Dynamics & Optimization of Natural Gas Systems

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Funded by DOE (GMLC & Grid Science)

Third Grid Science, Santa Fe, Jan 9 2019

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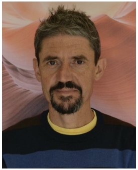
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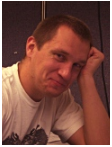
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Provocative pre-thoughts

- In energy systems, there is life beyond power systems ...

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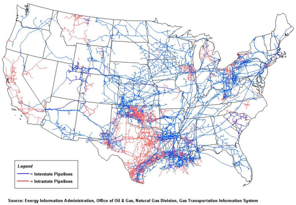
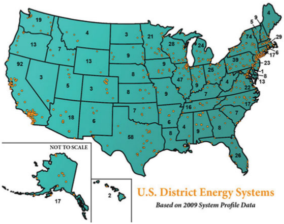
- In energy systems, there is life beyond power systems ...
- Optimizations need to be formulated & challenged ...
- For any formulation there may be many algorithms ...
- Physics/dynamics/statistics of the flows need to be integrated in ...
- ... be ready to leave your comfort zone

Energy (=Power, Gas, Heat) Systems



U.S. District Energy Systems Map



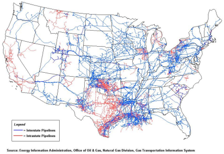
Gas & Heat systems undergo revolutionary changes too

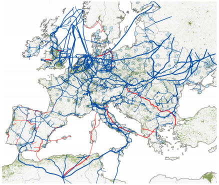
- significant recent (gas) & future (heat) growth
- power-gas-heat emerging interdependence

Towards Smart District Energy Systems

- Need to be smart in planning, operations, predictions
- Subject to uncertainty
- Inter-dependent = power, gas & heat

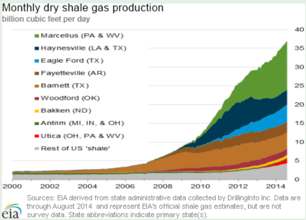
Natural Gas Systems

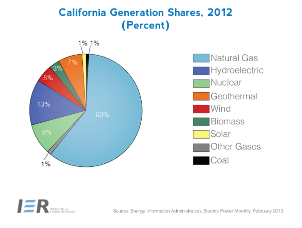




- needs to be smart too
- significant growth

Natural Gas Increasing Penetration





Applied Mathematics & Theoretical Engineering for Energy Systems

Methodology: posing & solving problems in

(1) **Direct** Models [Power/Gas/Heat Flows; Static → Dynamic; **Deterministic** → **Statistical**/Inference] ⇒

(2) Inverse Problems = **Machine Learning** & Data Analytics [State Estimation; Parameter Identification] ⇒

(3) **Optimization** & **Control** [Deterministic; Stochastic; Optimal Power/Gas/Heat Flows (Deterministic → Robust, Chance-Constrained → Distributionally Robust)] ⇒

(4) **Planning** & other multi-stage mixed problems

What are proper/interesting/relevant

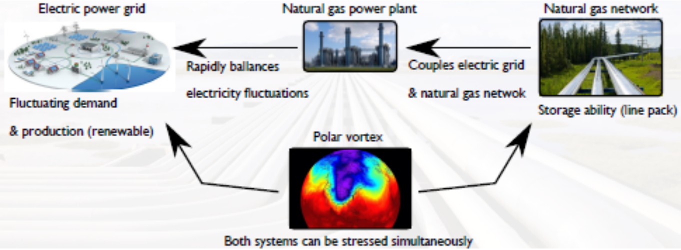
- problem formulations ?
- math-sound & physics-relevant methods, tools, solutions ?

Outline

1. Dynamics of Natural Gas Systems
   - Motivation. Structure.
   - Gas Dynamics & Line Pack
   - Uncertainty. Monotonicity.

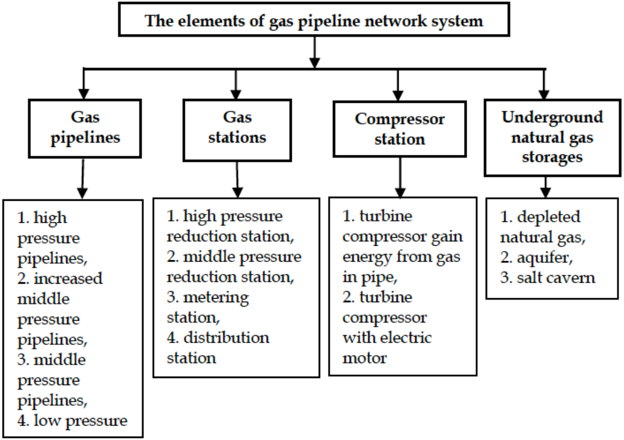
2. Optimization & Control of Natural Gas Systems
   - Static & Dynamic Optimal Gas Flow

Motivation: Gas-Grid Interdependence – Extreme Events



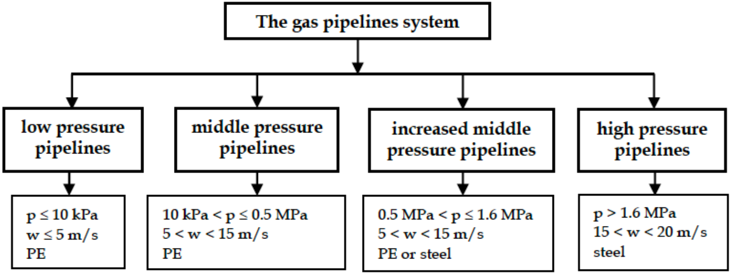
- The interdependence is amplified under extremes [NM - winter 2011, NE-ISO & PJM - winter 2014]
- Physics of Natural Gas systems, e.g. dynamics [line pack], adds complexity

Grid Structure & Regimes



[from The Gas Transportation in Natural Gas Networks, J. Szoplik (2014)]

