Robust policies for storage used to offset renewable variance

Daniel Bienstock, Gonzalo Muñoz and Shuoguang Yang,
Columbia University
Carsten Matke, NEXT ENERGY (Germany)

Talk outline

- Optimization framework for using storage to offset uncertainty in renewable forecast output
- "OPF-like" setup: minimize cost of generation
- Multiperiod model, with per-period renewable forecasts
- linear control for battery output
- nonconvex model for battery operation
- robust optimization used to handle forecast errors

Broader goal: dealing/using risky injections

- At some buses: controllable injections (e.g. generation) or known loads
- At some buses, uncontrollable (uncertain or adversarial) injections
- At other buses: controllable injection used to mitigate risk
- Multi-time period and 'OPF-like": minimize cost subject to being able to react to any (uncontrollable) event in each time period
- **Three-level** (not bilevel) optimization problem over multiple periods
- NP-hard in strong sense even for simple trees and one time period

Control setup outline

1. $\boldsymbol{T}$ time periods of equal length $\boldsymbol{\Delta}$. Assume $\boldsymbol{\Delta = 1}$.

2. $\boldsymbol{P_k^{g,t}}$ = output of generator at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (decision variable)

3. $\bar{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{t}} + \boldsymbol{w_i^t}$ = output of renewable at bus $\boldsymbol{i}$ at time $\boldsymbol{t}$.

   $\bar{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{t}}$ = forecast, $\boldsymbol{w_i^t}$ = error (uncertain).

4. **Control** used to set battery output; applied as a function of observations of the $\boldsymbol{w_i^t}$.

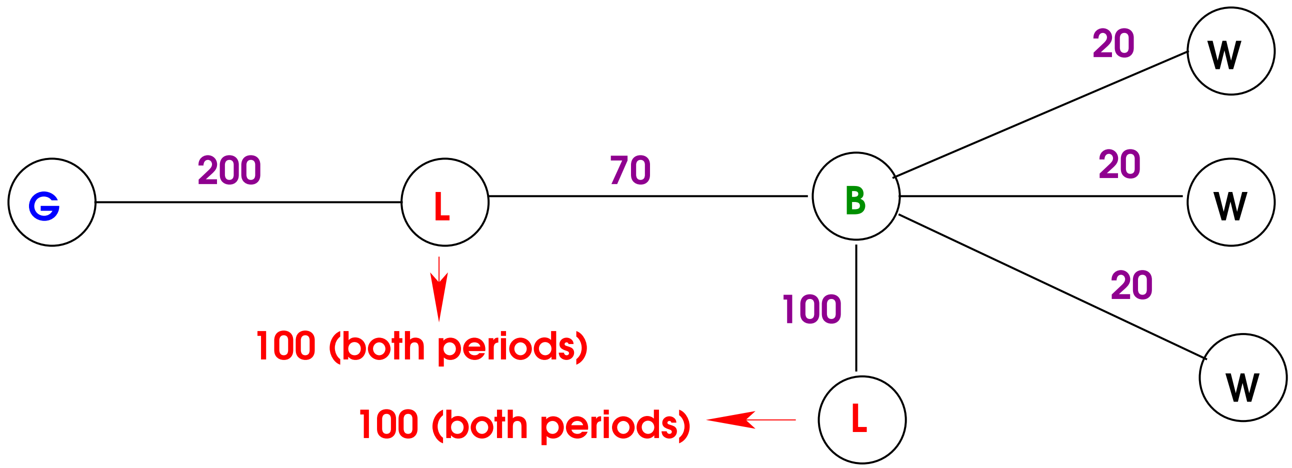
5. $\boldsymbol{w_i^t}$ **estimated** at start of period $\boldsymbol{t}$.

6. **Control** held constant during period $\boldsymbol{t}$.

7. **Control** does not handle additional real-time deviations.

