Probabilistic N-k Vulnerability Analysis for Power Systems

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Outline of the talk

- Introduction to the deterministic N-k problem
- Probabilistic variant
- Challenges
- Formulation
- Algorithm outline
- Preliminary computational results
- Possible extensions (work under progress)

Deterministic N-k problem

- Given: Transmission system and value of k
- Find: A set of k components
- Objective: Removal of these k components maximizes the minimum damage to the system
- Constraints: Physics of the transmission systems (power flow and physical limits)
- Damage - measured in terms of load shed

An alternate interpretation

- Bi-level Stackelberg game
- Attacker-defender model
- Attacker chooses k components
- Defender reacts optimally to minimize damage
- N-k problem: Attacker's view point - find optimal attack
- Network interdiction/inhibition, vulnerability evaluation problem

Probabilistic variant

- Each component is associated with probability of failure p_i
- Assumption: the probabilities are independent
- Probability of an N-k failure $p = p_1 \times p_2 \times \cdots \times p_k$
- Damage given by $p \times \text{Load shed}$

Challenges

- Number of possible solutions is N choose k
- For each solution we have to solve a power flow problem
- Power flow physics is non-convex
- Bi-level mixed integer non-linear problem

Literature

- Other applications: critical infrastructure, general capacitated flow networks - K. Wood, G. Brown, et. al; deterministic and stochastic versions
- Stochastic versions for transportation models - Cormican et. al (1998)
- Deterministic problem with DC approximation for power flow - Samleron et. al (2006)
- Many heuristic ranking schemes for flow based systems, capacity based algorithms - Qiang et. al (2008), A. Pinar et. al (2010)
- Bi-level formulations and algorithms for the deterministic problem - Samleron et. al (2009) and Arroyo et. al (2005)
- Min-cardinality version and Non-linear continuous version - Bienstock et. al (2010)
- Non-Linear continuous version - modify resistance of lines to maximize overload of any line

Goals of the current work

- Use more accurate representations for the physics and address the resulting computational challenges for the deterministic version of the problem
- Extension of the algorithms to probabilistic version; Does this break the algorithm?
- How different are the results when using more accurate models for physics of power flows; does it really matter, are current DC based models sufficient; under what conditions do the results diverge from each other?
- Case studies and detailed analysis

Work in progress

Problem formulation (deterministic variant)

$$\max_{\text{N-k failures}} \quad \eta$$ where, Outer problem

$\eta :=$ Minimum load shed for the N-k failure subject to power flow constraints and generation, line thermal capacity limits

Inner problem

Outer problem has binary variables

Inner problem has continuous variables and non-linear non-convex constraints

Bi-level, mixed-integer non-linear, non-convex problem

Inner problem

$$\min : \sum_{i \in \mathcal{N}} \Re(S_i^d)(1 - \ell_i) \text{ s.t.,}$$

Total load shed

$$v_i^l \leqslant |V_i| \leqslant v_i^u \ \forall i \in \mathcal{N},$$

$$S_i^{gl} \leqslant S_i^g \leqslant S_i^{gu} \ \forall i \in \mathcal{N},$$

$$|S_{ij}| \leqslant s_{ij}^u \ \forall (i,j) \in \mathcal{E}_s,$$

Voltage, generation and thermal limits

$$S_{ij} = Y_{ij}^*(V_i V_i^* - V_i V_j^*) \ \forall (i,j) \in \mathcal{E}_s,$$

$$S_i^g - S_i^d(1 - \ell_i) = \sum_{(i,j) \in \mathcal{E}} S_{ij} \ \forall i \in \mathcal{N},$$

AC-power flow

$$\theta^{\Delta l} \leqslant \angle V_i V_j^* \leqslant \theta^{\Delta u} \ \forall (i,j) \in \mathcal{E}_s, \text{ and}$$