Grid Structure & Regimes



+ **interstate pipe-lines** 1.6-14 MPa, 1MPa=145 psi

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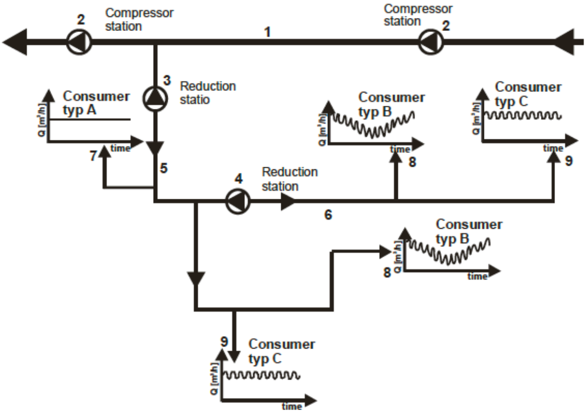
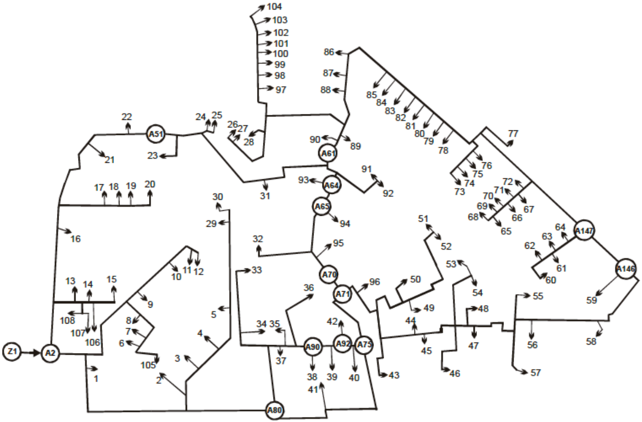
Grid Structure & Regimes



consumers are different, modeling is tricky

[from The Gas Transportation in Natural Gas Networks, J. Szoplik (2014)]

Grid Structure & Regimes



distribution is loopy! **transmission** is (largely, not always) not

[from The Gas Transportation in Natural Gas Networks, J. Szoplik (2014)]

Grid Structure & Regimes

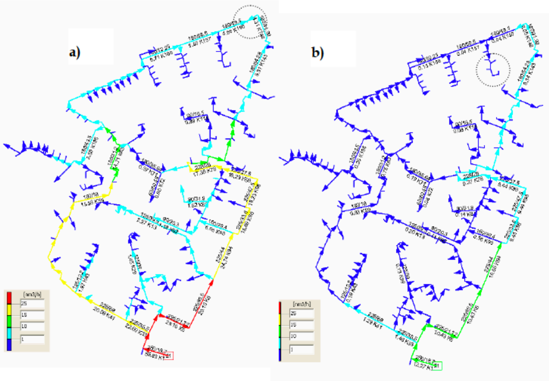
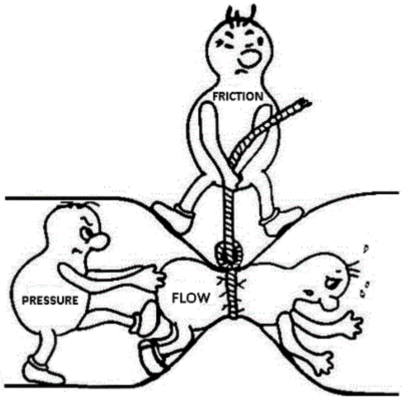


Fig. 11. Gas stream arrangement in pipelines network; t ≥ 18 °C; a) results for 7 am (p_{z1} = 1707 Pa; Q_{feed} = 55.83 m³/h); b) results for 3 am (p_{z1} = 1706 Pa; Q_{feed} = 12.27 m³/h).

gas flows change ... often (hours)

[from The Gas Transportation in Natural Gas Networks, J. Szoplik (2014)]

Crash course on the hydro (gas) dynamics



crash course on the hydro (gas) dynamics

- single pipe; not tilted (gravity is ignored); constant temperature
- ideal gas, $p \sim \rho$ – pressure and density are in a linear relation
- all fast transients are ignored – gas flow velocity is significantly slower than the speed of sound, $u \ll c_s$
- turbulence is modeled through turbulent friction; mass flow, $\phi = u\rho$, is averaged across the pipe's crossection

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Mainly interested in the regime of slow dynamics

- seconds-minutes and longer dynamics
  - driven = varying in time injection/consumption
  - sound waves are dumped [at sub-seconds]

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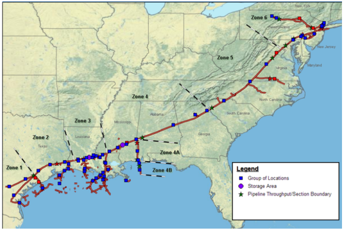
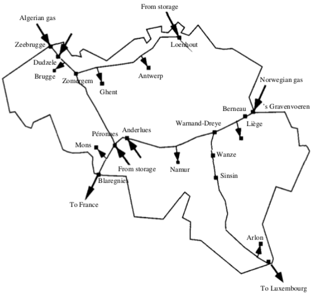
Balanced, stationary

Unbalanced, non-stationary, line pack

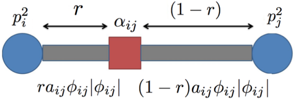
Stationary (balanced) Gas Flows



Belgian gas network.

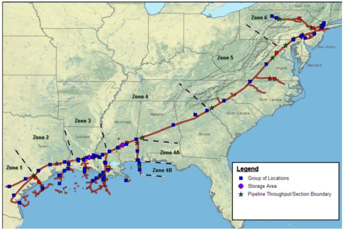


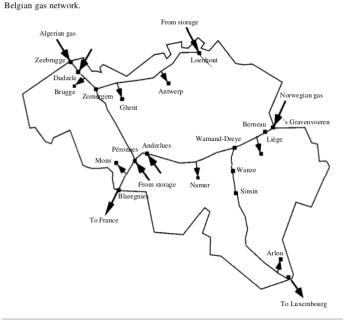
without compressors, $\alpha_{ij} = 1$



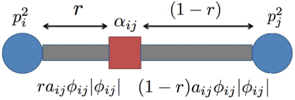
- Gas Flow Equations: $(\sum_i q_i = 0, \quad a_{ij} = L_{ij}\beta_{ij}/D_{ij})$
  $\forall (i,j): \quad p_i^2 - p_j^2 = a_{ij}\phi_{ij}^2$
  $\forall i: \quad q_i = \sum_{j:(i,j)\in\mathcal{E}} \phi_{ij} - \sum_{j:(j,i)\in\mathcal{E}} \phi_{ji}$

Stationary (balanced) Gas Flows





with compressors, $\alpha_{ij} \lessgtr 1$



- Gas Flow Equations: $\left(\sum_i q_i = 0, \quad a_{ij} = L_{ij}\beta_{ij}/D_{ij}\right)$

$$\forall (i,j): \quad \alpha_{ij}^2 = \frac{p_j^2 + (1-r)a_{ij}\phi_{ij}^2}{p_i^2 - r a_{ij}\phi_{ij}|\phi_{ij}|}$$

$$\forall i: \quad q_i = \sum_{j:(i,j)\in\mathcal{E}} \phi_{ij} - \sum_{j:(j,i)\in\mathcal{E}} \phi_{ji}$$

Approximating Transient Flows:

Computational Schemes [developed at LANL/ANSI]