A simple example on 2 periods

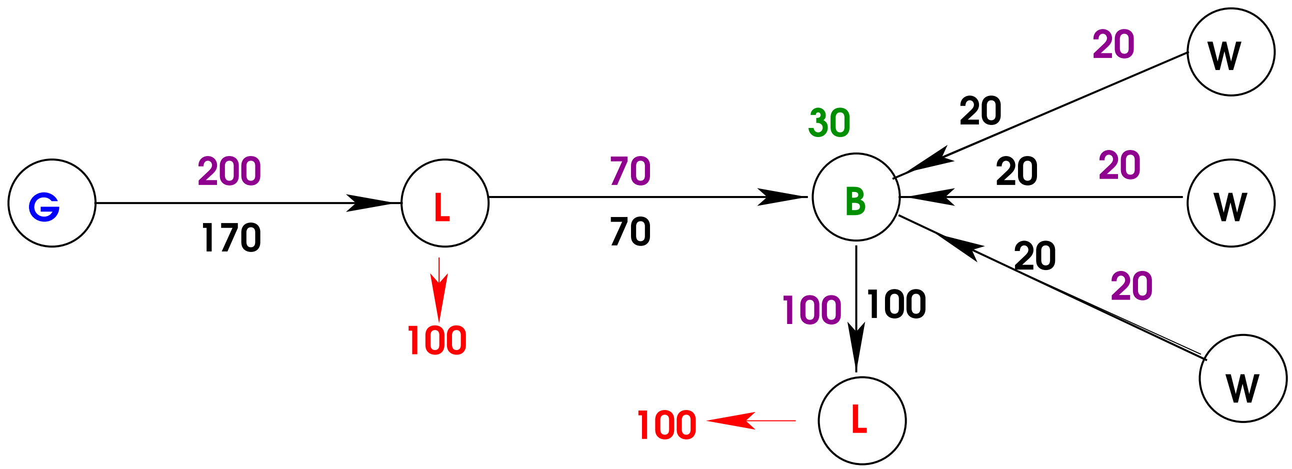
$\boldsymbol{G}$ = generator, $\boldsymbol{L}$ = load, $\boldsymbol{B}$ = battery, $\boldsymbol{W}$ = renewable



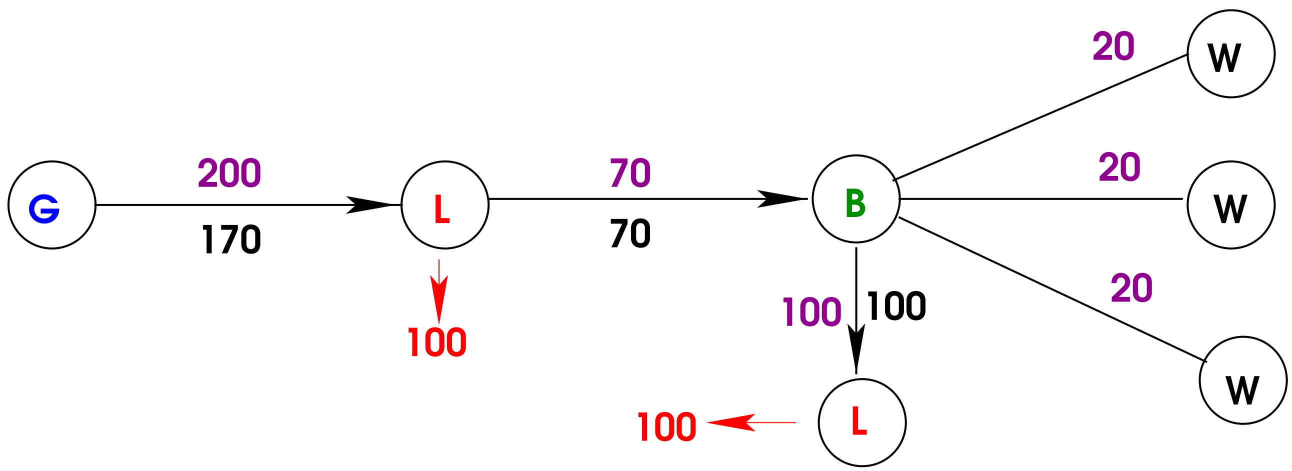
- **Period 1:** each renewable outputs **20**, no uncertainty.
- **Period 2:** each renewable outputs in the range $[\mathbf{0, 20}]$.
- Battery and generator are large, but battery starts **drained**.

- **Period 1:** each renewable outputs $\mathbf{20}$, no uncertainty.
- **Period 2:** each renewable outputs in the range $\mathbf{[0, 20]}$.