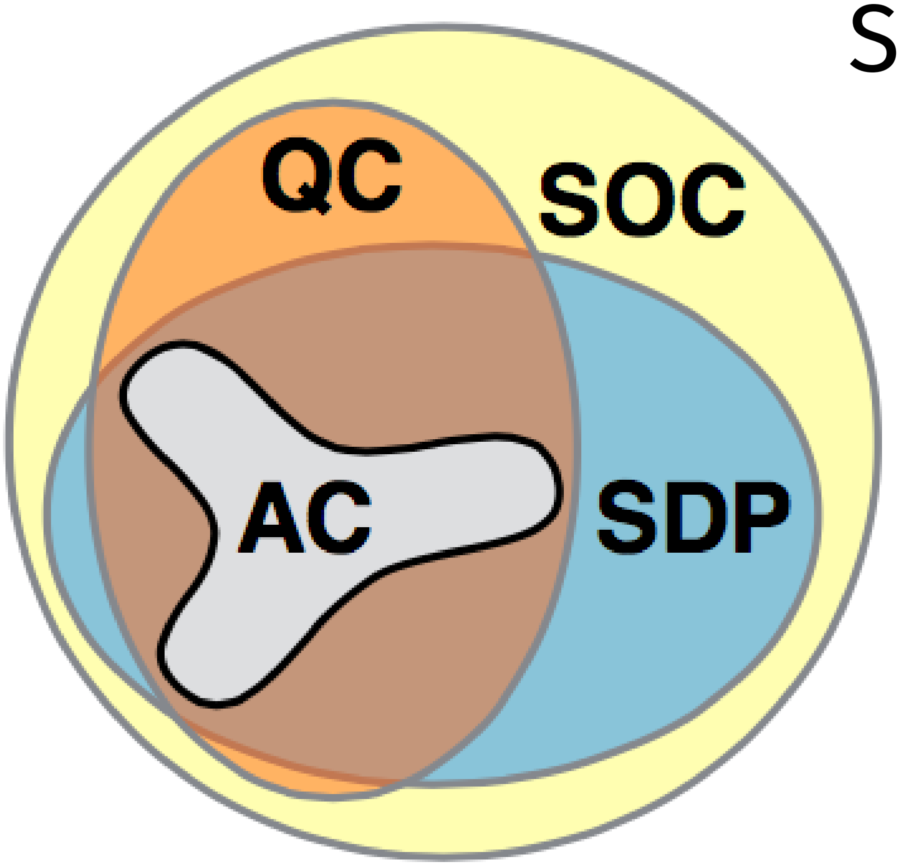
$$0 \leqslant \ell_i \leqslant 1 \ \forall i \in \mathcal{N}.$$

Phase angle limits and load-shed factor

First naive algorithm

- Enumerate all N-k failure scenarios
- For each failure scenario, solve the inner problem
- Pick the N-k failure scenario with maximum inner problem objective value
- Two issues: Enumeration and solving inner problem

Relaxations to the rescue!



