$\lambda_{13} + \lambda_{14} + \lambda_{15} + \lambda_{16} \leqslant z_2^3$$

Uniformly spaced partitions induce too many binaries. Hence, formulating tractable mixed-integer convex programs are crucial

Adaptive variable partitioning: Tightening gaps using mixed-integer convex programs

Local solvers (**IPOPT**) are amazing on ACOPF

Non-uniform, dynamically added partitions guided by **local** and **lower-bounding** solutions



H. Nagarajan, M. Lu, S. Wang, R. Bent, K. Sundar, "An Adaptive, Multivariate Partitioning Algorithm for Global Optimization of Nonconvex Programs," Journal of Global Optimization, 2018

H. Nagarajan, M. Lu, E. Yamangil, R. Bent, "Tightening McCormick Relaxations for Nonlinear Programs via Dynamic Multivariate Partitioning," Constraint Programming, 2016

Adaptive variable partitioning

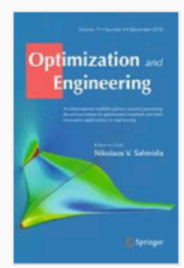
Was found useful in many applications

ELSEVIER
Computers & Chemical Engineering
Volume 95, 5 December 2016, Pages 38-48

An adaptive discretization MINLP algorithm for optimal synthesis of decentralized energy supply systems ☆

Sebastian Goderbauer [a, b], Björn Bahl [c], Philip Voll [c], Marco E. Lübbecke [b], André Bardow [c], Arie M.C.A. Koster [a]

[a] Lehrstuhl II für Mathematik, RWTH Aachen University, 52056 Aachen, Germany
[b] Operations Research, RWTH Aachen University, 52072 Aachen, Germany



Optimization and Engineering
pp 1–46 | Cite as

An adaptive discretization algorithm for the design of water usage and treatment networks

Authors | Authors and affiliations

Journal of Global Optimization
August 2018, Volume 71, Issue 4, pp 691–716 | Cite as

Global optimization of MIQCPs with dynamic piecewise relaxations

Authors | Authors and affiliations

Pedro A. Castillo Castillo, Pedro M. Castro, Vladimir Mahalec

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POD.jl: An open-source global MINLP solver

https://github.com/lanl-ansi/POD.jl

README.md

POD, A Global MINLP Solver

Dev: build passing codecov 67%

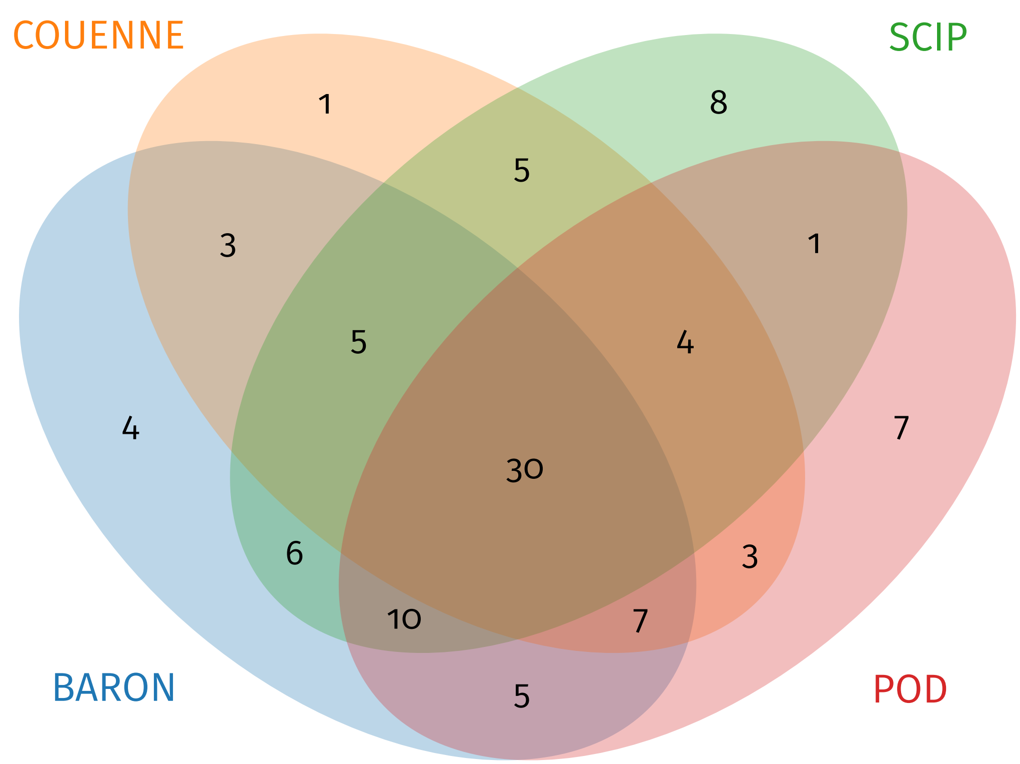
"POD: Piecewise convex relaxation (P), Outer-approximation (O), and Dynamic discretization (D)" is a novel global optimization algorithm that uses an adaptive convexification scheme and constraints programming methods to solve Mixed-Integer Non-Linear Programs (non-convex MINLPs) efficiently. MINLPs are famously known as the "hard" programming problems that exist in many applications (see this MINLPLibJuMP.jl for problem instances). POD is also a good fit for subsets of the MINLP family, e.g., Mixed-Integer Quadradic Convex Programming (MIQCP), Non-Linear Programming (NLP), etc.

