Los Alamos
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Topology Learning in Power Grids from Ambient Fluctuations

Deep Deka


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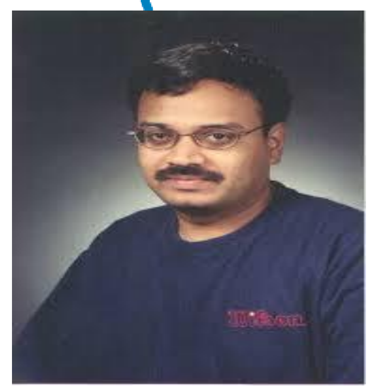
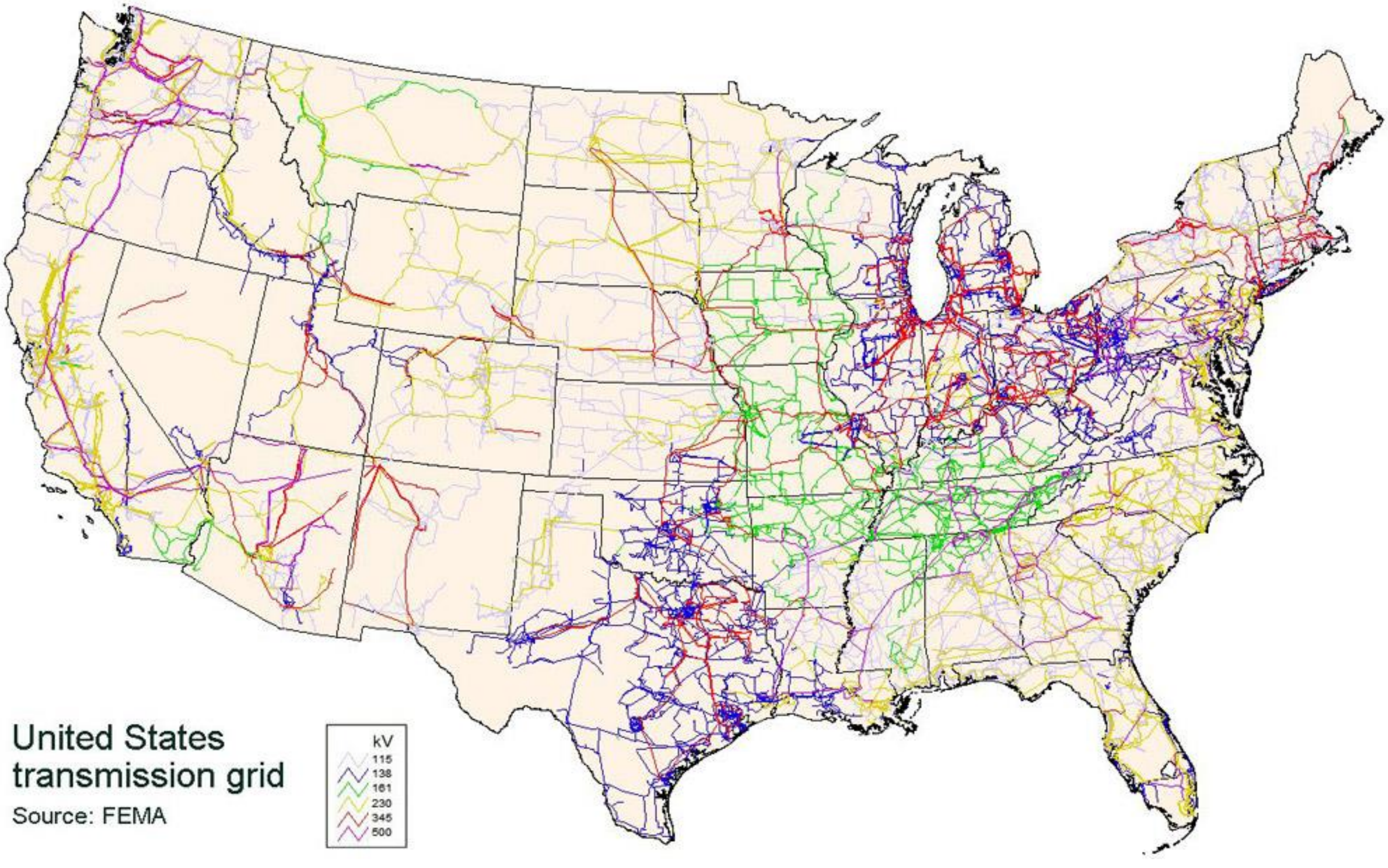

Murti (UMN)

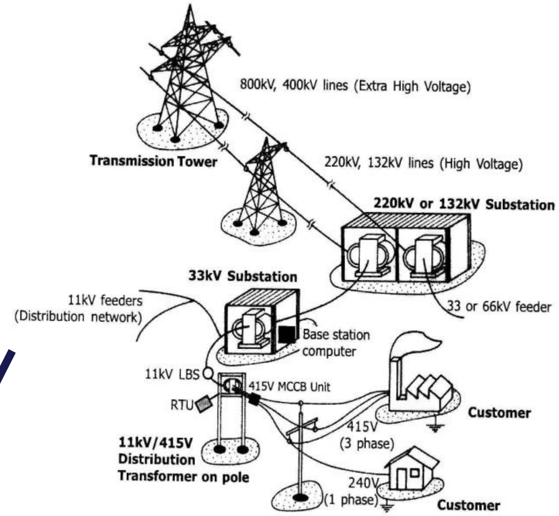
Things I will go over

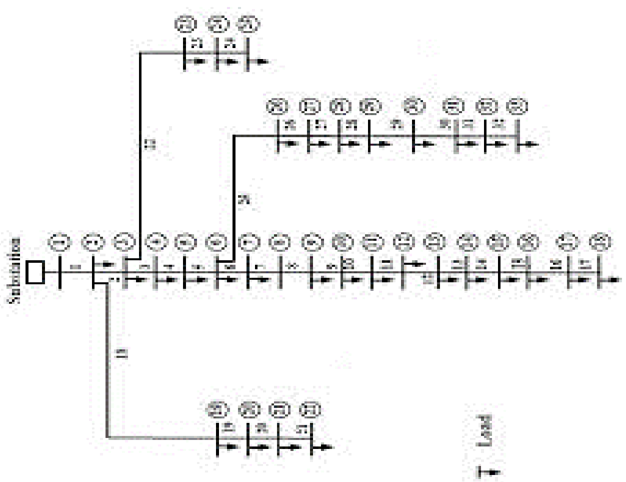
- Power Grid
  - Issues and approach
- Model Fundamentals: Static and Dynamic
  - Power flows
  - Graphical Model
- Topology Learning in *Radial and Loopy Grids*
- Extensions & Future Work

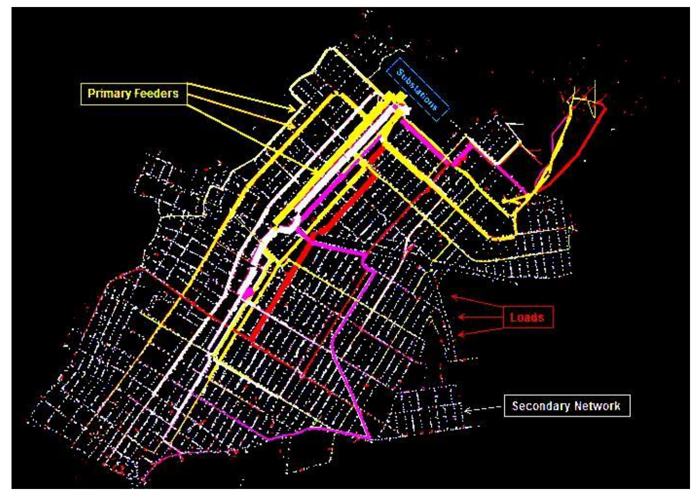
Power Grid


United States
transmission grid
Source: FEMA
kV
115
138
161
230
345
500

Grid Types







Radial/Tree

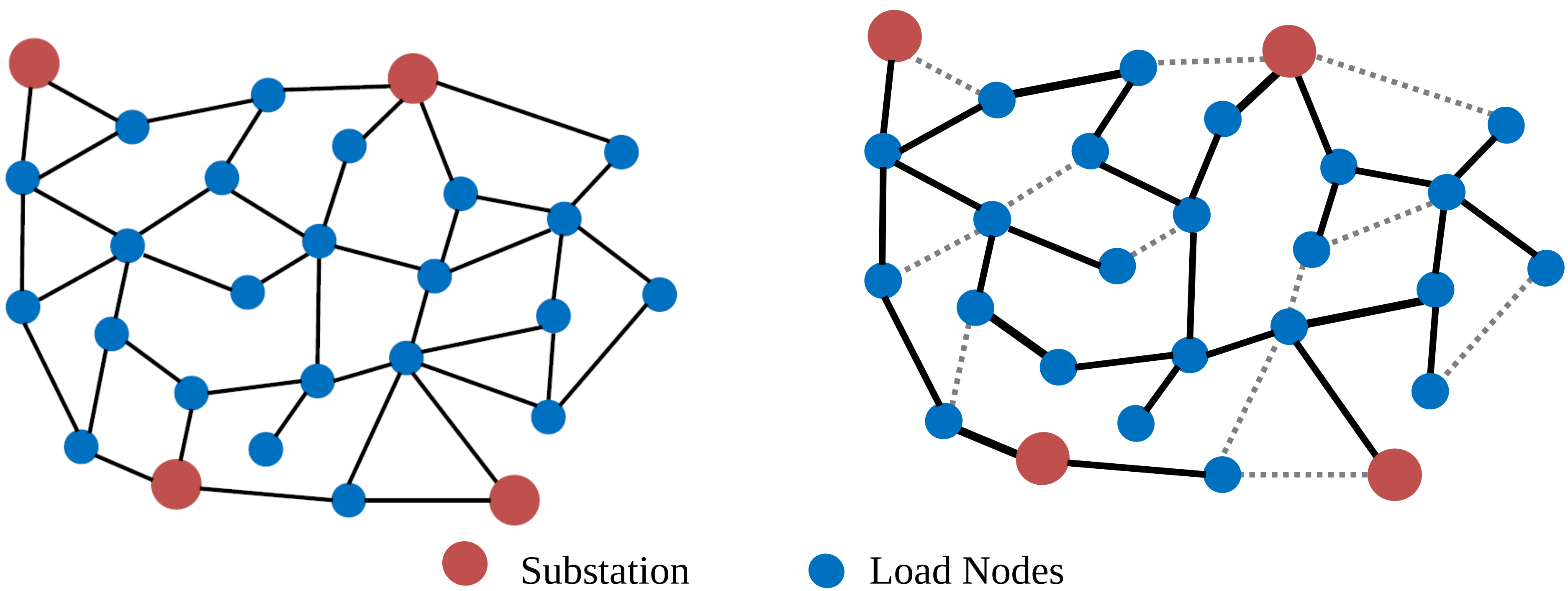
Interconnected/Meshed

Grid *'Operational'* Structure

- Underlying Loopy network
- Switches/Relays decide structure

Learning Problem:

- Estimate Configuration of Switches/Relays



Power Grid: Structure Learning

- Uses:
  - Real time control
  - Failure Identification
  - Optimizing flows

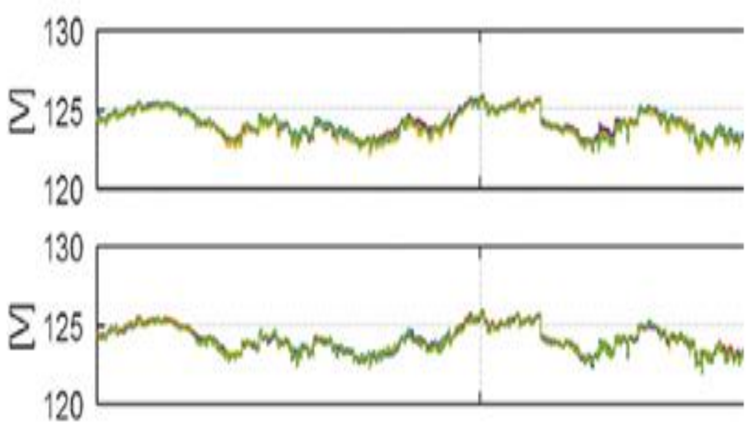
- Challenge:
  - Limited real-time breakers
  - Brute Force inefficient