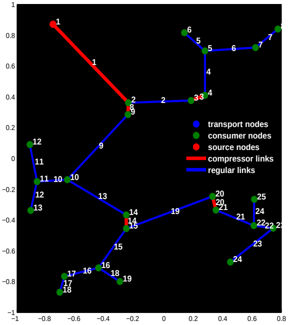
- Lump-elements ODE (our main tool for optimization constrained by PDEs). Divide pipe into segments for sufficient approximation of transients.
- Split-step method - new for gas systems [UNM-collaboration] most accurate = golden standard, allows extension to fast sound/shock transients [sub-seconds], e.g. physics modeling of turbulent dissipation and sound-scattering requires modification]
- Adiabatic method [new for gas systems (related to approach of M. Herty)] = approximate reduction from PDEs to nodal ODEs - much faster and sufficiently more accurate [at seconds and slower] - critical for extreme [e.g. large scale] simulations

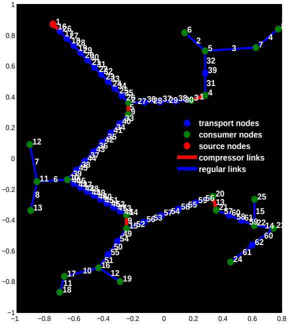
Developing all three

- To validate in the regime of overlap (slow transients)
- To access broader coverage of different extremes [physics]
- To extend applicability for mitigation (optimization & control)

Gas Network: Model Example

- A tree network with 25 joints and 24 pipelines, 1 source, 5 compressors
- Total length: 477km, $d_j = 36$ or 25 inches, $\lambda = 0.011$.





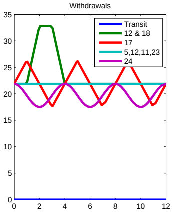
- auxiliary-nodes are added to spatially discretize pipelines longer than 10 km $\rightarrow$ 62 nodes [lump-element simulations]

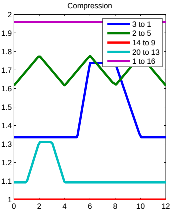
Dynamic Network Simulation [lump-elements ODEs]

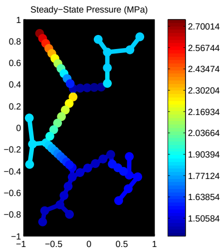
Line Pack

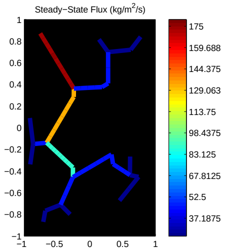
- total injection $\neq$ total consumption
- balance is achieved over day
- transient compressor also contributes to dynamics

- Withdrawals from 8 terminal nodes, 4 with transients
- Transient compressions at 5 joints
- A feasible steady state gas flow is computed
- Simulation using ode15s with adaptive step and relative tolerance 10^{-3}

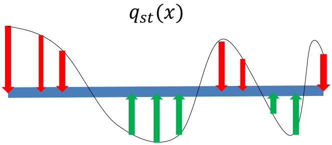








[MC, S. Backhaus, V. Lebedev + S. Dyachenko, A. Korotkevich, A. Zlotnik, 2014-16]



Steady (balanced) continuous profile
of gas injection/consumption

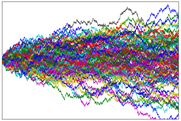
- $q(t,x) = q_{st}(x) + \xi(t,x)$
  - $q_{st}(x)$ is the forecasted consumption/injection of gas
  - $\xi(t,x)$ actual fluctuating/random profile of consumption/injection, e.g. **gas plant follows wind turbines**

Exemplary case: One dimensional (1+1) model – distributed injection/consumption and compression

- mass balance: $c_s^{-2}\partial_t p + \partial_x \phi = -q(t,x)$
- momentum balance: $\partial_x p + \frac{\beta}{2d}\frac{\phi|\phi|}{p} = \gamma(x)p$
- $\gamma(x)$ – distributed compression – assumed known

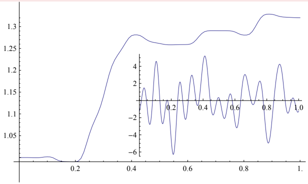
Diffusive Jitter $= \mathbb{E}\left[(p(t,x) - p_{st}(x))^2\right] \doteq \mathbb{E}\left[\delta p(t,x)^2\right] = t \cdot D(x)$

- **spatio-temporal fluctuations of actual pressure** (unbalanced/line pack) on the top of the steady/optimized/inhomogeneous forecast



$$\mathbb{E}\left[\delta p(t,x)^2\right] \to t \cdot D(x)$$

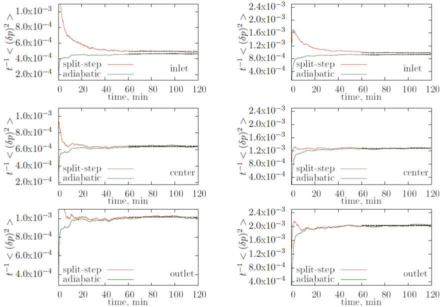
Diffusion coefficient shows local extrema at the points of flow reversals



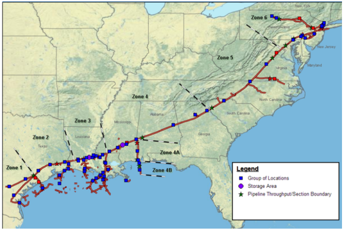
Adiabatic [perturbative]

- $q_{st}(x)$ is shown in inset – distributed injection/consumption, $q_{st}(0) = q_{st}(L) = 0$
- $\gamma(x)$ is chosen to get p_{st} = const (example)

Diffusive Pressure Jitter – Validation [split-step] & Confirmation [jitter]

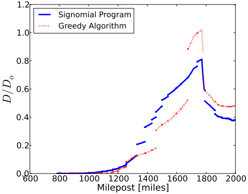


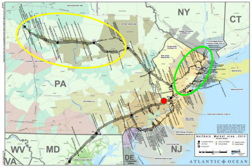
- Averaging over 4000 realizations
- Split-step vs Adiabatic
  - [non-perturbative]
- Adiabatic approach is validated
- Jitter phenomenon is qualitatively confirmed
- Extends to networks



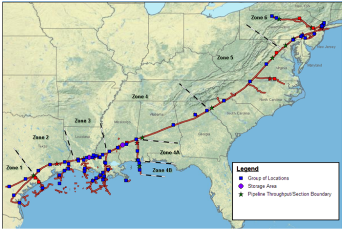
- Transco data - available online at `http://www.1line.williams.com/Transco/index.html`
- 24 hours period on Dec 27, 2012; $\phi_0 \approx 20kg/s$; ≈ 70 nodes; pressure range $500-800psi$
- mile post 1771 (large load in NJ, NYC)
- Marcellus shell (mile post 2000)
- mile post 1339 (large load in NC)

Diffusion coefficient as a function of distance along the Transco mainline





- peak at milepost 1771 (point of steady flow reversal)
- peak is at the same location for two distinct steady solutions (resulting from optimizations)
- peak is much higher for the greedy (steady) case



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Diffusion coefficient as a function of distance along the Transco mainline

Main points (qualitative):

- diffusive jitter (coefficient) is proper indicator/measure of the gas/pressure stress
- change in the steady/forecasted solution has a profound effect on the pressure fluctuations

Complications [additional to line pack]

- Uncertainty of loads +
  - gas generators [responding to fluctuations/uncertainty on the power side]
  - city-gates [primary customers]
  - both subject to failures/treats
  - compressors and (other) controls can also be uncertain

Can we deal with the uncertainty nested in optimization?

