A system-theoretic control framework for virtual power plants

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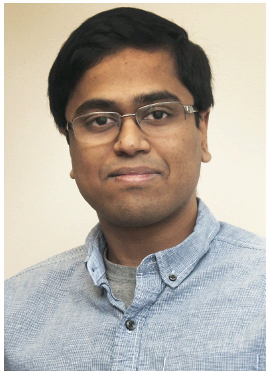

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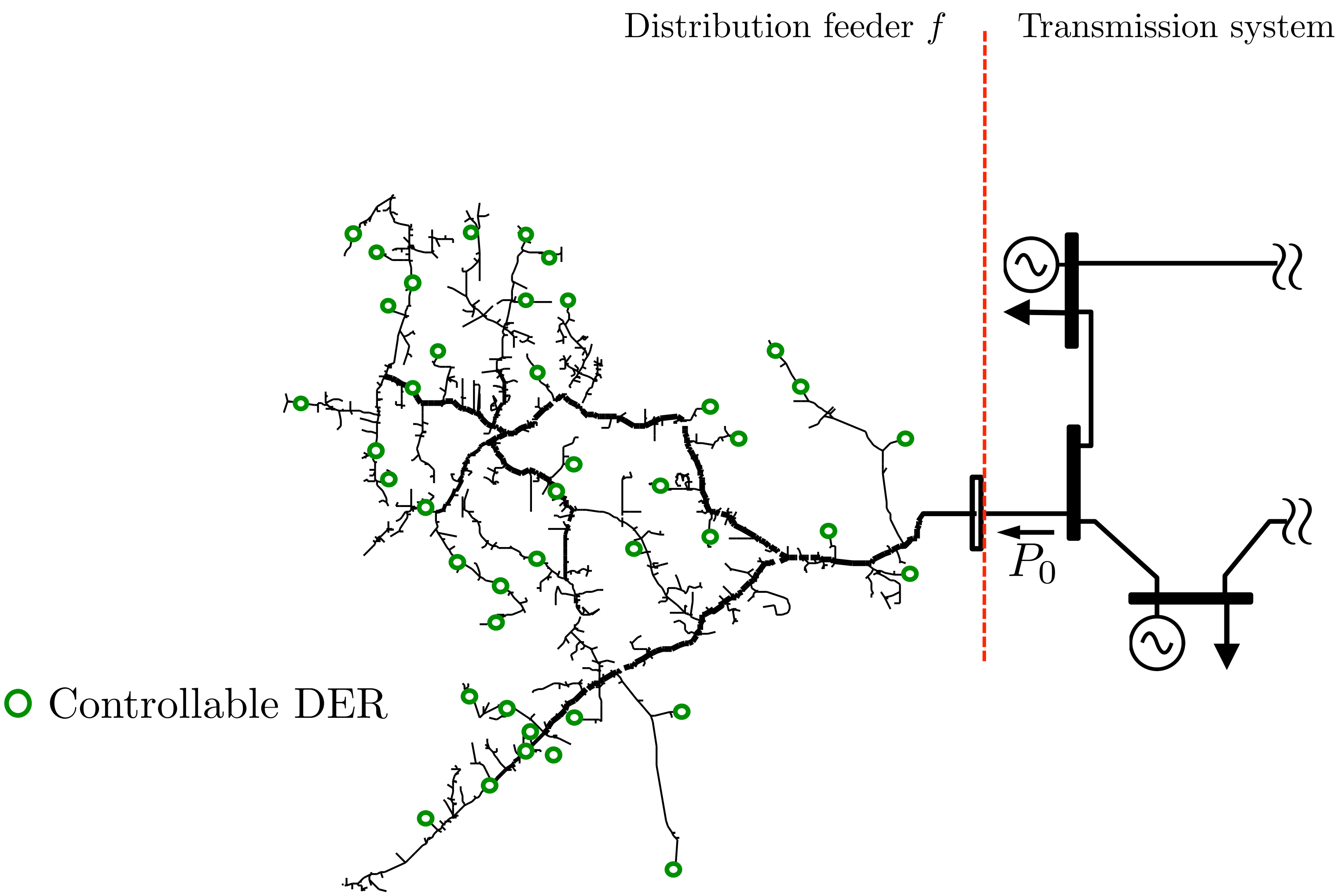
arpa·e CHANGING WHAT'S POSSIBLE

