Distributionally Robust Co-Optimization of Power Dispatch and Do-Not-Exceed Limits

Hongyan Ma and Ruiwei Jiang

Department of Industrial and Operations Engineering
University of Michigan

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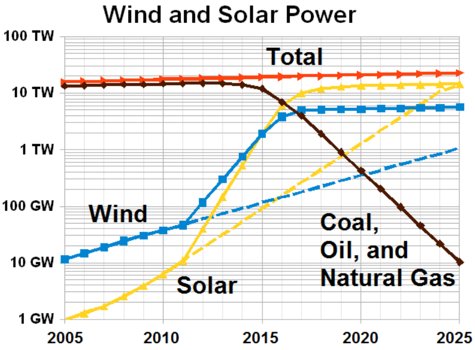
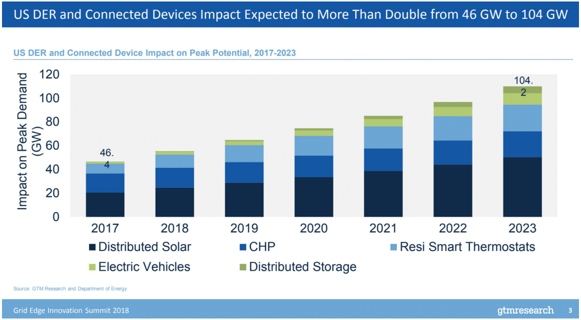
Growth of Renewable Energy

- Wind power increases 30% annually, and could reach 20% of total energy generation by 2030 (DoE).
- Solar energy increases the fastest (e.g., 60% annual increase for photovoltaic) (DoE).



Emerging Distributed Energy Resources

- Predicted for "explosive growth" (cf. `greentechmedia.com`).



Impact of Renewable Energy and DER

- Impact on system reliability and cost effectiveness.
  - Generation $>$ load: curtailment of renewable energy/DER.
  - Generation $<$ load: load shedding and blackouts.
  - Regulating dispatchable resources (e.g., fast-start units) $\rightarrow$ expensive.
  - Transmission line outages.

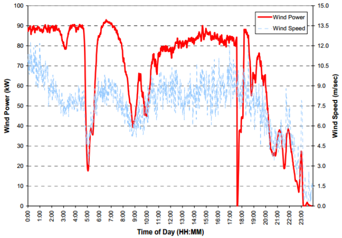


Figure: Example of wind power output in 24 hours [1]

[1] Source: http://www.nrel.gov/docs/fy04osti/36551.pdf/

Challenges to the ISO

1. Accommodating all renewable energy and DER inputs.
    - Potentially infeasible as the uncertainty grows.
    - Expensive to regulate other dispatchable resources.
2. Joint probability distribution of the nodal injection.
    - Typically imperfect understanding.
    - Historical data.

A Novel Solution

ISO-NE addresses Challenge #1 by

- Do-Not-Exceed (DNE) limits.
  - Zhao et al.'15–'16.
  - Power dispatch given.
  - Admissible range: in-accommodate, out-curtail/emergency regulations.
- Clear guideline for managing nodal injections.
- Convenient for measuring system flexibility.