Coffrin et. al (2016) - set sizes are for illustration and not to scale

SDP: Semi-Definite Programming

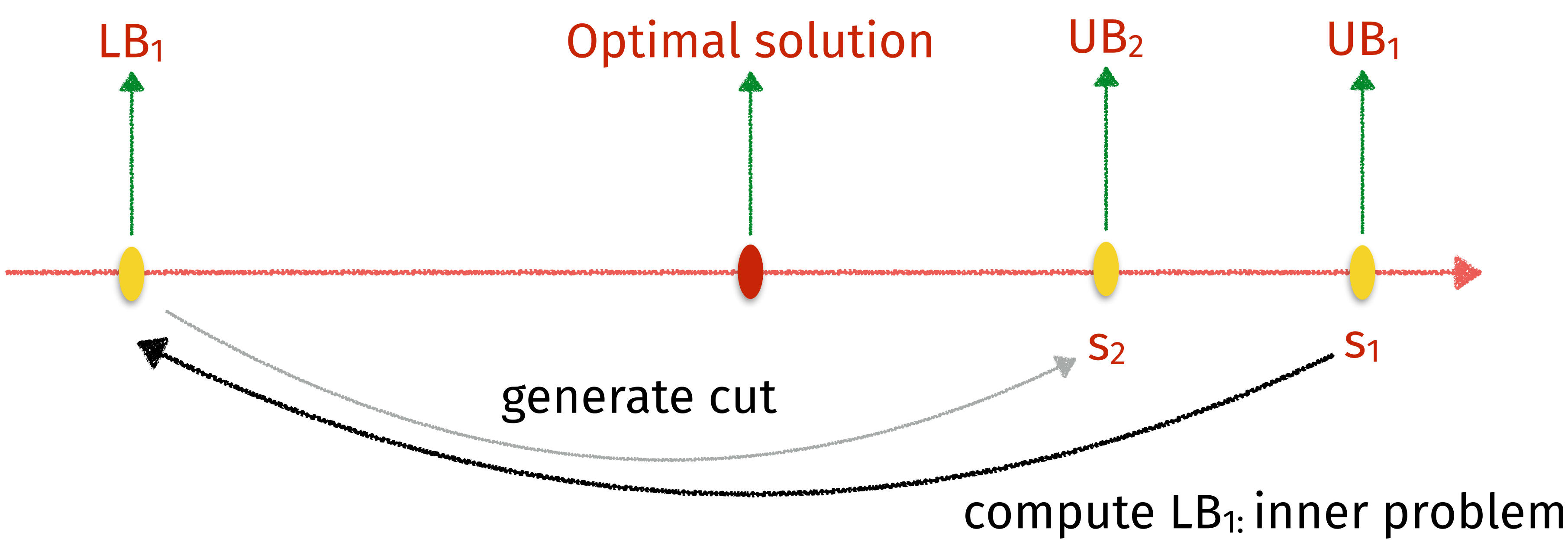
QC: Quadratic Convex

SOC: Second-Order Cone

Relaxations can be solved to optimality by off-the-shelf commercial and open-source solvers

Cutting-plane algorithm

solve the relaxed problem: $\eta^* \leftarrow \max_{\text{N-k failures}} \quad \eta$
$s^* \leftarrow$ optimal N-k failure to relaxed problem
upper bound $= \eta^*$
lower bound $=$ minimum load shed for s^*
while $gap > \varepsilon$ **do**
  add a cut to the relaxed problem using solution to minimum load shed for s^*
  resolve relaxed problem with cut and set η^* as the objective
  $s^* \leftarrow$ optimal N-k failure to relaxed problem
  upper bound $= \eta^*$
  lower bound $=$ minimum load shed for s^*
end



Generating the cut

- Notations: for each line i, x_i is a binary variable which takes a value 1 if line fails, 0 otherwise
- Relaxed problem $\eta^* := \max \ \eta \quad \text{s.t.} \sum_i x_i = k$
- Minimize the load shed for the optimal N-k failure for the relaxed problem
- Let p_i denote the real power flowing through line i
- Cut: $\eta \leqslant \eta^* + \sum_i p_i x_i$ (is not necessarily valid - Braess' paradox)

Extension to probabilistic case

- Each component is associated with probability of failure p_i
- Assumption: the probabilities are independent
- Probability of an N-k failure $p = p_1 \times p_2 \times \cdots \times p_k$

$$\max_{\text{N-k failures}} \quad (\text{probability of N-k failure}) \cdot \eta$$

Outer problem

$\eta :=$ Minimum load shed for the N-k failure subject to power flow constraints and generation, line thermal capacity limits

Inner problem

Convexification of the problem

$$\max \quad y \quad \text{s.t.},$$

$$\sum_i x_i = k,$$

$$p = \sum_i x_i \cdot \log p_i,$$

$$y \leq p + \log \eta,$$

$$y \leqslant p + \log(\eta^k) + \frac{1}{\eta^k}(\eta - \eta^k) \quad \forall (p^k, \eta^k)$$

Additional concave constraint

$\eta :=$ Minimum load shed for the N-k failure

subject to power flow constraints and generation, line thermal capacity limits

Relaxation

Cutting plane algorithm carries over without changes

Preliminary computational results

- Test case: IEEE RTS96 one area and three area systems - has line failure rates
- IEEE 14 bus system to compare enumeration results with cutting plane heuristic
- DC vs SOC relaxation (computation time)