- Solution
  - Smart meters: PMUs, micro-PMUs, IoT
  - Big Data: High fidelity measurements

Similar Approaches

- Power Systems based:
  - Flow equations
  - Regression, Relaxation
  - Reno et al, Rajagopal et al, Annaswamy et al
- Machine Learning based:
  - Empirical evidence based
  - Clustering, Greedy approaches
  - Bolognani et al, Arya et al, Deka et al., Rajagoapal et al, Low et al

- This Talk:
  – ***Probabilistic Graphical Model*** *for nodal voltages*



Similar Approaches

- Power Systems based:
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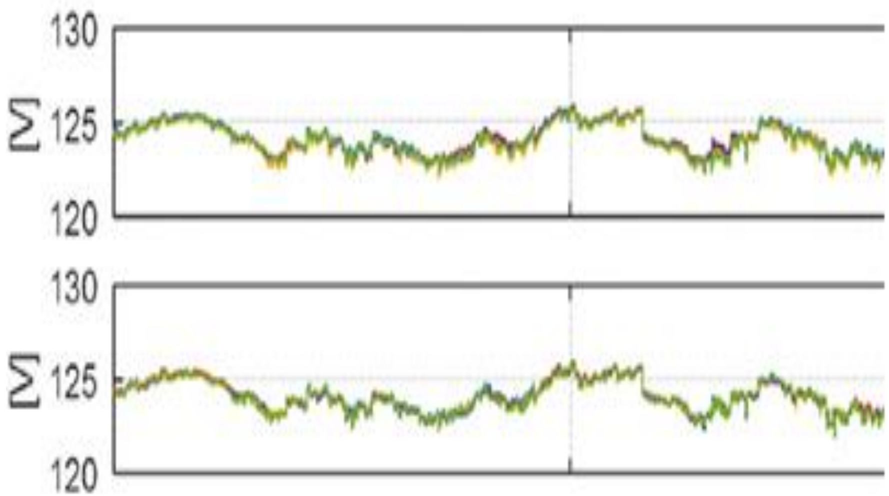
- This Talk:
  - ***Probabilistic Graphical Model*** *for nodal voltages*
  - Advantage:
    - **Provable** results
    - **No knowledge** of parameters or injections
    - Incorporates **Dynamics**

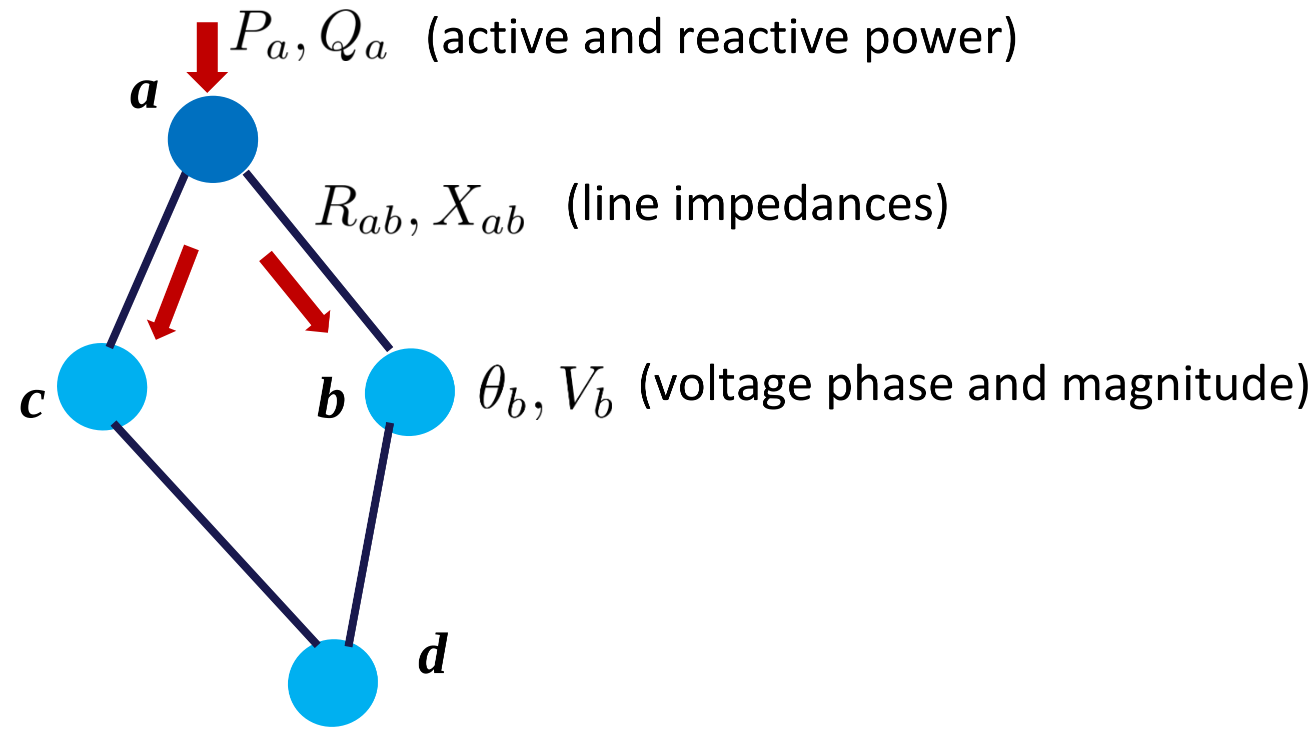
Learning Regime



Probabilistic Graphical Model *for the nodal voltages*

- Static Regime: *Power Flow Equations*
- Dynamic Regime: *Swing Equations*

Static Regime: AC power flow



$$P_a + \hat{i}Q_a = \sum_{(a,b)\ \text{is edge}} \underbrace{V_a e^{\hat{i}\theta_a}(V_a e^{-\hat{i}\theta_a} - V_b e^{-\hat{i}\theta_b})/(R_{ab} - \hat{i}X_{ab})}$$

Flow on line function of voltages

- Relaxation: ***One-one*** map from injections to voltages

Power Flow: Lossless Relaxations

- **DC** power flow: $V_a = 1,\ \theta_a - \theta_b \approx 0,\ R_{ab} = 0$

$$\theta = H_{1/X}^{-1} P$$

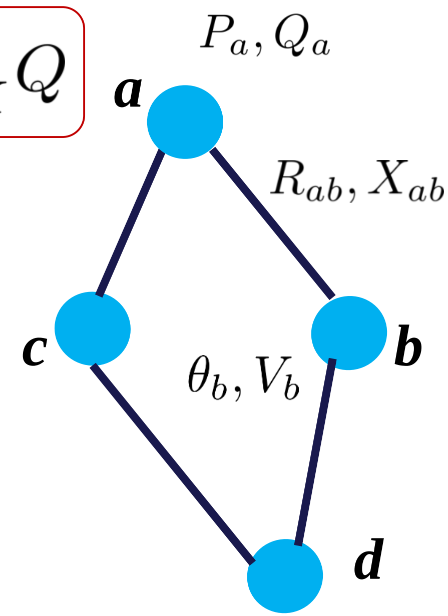
$H_{1/X}$ → wt. reduced Laplacian matrix

- **Linear-Coupled (LC)** power flow: $V_a \approx 1,\ \theta_a - \theta_b \approx 0$

$$\theta = H_{1/X}^{-1} P - H_{1/R}^{-1} Q, \quad V = H_{1/R}^{-1} P + H_{1/X}^{-1} Q$$

- **LinDist** flow (Baran-Wu): (radial networks)

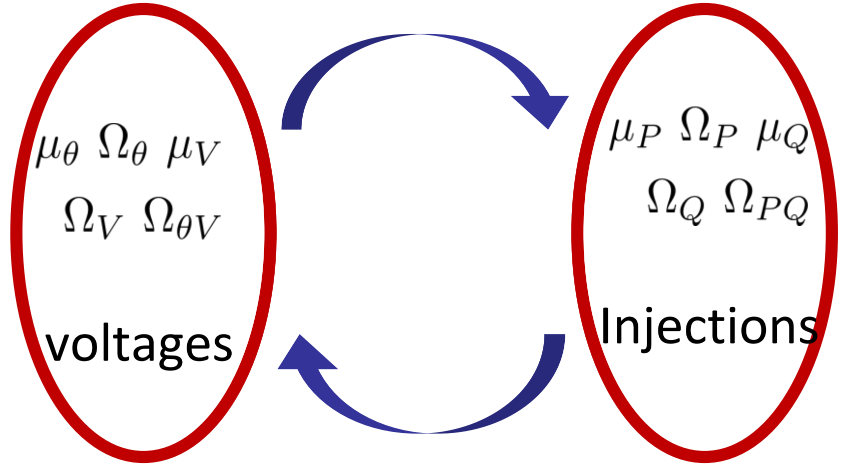
$$V^2/2 = H_{1/R}^{-1} P + H_{1/X}^{-1} Q$$



Probabilistic Distribution of Nodal Voltages

- Distribution of injections:

$$\mathcal{P}(P) = \prod_{a\in\mathcal{V}} \mathcal{P}_a(P_a)$$



- Distribution of voltages:

$$\mathcal{P}(V) = \frac{1}{|J_P(V)|} \prod_{a\in\mathcal{V}} \mathcal{P}_a(P_a)$$

$$= \frac{1}{|J_P(V)|} \prod_{a\in\mathcal{V}} \mathcal{P}_a\left(\sum_{b:\{a,b\}\in\mathcal{E}} V_a(V_a^* - V_b^*)/z_{ab}^* \right)$$

$J_P(V) = (\frac{\partial V}{\partial P})$ Jacobian

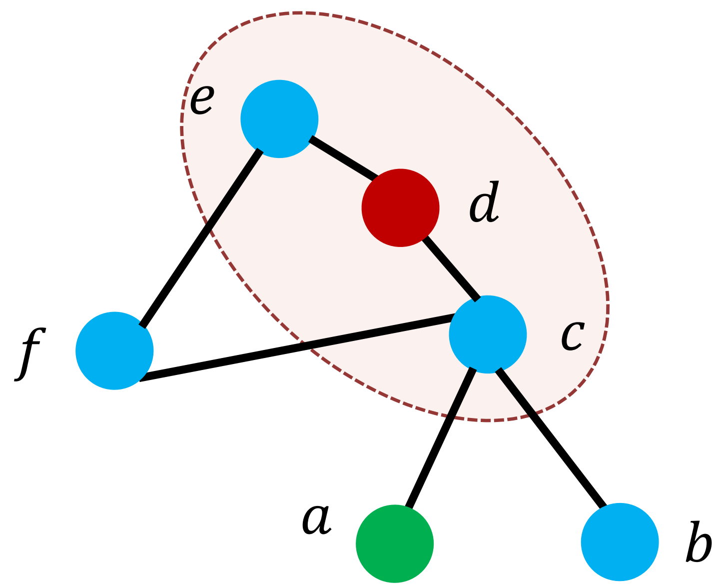
- Assumption : *Injection fluctuations are independent*