- Even when we set boundaries ... there is continuum of scenarios
- What to do with the continuum? Reduction to fewer scenarios?

Math & Physics Framework

Reminder:

Nodal Production/Consumption

$q(t)$

Flow in a Pipeline

$\phi(x,t)$

Density of Gas in a Pipeline

$\rho(x,t)$

Dynamic Flow Equations

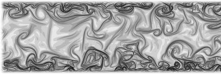
Mass Conservation

$$\partial_t \rho(x,t) + \partial_x \phi(x,t) = 0$$

Momentum Balance

$$\partial_x(\rho(x,t)^2) = \phi(x,t)\,|\phi(x,t)|$$

Nonlinear Partial Differential Equations



Steady State Flow Equations

$$\rho_{out}^2 - \rho_{in}^2 = \phi_{pipe}|\phi_{pipe}|$$

Algebraic Nonlinear Equations

Math & Physics Framework

Generalization – Compressible [Potential-Dissipative] Flows:

Nodal Production/Consumption

$q(t)$

Flow in a Pipeline

$\phi(x,t)$

Density of Gas in a Pipeline

$\rho(x,t)$

Dynamic Flow Equations

Mass Conservation

$$\partial_t \rho(x,t) + \partial_x \phi(x,t) = 0$$

Momentum Balance

$$\partial_x \rho(x,t) = F^d\big(\rho(x,t), \phi(x,t)\big)$$

Steady State Flow Equations

$$\rho_{out} = F^s(\rho_{in}, \phi)$$

Assumption

$$\frac{\partial}{\partial \phi} F(\rho, \phi) > 0$$

Monotonicity in Natural Gas (+) Networks

Steady State Flow Equations

$$\rho_{out} = F^s(\rho_{in}, \phi)$$

Ordered Consumptions at every node

$$\underline{q} \leq q \leq \bar{q}$$

Aquarius Theorem: Solutions Exists $\Rightarrow$ Order Preserved

$$\underline{\rho} \leq \rho \leq \bar{\rho}$$

Monotonicity in Natural Gas (+) Networks

Dissipative Flow Equations

Mass Conservation

$$\partial_t \rho(x,t) + \partial_x \phi(x,t) = 0$$

Generalized Momentum Balance

$$\partial_x \rho(x,t) = F^d\big(\rho(x,t), \phi(x,t)\big)$$

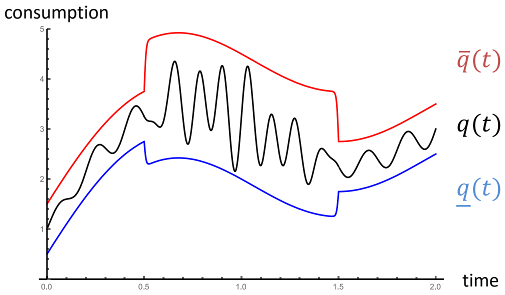
Ordered Initial Conditions

$$\underline{\rho}(x,0) \leq \rho(x,0) \leq \bar{\rho}(x,0)$$

Ordered Consumptions

$$\underline{q}(t) \leq q(t) \leq \bar{q}(t)$$

$$\Longrightarrow$$



Aquarius Theorem: Order Preserved

$$\underline{\rho}(x,t) \leq \rho(x,t) \leq \bar{\rho}(x,t)$$

Monotonicity ⇒ Robust Feasibility Tractable

Is a fixed operating point


safe (feasible)

$\underline{\rho}_i \leq \rho_i \leq \bar{\rho}_i$

under uncertain consumption?

$q_i \in [\underline{q}_i, \bar{q}_i]$

A priori hard problem

For infinite number of scenarios test existence of safe flows

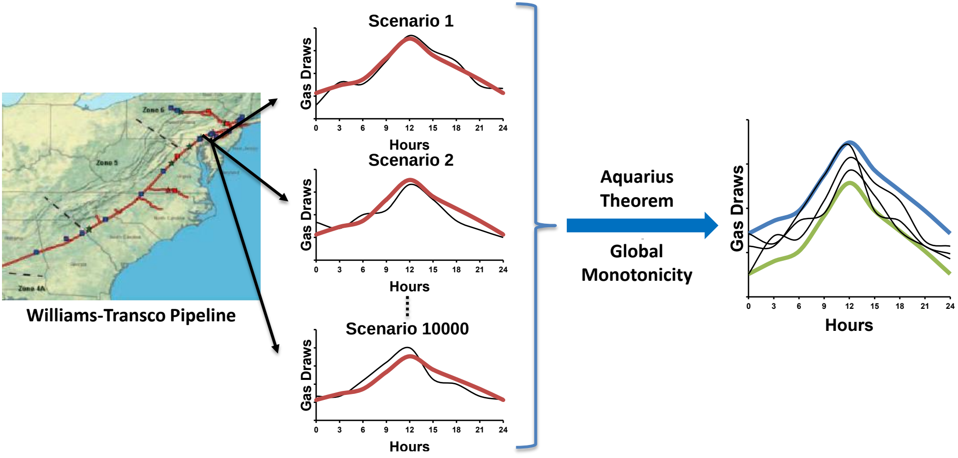
Aquarius Theorem (Dynamic & Static Dissipative Networks)

Densities are monotone with consumption

Only two extreme scenarios have to be tested for the entire network

$q_i = \underline{q}_i \qquad q_i = \bar{q}_i$

Example of Monotonicity for Scenario/Cases Reduction



Outline

Static Optimal Gas Flow

Minimizing the cost of compression ($\sim$ work applied externally to compress)

$$\min_{\alpha,p} \sum_{(i,j)} \frac{c_{ij}\phi_{ij}}{\eta_{ij}} \left(\alpha_{ij}^m - 1\right)^+ \Bigg| \begin{array}{l} \forall (i,j): \quad \alpha_{ij}^2 = \frac{p_j^2 + (1-r)a_{ij}\phi_{ij}^2}{p_i^2 - r a_{ij}\phi_{ij}^2} \\ \forall i: \quad 0 \le \underline{p}_i \le p_i \le \overline{p}_i \\ \forall (i,j): \alpha_{ij} \le \overline{\alpha}_{ij} \end{array}$$