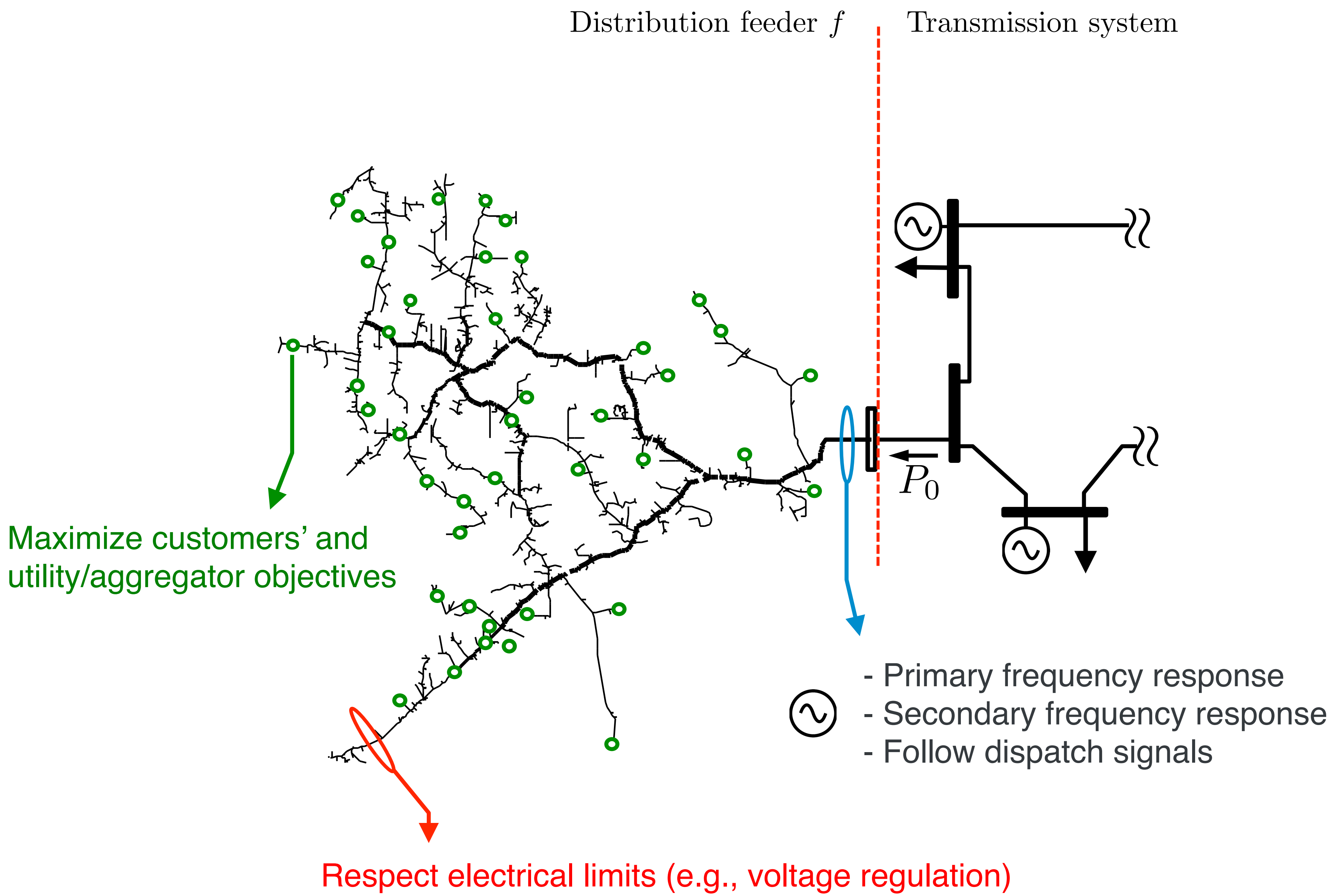
Network Optimized Distributed Energy Systems (NODES)