Graphical Model of Nodal Voltages

- Graphical Model: Graphical Factorization of Distribution
  - Nodes represent variables
  - Neighbors give conditional independence from all others

$$\mathcal{P}(X_d | X_a, X_c, X_e) = \mathcal{P}(X_d | X_c, X_e)$$

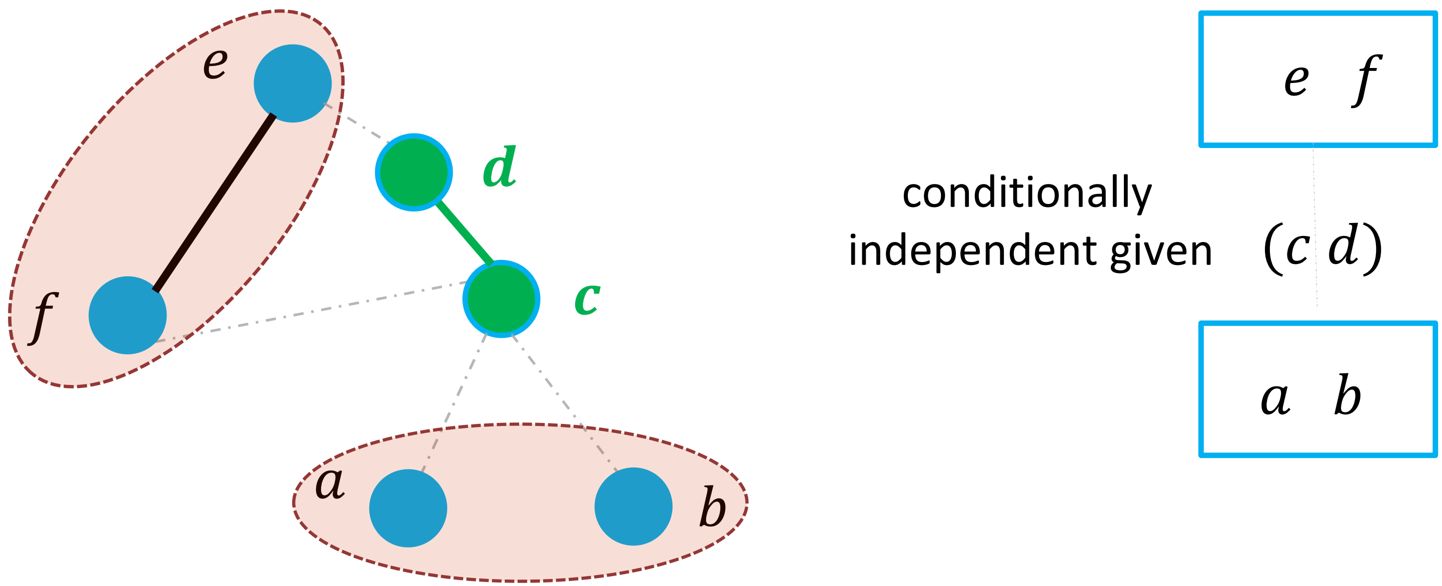
$$\mathcal{P}(X) \equiv$$



Graphical Model of Nodal Voltages

- Graphical Model: Graphical Factorization of Distribution
  - Nodes represent variables
  - Neighbors give conditional independence from all others
  - **Separator** sets make disjoint groups conditionally independent

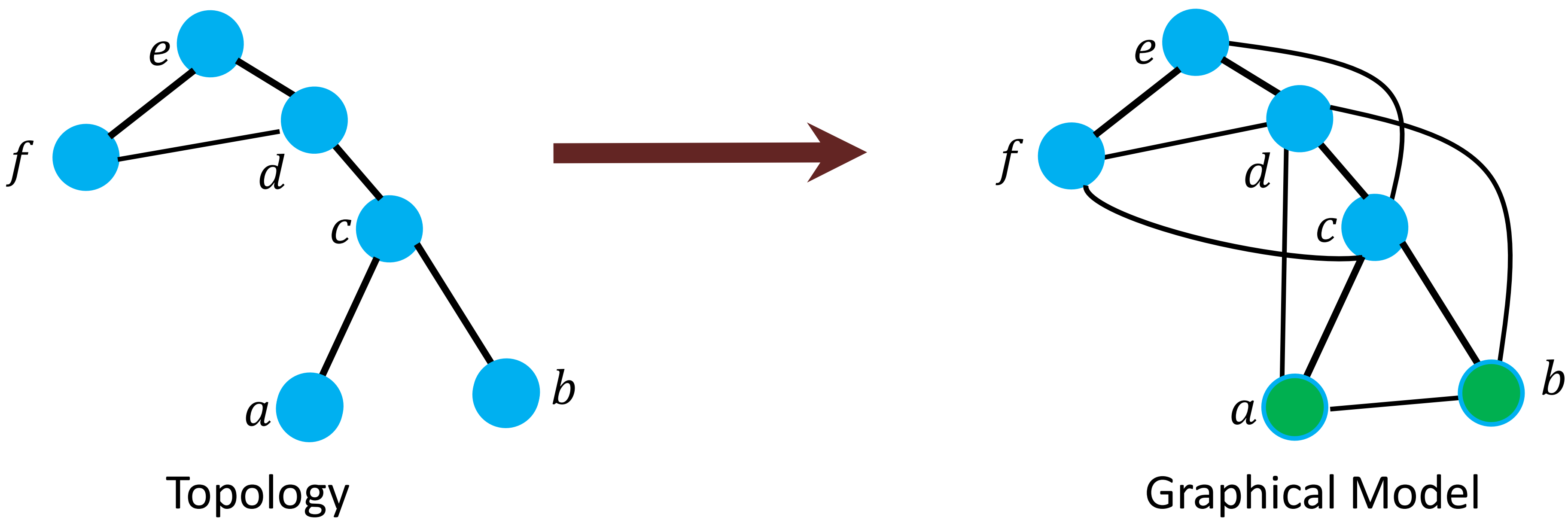
$\mathcal{P}(X) \equiv$



Graphical Model of Nodal Voltages

- Distribution $$\mathcal{P}(V) = \frac{1}{|J_P(V)|}\prod_{a\in\mathcal{V}}\mathcal{P}_a\left(\sum_{b:\{a,b\}\in\mathcal{E}} V_a(V_a^* - V_b^*)/z_{ab}^*\right)$$

- If Jacobian $J_P(V)$ is separable:
  - Graphical Model : Topology Edges + *2-hop neighbors*



Graphical Model of Nodal Voltages

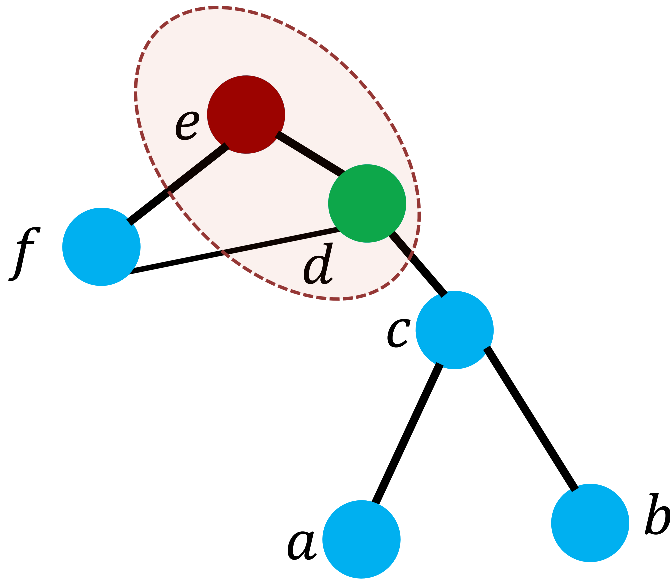
- Distribution $$\mathcal{P}(V) = \frac{1}{|J_P(V)|} \prod_{a\in\mathcal{V}} \mathcal{P}_a \left(\sum_{b:\{a,b\}\in\mathcal{E}} V_a(V_a^* - V_b^*)/z_{ab}^* \right)$$

- If Jacobian $J_P(V)$ is separable:
  - Graphical Model : Topology Edges + *2-hop neighbors*

- Proof:
  - Factorize $\mathcal{P}(V)$
  - Terms including node f

$$\mathcal{P}_f\left(V_f(V_f^* - V_e^*)/z_{ef}^*\right)$$

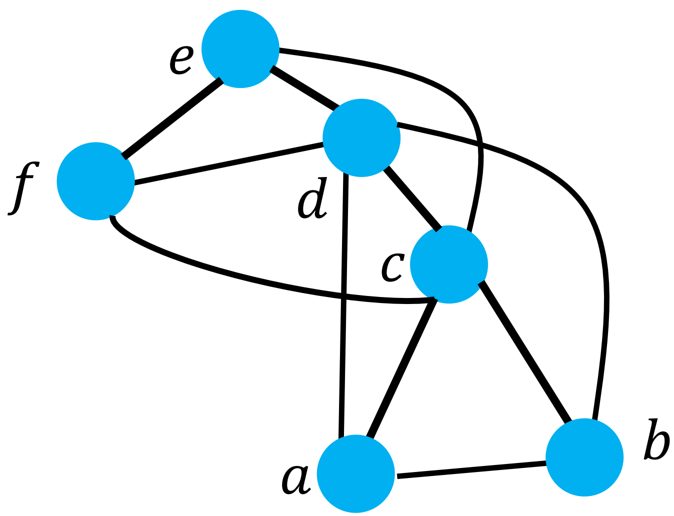
$$\mathcal{P}_e\left(V_e(V_e^* - V_f^*)/z_{ef}^* + V_e(V_e^* - V_d^*)/z_{ed}^*\right)$$



Graphical Model → Topology estimation

- Distribution $$\mathcal{P}(V) = \frac{1}{|J_P(V)|}\prod_{a\in\mathcal{V}}\mathcal{P}_a\left(\sum_{b:\{a,b\}\in\mathcal{E}} V_a(V_a^* - V_b^*)/z_{ab}^*\right)$$

- How to distinguish true edges??

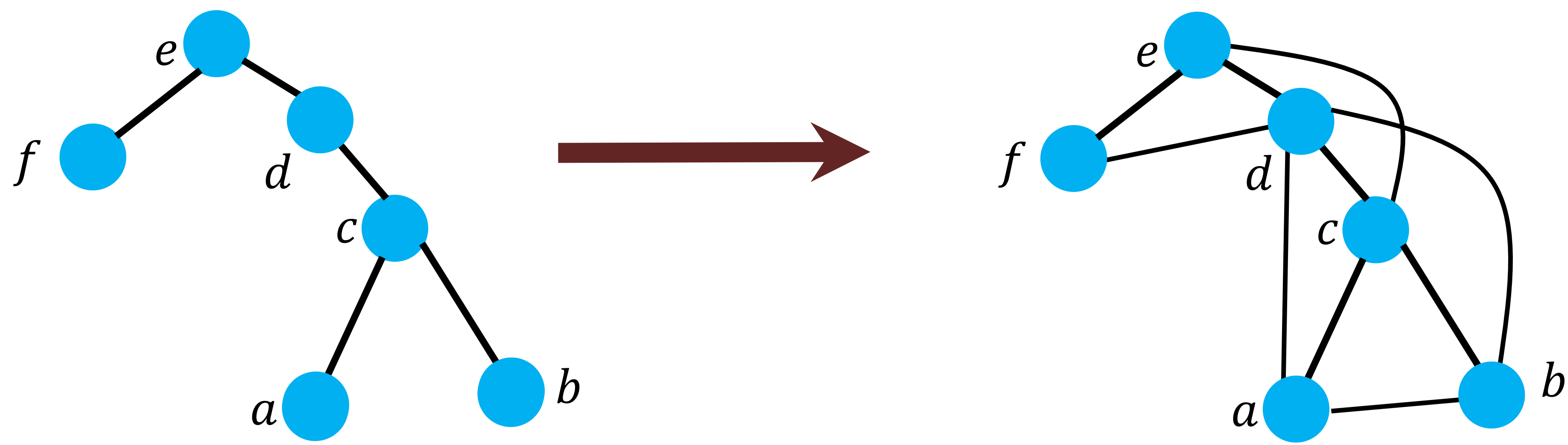


Graphical Model → Topology estimation

- Radial Grids : Separation Possible

Only **topology edges** are separators

(PSCC 2016)