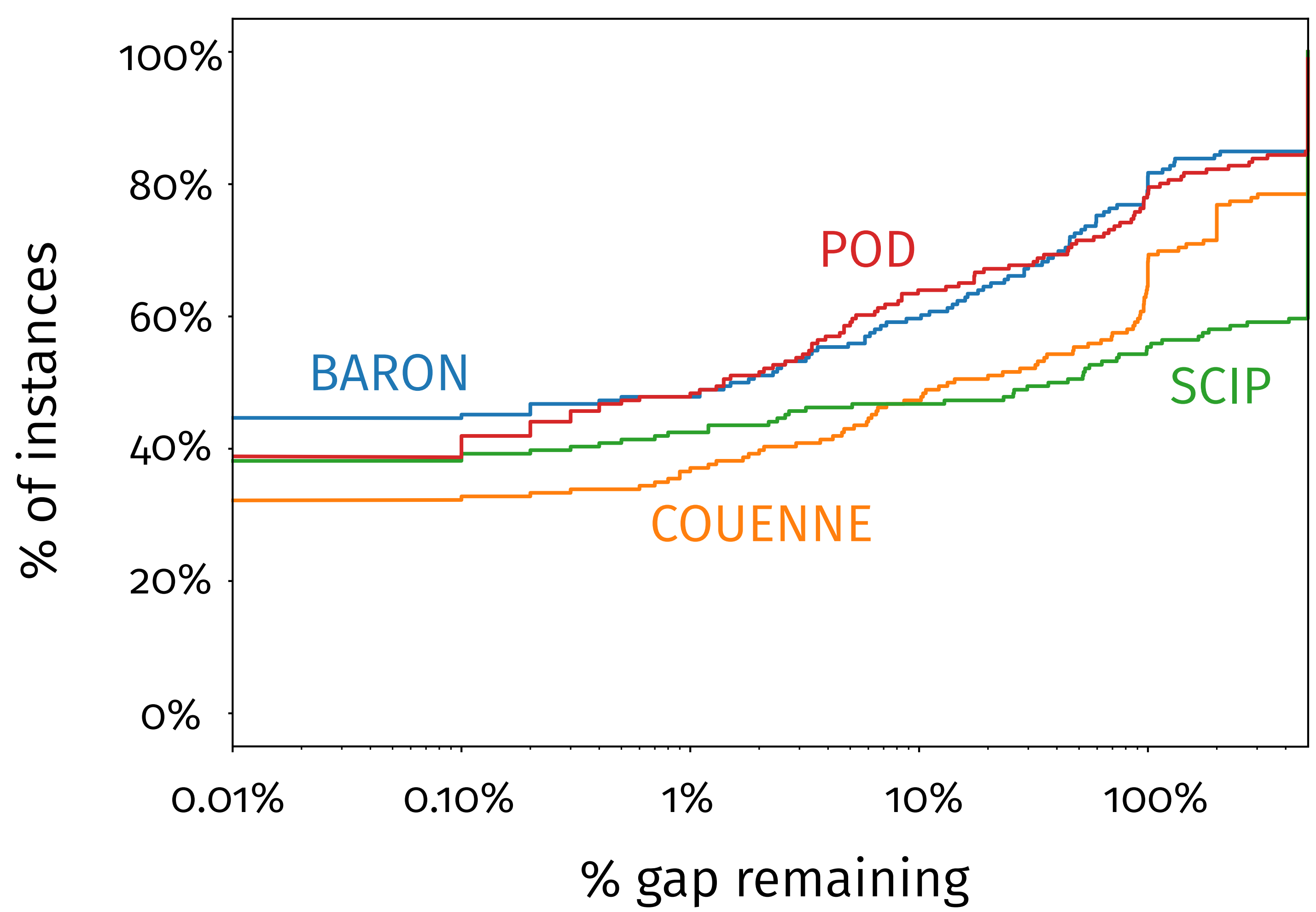
Unlike many other state-of-the-art MINLP solvers, POD is entirely built upon JuMP and MathProgBase Interface in Julia, which provides incredible flexibility for usage and further development.