Optimality gap for small instances

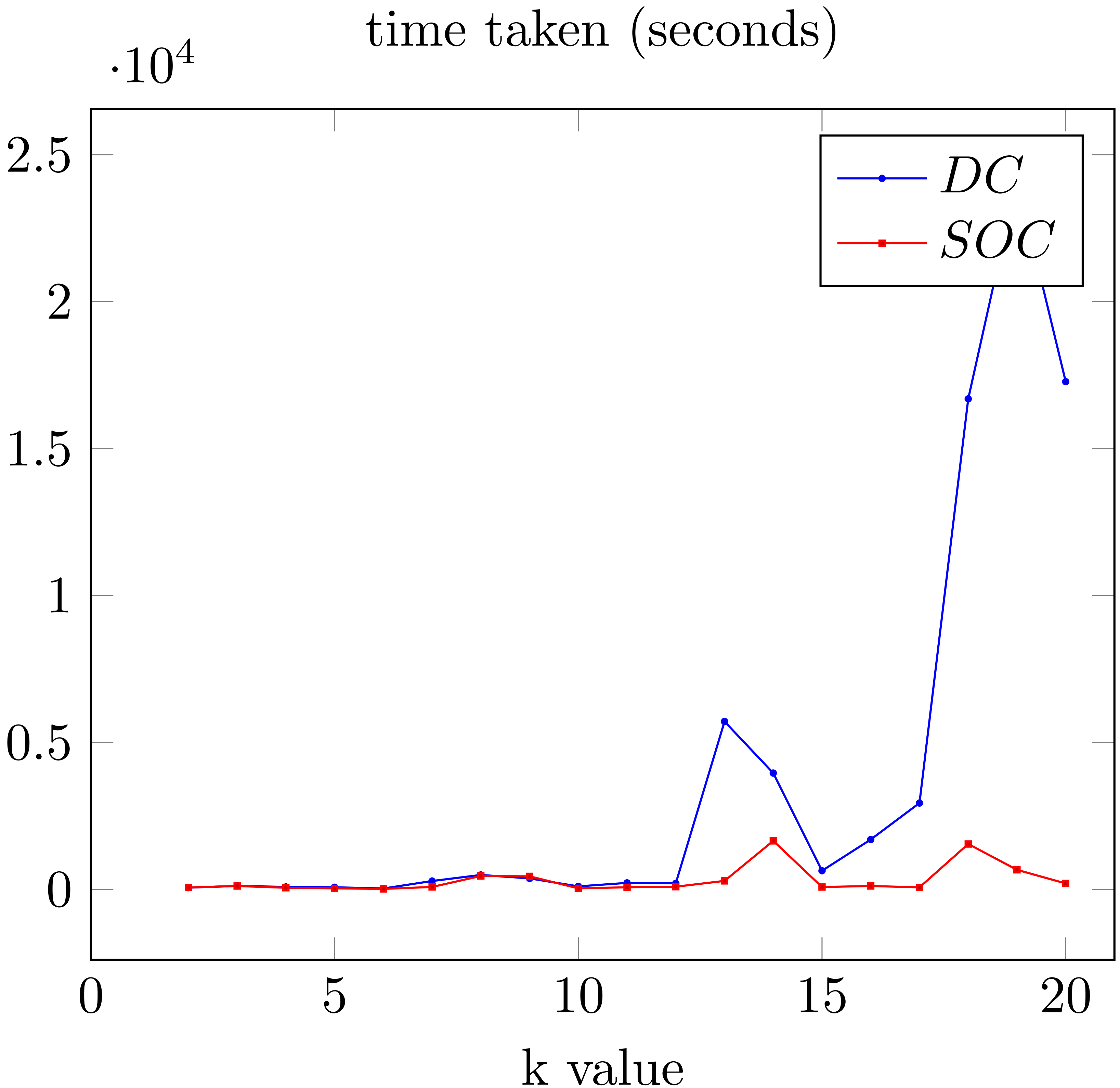
- Cutting plane heuristic converges to optimal solutions for the test cases IEEE RTS96 one area and IEEE 14 bus test systems for k values {2,3,4}