Illustration of DNE Limits

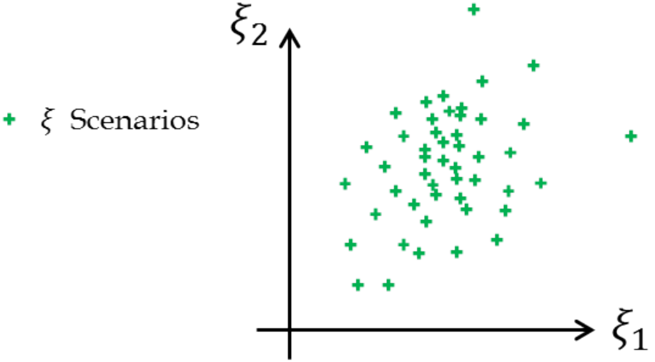
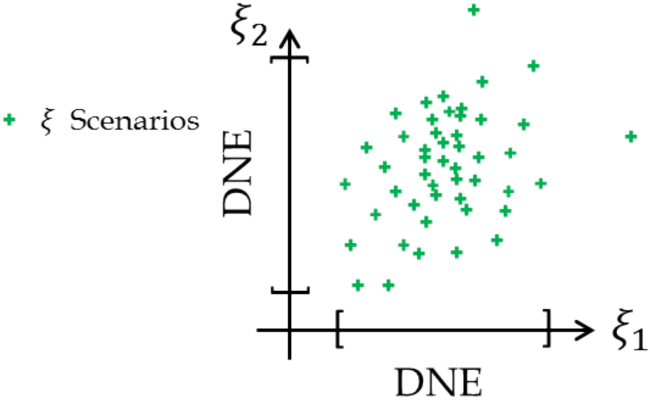


Illustration of DNE Limits



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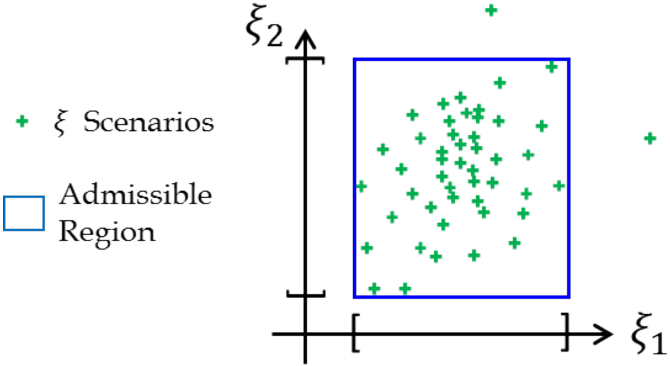

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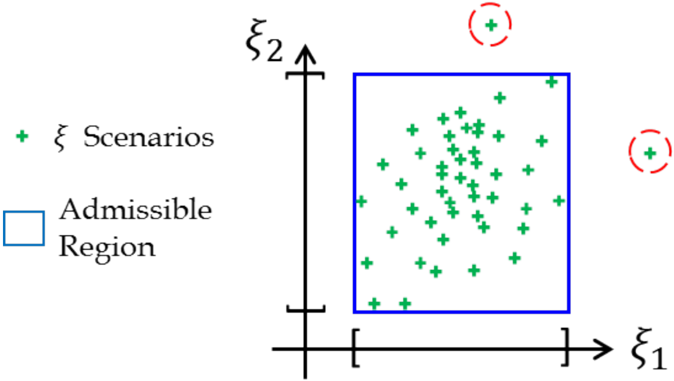

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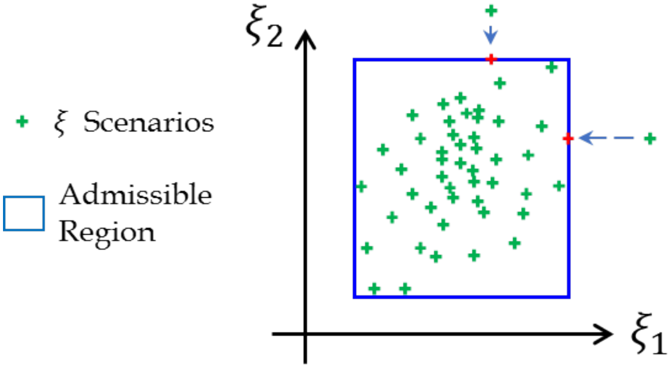
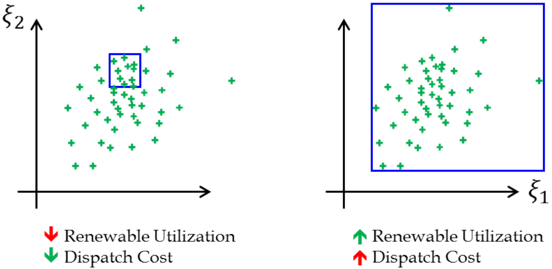