POD globally solves a given MINLP by:

- Analyzing the problem's expressions (objective & constraints) and applies approporite convex relaxations
- Performing novel adaptive/dynamic partitioning methods to create piecewise relaxations, bound tightening and polyhedral outer-approximations to guarantee global convergence

POD.jl: An open-source global MINLP solver





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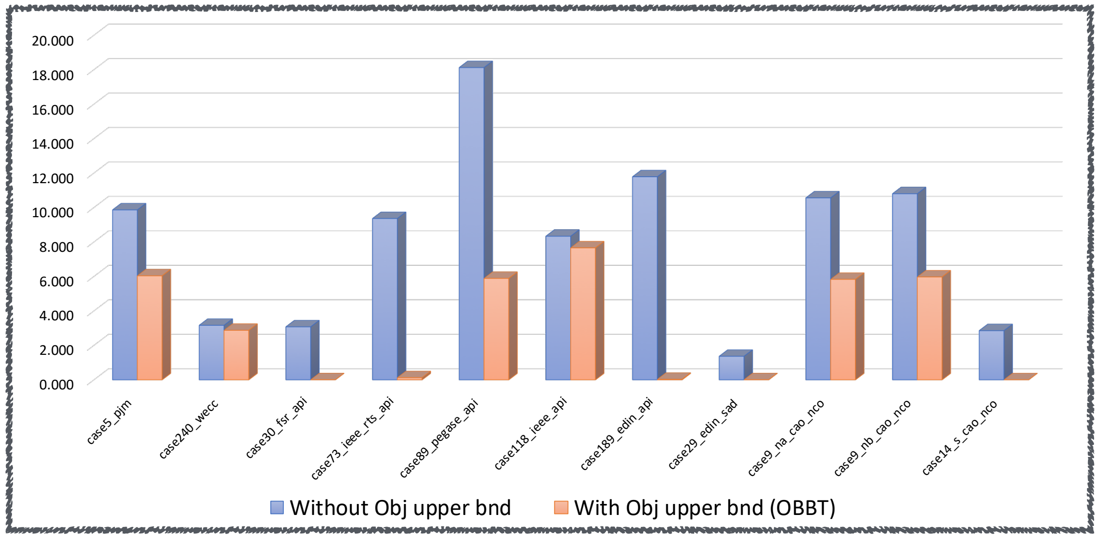
Numerical results on ACOPF



Global optimum - 80%
of hard instances

+

3 hours time limit for piecewise, dynamic partitioning algorithm

Global optimum - 95%
of hard instances (N <= 300)

Challenging Instances

case89_pegase_api (4.5%)
case118_ieee_api (1.1%)
case9_bgm_nco (3.5%)
case39_1_bgm_nco (3.3%)

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Revisiting extreme-point formulation

Instances	QC^{rmc} (%)	QC^{conv} (%)
case3_lmbd	1.21	0.96
case30_ieee	15.64	15.20
case3_lmbd_api	1.79	1.59
case24_ieee_rts_api	11.88	8.78
case73_ieee_rts_api	10.97	9.64
case3_lmbd_sad	1.42	1.37
case4_gs_sad	1.53	0.96
case5_pjm_sad	0.99	0.77
case24_ieee_rts_sad	2.93	2.77
case73_ieee_rts_sad	2.53	2.38
case118_ieee_sad	4.61	4.14
case179_goc_api	7.18	7.21

Unexpected

Revisiting extreme-point formulation

$$p_{ij} = \boldsymbol{g}_{ij} v_i^2 - \boxed{\boldsymbol{g}_{ij} v_i v_j \widecheck{cs}_{ij} - \boldsymbol{b}_{ij} v_i v_j \widecheck{sn}_{ij}}$$
$$q_{ij} = -\boldsymbol{b}_{ij} v_i^2 - \boldsymbol{g}_{ij} v_i v_j \widecheck{cs}_{ij} + \boldsymbol{b}_{ij} v_i v_j \widecheck{sn}_{ij}$$

$$-\boldsymbol{g}_{ij} \boxed{v_i v_j \widecheck{cs}_{ij}} - \boldsymbol{b}_{ij} \boxed{v_i v_j \widecheck{sn}_{ij}}$$

Extreme point captures the convex hull locally on trilinear monomials

$$-\boldsymbol{g}_{ij} \boxed{v_i v_j} \widecheck{cs}_{ij} - \boldsymbol{b}_{ij} \boxed{v_i v_j} \widecheck{sn}_{ij}$$

Recursive McCormick shares the lifted variable across the trilinear monomials

But neither captures the **convex hull** of $-\boldsymbol{g}_{ij} v_i v_j \widecheck{cs}_{ij} - \boldsymbol{b}_{ij} v_i v_j \widecheck{sn}_{ij}$