Topology

Graphical Model

Graphical Model → Topology estimation

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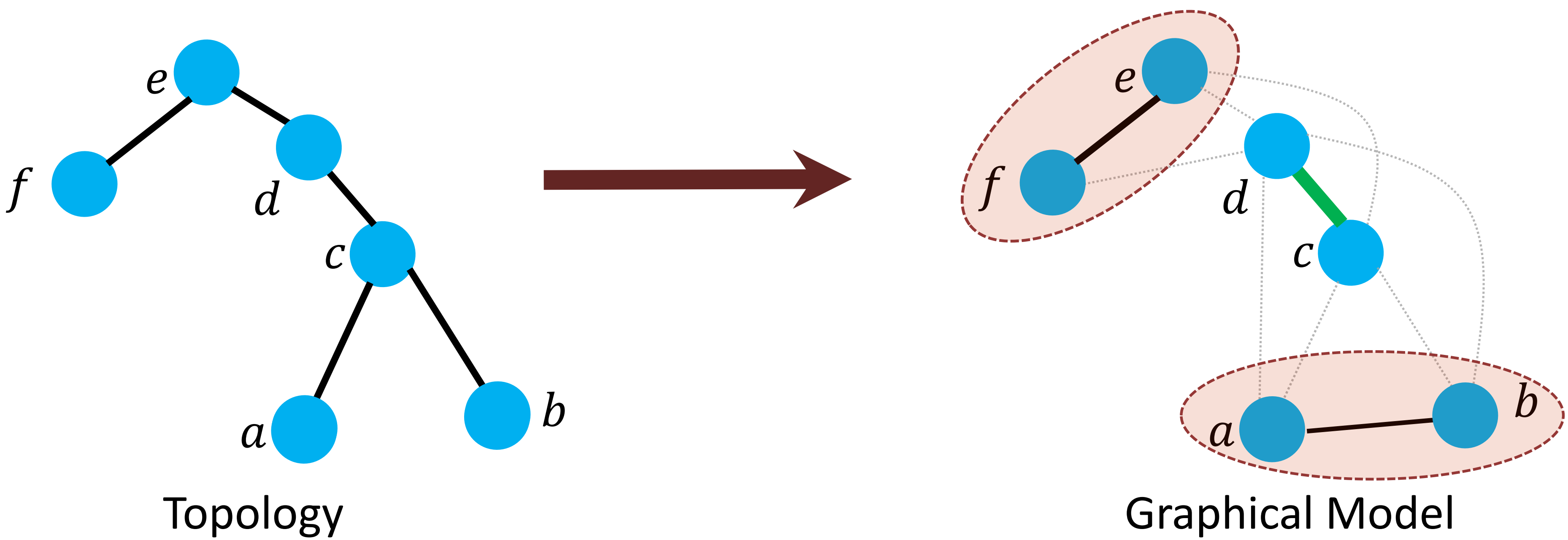

Graphical Model → Topology estimation

- Radial Grids : Separation Possible

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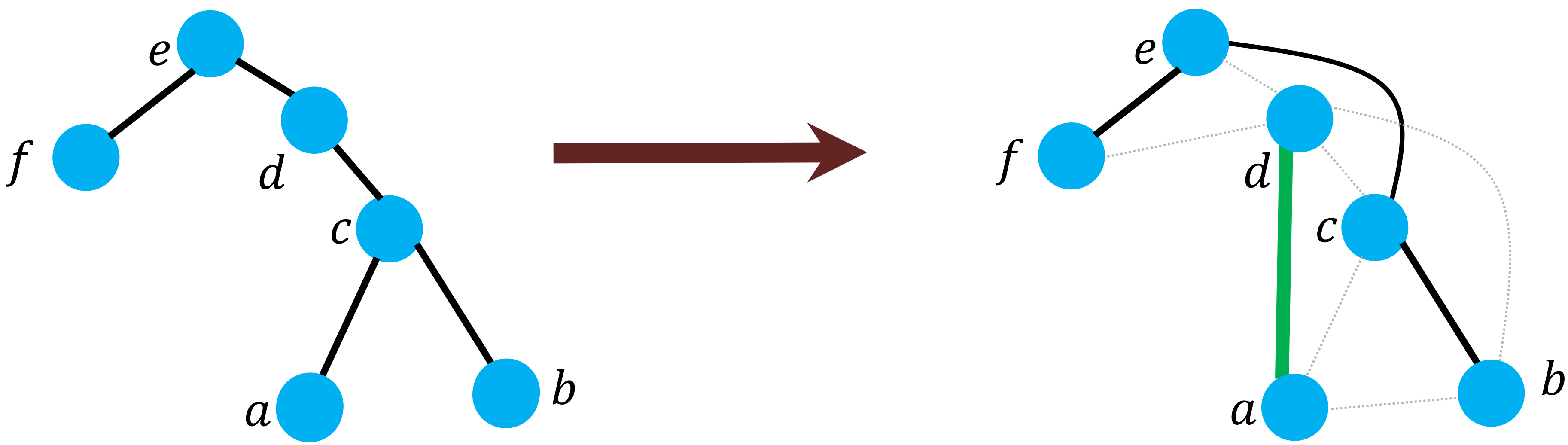


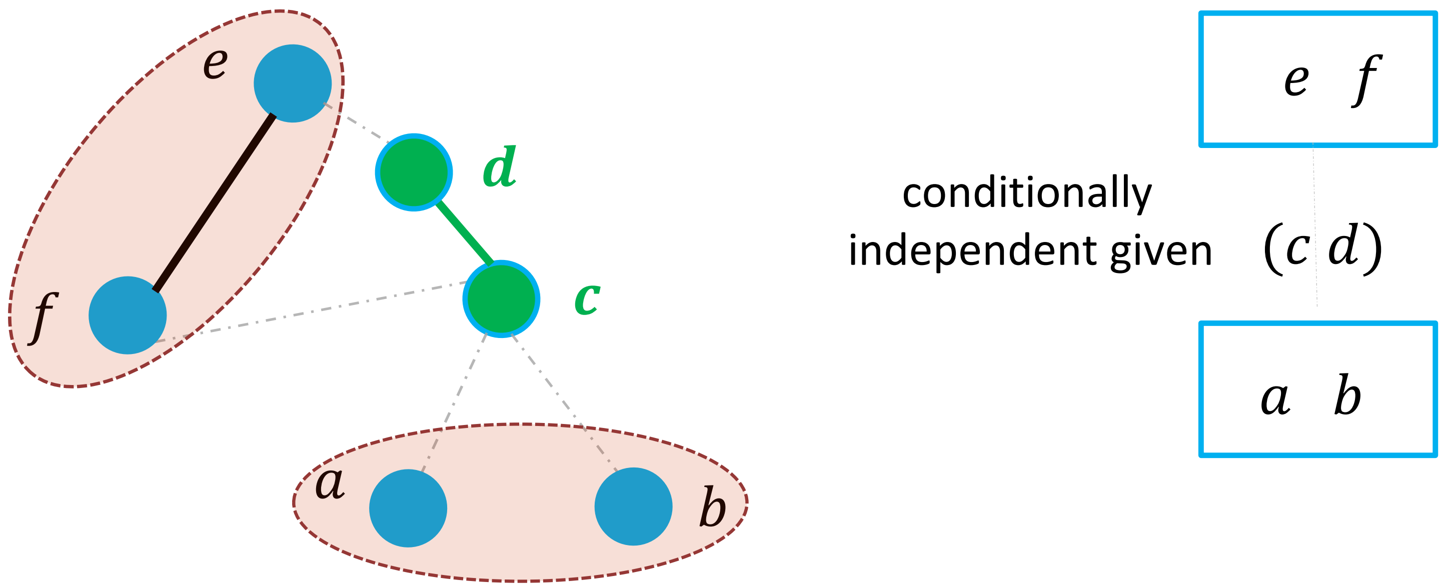
Topology

Graphical Model

Previously…..

- **Separator** sets make disjoint groups conditionally independent

$\mathcal{P}(X) \equiv$



Structure Learning Algorithm

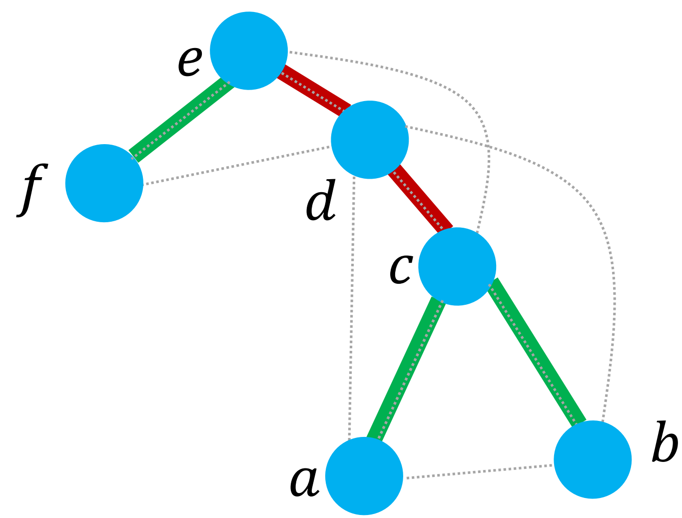
- Learn the *intermediate* edges (not connected to leaves)

$$\mathcal{P}(V_f, V_a | V_c, V_d) = \mathcal{P}(V_f | V_c, V_d)\mathcal{P}(V_a | V_c, V_d)$$

Conditional Independence test

- Learn edges to *leaves* using known edge pairs.

$$\mathcal{P}(V_f, V_c | V_e, V_d) = \mathcal{P}(V_f | V_e, V_d)\mathcal{P}(V_c | V_e, V_d)$$



Additional information needed:

- **None** (no impedance/injection statistics)
- No knowledge of distribution type

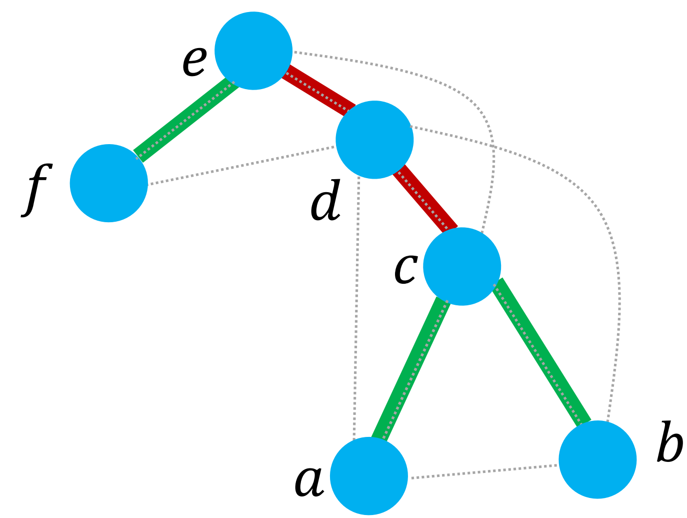
Structure Learning Algorithm: Computation

Computational Complexity:

- $O(N^4)$ conditional probability checks (all edges permissible)