Period 1:



Period 2:



Notation

1. $\boldsymbol{T}$ time periods of equal length $\boldsymbol{\Delta}$. Assume $\boldsymbol{\Delta = 1}$ for this talk.
2. $\boldsymbol{P_k^{g,t}}$ = output of generator at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (decision variable)
3. $\boldsymbol{\bar{w}_i^t}$ + $\boldsymbol{w_i^t}$ = output of renewable at bus $\boldsymbol{i}$ at time $\boldsymbol{t}$.
   $\boldsymbol{\bar{w}_i^t}$ = forecast, $\boldsymbol{w_i^t}$ = error (uncertain).
4. $\boldsymbol{\delta_j^t}$ = output of battery at bus $\boldsymbol{j}$ at time $\boldsymbol{t}$.
   Assumption: all batteries at a given bus $\boldsymbol{j}$ are of similar type.
5. $\boldsymbol{P_k^{d,t}}$ = load at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (data).
6. **DC** power flow $\rightarrow$ for all $\boldsymbol{t}$, and all $\boldsymbol{w}$,

$$\sum_k P_k^{g,t} \; + \; \sum_i (\bar{w}_i^t + \; \boldsymbol{w_i^t}) \; + \; \sum_j \delta_j^t \; = \; \sum_k P_k^{d,t}$$

1. $\boldsymbol{T}$ time periods of equal length $\boldsymbol{\Delta}$. Assume $\boldsymbol{\Delta = 1}$ for this talk.
2. $\boldsymbol{P_k^{g,t}}$ = output of generator at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (decision variable)
3. $\boldsymbol{\bar{w}_i^t}$ + $\boldsymbol{w_i^t}$ = output of renewable at bus $\boldsymbol{i}$ at time $\boldsymbol{t}$. $\boldsymbol{\bar{w}_i^t}$ = forecast, $\boldsymbol{w_i^t}$ = error (uncertain).
4. $\boldsymbol{\delta_j^t}$ = output of battery at bus $\boldsymbol{j}$ at time $\boldsymbol{t}$.
5. $\boldsymbol{P_k^{d,t}}$ = load at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (data).
6. For all $\boldsymbol{t}$, and all $\boldsymbol{w}$, $\sum_k P_k^{g,t} + \sum_i (\bar{w}_i^t + \boldsymbol{w_i^t}) + \sum_j \delta_j^t = \sum_k P_k^{d,t}$

Generic linear control:

$$\boldsymbol{\delta_j^t} = - \sum_i \lambda_{ji}^t \, \boldsymbol{w_i^t}$$

- λ^t: decision variables
- Complex? Requires many (accurate?) real-time observations

1. $\boldsymbol{T}$ time periods of equal length $\boldsymbol{\Delta}$. Assume $\boldsymbol{\Delta = 1}$.
2. $\boldsymbol{P_k^{g,t}}$ = output of generator at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (decision variable)
3. $\boldsymbol{\bar{w}_i^t}$ + $\boldsymbol{w_i^t}$ = output of renewable at bus $\boldsymbol{i}$ at time $\boldsymbol{t}$. $\boldsymbol{\bar{w}_i^t}$ = forecast, $\boldsymbol{w_i^t}$ = error (uncertain).
4. $\boldsymbol{\delta_j^t}$ = output of battery at bus $\boldsymbol{j}$ at time $\boldsymbol{t}$.
5. $\boldsymbol{P_k^{d,t}}$ = load at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (data).
6. For all $\boldsymbol{t}$, and all $\boldsymbol{w}$, $\sum_k P_k^{g,t} + \sum_i(\bar{w}_i^t + \boldsymbol{w_i^t}) + \sum_j \delta_j^t = \sum_k P_k^{d,t}$

Specialized linear control:

$$\boldsymbol{\delta_j^t} = -\lambda_j^t \sum_{i \in R(j)} \boldsymbol{w_i}$$

- λ_j^t: decision variables
- $\boldsymbol{R(j)}$: set of buses that battery at $\boldsymbol{j}$ **responds to**

1. $\boldsymbol{T}$ time periods of equal length $\boldsymbol{\Delta}$. Assume $\boldsymbol{\Delta = 1}$.
2. $\boldsymbol{P_k^{g,t}}$ = output of generator at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (decision variable)
3. $\boldsymbol{\bar{w}_i^t}$ + $\boldsymbol{w_i^t}$ = output of renewable at bus $\boldsymbol{i}$ at time $\boldsymbol{t}$. $\boldsymbol{\bar{w}_i^t}$ = forecast, $\boldsymbol{w_i^t}$ = error (uncertain).
4. $\boldsymbol{\delta_j^t}$ = output of battery at bus $\boldsymbol{j}$ at time $\boldsymbol{t}$.
5. $\boldsymbol{P_k^{d,t}}$ = load at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (data).
6. For all $\boldsymbol{t}$, and all $\boldsymbol{w}$, $\sum_k P_k^{g,t} + \sum_i (\bar{w}_i^t + \boldsymbol{w_i^t}) + \sum_j \delta_j^t = \sum_k P_k^{d,t}$