Objective

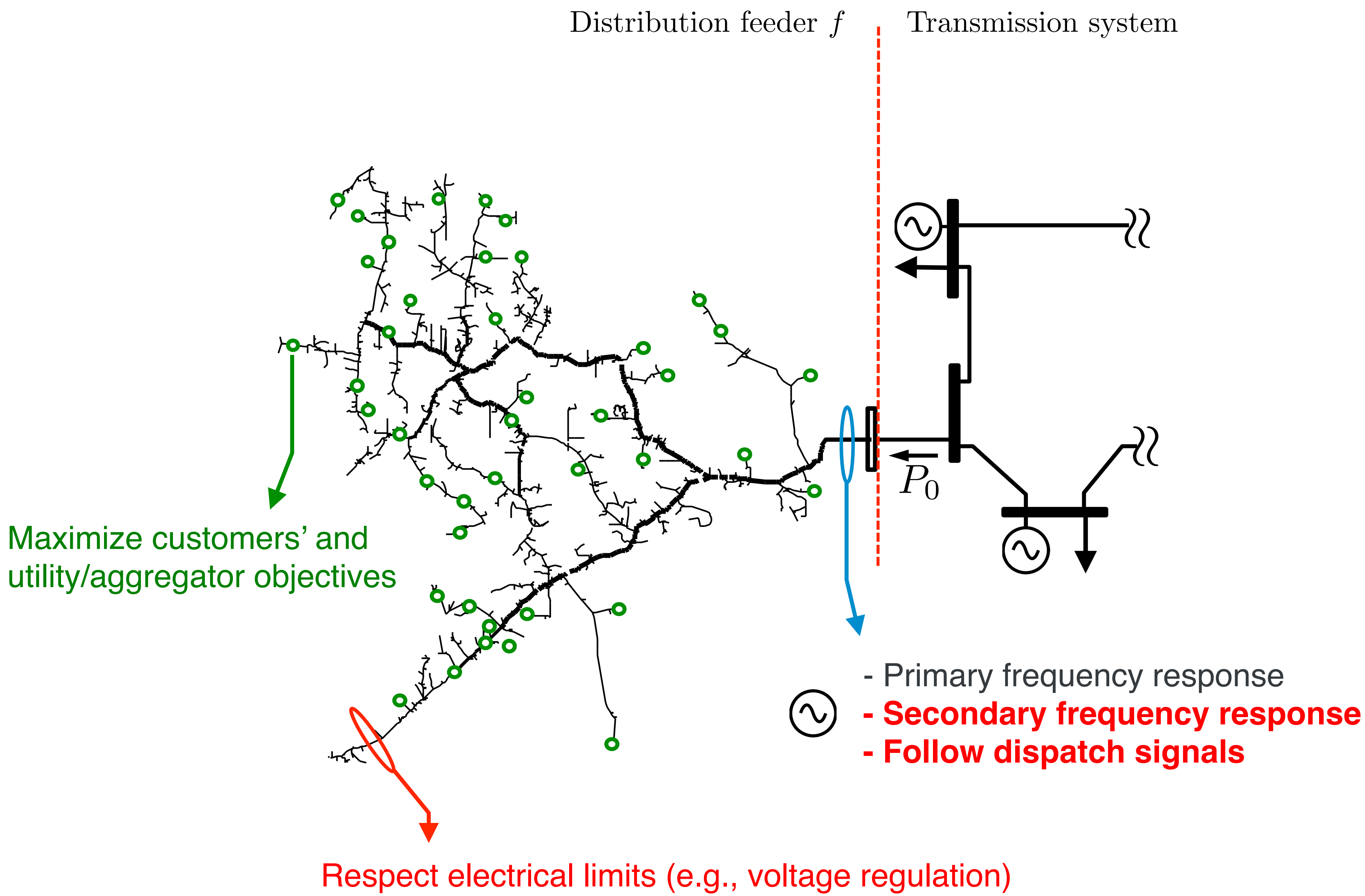


- Enable DER coordination to pursue objectives of customers and utility/aggregator
- Enable feeder to emulate a *virtual power plant* providing services to the main grid

