Topology Learning in Power Grids from Ambient Fluctuations

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Things I will go over

• Power Grid
  • Issues and approach

• Model Fundamentals: Static and Dynamic
  • Power flows
  • Graphical Model

• Topology Learning in *Radial and Loopy Grids*

• Extensions & Future Work
Power Grid
Grid Types

Radial/Tree

Interconnected/Meshed
Grid ‘Operational’ Structure

- Underlying Loopy network
- Switches/Relays decide structure

Learning Problem:
- Estimate Configuration of Switches/Relays

Substation
Load Nodes
Power Grid: Structure Learning

• Uses:
  • Real time control
  • Failure Identification
  • Optimizing flows

• Challenge:
  • Limited real-time breakers
  • Brute Force inefficient

• Solution
  • Smart meters: PMUs, micro-PMUs, IoT
  • Big Data: High fidelity measurements
Similar Approaches

- Power Systems based:
  - Flow equations
  - Regression, Relaxation
  - Reno et al, Rajagopal et al, Annaswamy et al

- Machine Learning based:
  - Empirical evidence based
  - Clustering, Greedy approaches
  - Bolognani et al, Arya et al, Deka et al., Rajagoapal et al, Low et al

- This Talk:
  - *Probabilistic Graphical Model* for nodal voltages
Similar Approaches

- **Power Systems based:**
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- **This Talk:**
  - *Probabilistic Graphical Model* for nodal voltages

- Advantage:
  - **Provable** results
  - **No knowledge** of parameters or injections
  - Incorporates **Dynamics**
Learning Regime

Probabilistic Graphical Model for the nodal voltages

- Static Regime: *Power Flow Equations*

- Dynamic Regime: *Swing Equations*
Static Regime: AC power flow

\[ P_a + iQ_a = \sum_{(a,b) \text{ is edge}} V_a e^{i\theta_a} (V_a e^{-i\theta_a} - V_b e^{-i\theta_b})/(R_{ab} - iX_{ab}) \]

Flow on line function of voltages

Relaxation: One-one map from injections to voltages
Power Flow: **Lossless Relaxations**

- **DC power flow:** \( V_a = 1, \ \theta_a - \theta_b \approx 0, \ R_{ab} = 0 \)

\[ \theta = H_{1/X}^{-1} P \]

\( H_{1/X} \rightarrow \) wt. reduced Laplacian matrix

- **Linear-Coupled (LC) power flow:** \( V_a \approx 1, \ \theta_a - \theta_b \approx 0 \)

\[ \theta = H_{1/X}^{-1} P - H_{1/R}^{-1} Q, \ \ V = H_{1/R}^{-1} P + H_{1/X}^{-1} Q \]

- **LinDist flow (Baran-Wu):** (radial networks)

\[ V^2/2 = H_{1/R}^{-1} P + H_{1/X}^{-1} Q \]
Probabilistic Distribution of Nodal Voltages

- Distribution of injections:
  \[ \mathcal{P}(P) = \prod_{a \in \mathcal{V}} \mathcal{P}_a(P_a) \]

- Distribution of voltages:
  \[ \mathcal{P}(V) = \frac{1}{|J_P(V)|} \prod_{a \in \mathcal{V}} \mathcal{P}_a(P_a) \]
  \[ = \frac{1}{|J_P(V)|} \prod_{a \in \mathcal{V}} \mathcal{P}_a \left( \sum_{b: \{a,b\} \in \mathcal{E}} V_a(V_a^* - V_b^*) / \tilde{z}_{ab}^* \right) \]
  \[ J_P(V) = \left( \frac{\partial V}{\partial P} \right) \text{ Jacobian} \]

- Assumption: Injection fluctuations are independent
Graphical Model of Nodal Voltages

- **Graphical Model**: Graphical Factorization of Distribution
  - Nodes represent variables
  - Neighbors give conditional independence from all others

\[
P(X_d | X_a, X_c, X_e) = P(X_d | X_c, X_e)
\]

\[
P(X) \equiv f
\]
Graphical Model of Nodal Voltages

- **Graphical Model**: Graphical Factorization of Distribution
  - Nodes represent variables
  - Neighbors give conditional independence from all others
  - **Separator** sets make disjoint groups conditionally independent

\[ \mathcal{P}(X) \equiv \]

\[
\begin{array}{c}
\text{Separator sets make disjoint groups conditionally independent}
\end{array}
\]

\[
\begin{array}{c}
e f \\
(c d) \\
a b
\end{array}
\]
Graphical Model of Nodal Voltages

• Distribution

\[ \mathcal{P}(V) = \frac{1}{|J_P(V)|} \prod_{a \in \mathcal{V}} \mathcal{P}_a \left( \sum_{b: \{a, b\} \in \mathcal{E}} V_a (V_a^* - V_b^*) / \tilde{z}_{ab}^* \right) \]