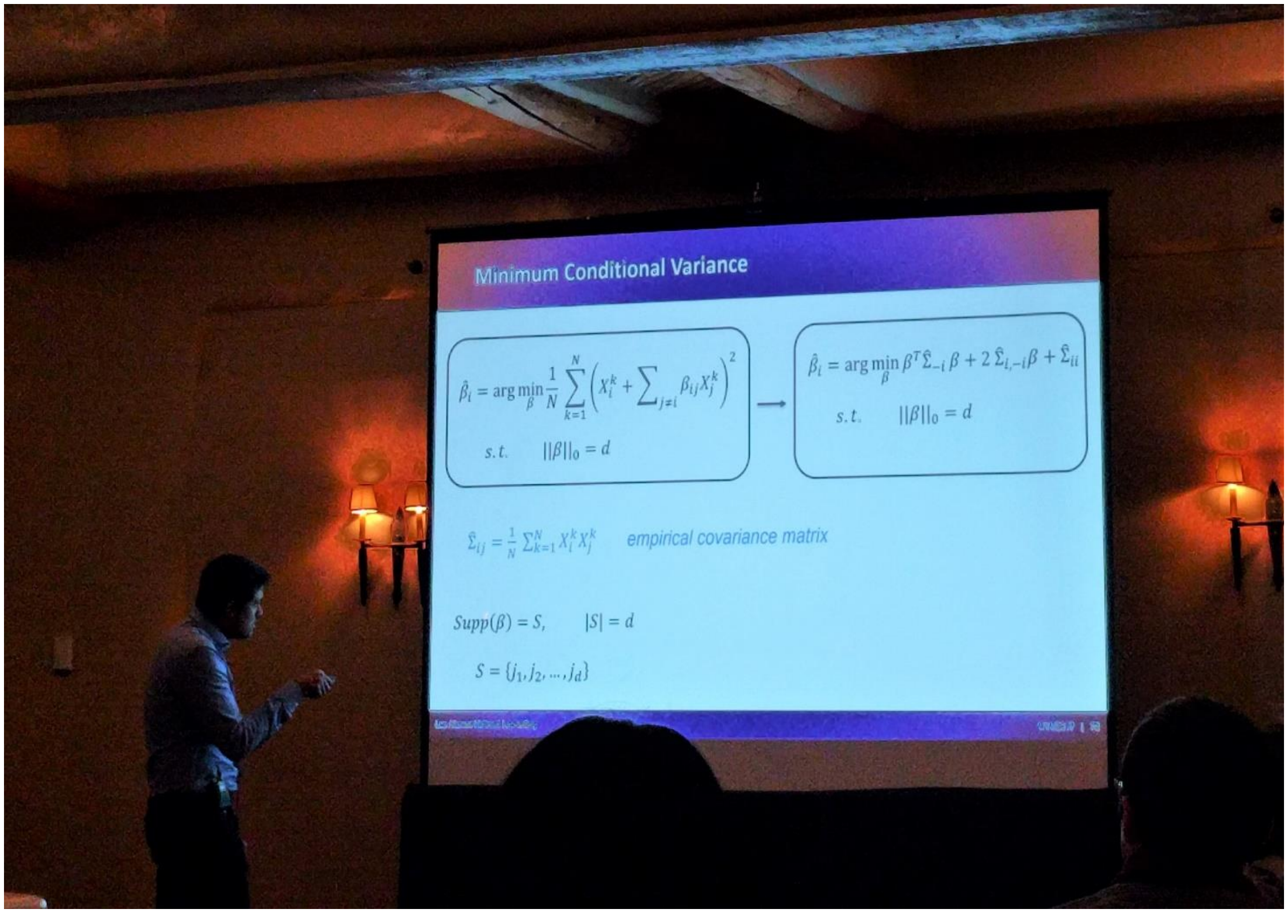
Complexity of conditional probability test:

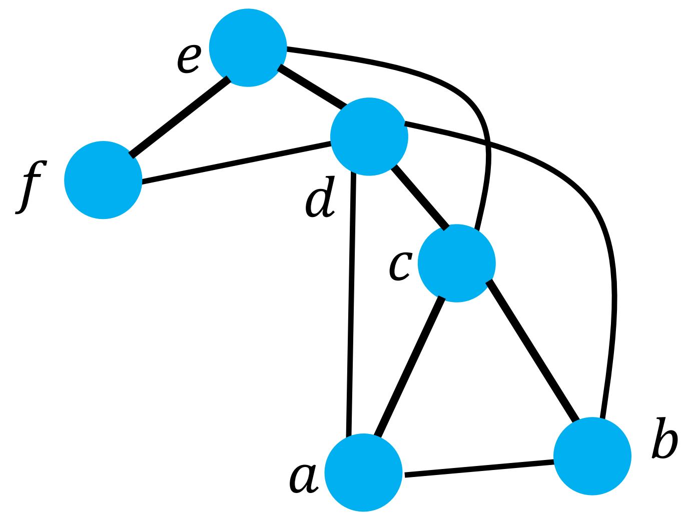
- Discrete complex voltages: $O(p^6)$
- Continuous voltages:
  - Kernel based non-parametric checks
  - Hilbert-Schmidt norm for covariance
- **Independent of Network Size**



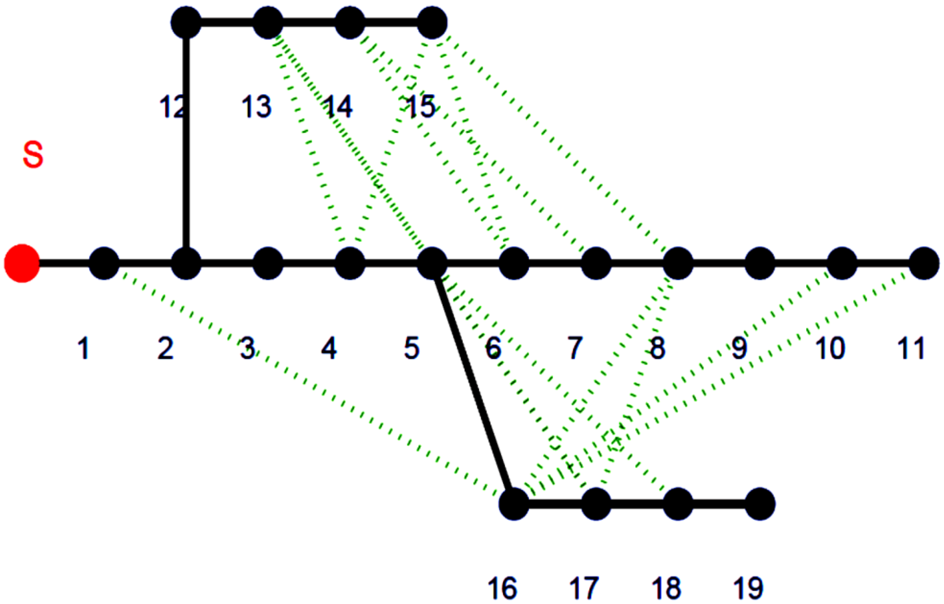
Special Case: Gaussian random variables

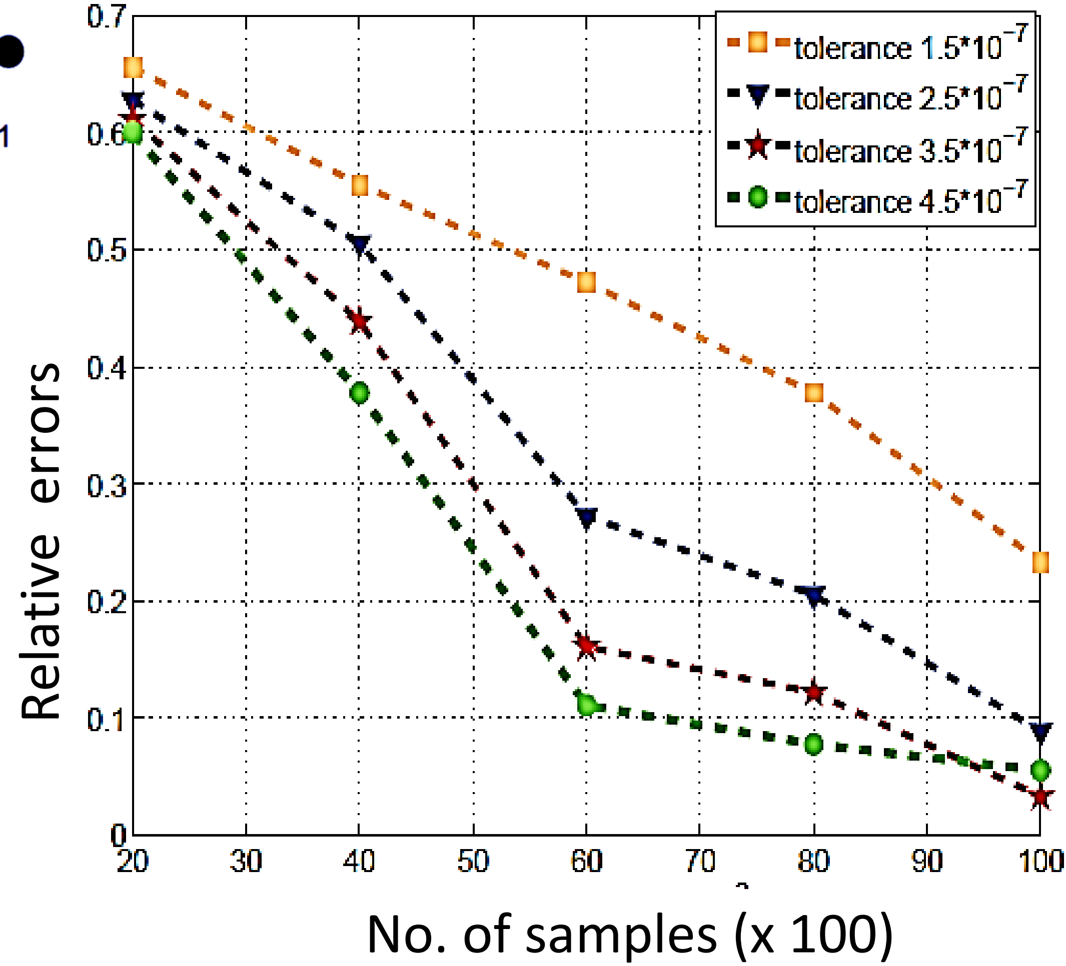
- Inverse covariance Σ_V^{-1} gives graphical model
- Learn the graphical model directly: Graphical Lasso, Lasso etc.





Simulations



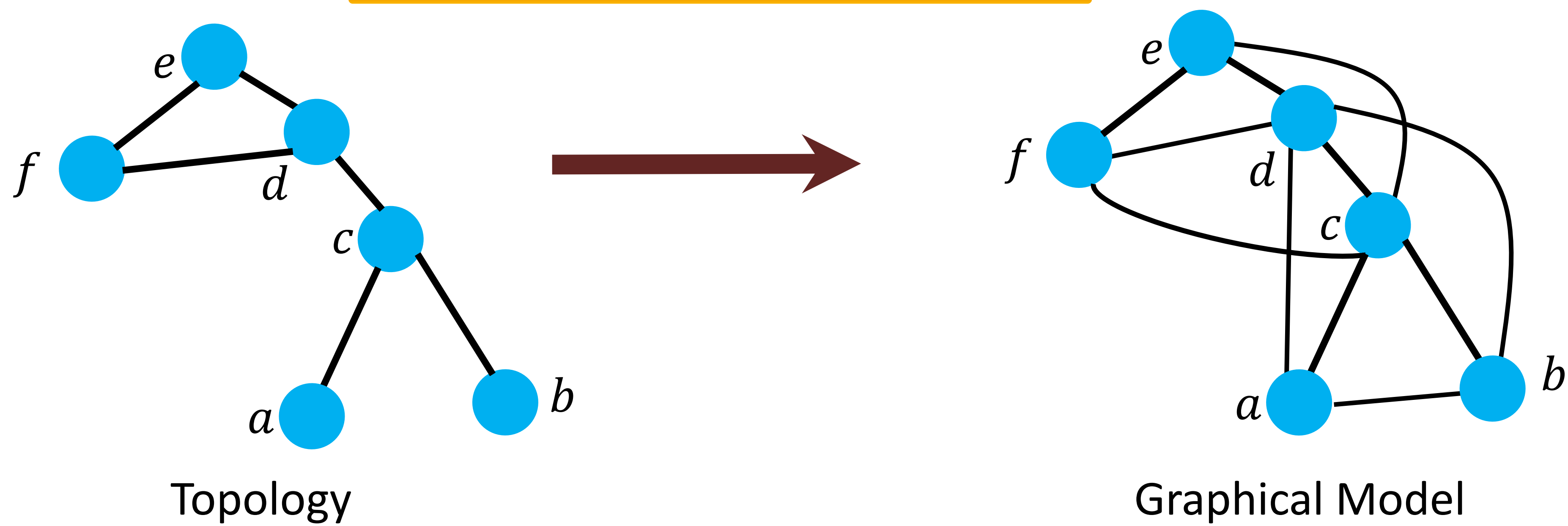


What about loopy grids??

- How to distinguish true edges??
- Separation Results do not hold …. **hence not possible**

Not Really

Use DYNAMICS!!