Specialized linear control:

$$\boldsymbol{\delta_j^t} = -\lambda_j^t \sum_{i \in R(j)} \boldsymbol{w_i}$$

- λ_j^t: decision variables
- $\boldsymbol{R(j)}$: set of buses that battery at $\boldsymbol{j}$ **responds to**

Example 1: (bad?) a battery at each renewable, $\boldsymbol{R(j) = j}$ for all j

Example 2: for all j, $\boldsymbol{R(j) =}$ all renewables

Regardless of control, **nominal case:** must hold

$$\sum_k P_k^{g,t} + \sum_i \bar{w}_i^t = \sum_k P_k^{d,t}$$

Balance: $\sum_k P_k^{g,t} + \sum_i(\bar{w}_i^t + \boldsymbol{w_i^t}) + \sum_j \delta_j^t = \sum_k P_k^{d,t}$

Nominal case: $\sum_k P_k^{g,t} + \sum_i \bar{w}_i^t = \sum_k P_k^{d,t}$

Generic linear control: $\boldsymbol{\delta_j^t} = -\sum_i \lambda_{ij}^t \boldsymbol{w_i^t}$

Rewrite balance:

$$\sum_k P_k^{g,t} + \sum_i \bar{w}_i^t + \sum_i \left[\left(1 - \sum_j \lambda_{ji}^t \right) \boldsymbol{w_i^t} \right] = \sum_k P_k^{d,t}$$

Together with nomimal case, implies:

$$\boldsymbol{\sum_i \left[\left(1 - \sum_j \lambda_{ji}^t \right) w_i^t \right] = 0}$$

Balance: $\sum_k P_k^{g,t} + \sum_i (\bar{w}_i^t + \boldsymbol{w_i^t}) + \sum_j \delta_j^t = \sum_k P_k^{d,t}$

Nominal case: $\sum_k P_k^{g,t} + \sum_i \bar{w}_i^t = \sum_k P_k^{d,t}$

Generic linear control: $\boldsymbol{\delta_j^t} = - \sum_i \lambda_{ij}^t \, \boldsymbol{w_i^t}$

Rewrite balance:

$$\sum_k P_k^{g,t} + \sum_i \bar{w}_i^t + \sum_i \left[\left(1 - \sum_j \lambda_{ji}^t \right) \, \boldsymbol{w_i^t} \right] = \sum_k P_k^{d,t}$$

Together with nomimal case, implies:

$$\sum_i \left[\left(1 - \sum_j \boldsymbol{\lambda_{ji}^t} \right) \, \boldsymbol{w_i^t} \right] = \mathbf{0}$$

"Full dimensional" uncertainty set $\Rightarrow$

$$\sum_j \boldsymbol{\lambda_{ji}^t} = \mathbf{1} \quad \text{for all } \boldsymbol{i} \text{ (and } \boldsymbol{t}\text{)}$$

Balance: $\sum_k P_k^{g,t} + \sum_i (\bar{w}_i^t + \boldsymbol{w_i^t}) + \sum_j \delta_j^t = \sum_k P_k^{d,t}$

Nominal case: $\sum_k P_k^{g,t} + \sum_i \bar{w}_i^t = \sum_k P_k^{d,t}$

Specialized linear control: $\boldsymbol{\delta_j^t} = -\lambda_j^t \sum_{i \in R(j)} \boldsymbol{w_i^t}$

Rewrite balance:

$$\sum_k P_k^{g,t} + \sum_i \bar{w}_i^t + \sum_i \left[\left(1 - \sum_{j\,:\, i \in R(j)} \lambda_j^t \right) \boldsymbol{w_i^t} \right] = \sum_k P_k^{d,t}$$

Together with nomimal case, implies:

$$\left(1 - \sum_{j\,:\, i \in R(j)} \lambda_j^t \right) \boldsymbol{w_i^t} = 0$$

"Full dimensional" uncertainty set $\Rightarrow$

$$\sum_{j\,:\, i \in R(j)} \boldsymbol{\lambda_j^t} = \mathbf{1} \text{ for all } \boldsymbol{i} \text{ (and } \boldsymbol{t}\text{)}$$

Sign of $\boldsymbol{\lambda^t}$?

Battery model

- Energy state bounds. Let E_j^0 = initial energy state of battery at site i. Then

$$\boldsymbol{E_j^t} = E_j^0 + \Delta \sum_{h=1}^{t} \boldsymbol{\delta_j^h}$$

is the energy state at the end of period t. (Δ = length of time periods)
Must have lower- and upper-bounds on $\boldsymbol{E_j^t}$.