• If Jacobian \( J_P(V) \) is separable:
  • Graphical Model: Topology Edges + 2-hop neighbors
Graphical Model of Nodal Voltages

- Distribution
  \[ \mathcal{P}(V) = \frac{1}{|J_P(V)|} \prod_{a \in \mathcal{V}} \mathcal{P}_a \left( \sum_{b: \{a,b\} \in \mathcal{E}} V_a (V_a^* - V_b^*) / \tilde{z}_{ab}^* \right) \]

- If Jacobian \( J_P(V) \) is separable:
  - Graphical Model: Topology Edges + 2-hop neighbors

- Proof:
  - Factorize \( \mathcal{P}(V) \)
  - Terms including node \( f \)

\[ \mathcal{P}_f \left( V_f (V_f^* - V_e^*) / \tilde{z}_{ef}^* \right) \]
\[ \mathcal{P}_e \left( V_e (V_e^* - V_f^*) / \tilde{z}_{ef}^* + V_e (V_e^* - V_d^*) / \tilde{z}_{ed}^* \right) \]
Graphical Model $\rightarrow$ Topology estimation

• Distribution \[ \mathcal{P}(V) = \frac{1}{|J_P(V)|} \prod_{a \in V} \mathcal{P}_a \left( \sum_{b: \{a, b\} \in \mathcal{E}} V_a (V_a^* - V_b^*) / \tilde{z}_{ab}^* \right) \]

• How to distinguish true edges??
Graphical Model $\rightarrow$ Topology estimation

- Radial Grids: Separation Possible

Only **topology edges** are separators

(PSCC 2016)
Graphical Model $\rightarrow$ Topology estimation

- Radial Grids: Separation Possible

Only **topology edges** are separators

Topology

![Graphical Model](image.png)
Graphical Model $\rightarrow$ Topology estimation

- Radial Grids: Separation Possible

Only **topology edges** are separators
Previously.....

- **Separator** sets make disjoint groups conditionally independent

\[ \mathcal{P}(X) \equiv \]
Structure Learning Algorithm

• Learn the *intermediate* edges (not connected to leaves)

\[
P(V_f, V_a | V_c, V_d) = P(V_f | V_c, V_d) P(V_a | V_c, V_d)
\]

Conditional Independence test

• Learn edges to *leaves* using known edge pairs.

\[
P(V_f, V_c | V_e, V_d) = P(V_f | V_e, V_d) P(V_c | V_e, V_d)
\]

Additional information needed:
• **None** (no impedance/injection statistics)
• No knowledge of distribution type
Computational Complexity:

- $O(N^4)$ conditional probability checks (all edges permissible)

Complexity of conditional probability test:

- Discrete complex voltages: $O(p^6)$
- Continuous voltages:
  - Kernel based non-parametric checks
  - Hilbert-Schmidt norm for covariance
- Independent of Network Size
Special Case: **Gaussian** random variables

- Inverse covariance $\Sigma^{-1}$ gives graphical model
- Learn the graphical model directly: **Graphical Lasso, Lasso etc.**

Minimum Conditional Variance

$$
\beta_i = \arg \min_{\beta} \frac{1}{N} \sum_{i=1}^{n} (x_i^T \beta + \sum_{j \neq i} \beta_j x_j)^2 \\
\text{s.t.} \quad ||\beta||_0 = d
$$

$$
\hat{\beta}_i = \arg \min_{\beta} \beta^T \Sigma_{-i} \beta + 2 \Sigma_{-i} x_i \beta + \Sigma_{-i} x_i \\
\text{s.t.} \quad ||\beta||_0 = d
$$

$$
\hat{\varepsilon}_{ij} = \frac{1}{2} \sum_{k=1}^{p} x_i^k x_j^k \quad \text{empirical covariance matrix}
$$

$\text{Supp}(\beta) = S, \quad |S| = d$

$S = \{i_j, j_2, \ldots, j_d\}$
What about loopy grids??