Feeder as a virtual power plant

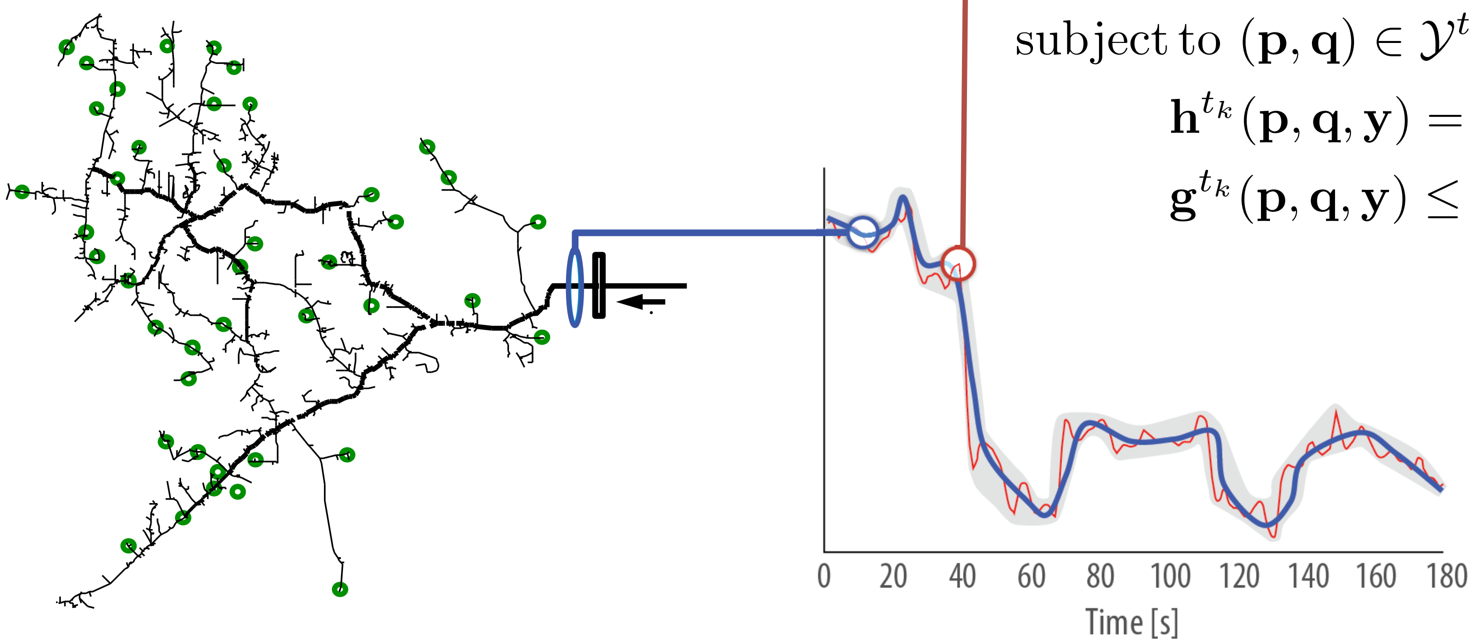


Feeder as a virtual power plant



Design principles

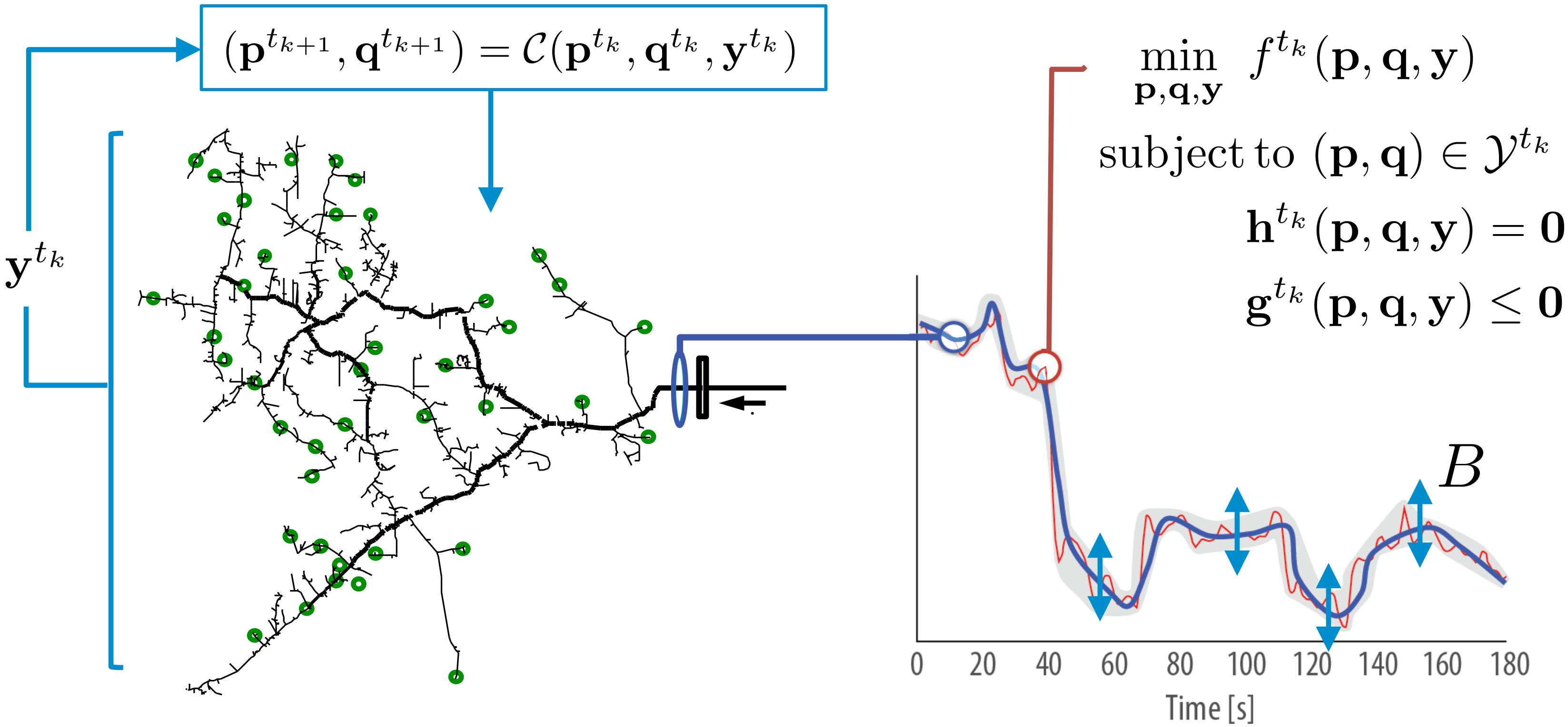
- Leverage the time-varying optimization formalism [Simonetto-Leus’14, Simonetto-Dall’Anese’16]



$$\min_{\mathbf{p},\mathbf{q},\mathbf{y}} \; f^{t_k}(\mathbf{p},\mathbf{q},\mathbf{y})$$
$$\text{subject to } (\mathbf{p},\mathbf{q}) \in \mathcal{Y}^{t_k}$$
$$\mathbf{h}^{t_k}(\mathbf{p},\mathbf{q},\mathbf{y}) = \mathbf{0}$$
$$\mathbf{g}^{t_k}(\mathbf{p},\mathbf{q},\mathbf{y}) \leq \mathbf{0}$$