- Special bounds for $\boldsymbol{E_j^T}$?

- Discharge rate bounds. We will want to lower- and upper-bound

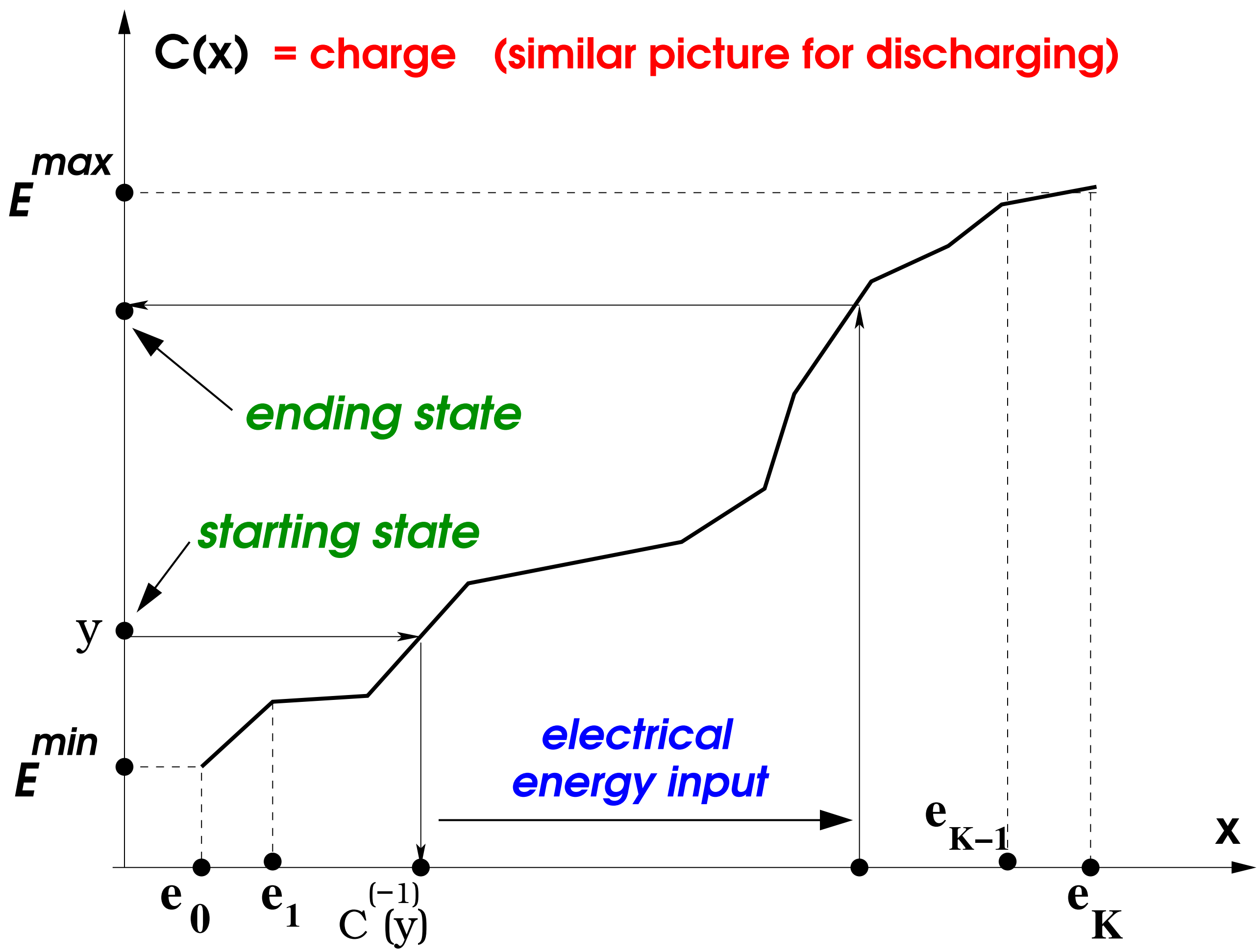
$$\boldsymbol{\delta_j^t} = \bar{D}_j^t - \lambda_j^t \left(\sum_{i \in R(j)} \boldsymbol{w_i^t} \right)$$

for all batteries $\boldsymbol{i}$, time $\boldsymbol{t}$ and all $\boldsymbol{w}$.