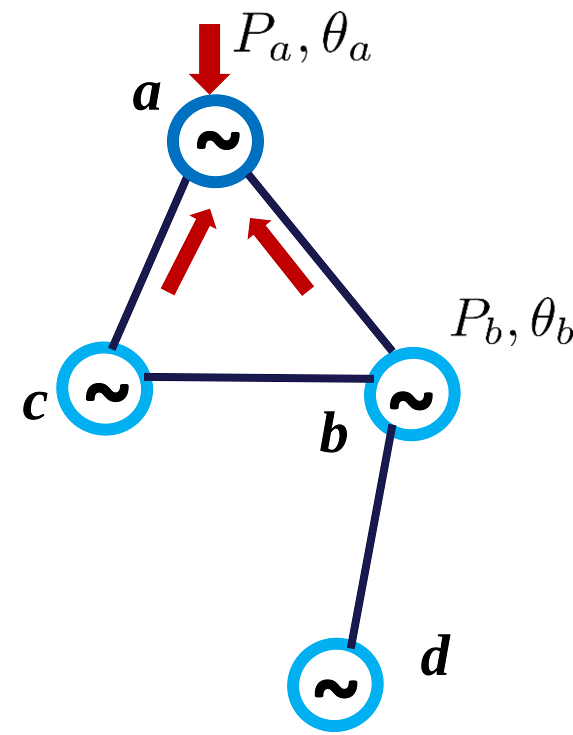


Dynamic Regime: Swing Equations

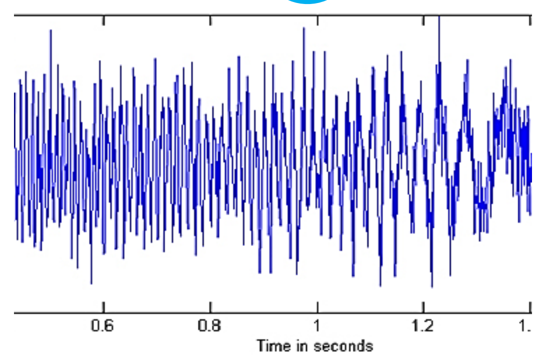
- Fluctuations due to ambient noise in injections:

$$M_a \ddot{\theta}_a + D_a \dot{\theta}_a = \sum_{(a,b) \text{ is edge}} B_{ab}(\theta_b - \theta_a) + P_a$$

Dynamics of phase angles — Net power imbalance



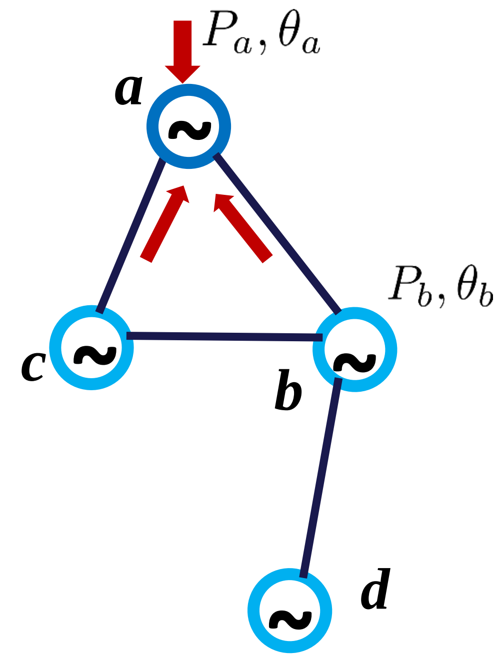
- Frequency $\omega_a = \dot{\theta}_a$
- Inertia (M) and Damping (D) from synchronous machines.
- Stochastic noise P_a



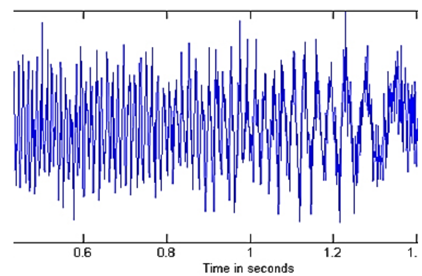
Linear Dynamical System for Swing Equations

- Swing Equations in z-domain:

$$M_a\ddot{\theta}_a + D_a\dot{\theta}_a = \sum_{(a,b)\text{ is edge}} B_{ab}(\theta_b - \theta_a) + P_a$$



$$\theta(z) = H(z)\theta(z) + \hat{P}(z)$$



- Noise Model:
  - Stochastic Wide-Sense Stationary (WSS)
  - Uncorrelated
  - Diagonal Power Spectral Density: $\Phi_{\hat{P}} = \mathcal{Z}\left(\mathbb{E}[\hat{P}\hat{P}^T]\right)$

Linear Dynamical System for Swing Equations

- Swing Equations in z-domain:

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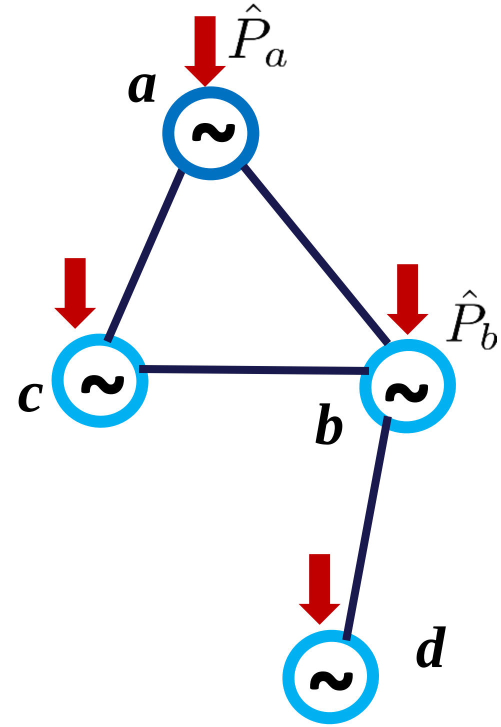
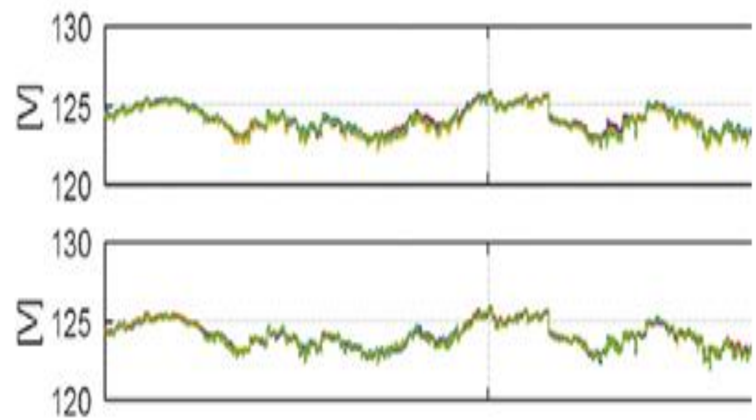
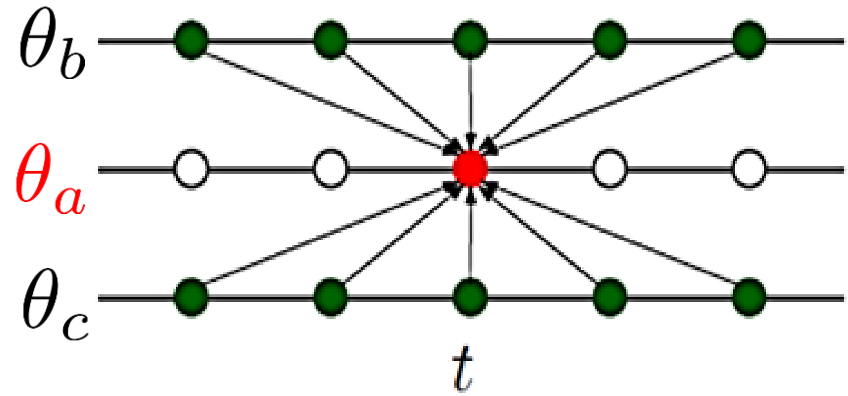


- Measurements:

  Time-series of ***phase angle dynamics***



Wiener Filter for phase angles



- Wiener Filter:

  (Wiener, Kolmogorov ,1950)

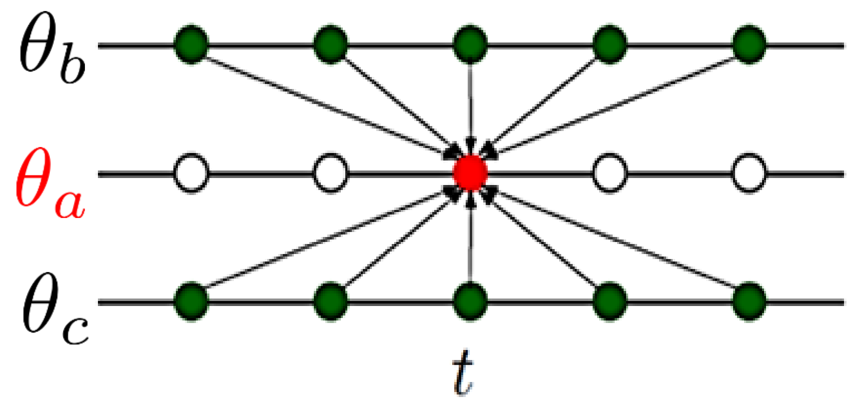
$$\min_{X \in tf\text{-}span\{\theta_i\}_{i \neq a}} \|\theta_a - X\|^2$$

- Solution: $$X_a = \sum_{i \neq a} W_{ai}(z)\theta_i$$

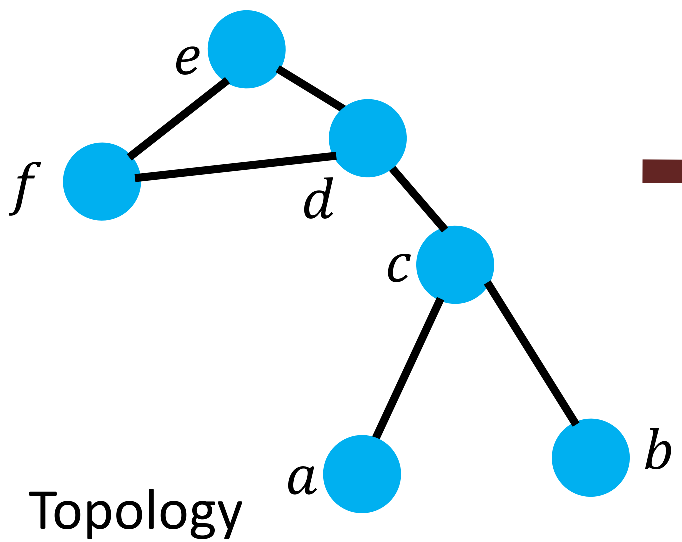
- Optimal ***non-causal projection***
- Related to ***Power-Spectral density*** and transfer function $H(z)$
- Computation Complexity: $O(N^3)$