- “*Sample and solve*”: series of time-invariant optimization problems, one every $\tau := t_k - t_{k-1}$
- Not practical: computational/operational limits; convergence; model mismatches

Design principles



- Low-complexity *online* algorithms to find and track optimal solutions
- *Feedback* from the system to cope with model mismatches and promote adaptability [Dall'Anese at al'15, Bernstein at al'16, Dall'Anese-Simonetto'16, Gan-Low'16]
- Establish analytical results for tracking capabilities

Formalizing operational target

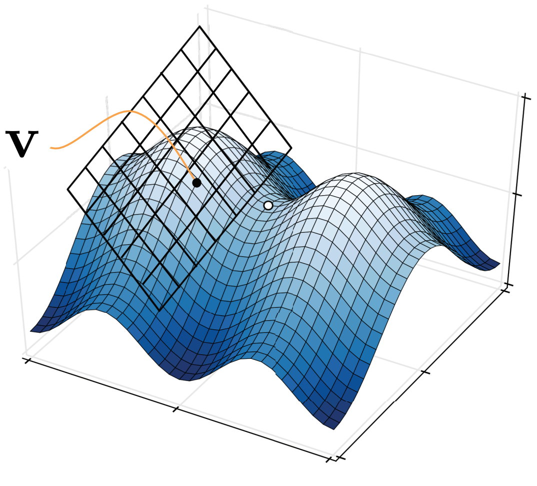
- Nonlinear AC power flows

$$\begin{bmatrix} I_0 \\ \mathbf{i} \end{bmatrix} = \underbrace{\begin{bmatrix} y_{00} & \bar{\mathbf{y}}^\mathsf{T} \\ \bar{\mathbf{y}} & \mathbf{Y} \end{bmatrix}}_{:=\mathbf{Y}_{\text{net}}} \begin{bmatrix} V_0 \\ \mathbf{v} \end{bmatrix} \longrightarrow \begin{array}{l} \mathbf{s}_{\text{inj}} = \text{diag}(\mathbf{v})\, \mathbf{i}^* = \text{diag}(\mathbf{v})\left(\mathbf{Y}^*\mathbf{v}^* + \bar{\mathbf{y}}^* V_0^*\right) \\ S_0 = |V_0|^2(y_{01}^* + y_0^*) - V_0(y_{01}^* V_1^*) \end{array}$$

- (Approximate) linear relationships

$$|\mathbf{v}| \approx \mathbf{R}\mathbf{p}_{\text{inj}} + \mathbf{B}\mathbf{q}_{\text{inj}} + \mathbf{a}$$

$$\begin{bmatrix} P_0 \\ Q_0 \end{bmatrix} \approx \mathbf{M}\mathbf{p}_{\text{inj}} + \mathbf{N}\mathbf{q}_{\text{inj}} + \mathbf{c}$$



- How to obtain (and update) the model parameters?
  - *Regression-based*; e.g., online recursive least squares [Angelosante-Giannakis'09]
  - *Model-based* [Baran-Wu'89, Dhople at al'15, Bolognani-Dorfler'15]

Formalizing operational target

- Targeted setpoint at feeder head: $h^{t_k}|P_0^{t_k}(\mathbf{p},\mathbf{q}) - P_{0,\text{set}}^{t_k}| \leq E^{t_k}$

- Targeted problem:

$$
\begin{aligned}
(\text{P1}^{t_k}) \quad \min_{\mathbf{p},\mathbf{q}} \; & \sum_{i\in\mathcal{G}} f_i^{t_k}(P_i, Q_i) && \text{Cost/objective} \\
\text{subject to } & P_i, Q_i \in \mathcal{Y}_i^{t_k}, \quad \forall\, i && \text{Hardware constraints} \\
& h^{t_k}(P_0^{t_k}(\mathbf{p},\mathbf{q}) - P_{0,\text{set}}^{t_k}) \leq E^{t_k}, && \text{Tracking setpoints} \\
& h^{t_k}(P_0^{t_k}(\mathbf{p},\mathbf{q}) - P_{0,\text{set}}^{t_k}) \geq -E^{t_k}, && \\
& g_n^{t_k}(\mathbf{p},\mathbf{q}) \leq 0, \quad \forall n \in \mathcal{M} && \text{Voltage regulation} \\
& \bar{g}_n^{t_k}(\mathbf{p},\mathbf{q}) \leq 0, \quad \forall n \in \mathcal{M} &&
\end{aligned}
$$