Literature Review: DNE Limits

- Sequential decisions: power dispatch first, then DNE limits.
  - Zhao et al.'15, Wei et al.'15a-'15b.
  - Challenge: might underestimate the dispatch capability.
- Co-optimization.
  - Li et al.'15, Wei et al.'16, Shao et al.'17a-'17b, Wang et al.'17.
  - Challenge: two-stage RO with an endogenous uncertainty set.
  - Column-and-constraint generation (cf. Zeng & Zhao'13).
- Risk of wasting renewable/DER.
  - Wang et al.'16: risk of curtailment.
  - Qiu et al.'17: chance constraint on curtailment.
  - Monte Carlo sampling-based solution approaches.
  - Challenge: intractable as # of samples grows; imperfect understanding on $\mathbb{P}$.
  - Challenge: difficult trade-off.

Chance Constraint on Curtailment

- Chance constraint: $\mathbb{P}\{\xi \in \text{Admissible Range}\} \geq$ Threshold.
  - Threshold too low: low utilization.
  - Threshold too high: large dispatch cost.



Chance Constraint on Curtailment

- Chance constraint: $\mathbb{P}\{\xi \in \text{Admissible Range}\} \geq \text{Threshold}$.
  - Threshold too low: low utilization.
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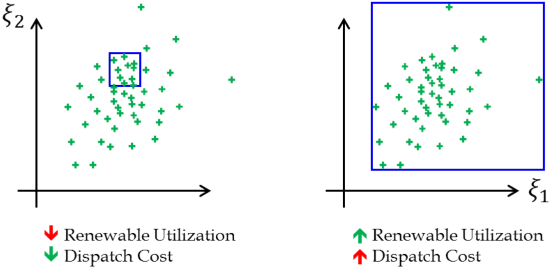
- Trade-off between:
  - Renewable utilization.
  - Dispatch cost.



Distributional Ambiguity

- Distributionally Robust Optimization.
- Ambiguity set $\mathcal{D}$.
  - A family of "plausible" probability distributions.
  - Data-driven.
  - Moment-based ambiguity set:

$$\mathcal{D} \;=\; \left\{\mathbb{P}:\; \mathbb{E}_{\mathbb{P}}[\xi] = \mu,\; \mathrm{Var}(\xi_i) = \sigma_i^2,\; \forall i\right\}.$$

- DR risk measures.

$$\mathbb{E}_{\mathbb{P}}[f(x,\xi)] \quad \rightarrow \quad \sup_{\mathbb{P}\in\mathcal{D}} \mathbb{E}_{\mathbb{P}}[f(x,\xi)]$$

$$\mathbb{P}\left\{f(x,\xi) \leq 0\right\} \geq 1-\epsilon \quad \rightarrow \quad \inf_{\mathbb{P}\in\mathcal{D}} \mathbb{P}\left\{f(x,\xi) \leq 0\right\} \geq 1-\epsilon$$

Our Proposal

$\forall \varepsilon_t \in [\varepsilon_t^{\mathsf{L}}, \varepsilon_t^{\mathsf{U}}]$, there exist $\{p_{it}(\varepsilon_t)\}_{i \in [I]}$ such that:

Power supply-demand balance,

Transmission line capacity (dc power flow),

Bounds on power supply,

Ramp-rate limits, ramping capacity within response time windows.

- Hypercube DNE limits $[\varepsilon_t^{\mathsf{L}}, \varepsilon_t^{\mathsf{U}}]$.