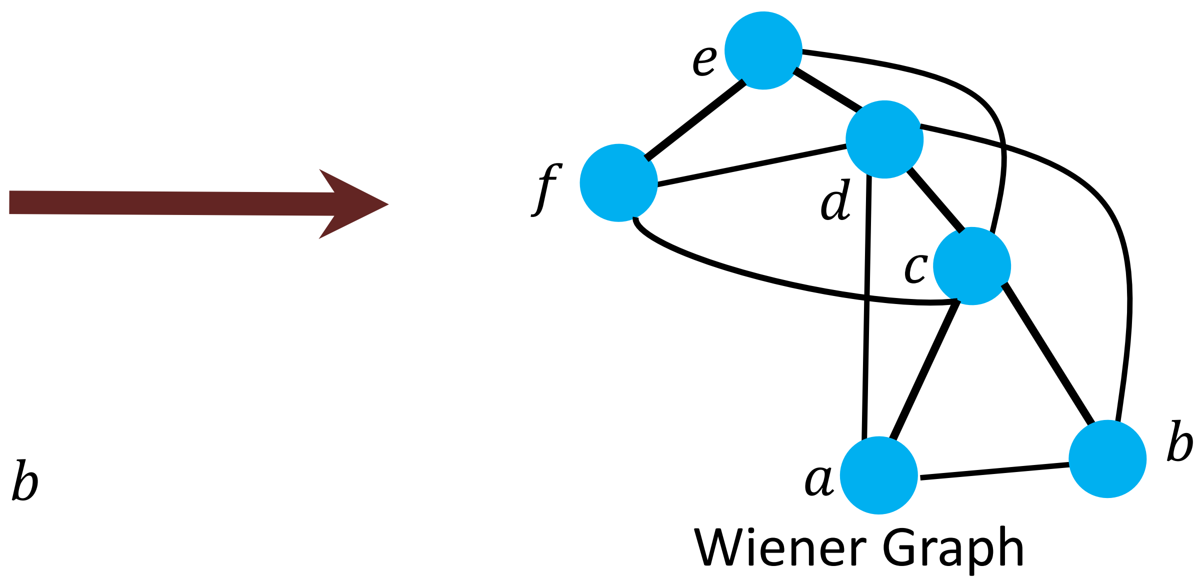
Wiener Filter for phase angles

- Wiener Filter:
- Solution: $X_a = \sum_{i \neq a} W_{ai}(z)\theta_i$



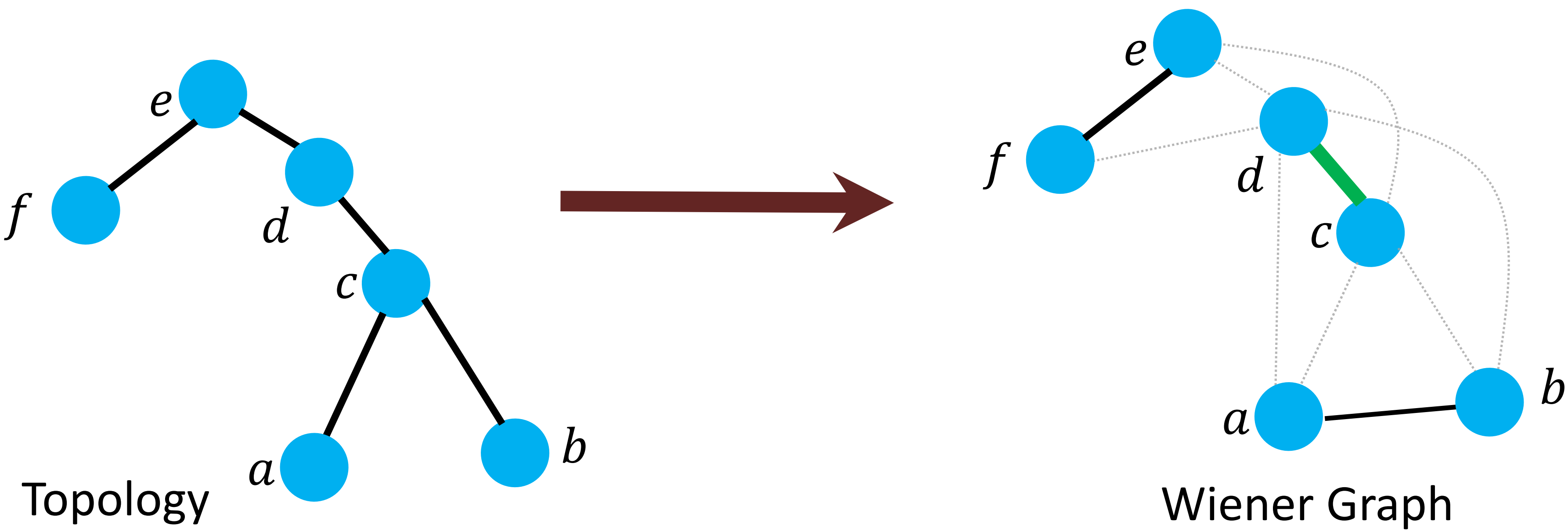
Wiener Graph of phase angles: for non-zero Wiener coefficients





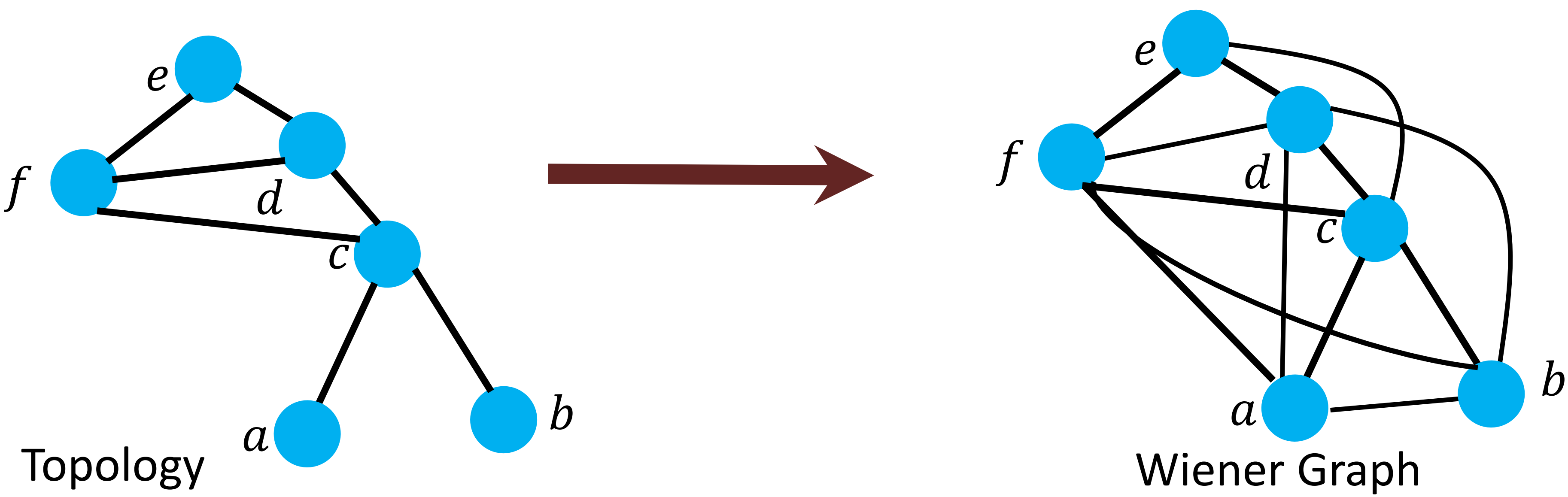
Wiener Graph for phase angles

- Two-hop neighbors are edges
- Topology Estimation for ***radial networks***:
  - Use separability tests on Wiener graph
  - ACC 2017 (submitted)



Wiener Graph for phase angles

- Topology Estimation for **loopy networks??**
- Use information in the Wiener coefficients:
  - $W_{ab}(z)$ function of frequency
  - Not scalar (different from scalar models)

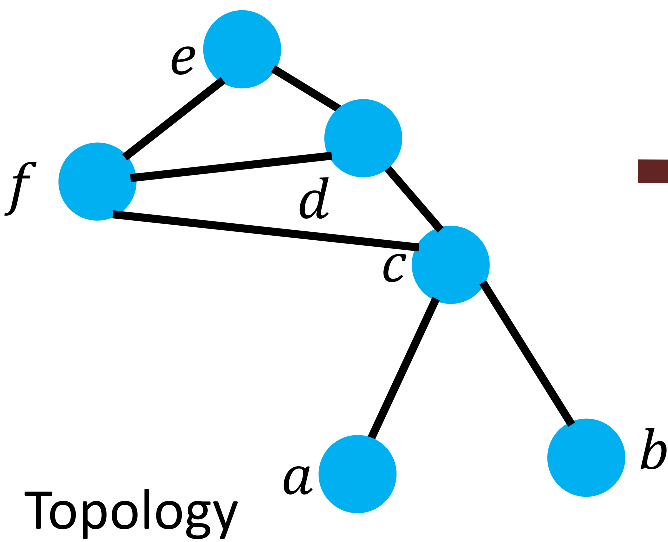


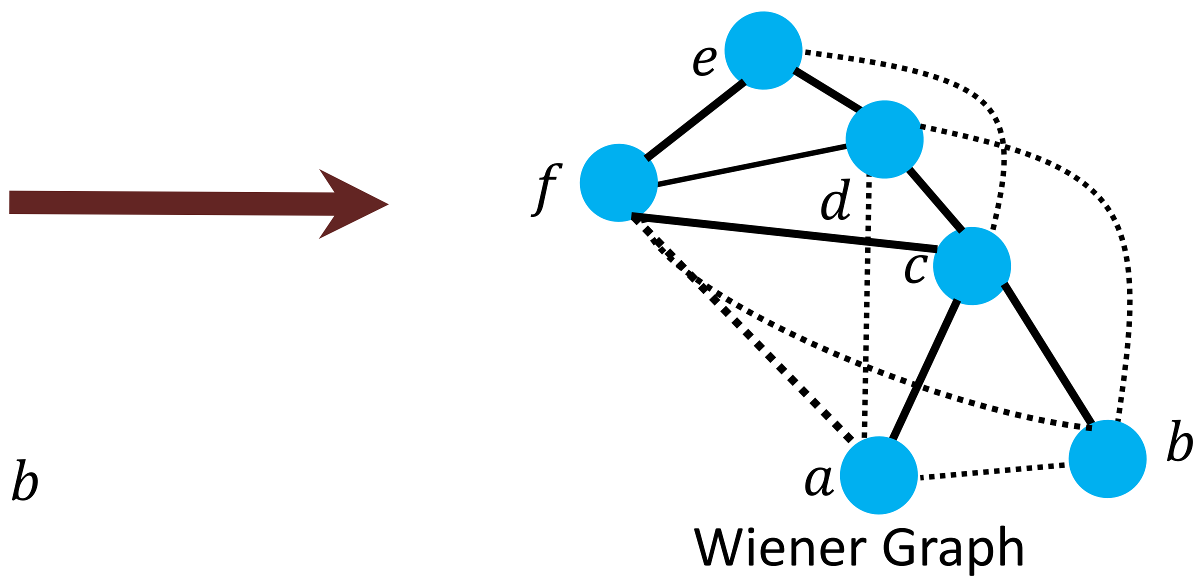
Wiener Graph for phase angles

- Topology Estimation for **loopy networks??**
- Use information in the Wiener coefficients:

Pruning Result: *Phase Response* of complex Wiener coefficient $W_{ab}(z)$ is **constant** for spurious edges between two-hop neighbors