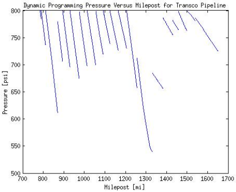
- $0 < m = (\gamma-1)/\gamma < 1$, γ- gas heat capacity ratio (thermodynamics)

- The problem is **convex on trees** (many existing gas transmission systems are trees) $\Leftarrow$ through GeometricProgramming (log-function transformation)

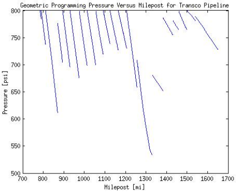
- **S. Misra, M. W. Fisher, S. Backhaus, R. Bent, MC, F. Pan**, *Optimal compression in natural gas networks: a geometric programming approach*, IEEE CONES 2014

Static OGF experiments (Transco pipeline)

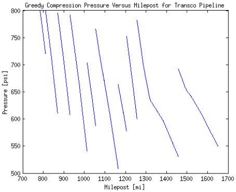
Dynamic Programming of (Wong, Larson '68)



Geometric Programming (ours)



Greedy Compression (current practice)



GP is advantageous over DP

- Exact = no-need to discretize.
- Faster. Allows distributed (ADMM) implementation.

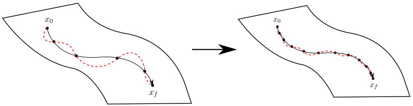
- Convexity is lost in the loopy case. However, efficient heuristics is available.
- This is only one of many possible OGF formulations. Another (Norvegian/European) example – maximize throughput.
- Major handicap of the formulation (ok for scheduling but) = did not account for the **line pack** (dynamics/storage in lines for hours) $\Rightarrow$

Optimal Control: Economic Transient Compression (ETC)

- Minimize cost of compression such that pressure is maintained within bounds
- State functions are $\rho(t) \in \mathbb{R}^{V-1}$ and $\varphi(t) \in \mathbb{R}^{E}$
- Control functions are $\alpha(t) \in \mathbb{R}^{C}$ where $C < 2E$

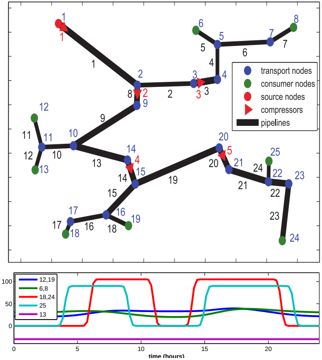
$$
\begin{array}{lll}
\textbf{ETC}: & \min\limits_{\alpha} & \displaystyle\int_0^T \sum_{\{i,j\}\in\mathcal{E}} \frac{1}{\eta_{ij}} |\varphi_{ij}(t)|((\alpha_{ij}(t))^m - 1)\mathrm{d}t \\
& s.t. & |A_d|\Lambda|B_d^T|\dot{\rho} = 4(A_d\varphi - d) - |A_d|\Lambda|B_s^T|\dot{s} \\
& & \dot{\varphi} = -\Lambda^{-1}(B_s^T s + B_d^T \rho) - Kg(\varphi, |B_s^T|s + |B_d^T|\rho) \\
& & \rho^i_{\min} \le \alpha_{ij}(t)\rho_i(t) \le \rho^i_{\max}, \qquad \forall\{i,j\}\in\mathcal{E} \\
& & \mathbf{1} \le \alpha_{ij}(t) \le \alpha_{ij}^{\max} \qquad \forall\{i,j\}\in\mathcal{E} \\
& & \rho(0) = \rho(T), \qquad \varphi(0) = \varphi(T)
\end{array}
$$

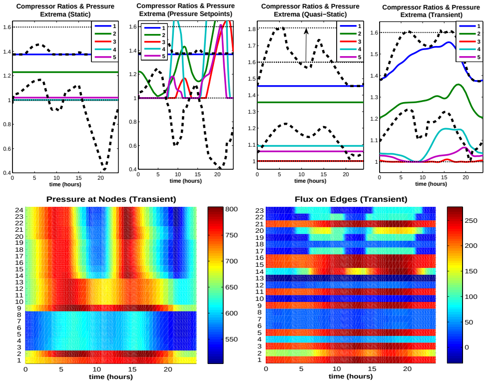
- State at each time instance is function with compact support
- Convert into algebraic equations using spectral approximation



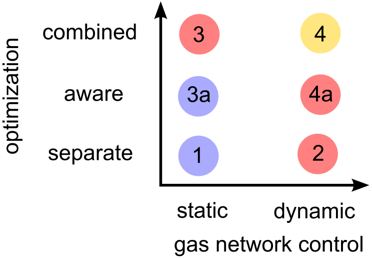
Dynamic Optimal Gas Flow: Control of Compression – ETC

- Thick and thinner lines indicate 36” and 25” pipes, respectively. Pressure is bounded between 500 and 800 psi on all pipes. Friction factor and sound speed are $\lambda = 0.01$ and $c_s = 377.968$ m/s.





Gas-Grid Coordination Scenarios



1 Optimal Power Flow (OPF) and Static Optimal Gas Flow (OGF) solved separately.

2 OPF and Dynamic OGF solved separately.

3a OPF solved with gas dynamics constraints following from static OGF.

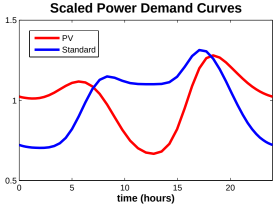
4a OPF solved with gas dynamics constraints following from dynamic OGF.

3 OPF and OGF solved together with a limited overlap (gas compressor setpoints).

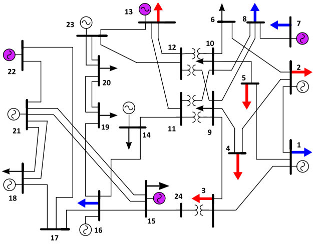
4 OPF and dynamic OGF solved together.