Our Proposal

$$\mathbb{P}\left(\varepsilon_t \in [\varepsilon_t^{\mathsf{L}}, \varepsilon_t^{\mathsf{U}}]\right) \geq u, \ \forall t \in [T],$$

$$u_0 \leq u \leq 1.$$

- Hypercube DNE limits $[\varepsilon_t^{\mathsf{L}}, \varepsilon_t^{\mathsf{U}}]$.
- Chance constraint with adjustable risk threshold u.

Our Proposal

$$\inf_{\mathbb{P}\in\mathcal{D}} \mathbb{P}\left(\varepsilon_t \in [\varepsilon_t^{\mathsf{L}}, \varepsilon_t^{\mathsf{U}}]\right) \geq u, \ \forall t \in [T],$$

$$u_0 \leq u \leq 1.$$

Ambiguity set:

$$\mathcal{D} := \left\{ \mathbb{P} : \begin{array}{l} \mathbb{E}_{\mathbb{P}}[\varepsilon_{kt}] = \mu_{kt}, \ \mathrm{Var}(\varepsilon_{kt}) = \sigma_{kt}^2, \\ \varepsilon_{kt} \text{ is unimodal about } \mu_{kt}, \ \forall k \in [K], \forall t \in [T] \end{array} \right\}.$$

- Hypercube DNE limits $[\varepsilon_t^{\mathsf{L}}, \varepsilon_t^{\mathsf{U}}]$.
- DR chance constraint with adjustable risk threshold u.
- Alternative DR operational risks. (omit)

Our Proposal

The model:

$$\min_{\hat{p},B,b,\varepsilon^{\mathsf{L}},\varepsilon^{\mathsf{U}},u} \quad \sum_{t\in[T]} \sum_{i\in[I]} C_i(\hat{p}_{it}) - \delta u$$

$$\text{s.t.} \quad \text{Power pre-distach}, \quad \text{DNE limits}, \quad \text{DR chance constraints.}$$

- Power pre-dispatch cost: $\sum_{t\in[T]} \sum_{i\in[I]} C_i(\hat{p}_{it})$.
- δ: trade-off between renewable utilization and dispatch cost.
- $\delta \uparrow$: renewable utilization $\uparrow$, dispatch cost $\uparrow$.
- Sweeping $\delta \Rightarrow$ cost-utilization frontier.

Conservative Approximation, DNE Limits

$$\exists\ \varepsilon^{\mathsf{L}}, \varepsilon^{\mathsf{U}}, p(\varepsilon):$$
$$Tx + Wp(\varepsilon) \leq H\varepsilon,\ \forall \varepsilon \in [\varepsilon^{\mathsf{L}}, \varepsilon^{\mathsf{U}}],$$

- Abstract form.

Conservative Approximation, DNE Limits

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&Tx + Wp(\varepsilon) \le H\varepsilon,\ \forall \varepsilon \in [\varepsilon^{\mathsf{L}}, \varepsilon^{\mathsf{U}}],\\
&p(\varepsilon) = B\varepsilon + b,
\end{aligned}$$

- Abstract form.
- Affine decision rule (ADR).

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&p(\varepsilon) = B\varepsilon + b,
\end{aligned}$$

- Abstract form.
- Affine decision rule (ADR).
- Define $E = \mathrm{diag}(\varepsilon^{\mathsf{U}} - \varepsilon^{\mathsf{L}})$.
- Then,

$$[\varepsilon^{\mathsf{L}}, \varepsilon^{\mathsf{U}}] \ = \ \{\varepsilon^{\mathsf{L}} + Ev :\ v \in [0, e]\}\,.$$

Conservative Approximation, DNE Limits

$$\exists\ \varepsilon^{\mathsf{L}}, \varepsilon^{\mathsf{U}}, B, b:$$
$$Tx + W\big(BEv + B\varepsilon^{\mathsf{L}} + b\big) \le H\varepsilon^{\mathsf{L}} + HEv, \quad \forall v \in [0, e].$$