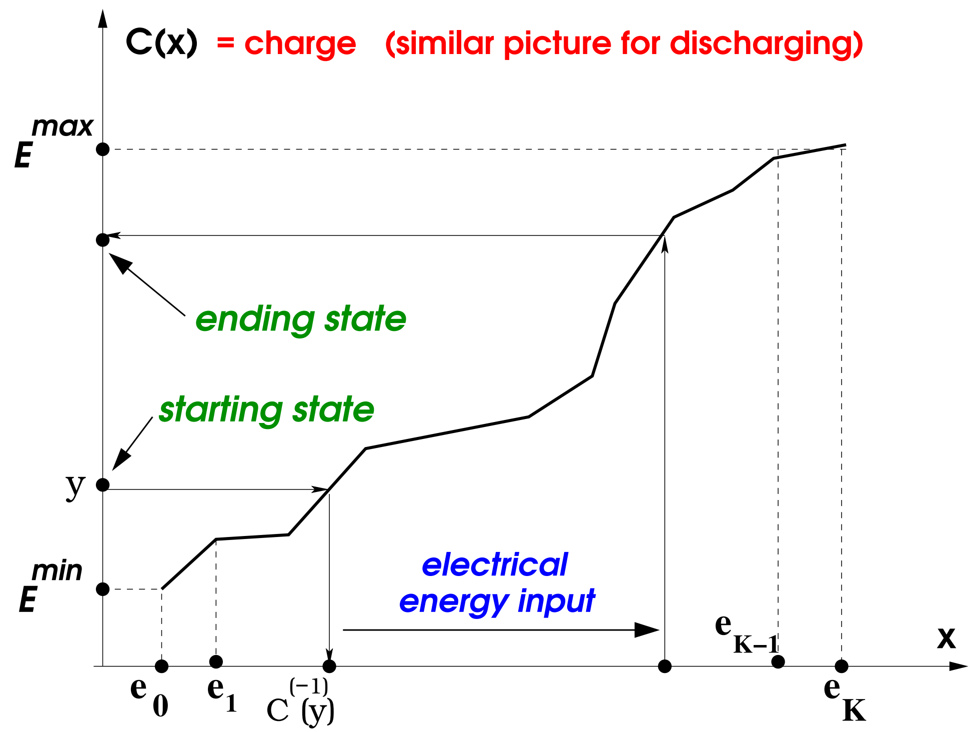
Battery chemistry is complex. Traditional model:

1. $\boldsymbol{0 < \eta_i^c \leq 1}$, $\boldsymbol{0 < \eta_i^d \leq 1}$: charging, discharging efficiency for battery $\boldsymbol{i}$
2. If we **inject electrical power** $\boldsymbol{D_i \geq 0}$ into battery $\boldsymbol{i}$, energy state increases by $\boldsymbol{\eta_i^c D_i}$
3. If we **extract electrical power** $\boldsymbol{G_i \geq 0}$ from battery $\boldsymbol{i}$, energy state decreases by $\boldsymbol{(\eta_i^d)^{-1} G_i}$
4. **Summary:** if the **electrical power injection** into battery $\boldsymbol{i}$ is $\boldsymbol{P_i}$, energy state changes by
$$(-\eta_i^d)^{-1} [P_i]^- + \eta_i^c [P_i]^+$$
5. A **nonconvex** model: if $\boldsymbol{P_i}$ is a decision variable, must enforce
$$[P_i]^- [P_i]^+ = 0$$

Piecewise-constant battery efficiency model



- Slope = charge (or discharge) efficiency



- **Constraint**: if the battery charge lies in some interval

$$[\, C(e_s)\, ,\, C(e_{s+1})\,]$$

then can inject charge **at most**

$$\sigma_s \quad (= \text{slope}) \text{ per unit time}$$

- Generalizes traditional constraint
- But very nonconvex

Renewable forecast error model

Should we rely on chance-constrained models?

Renewable forecast error model

Should we rely on chance-constrained models?

Suppose that $\boldsymbol{x(t)}$, $\boldsymbol{t = 1, 2, \ldots, T}$ is a stochastic process:

- Even if we understand each $\boldsymbol{x(t)}$ well enough to provide tail probabilities ...

Renewable forecast error model

Should we rely on chance-constrained models?

Suppose that $\boldsymbol{x(t)}$, $\boldsymbol{t = 1, 2, \ldots, T}$ is a stochastic process:

- Even if we understand each $\boldsymbol{x(t)}$ well enough to provide tail probabilities ...
- **Partial sums** $\sum_{t=1}^{k} \boldsymbol{x(t)}$ are a different story

Except e.g. in the gaussian case

Renewable forecast error model

Output of renewable $\boldsymbol{i}$ at time $\boldsymbol{t}$: $\bar{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{t}} + \boldsymbol{w}_{\boldsymbol{i}}^{\boldsymbol{t}}$

- $\bar{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{t}}$ = forecast output
- $\boldsymbol{w}_{\boldsymbol{i}}^{\boldsymbol{t}}$ = error; $\boldsymbol{w} \in \mathcal{W}$, where $\mathcal{W}$ = uncertainty model.