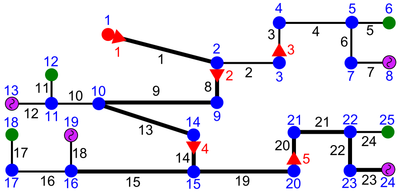
Gas-Grid Coupling

- Gas generators fuel use is a quadratic function of power: $h_i(t) = q(p_i(t)) = q_0 + q_1 p_i(t) + q_2 p_i^2(t)$
- Minimize combined OPF and Dynamic OGF objectives, satisfy all constraints
- 40%-60% of gas for power
- Local Distribution Companies (LDCs) located at gas nodes 6, 12, 18, 25 [green]



- Gas units at power/gas nodes (22/8), (15/13), (13/24), (7,19) [purple]
- 2724 MW prod. capacity, approx 550,000 mmBTU gas moved (base case)

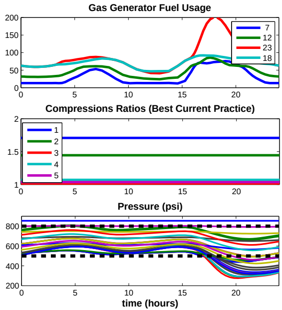


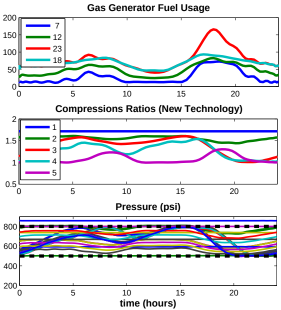


Current Practice vs New Technology – High Stress Case

1: OPF is solved, 15% extra capacity requested. Static OGF solved.

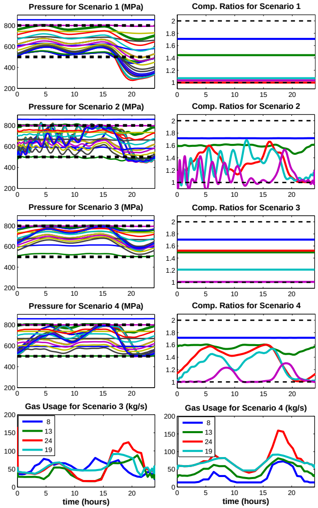
4: OPF and dynamic OGF with full coordination





- 300 psi-days pressure violation (certain supply disruption) vs. 1 psi-day
- Co-optimization yields alternative day-ahead generation dispatch and transient compression protocols that keep gas system pressure within bounds

Comparison of Scenarios and Stress Cases



DC OPF objective ($ $\times 10^6$)

	1	2	3	4
low	0.5972	0.5972	0.5971	0.5971
base	0.7316	0.7316	0.7532	0.7316
high	0.8256	0.8256	1.0250	0.8883

Gas Usage for Generation (mmBTU $\times 10^3$)

	1	2	3	4
low	214.46	214.46	214.15	214.14
base	306.08	306.08	309.81	305.93
high	380.15	380.15	340.75	362.44

Pressure Violation Norm (psi-days)

	1	2	3	4
low	4.5794	0.1146	0.1309	0.0843
base	83.751	0.1923	0	0.0255
high	303.61	56.925	0	1.0802

Take home messages [from LANL + about natural gas systems]

Developed [practical/algorithmic] approaches, tools and capabilities in

- modeling and simulating line pack efficiently
- dealing with uncertainty $\infty \to 2$
- describing and developing [projecting to future] dynamic optimization/control schemes
- accessing consequences of gas-grid interaction [analysis, optimization, control, mitigation of stesses/uncertainty]

LANL/ANSI papers on "gas" + "gas-grid" so far [arxiv]

- M. Chertkov, A. Korotkevich, Adiabatic approach for natural gas pipeline computations, CDC 2017.
- S. Dyachenko, A. Zlotnik, A. Korotkevich, M. Chertkov, Operator Splitting Method for Dynamic Simulations of Flows in Natural Gas Transportation Networks, Physica D 2017.
- M. Chertkov, V. Lebedev, S. Backhaus, D. Dyachenko, A. Korotkevich, A. Zlotnik, Nonlinear Stochastic Dynamics of Pressure in Natural Gas Systems, in preparation.
- A. Zlotnik, L. Roald, S. Backhaus, M. Chertkov, and G. Andersson, Control Policies for Operational Coordination of Electric Power and Natural Gas Transmission Systems, ACC (American Control Conference) 2016.
- A. Zlotnik, S. Misra, M. Vuffray, and M. Chertkov, Monotonicity of Actuated Flows on Dissipative Transport Networks, ECC (European Control Conference) 2016.
- A. Zlotnik, M. Chertkov, K. Turitsyn, Assessing risk of gas shortage in coupled gas-electricity infrastructures, HICSS-49/2016.
- A. Zlotnik, L. Roald, M. Chertkov, S. Backhaus, Coordinated Operational Planning for Integrated Electric Power and Natural Gas Infrastructures, IEEE Power Systems 2016.
- A. Zlotnik, M. Chertkov, S. Backhaus, Optimal Control of Transient Flow in Natural Gas Networks, CDC 2015.
- K. Dvijotham, M. Vuffray, S. Misra, M. Chertkov, Natural Gas Flow Solutions with Guarantees: A Monotone Operator Theory Approach, arXiv:1506.06075.
- M. Vuffray, S. Misra, M. Chertkov, Monotonicity of Dissipative Flow Networks Renders Robust Maximum Profit Problem Tractable: General Analysis and Application to Natural Gas Flows, CDC 2015.
- S. Misra, M. Vuffray, M. Chertkov, Maximum Throughput Problem in Dissipative Flow Networks with Application to Natural Gas Systems, CDC 2015.
- M. Chertkov, M. Fisher, S. Backhaus, R. Bent, S. Misra, Pressure Fluctuations in Natural Gas Networks caused by Gas-Electric Coupling, HICSS-48/2015.
- M. Chertkov, V. Lebedev, S. Backhaus, Cascading of Fluctuations in Interdependent Energy InfrastructuresGas-Grid Coupling, Applied Energy 2015.
- S. Misra, M. W. Fisher, S. Backhaus, and R. Bent, M. Chertkov, F. Pan, Optimal compression in natural gas networks: a geometric programming approach, IEEE Transactions on Control of Networks 2015.