Conservative Approximation, DNE Limits

$$
\begin{aligned}
&\exists\ \varepsilon^{\mathsf{L}}, \varepsilon^{\mathsf{U}}, B, b :\\
&Tx + W\big(BEv + B\varepsilon^{\mathsf{L}} + b\big) \le H\varepsilon^{\mathsf{L}} + HEv, \quad \forall v \in [0, e].
\end{aligned}
$$

is equivalent to

$$
\begin{aligned}
&\exists\ \varepsilon^{\mathsf{L}}, \varepsilon^{\mathsf{U}}, S, s_0 :\\
&Tx + W(Sv + s_0) \le H\varepsilon^{\mathsf{L}} + HEv, \quad \forall v \in [0, e].
\end{aligned}
$$

- Forward transformation: $S = BE$, $s_0 = B\varepsilon^{\mathsf{L}} + b$.
- Backward transformation: $B = SE^{-1}$, $b_0 = s_0 - SE^{-1}\varepsilon^{\mathsf{L}}$.

Reformulation, DR Chance Constraint

Theorem

If $u > 2/3$, then, for all $t \in [T]$, DR chance constraint is equivalent to the following second-order conic constraints:

$$\begin{aligned}
&\left\| \begin{bmatrix} \sqrt{\frac{8}{3}} \\ r_{kt} - z_{kt} \end{bmatrix} \right\|_2 \leq r_{kt} + z_{kt}, \ \forall k \in [K], \\
&\left\| \begin{bmatrix} s_{kt} - 1 \\ 2z_{kt} \end{bmatrix} \right\|_2 \leq s_{kt} + 1, \ \forall k \in [K], \\
&\sigma_{kt} r_{kt} \leq \mu_{kt} - \varepsilon^{\mathrm{L}}_{kt}, \ \forall k \in [K], \\
&\sigma_{kt} r_{kt} \leq \varepsilon^{\mathrm{U}}_{kt} - \mu_{kt}, \ \forall k \in [K], \\
&\sum_{k \in [K]} s_{kt} \leq 1 - u, \\
&r_{kt}, s_{kt}, z_{kt} \geq 0, \ \forall k \in [K].
\end{aligned}$$

Proof: based on Xie-Ahmed-Jiang '17.

Two IEEE Systems

- $T = 24$ hours.
- IEEE 14-bus system.
  - Generator and network characteristics from MATPOWER.
- IEEE 118-bus system.
  - Generator and network characteristics from MATPOWER.

Two IEEE Systems

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  - 2 wind farms: 80MW at node 5, 100MW at node 7.
- IEEE 118-bus system.
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  - 3 wind farms: 300MW at nodes 18, 32, 88.

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- IEEE 14-bus system.
  - Generator and network characteristics from MATPOWER.
  - 2 wind farms: 80MW at node 5, 100MW at node 7.
  - $\mathcal{D}$: (hypothetical) Gaussian data.
  - CPU time: 30.80s (average), 33.42s (max).
- IEEE 118-bus system.
  - Generator and network characteristics from MATPOWER.
  - 3 wind farms: 300MW at nodes 18, 32, 88.
  - $\mathcal{D}$: NREL Eastern Wind Dataset.
  - CPU time: 296.38s (average), 307.51s (max).