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Revisiting extreme-point formulation

$$\varphi(v_i, v_j, \widecheck{cs}_{ij}, \widecheck{sn}_{ij}) = -\boldsymbol{g}_{ij} \underbrace{v_i v_j \widecheck{cs}_{ij}}_{\boldsymbol{\lambda^c}} - \boldsymbol{b}_{ij} \underbrace{v_i v_j \widecheck{sn}_{ij}}_{\boldsymbol{\lambda^s}}$$

$$\begin{pmatrix} \lambda^c_{ij,1} + \lambda^c_{ij,2} - \lambda^s_{ij,1} - \lambda^s_{ij,2} \\ \lambda^c_{ij,3} + \lambda^c_{ij,4} - \lambda^s_{ij,3} - \lambda^s_{ij,4} \\ \lambda^c_{ij,5} + \lambda^c_{ij,6} - \lambda^s_{ij,5} - \lambda^s_{ij,6} \\ \lambda^c_{ij,7} + \lambda^c_{ij,8} - \lambda^s_{ij,7} - \lambda^s_{ij,8} \end{pmatrix}^T \begin{pmatrix} \boldsymbol{v_i^l} \cdot \boldsymbol{v_j^l} \\ \boldsymbol{v_i^l} \cdot \boldsymbol{v_j^u} \\ \boldsymbol{v_i^u} \cdot \boldsymbol{v_j^l} \\ \boldsymbol{v_i^u} \cdot \boldsymbol{v_j^u} \end{pmatrix} = 0$$

Tying constraint

Theorem

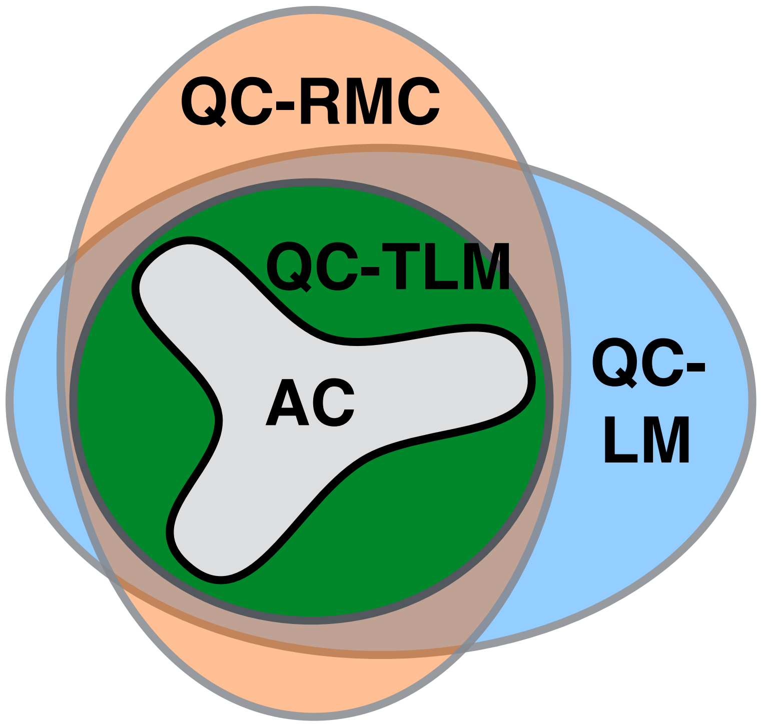
Term-wise convex-hull formulation, intersected with the constraint tying $\boldsymbol{\lambda^c}$ and $\boldsymbol{\lambda^s}$ captures the convex hull of $\varphi(v_i, v_j, \check{cs}_{ij}, \check{sn}_{ij})$ in the extended space $(v_i, v_j, \check{cs}_{ij}, \check{sn}_{ij}, \boldsymbol{\lambda^c}, \boldsymbol{\lambda^s}.)$

Computational results on bound tightening with improved relaxations
https://arxiv.org/abs/1809.04565

Revisiting extreme-point formulation

Instances	QC^{rmc} (%)	QC^{conv} (%)
case3_lmbd	1.21	0.96
case30_ieee	15.64	15.20
case3_lmbd_api	1.79	1.59
case24_ieee_rts_api	11.88	8.78
case73_ieee_rts_api	10.97	9.64
case3_lmbd_sad	1.42	1.37
case4_gs_sad	1.53	0.96
case5_pjm_sad	0.99	0.77
case24_ieee_rts_sad	2.93	2.77
case73_ieee_rts_sad	2.53	2.38
case118_ieee_sad	4.61	4.14
case179_goc_api	7.18	7.21

→ 6.98%



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Future directions

- Extension of tying constraints to the partitioned case
- Other OPF formulations (Rectangular, Current-Voltage-based, Tangent)
- Incorporating SDP-based cuts by exploiting graph sparsity
- Scaling to large-scale networks?

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Thank you!

Questions?