What is next – plans/what? [energy systems]

- continue to work on **modeling & controlling uncertainty** in interacting energy + infrastructures [power, gas, district heating, water, communications]
  - specific networks [real models, data driven]
  - modeling: physics of flows $\to$ proper model reduction $\to$ state-of-the-art dynamic simulations
  - statistical tools: Graphical Models, Machine Learning [physics informed, algorithms]
  - optimization & control [algorithms]
- analysis of fast (sub-seconds) **wave phenomena** ... propagation of
  - sound/shock waves (natural gas, district heating, water)
  - electro-mechanical waves (power)

  through energy + networks/systems
  - Damping & Interaction.
  - Localization & Timing of a Source/Perturbation
  - Analysis & Prevention of Damage
- **District** level optimization & control including **electric- & gas-** systems

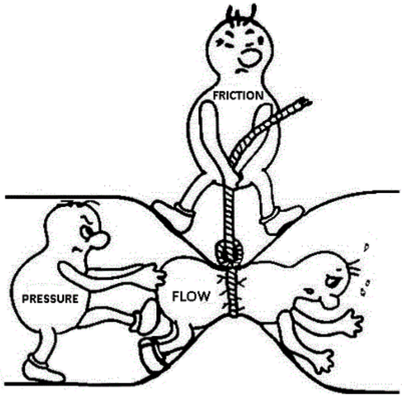
What is next – tactics/how? [energy systems]

What? Energy Systems
Focus? District Level

- Power Grids
- Natural Gas Systems
- District Heating Systems
- Inter-dependent Infrastructures
- + Cyber

How? Theoretical Engineering

- Physical Network Flow
  - Static/Feasibility
  - Dynamical Systems/Tranasients
  - Uncertainty, Fluctuations
- Applied Statistics/Probability
  - Statistical Physics/Mechanics
  - Graphical Models
  - Machine Learning
- Optimization
  - Stochastic (Chance-Constrained)
  - Robust
  - Optimal Control



Thank You!

Appendix Outline

Outline

[**A. Zlotnik**, MC, S. Backhaus, 2015-2016]

- Nondimensionalization: $\tilde{t}=\frac{t}{\ell/a}$, $\tilde{x}=\frac{x}{\ell}$, $\tilde{\rho}=\frac{\rho}{\rho_0}$, $\tilde{\phi}=\frac{\phi}{a\rho_0}$
- Nondimensional equations: $\partial_t\rho+\partial_x\phi=0$, $\partial_t\phi+\partial_x\rho=-\frac{\lambda\ell}{2D}\frac{\phi|\phi|}{\rho}$
- Lumped-element approximation: integrate equations along the pipe

$$\int_0^L(\partial_t\rho+\partial_x\phi)dx=0,\qquad \int_0^L(\partial_t q+\partial_x\rho)dx=-\frac{\lambda\ell}{2D}\int_0^L\frac{\phi|\phi|}{\rho}dx$$

- Evaluate and approximate integrals:

$$\frac{L}{2}(\rho_0+\rho_L)_t=\phi_0-\phi_L,\qquad \frac{L}{2}(\phi_0+\phi_L)_t=(\rho_0-\rho_L)-\frac{\lambda\ell L}{4D}\frac{(\phi_0+\phi_L)|\phi_0+\phi_L|}{\rho_0+\rho_L}$$

- Divide pipe into segments for sufficient approximation of transients of interest

gas-dynamic

[**S. Dyachenko, A. Korotkevich**, A. Zlotnik, MC, S. Backhaus, 2015-16]

$$c_s^{-2}\partial_t p + \underbrace{\partial_x \phi}_{linear} = 0$$

$$\partial_t \phi + \underbrace{\partial_x p}_{linear} + \underbrace{\frac{\beta}{2pD}\phi|\phi|}_{nonlinear} = 0$$

- solved alternating linear and nonlinear steps

Properties of the Split-step scheme

- the scheme is exact = conserves the total amount of gas in the pipe
- unconditionally stable
- explicit (both linear and nonlinear)
- second order accurate (high order generalizations are possible)

- First application of the split-step in natural gas dynamics
- Expected to be uniquely suitable/appropriate to describe fast transients – equations need to be modified to account for scattering/dissipation of high frequency waves on turbulence (pipe boundary layer) proper

◂ gas-dynamic

[**MC, V. Lebedev**, S. Backhaus + S. Dyachenko, A. Korotkevich, A. Zlotnik, 2014-16]

Allows reduced description = only network-nodal ODEs

$$\forall (i\to j)\in\mathcal{E}:\ \ \phi_{i\to j}=\mathrm{sgn}(p_i-p_j)\sqrt{\frac{|p_i^2-p_j^2|}{\alpha_{ij}L_{ij}}+\frac{2L_{ij}}{15c_s^2}}$$

$$*\left(F^{(0,1)}(p_i,p_j)\frac{d}{dt}p_i+F^{(1,0)}(p_i,p_j)\frac{d}{dt}p_j\right)$$

$$\forall i\in\mathcal{V}:\ \ q_i=\sum_{j:(i,j)\in\mathcal{E}}\phi_{i\to j}$$

$$\forall i\in\mathcal{V}_c,\ \ (i,j),(i,k)\in\mathcal{E}:\ \ p_{i\to j}=\gamma_{j\to k}p_{i\to k}$$

$$F(p_1,p_2)\doteq\frac{3p_1^3+6p_1^2p_2+4p_1p_2^2+3p_2^3}{(p_1+p_2)^2}$$

$$F^{(0,1)}(p_1,p_2)\doteq\partial_{p_1}F(p_1,p_2),\quad F^{(1,0)}(p_1,p_2)\doteq\partial_{p_2}F(p_1,p_2)$$

- derived under assumptions that changes are slow, driven by consumption/production
- allows efficient (explicit) high-order network scheme
- validated against split-step & lump-element

gas-dynamic

Long pipe with distributed compression + injection/consumption

$$c_s^{-2}\partial_t p+\partial_x\phi=q_{st}(x)+\xi(t,x),\ \ \partial_x p+\alpha\frac{\phi|\phi|}{2p}=\gamma(x)p$$

Stationary Solution

$$\phi_{st}(x)=\phi_0+\int_0^x dx'\, q_{st}(x')$$

$$(p_{st}(x))^2=\mathcal{Z}(x)\left(p_L^2+\int_x^L\frac{dx'\,\alpha\phi_{st}(x')|\phi_{st}(x')|}{\mathcal{Z}(x')}\right)$$

$$\mathcal{Z}(x)\doteq\exp\left(-2\int_x^L dx'\,\gamma(x')\right)$$

Full solution – in adiabatic approximation

$$\frac{p(t,x)^2}{\mathcal{Z}(x)}\approx(p_L(t))^2+\int_x^L\frac{dx'\,\alpha\phi_{st}(x')|\phi_{st}(x')|}{\mathcal{Z}(x')}$$

$$\frac{d}{dt}(p_L(t))^2\int_0^L\frac{\exp\left(-\int_x^L dx'\,\gamma(x')\right)dx}{2\sqrt{(p_L(t))^2+\int_x^L dx'\,\alpha\phi_{st}(x')|\phi_{st}(x')|}}$$

$$=c_s^2\int_0^L dx\,\xi(t,x)$$

- $\Rightarrow$ Diffusive Jitter in the result of averaging over $\xi(t,x)$

◂ distributed pipe/diffusive jitter

Adiabatic Approach

Setting discussed in CDC 2017 by MC & Korotkevich

- Single pipe of length L, $x \in [0, L]$
- Dynamics of flow, ϕ, and pressure, p, is governed by
  - $c_s^{-2}\partial_t p + \partial_x \phi = 0, \quad \partial_x p + \alpha \frac{\phi|\phi|}{2p} = 0$
  - initial conditions $(t = 0) =$ steady, balanced solution
  - boundary conditions – e.g. $p(t; 0)$ and $p(t; L)$ are fixed - introduce time scale (slower then speed of sound)