Results - 14 Buses

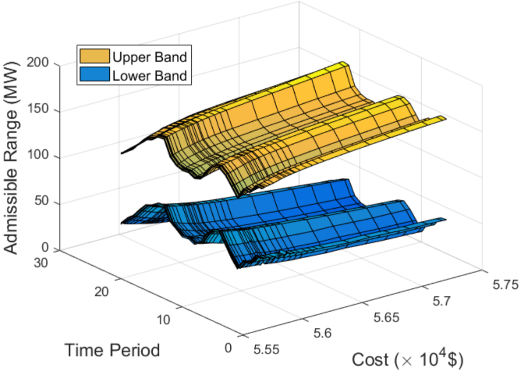

Figure: Minimum Dispatch Costs vs. DNE Limits of Wind Power

Results - 14 Buses

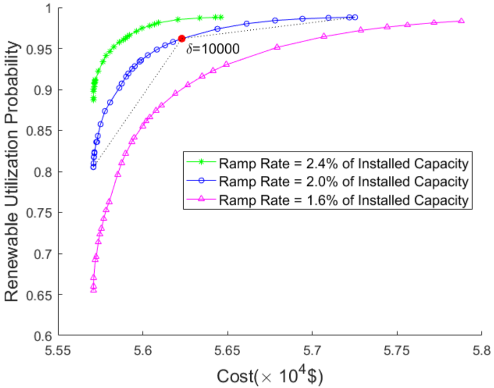

Figure: Cost-Utilization Frontier under Various Ramping Capabilities

Sequential Approach vs. Co-Optimization

- ODNE = power dispatch first, then DNE limits (cf. Zhao et al.'16).
- IDNE = our co-optimization approach.
- Out-of-sample comparison.

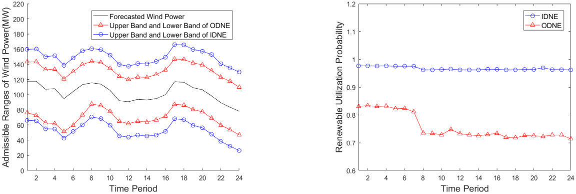

Figure: Comparison on (a) Admissible Range and (b) Renewable Utilization Probability

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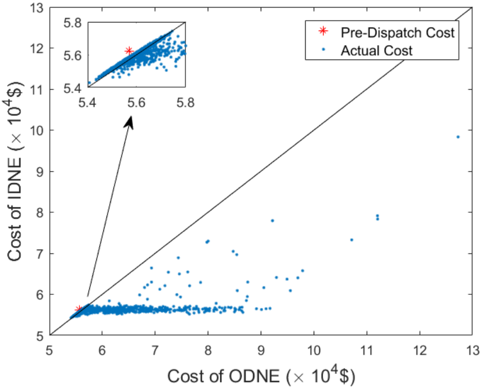

Figure: Comparison on Actual Cost

Results - 118 Buses

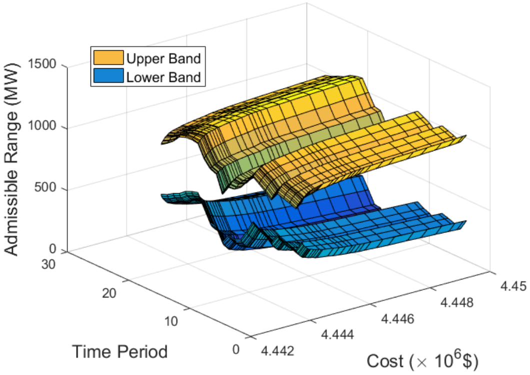

Figure: Minimum Dispatch Cost vs. Admissible Ranges of Wind Power

Results - 118 Buses

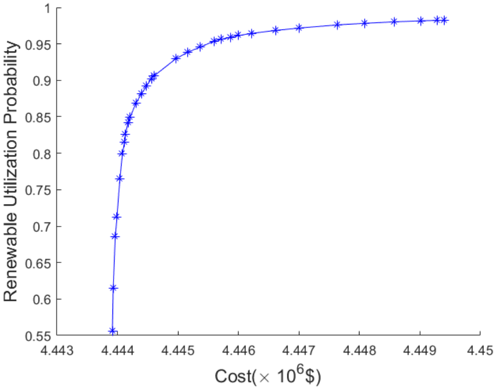

Figure: Cost-Utilization Frontier

Takeaways

- Co-optimizing power dispatch and DNE limits benefits significantly.
- Ramping capability $\propto$ system flexibility.

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- Manuscript available online:
  `https://arxiv.org/abs/1808.02007`

THANK YOU!