Where:

$$g_n^{t_k}(\mathbf{p},\mathbf{q}) := V^{\min} - \bar{a}_n^{t_k} - \sum_{i\in\mathcal{G}}(r_{n,i}^{t_k} P_i + b_{n,i}^{t_k} Q_i) \qquad \approx V^{\min} - |V_n^{t_k}|$$

$$\bar{g}_n^{t_k}(\mathbf{p},\mathbf{q}) := \sum_{i\in\mathcal{G}}(r_{n,i}^{t_k} P_i + b_{n,i}^{t_k} Q_i) + \bar{a}_n^{t_k} - V^{\max} \qquad \approx |V_n^{t_k}| - V^{\max}$$

$$P_0^{t_k}(\mathbf{p},\mathbf{q}) := \sum_{i\in\mathcal{G}}(m_{1,i}^{t_k} P_i + n_{1,i}^{t_k} Q_i) + \bar{c}_1^{t_k} \qquad \approx P_0^{t_k}$$

Formalizing operational target

- Targeted setpoint at feeder head: $h^{t_k}|P_0^{t_k}(\mathbf{p},\mathbf{q}) - P_{0,\text{set}}^{t_k}| \leq E^{t_k}$

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& \bar{g}_n^{t_k}(\mathbf{p},\mathbf{q}) \leq 0, \qquad \forall n \in \mathcal{M} && \text{Voltage regulation}
\end{aligned}
$$

- *(Ass. 1)* $f_i^t(\mathbf{u}_i)$ convex and continuously differentiable
- *(Ass. 2)* The map $\mathbf{g}^t(\mathbf{u}) := [\nabla_{\mathbf{u}_1}^{\mathsf{T}} f_1^t(\mathbf{u}_1), \ldots, \nabla_{\mathbf{u}_G}^{\mathsf{T}} f_G^t(\mathbf{u}_G)]^{\mathsf{T}}$ is Lipschitz continuous
- *(Ass. 3)* Problem is feasible

Controller design

$$
\begin{aligned}
(\mathrm{P1}^{t_k}) \ \min_{\mathbf{p},\mathbf{q}} \ & \sum_{i \in \mathcal{G}} f_i^{t_k}(P_i, Q_i) \\
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& h^{t_k}(P_0^{t_k}(\mathbf{p},\mathbf{q}) - P_{0,\mathrm{set}}^{t_k}) \leq E^{t_k}, \\
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\end{aligned}
$$

- Primal-dual gradient method for regularized Lagrangian **+ *online* + *feedback***

- $\mathcal{L}_r^{t_k}(\mathbf{p},\mathbf{q},\mathbf{d}) := \mathcal{L}^{t_k}(\mathbf{p},\mathbf{q},\mathbf{d}) + \frac{\nu}{2}\sum_{i\in\mathcal{G}}(P_i^2 + Q_i^2) - \frac{\epsilon}{2}\|\mathbf{d}\|_2^2$

- $\max_{\mathbf{d}\in\mathbb{R}_+^{2M+2}} \min_{\mathbf{p},\mathbf{q}\in\mathcal{Y}^{t_k}} \quad \mathcal{L}_r^{t_k}(\mathbf{p},\mathbf{q},\mathbf{d})$

  $\mathbf{z}^{*,t_k} := \{\mathbf{p}^{*,t_k}, \mathbf{q}^{*,t_k}, \mathbf{d}^{*,t_k}\}$: unique primal-dual (time-varying) solutions

Controller design

$$(\mathrm{P1}^{t_k}) \ \min_{\mathbf{p},\mathbf{q}} \sum_{i \in \mathcal{G}} f_i^{t_k}(P_i, Q_i)$$

$$\text{subject to } P_i, Q_i \in \mathcal{Y}_i^{t_k}, \quad \forall\, i$$

$$h^{t_k}(P_0^{t_k}(\mathbf{p},\mathbf{q}) - P_{0,\mathrm{set}}^{t_k}) \leq E^{t_k},$$

$$h^{t_k}(P_0^{t_k}(\mathbf{p},\mathbf{q}) - P_{0,\mathrm{set}}^{t_k}) \geq -E^{t_k},$$