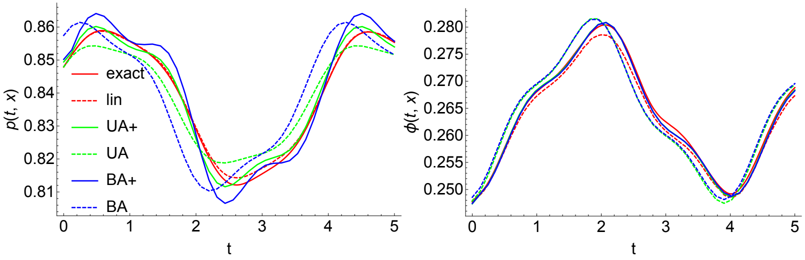
Adiabatic Approach

- $c_s^{-2}\partial_t p + \partial_x \phi = 0, \quad \partial_x p + \alpha \frac{\phi|\phi|}{2p} = 0$

Main Idea: Adiabaticity = separation of time scales

- Suppose that **a parameterized family** of exact solutions, $\phi_{exact}(t;x;\xi)$, $p_{exact}(t;x;\xi)$, is known
  - $\cdots(\underbrace{t}_{\text{time}};\underbrace{x}_{\text{space}};_{\text{const.}}\underbrace{\xi}_{\text{paramters}})$
- Seeking for solution in the form [adiabatically=**slowly evolving** (parametrized) solution + **perturbative correction**]:
  - $\phi(t;x) = \phi_{exact}(t;x;\xi(t)) + \delta\phi(t;x),\ p(t;x) = p_{exact}(t;x;\xi(t)) + \delta p(t;x)$
  - $\delta p(t;0) = \delta p(t;L) = 0$
  - $p(t;0) = p_{exact}(t;0;\xi(0)),\ p(t;L) = p_{exact}(t;L;\xi(0))$
  - $\underbrace{\delta p(t;x) \ll p(t;x),\ \delta\phi(t;x) \ll \phi(t;x)}_{\text{corrections are small}} \rightarrow$ linearization
  - in addition $c_s^{-2}\partial_t \delta p \ll \delta_x \delta\phi$
- a version of **singular perturbation** analysis/method

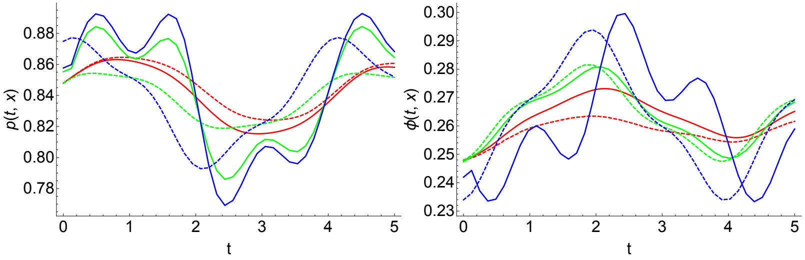
Experiments: Pressure-Pressure Boundary Conditions



- $p(t; 0)$ and $p(t; L)$ are fixed/given
- $p(t; L/2)$ and $\phi(t; L/2)$ are shown

- Unbalanced Adiabatic = Adiabatic profile build about Unbalanced Exact Solution
- Balanced Adiabatic = Adiabatic profile build about Balanced Exact Solution
- UA+ = UA + linear correction
- BA+ = BA + linear correction

Experiments: Pressure-Mass Flow Boundary Conditions



- $p(t;0)$ and $\phi(t;L)$ are fixed/given
- $p(t;L/2)$ and $\phi(t;L/2)$ are shown

- Exact – solid red; Linearized – dashed red
- Unbalanced Adiabatic - green dashed
- Balanced Adiabatic - blue dashed
- UA+ - green solid
- BA+ - blue solid

Exact Balanced solution

Steady (time independent) Solution of

$$c_s^{-2}\partial_t p+\partial_x\phi=0,\quad \partial_x p+\alpha\frac{\phi|\phi|}{2p}=0$$

$$p_{\mathrm{BA}}(x;p_{in},p_{out})=\sqrt{(p_{in})^2-\frac{x}{L}\left((p_{in})^2-(p_{out})^2\right)}$$

$$\phi_{\mathrm{BA}}(p_{in},p_{out})=\sqrt{\frac{(p_{in})^2-(p_{out})^2}{\alpha}}$$

adiabatic

- $p_{in}=\text{const}\to p_{in}(t),\quad p_{out}=\text{const}\to p_{out}(t)$
- **Model Reduction**: from PDEs $\Rightarrow$ explicit ODEs for the parameters $(p_{in}(t),\ p_{out}(t))$

Exact Unbalanced solution

New family of Exact time dependent solutions of

$$c_s^{-2}\partial_t p + \partial_x \phi = 0, \quad \partial_x p + \alpha \frac{\phi|\phi|}{2p} = 0$$

$$p(t,x) = p_0 \exp\left(\frac{\lambda c_s^2}{\sqrt{2\alpha}} t + \psi_\lambda(x)\right), \quad \phi(t,x) = \sqrt{-\frac{2p_0^2}{\alpha}\psi'_\lambda(x)} \exp\left(\frac{\lambda c_s^2}{\sqrt{2\alpha}} t + \psi_\lambda(x)\right)$$

$$\psi_\lambda(x) = -\int_0^x dx' G(x';\lambda), \quad \int\limits_{G(x;\lambda,G_0)}^{G_0} \frac{dz}{\lambda\sqrt{z - 2z^2}} = x \text{ [implicit]}$$

adiabatic

- $G_0 = \text{const} \to G_0(t), \quad \lambda = \text{const} \to \lambda(t)$
- **Model Reduction**: from PDEs $\Rightarrow$ implicit ODEs for the parameters $(G_0(t), \lambda(t))$ [implicit = tabulation is involved]

Conclusions [Adiabatic Approach to Gas Systems]

- New family of **exact solutions** (growing, decreasing) for a single pipe
- **Adiabatic** solutions: explicit (balanced), implicit (**unbalanced**)
- Experiments: **emprirical validation** of the adiabaticity (scale separation + control of corrections)
- **Model reduction**: from PDE(s) to ODE(s) = tractability & scalability

Path Forward [Adiabatic Approach to Gas Systems]

... adiabatic approach ...

- Other exact solutions (oscillating) ?
- Demonstration on (large) networks (efficiency, scalability)
- Integration into network-wide control & optimization