Concentration models: $\mathcal{W}$ is the set of all $\boldsymbol{w}$ such that

$$C^1 w^+ + C^2 w^- \leq b, \tag{1a}$$

$$w_{k,t}^{\min} \leq w_k^t \leq w_{k,t}^{\max} \quad \text{for all } t \text{ and } k, \tag{1b}$$

- $\boldsymbol{C^1}, \boldsymbol{C^2}$ are **nonnegative** matrices
- $\boldsymbol{b}, \ \boldsymbol{w}^{\mathbf{min}}, \boldsymbol{w}^{\mathbf{max}}$: parameters of model
- Caution! (1a) is **not** a linear model
- Allows for spatial and temporal correlation

Renewable forecast error model

Output of renewable $\boldsymbol{i}$ at time $\boldsymbol{t}$: $\bar{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{t}} + \boldsymbol{w}_{\boldsymbol{i}}^{\boldsymbol{t}}$

- $\bar{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{t}}$ = forecast output
- $\boldsymbol{w}_{\boldsymbol{i}}^{\boldsymbol{t}}$ = error; $\boldsymbol{w} \in \mathcal{W}$, where $\mathcal{W}$ = uncertainty model.

Special example: uncertainty budgets

$$|w_i^t| \le \gamma_i^t, \quad \text{all } t \text{ and } i$$
$$\sum_i \alpha_i^t |w_i^t| \le \Gamma^t \quad \text{all } t$$

Here, the $\boldsymbol{\gamma_i^t}$, $\boldsymbol{\alpha_i^t}$ and $\boldsymbol{\Gamma^t}$ are parameters.
Extra special case: $\boldsymbol{\alpha_i^t = 1/\gamma_i^t}$

Many variations, e.g. time-correlated models

$$\sum_t \alpha_i^t |w_i^t| \le \Gamma_i, \quad \sum_i \sum_t \alpha_i^t |w_i^t| \le \Gamma$$

Optimization Problem

Minimize generation cost subject to being feasible under all modeled renewable outputs.

- **Variables:** $\boldsymbol{P^g}, \boldsymbol{\lambda}$ (both time-indexed)

- **Constraints:** feasibility for all $\boldsymbol{w} \in \mathcal{W}$ (soft-robustness for lines)

Optimization Problem

$$\min_{P^g,\lambda} \quad \sum_t \sum_k c_k^t(P_k^{g,t})$$

s.t. the following system is feasible at all times t, for all $\boldsymbol{w} \in \mathcal{W}$:

Flow balance:

$$B\,\boldsymbol{\theta^t} \;=\; P^{g,t} + \overbrace{\bar{w}^t + \boldsymbol{w^t}}^{\text{renewables}} \;-\; \overbrace{\left(\sum_i \boldsymbol{w_i^t}\right)\lambda^t}^{\text{batteries}} - P^{d,t}$$

Energy state constraints

Hard (?) line limits:

$$\frac{|\,\boldsymbol{\theta_k^t} - \boldsymbol{\theta_m^t}|}{x_{km}} \leq u_{km} \quad \forall \quad km, \quad t$$

Soft line limits are equally tractable and might be preferred

Soft line limits

Let $\boldsymbol{\mathcal{W}^h}$ and $\boldsymbol{\mathcal{W}}$ be **two** uncertainty models with $\boldsymbol{\mathcal{W}^h} \subseteq \boldsymbol{\mathcal{W}}$

- Require that

$$\frac{|\,\boldsymbol{\theta_k^t} - \boldsymbol{\theta_m^t}|}{x_{km}} \;\leq\; u_{km}$$

  for all km, $\quad t$ **and** $\boldsymbol{w} \in \mathcal{W}^h$

- Require that

$$\frac{|\,\boldsymbol{\theta_k^t} - \boldsymbol{\theta_m^t}|}{x_{km}} \;\leq\; u_{km}(1+\epsilon)$$

  for all km, t **and** $\boldsymbol{w} \in \mathcal{W}$

Ben-Tal, Boyd, Nemirovski, "Comprehensive Robust" models (2005)

(Familiar?) cutting-plane procedure

Start with a **relaxation** for the robust problem, e.g. the nominal problem (no errors), and then