- Checked against total enumeration

- Convergence in less than 30 iterations for all values of k

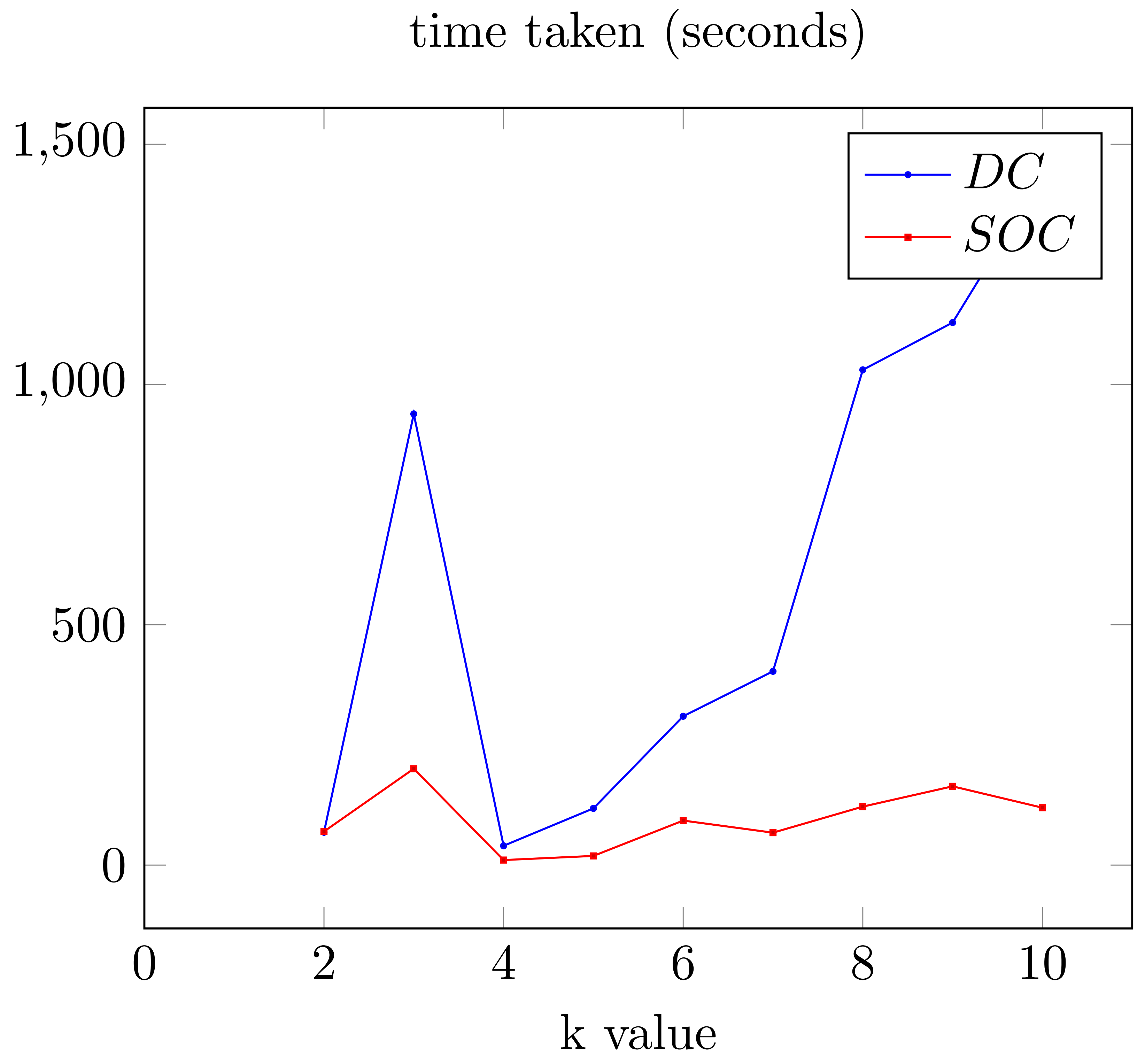
- For larger instances, enumeration becomes difficult

Computation time - deterministic



IEEE RTS96
3 area

Computation time - stochastic



IEEE RTS96
3 area

Work under progress

- Develop algorithms for estimating upper bounds on the optimal solution
- When do we need to use these relaxations instead of DC approximation for the AC power flow physics?
- Topology and capacity based heuristics to compute lower bounds
- AC feasibility analysis
- Scalability studies

Thank you ! Questions?