- Doesn't depend on noise model

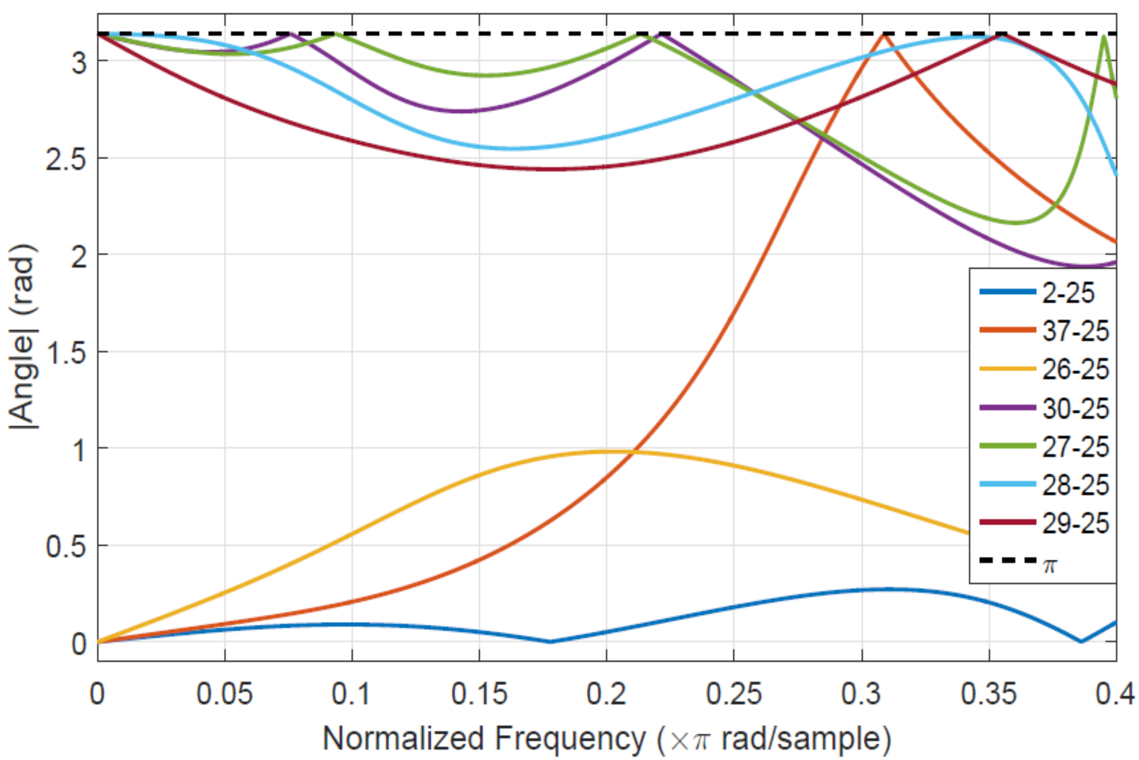


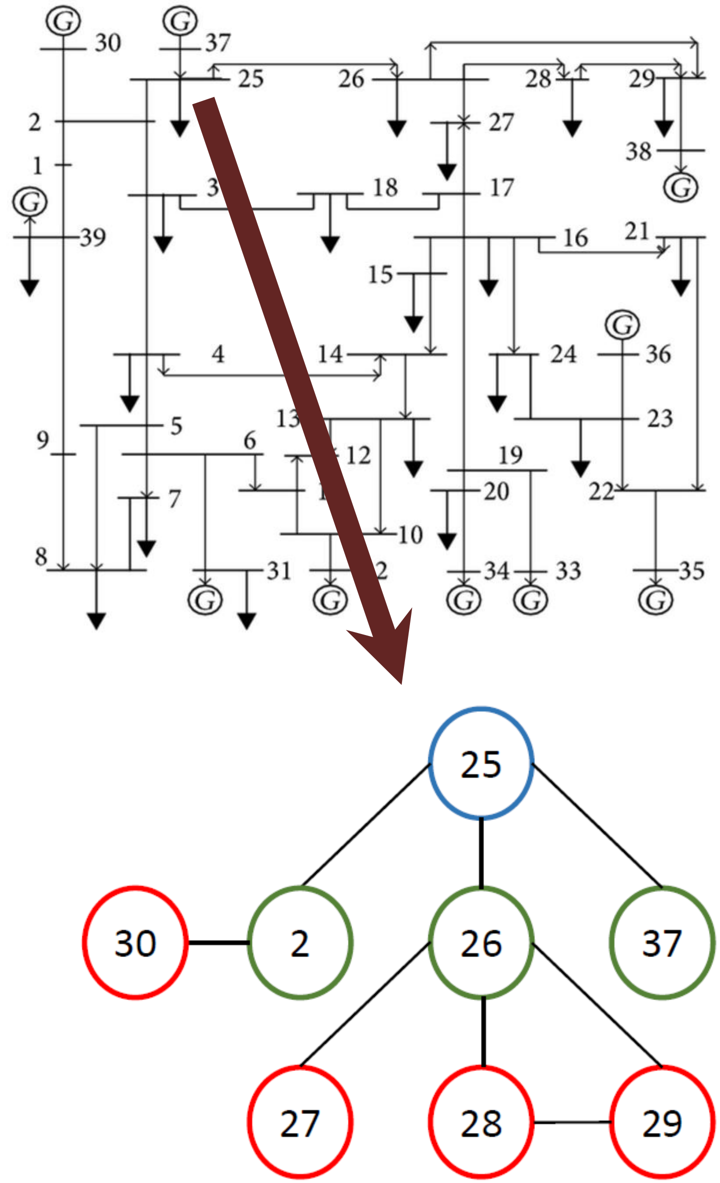


Wiener Graph for phase angles

- Edge pruning Example:

Absolute values of Phase angles of $W_{25,2}, W_{25,37}, W_{25,26}, W_{25,30}, W_{25,27}, W_{25,28}, W_{25,29}$





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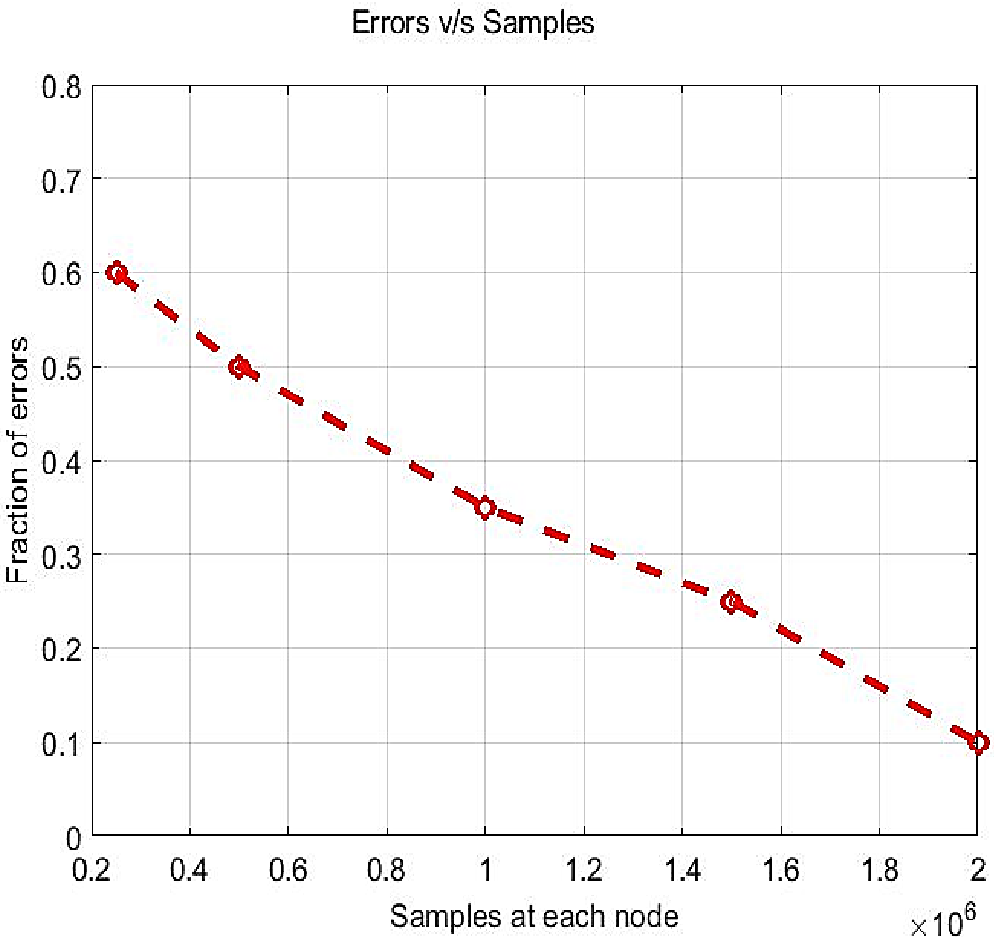
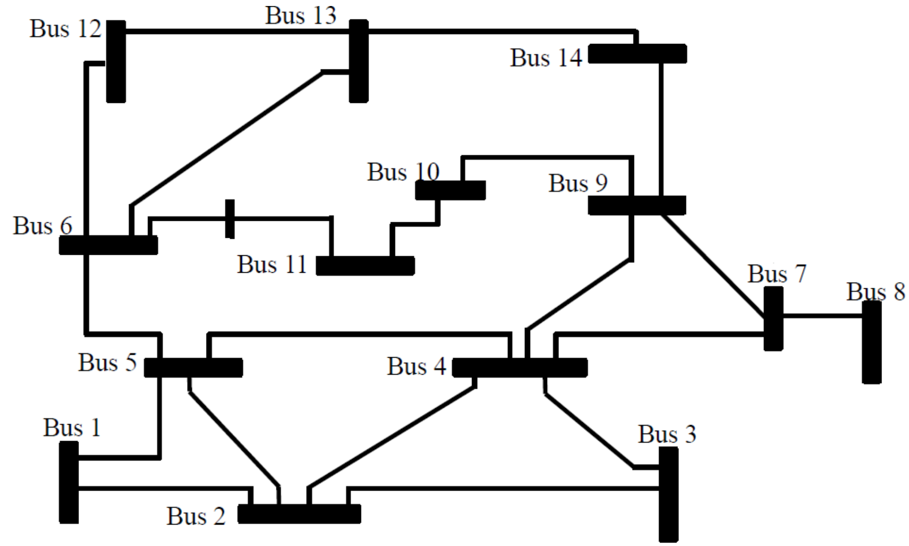
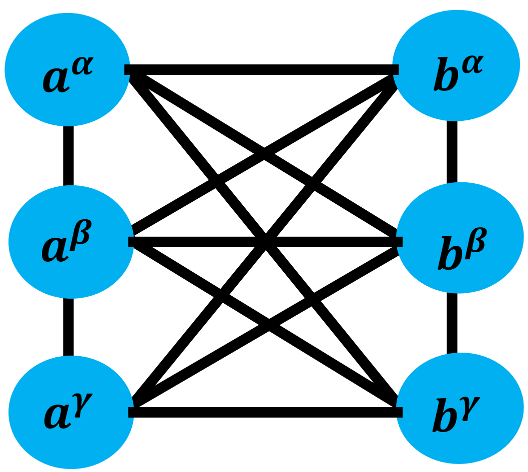




Extensions

- Radial networks **(Static Case):**
  - 3 phase unbalanced network

$$P_a + \hat{i}Q_a = \sum_{b:(a,b)\in\mathcal{E}^{\mathcal{F}}} \frac{V_a e^{\hat{i}\theta_a}\left(V_a e^{-\hat{i}\theta_a} - V_b e^{-\hat{i}\theta_b}\right)}{R_{ab} - \hat{i}X_{ab}}$$



$$\hat{P}_a = \sum_{b:(a,b)\in\mathcal{E}^{\mathcal{F}}} diag\left(\hat{V}_a \hat{I}_{ab}^H\right)$$

$$= \sum_{b:(a,b)\in\mathcal{E}^{\mathcal{F}}} diag\left(\hat{V}_a (\hat{V}_a^H - \hat{V}_b^H) {\hat{Z}^{H}}_{ab}^{-1}\right)$$

Extensions

- Loopy networks **(Dynamic Case):**
  - General Linear Dynamical Systems with directed/undirected edges
  - Change Detection of networks

Future Questions:

- **Sample Optimal** Wiener Filter : Regression?? Lasso??
- Presence of **hidden nodes:** order based separation
- Higher order control, **AC flow** equations in dynamics?
- Effect of **sampling frequency**
- **Parameter** estimation

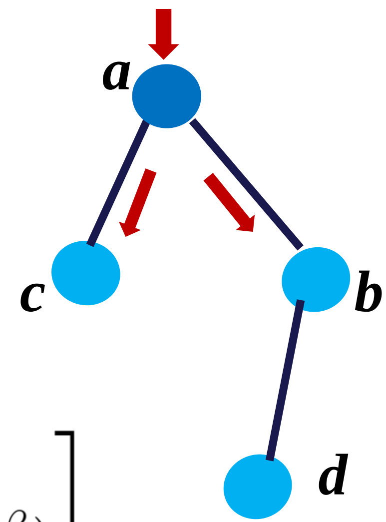
Thank You

Questions!

Extensions - 3 phase unbalanced network:

- 3 phase Linear Coupled power flow model:

$$V = \hat{M}^{-1} Z^H \hat{M}^{-T} P$$



$$\text{where } P = \begin{bmatrix} P^\alpha + iQ^\alpha \\ e^{-i2\pi/3}(P^\beta + iQ^\beta) \\ e^{i2\pi/3}(P^\gamma + iQ^\gamma) \end{bmatrix}, V = \begin{bmatrix} V^\alpha - i\theta^\alpha \\ e^{-i2\pi/3}(V^\beta - i\theta^\beta) \\ e^{i2\pi/3}(V^\gamma - i\theta^\gamma) \end{bmatrix}$$

$$\hat{M} = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix}, Z = \begin{bmatrix} Z^{\alpha\alpha} & Z^{\alpha\beta} & Z^{\alpha\gamma} \\ Z^{\alpha\beta} & Z^{\beta\beta} & Z^{\beta\gamma} \\ Z^{\alpha\gamma} & Z^{\beta\gamma} & Z^{\gamma\gamma} \end{bmatrix}$$

Every block is a diagonal matrix