• How to distinguish true edges??
• Separation Results do not hold …. **hence not possible**

![Diagram showing topological and graphical models](image)
Dynamic Regime: Swing Equations

- Fluctuations due to ambient noise in injections:

\[ M_a \ddot{\theta}_a + D_a \dot{\theta}_a = \sum_{(a,b) \text{ is edge}} B_{ab}(\theta_b - \theta_a) + P_a \]

Dynamics of phase angles

Net power imbalance

- Frequency \( \omega_a = \dot{\theta}_a \)
- Inertia \( M \) and Damping \( D \) from synchronous machines.
- Stochastic noise \( P_a \)
Linear Dynamical System for Swing Equations

- Swing Equations in z-domain:

\[ M_a \ddot{\theta}_a + D_a \dot{\theta}_a = \sum_{(a,b) \text{ is edge}} B_{ab}(\theta_b - \theta_a) + P_a \]

\[ \theta(z) = H(z)\theta(z) + \hat{P}(z) \]

- Noise Model:
  - Stochastic Wide-Sense Stationary (WSS)
  - Uncorrelated
  - Diagonal Power Spectral Density: \( \Phi_{\hat{P}} = \mathcal{Z} \left( \mathbb{E}[\hat{P}\hat{P}^T] \right) \)
Linear Dynamical System for Swing Equations

• Swing Equations in z-domain:

\[ \theta(z) = H(z)\theta(z) + \hat{P}(z) \]

• Measurements:

Time-series of phase angle dynamics
**Wiener Filter for phase angles**

- **Wiener Filter:**
  \[(\text{Wiener, Kolmogorov, 1950})\]
  \[
  \min_{X \in \text{tf-span}\{\theta_i\}_{i \neq a}} \|\theta_a - X\|^2
  \]

- **Solution:**
  \[X_a = \sum_{i \neq a} W_{ai}(z)\theta_i\]

- **Optimal non-causal projection**
- **Related to Power-Spectral density** and transfer function \(H(z)\)
- **Computation Complexity:** \(O(N^3)\)
Wiener Filter for phase angles

- Wiener Filter:
- Solution: $X_a = \sum_{i \neq a} W_{ai}(z) \theta_i$
Wiener Graph for phase angles

- Two-hop neighbors are edges
- Topology Estimation for *radial networks*:
  - Use separability tests on Wiener graph
  - ACC 2017 (submitted)
**Wiener Graph for phase angles**

- Topology Estimation for **loopy networks**
- Use information in the Wiener coefficients:
  - $W_{ab}(z)$ function of frequency
  - Not scalar (different from scalar models)
Topology Estimation for loopy networks?

- Use information in the Wiener coefficients:

**Pruning Result:** *Phase Response* of complex Wiener coefficient $W_{ab}(z)$ is **constant** for spurious edges between two-hop neighbors

- Doesn’t depend on noise model
Wiener Graph for phase angles

- Edge pruning Example:

![Graph and Curve Plot Illustrating Phase Angles and Frequency Analysis]
Simulations

Errors v/s Samples

Fraction of errors vs Samples at each node
• Radial networks (Static Case):
  – 3 phase unbalanced network

\[
P_a + iQ_a = \sum_{b: (a,b) \in \mathcal{E}} \frac{V_a e^{i\theta_a} (V_a e^{-i\theta_a} - V_b e^{-i\theta_b})}{R_{ab} - iX_{ab}}
\]

\[
\hat{P}_a = \sum_{b: (a,b) \in \mathcal{E}} \text{diag} \left( \hat{V}_a \hat{I}_{ab}^H \right)
\]

\[
= \sum_{b: (a,b) \in \mathcal{E}} \text{diag} \left( \hat{V}_a (\hat{V}_a^H - \hat{V}_b^H) \hat{Z}_{ab} H^{-1} \right)
\]
Extensions

- Loopy networks **(Dynamic Case)**:
  - General Linear Dynamical Systems with directed/undirected edges
  - Change Detection of networks

Future Questions:
- **Sample Optimal** Wiener Filter: Regression?? Lasso??
- Presence of **hidden nodes**: order based separation
- Higher order control, **AC flow** equations in dynamics?
- Effect of **sampling frequency**
- **Parameter** estimation
Thank You

Questions!
Extensions - 3 phase unbalanced network:

3 phase Linear Coupled power flow model:

\[ V = \hat{M}^{-1} Z^H \hat{M}^{-T} P \]

where \( P = \begin{bmatrix} P^\alpha + iQ^\alpha \\ e^{-i2\pi/3}(P^\beta + iQ^\beta) \\ e^{i2\pi/3}(P^\gamma + iQ^\gamma) \end{bmatrix}, V = \begin{bmatrix} V^\alpha - i\theta^\alpha \\ e^{-i2\pi/3}(V^\beta - i\theta^\beta) \\ e^{i2\pi/3}(V^\gamma - i\theta^\gamma) \end{bmatrix} \)

\[ \hat{M} = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix}, Z = \begin{bmatrix} Z^{\alpha\alpha} & Z^{\alpha\beta} & Z^{\alpha\gamma} \\ Z^{\alpha\beta} & Z^{\beta\beta} & Z^{\beta\gamma} \\ Z^{\alpha\gamma} & Z^{\beta\gamma} & Z^{\gamma\gamma} \end{bmatrix} \]

Every block is a diagonal matrix