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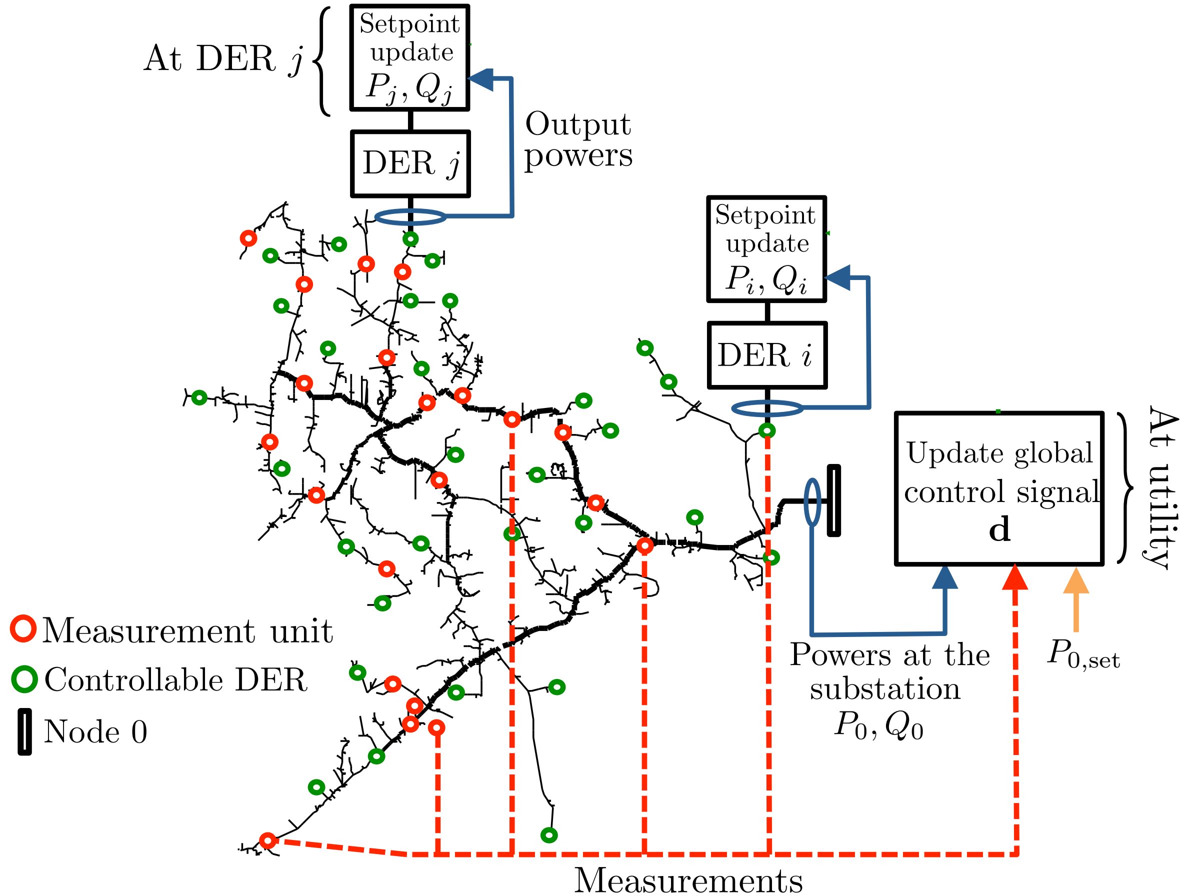
- $\max_{\mathbf{d}\in\mathbb{R}_+^{2M+2}} \min_{\mathbf{p},\mathbf{q}\in\mathcal{Y}^{t_k}} \quad \mathcal{L}_r^{t_k}(\mathbf{p},\mathbf{q},\mathbf{d})$