1. Solve relaxation, with solution $(\boldsymbol{P^{g*}}, \boldsymbol{\lambda^*})$
2. **Play adversary:** find a worst-case distribution $\hat{\boldsymbol{w}}$ for $(\boldsymbol{P^{g*}}, \boldsymbol{\lambda^*})$

   Comment: Requires solving small LPs
3. Procedure detects if a battery constraint (or line limit constraint) is violated.
4. If so – add corresponding cut (**a disjunctive** cut).
5. Else if the adversary fails, $(\boldsymbol{P^{g*}}, \boldsymbol{\lambda^*})$ is **optimal**

Adversarial procedure, batteries

Concentration models for renewable uncertainty:

$\mathcal{W}$ is the set of all $\boldsymbol{w}$ such that

$$C^1 w^+ \; + \; C^2 w^- \leq \; b,$$
$$w_{k,t}^{\min} \; \leq \; w_k^t \; \leq \; w_{k,t}^{\max} \quad \text{for all } t \text{ and } k$$

- $\boldsymbol{C^1, C^2}$ are **nonnegative** matrices

What is the problem?

Already in "simple" charge/discharge efficiency model, want for all k and t,

$$\Delta \max_{\boldsymbol{w}\in\mathcal{W}} \left\{ \sum_{i=1}^{t} \lambda_{k,i} \left(-(\eta^d_{k,i})^{-1} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^- + \eta^c_{k,i} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^+ \right) \right\} \le E^{\max}_{k,t} - E_{k,0},$$

and

$$\Delta \min_{\boldsymbol{w}\in\mathcal{W}} \left\{ \sum_{i=1}^{t} \Delta_i \lambda_{k,i} \left(-(\eta^d_{k,i})^{-1} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^- + \eta^c_{k,i} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^+ \right) \right\} \ge E^{\min}_{k,t} - E_{k,0}.$$

What is the problem?

Already in "simple" charge/discharge efficiency model, want for all k and t,

$$\Delta \max_{\boldsymbol{w}\in\mathcal{W}} \left\{ \sum_{i=1}^{t} \lambda_{k,i} \left(-(\eta^d_{k,i})^{-1} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^- + \eta^c_{k,i} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^+ \right) \right\} \leq E^{\max}_{k,t} - E_{k,0},$$

and

$$\Delta \min_{\boldsymbol{w}\in\mathcal{W}} \left\{ \sum_{i=1}^{t} \Delta_i \lambda_{k,i} \left(-(\eta^d_{k,i})^{-1} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^- + \eta^c_{k,i} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^+ \right) \right\} \geq E^{\min}_{k,t} - E_{k,0}.$$

Adversarial problem:

choose $\boldsymbol{w} \in \boldsymbol{\mathcal{W}}$ to break either constraint.

Adversarial problem appears to be NON convex!

What is the problem?

Already in "simple" charge/discharge efficiency model, want for all k and t,

$$\Delta \max_{\boldsymbol{w}\in\mathcal{W}} \left\{ \sum_{i=1}^{t} \lambda_{k,i} \left(-(\eta^d_{k,i})^{-1} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^- + \eta^c_{k,i} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^+ \right) \right\} \leq E^{\max}_{k,t} - E_{k,0},$$

and

$$\Delta \min_{\boldsymbol{w}\in\mathcal{W}} \left\{ \sum_{i=1}^{t} \Delta_i \lambda_{k,i} \left(-(\eta^d_{k,i})^{-1} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^- + \eta^c_{k,i} \left[\sum_j \boldsymbol{w}^t_{j,i} \right]^+ \right) \right\} \geq E^{\min}_{k,t} - E_{k,0}.$$

Adversarial problem:

choose $\boldsymbol{w} \in \boldsymbol{\mathcal{W}}$ to break either constraint.

Lemma!

An optimal solution to adversarial problem is **sign-consistent**.

Yields efficient routine!

Six-period results on Polish Grid (2746wp)

penetration	max error	Cost	Iterations	Time (s)
18 %	9 %	7236431	18	348
18 %	14 %	7257286	28	515
18 %	17 %	7263172	25	498

- base load $\approx$ 24.8 GW
- 32 wind farms, average predicted output $\approx$ 4.5 GW
- 50 batteries, total initial energy state $\approx$ 3.2 GW

Interesting results?

- Solution is **sparse**! **50** batteries, but only $\sim$ **14** participate in the optimal control in any time period
- Solution is **non-uniform**! Different batteries participate to different **degree**, even though batteries are all identical
- **Which** batteries **participate** depends on the time period! Why?