- $\sigma := \sup \left\|\mathbf{z}^{*,t_{k+1}} - \mathbf{z}^{*,t_k}\right\|$

Real-time control strategy

[S1] Measure voltages and power at the substation

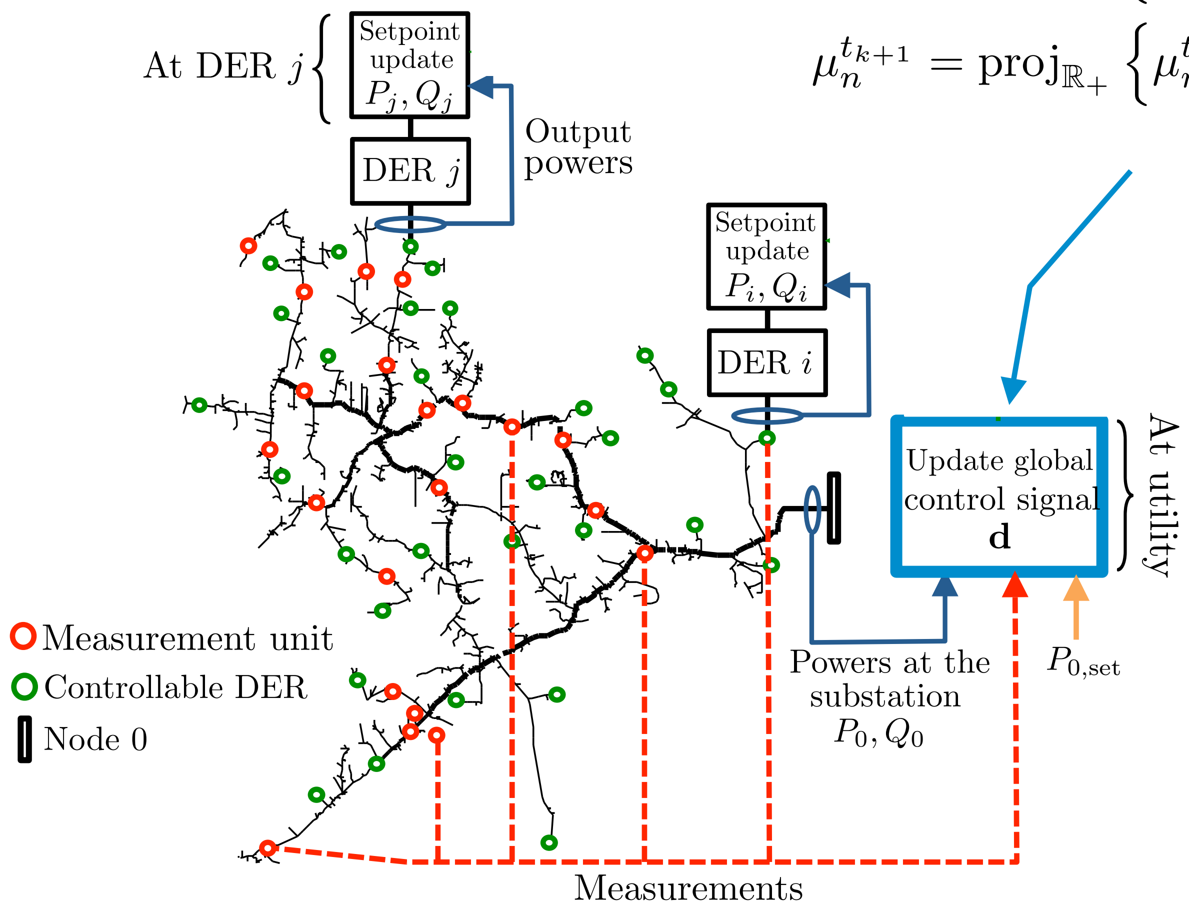


Real-time control strategy

[S2a] Update dual variables using measurements

$$\gamma_n^{t_{k+1}} = \mathrm{proj}_{\mathbb{R}_+}\left\{\gamma_n^{t_k} + \alpha\left(V^{\min} - |\hat{V}_n^{t_k}| - \epsilon\gamma_n^{t_k}\right)\right\}$$

$$\mu_n^{t_{k+1}} = \mathrm{proj}_{\mathbb{R}_+}\left\{\mu_n^{t_k} + \alpha\left(|\hat{V}_n^{t_k}| - V^{\max} - \epsilon\mu_n^{t_k}\right)\right\}$$

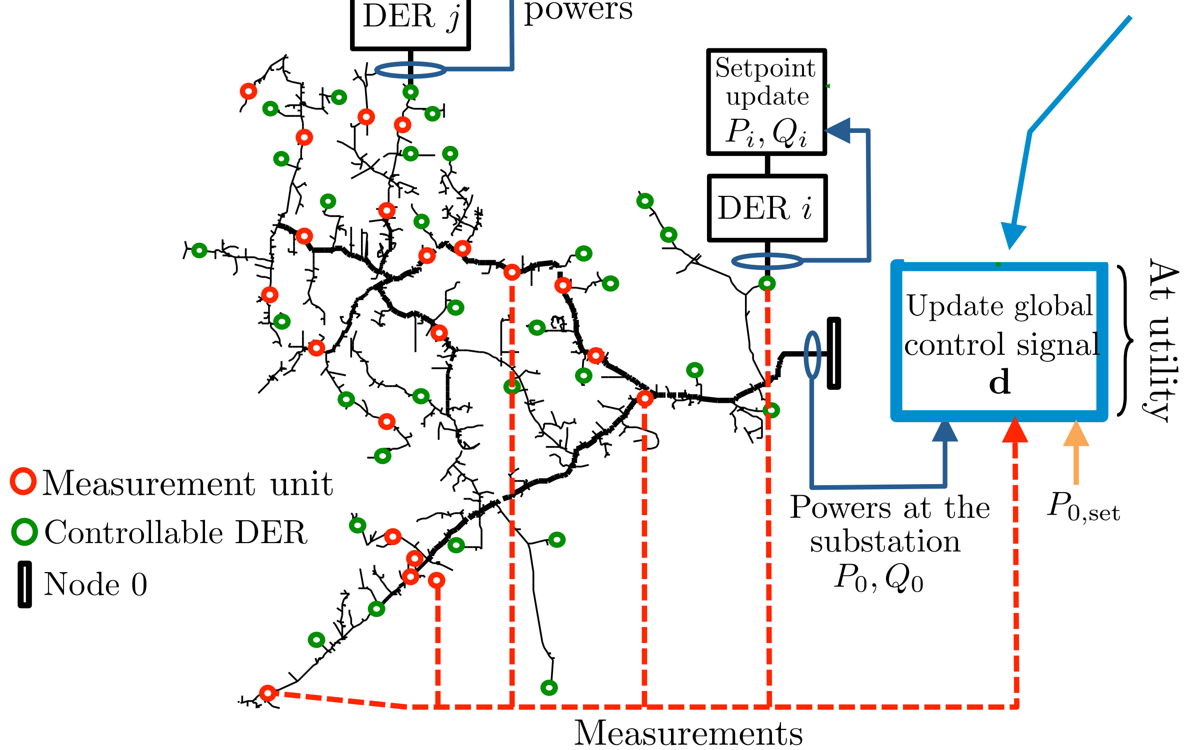


Real-time control strategy

[S2a] Update dual variables using measurements

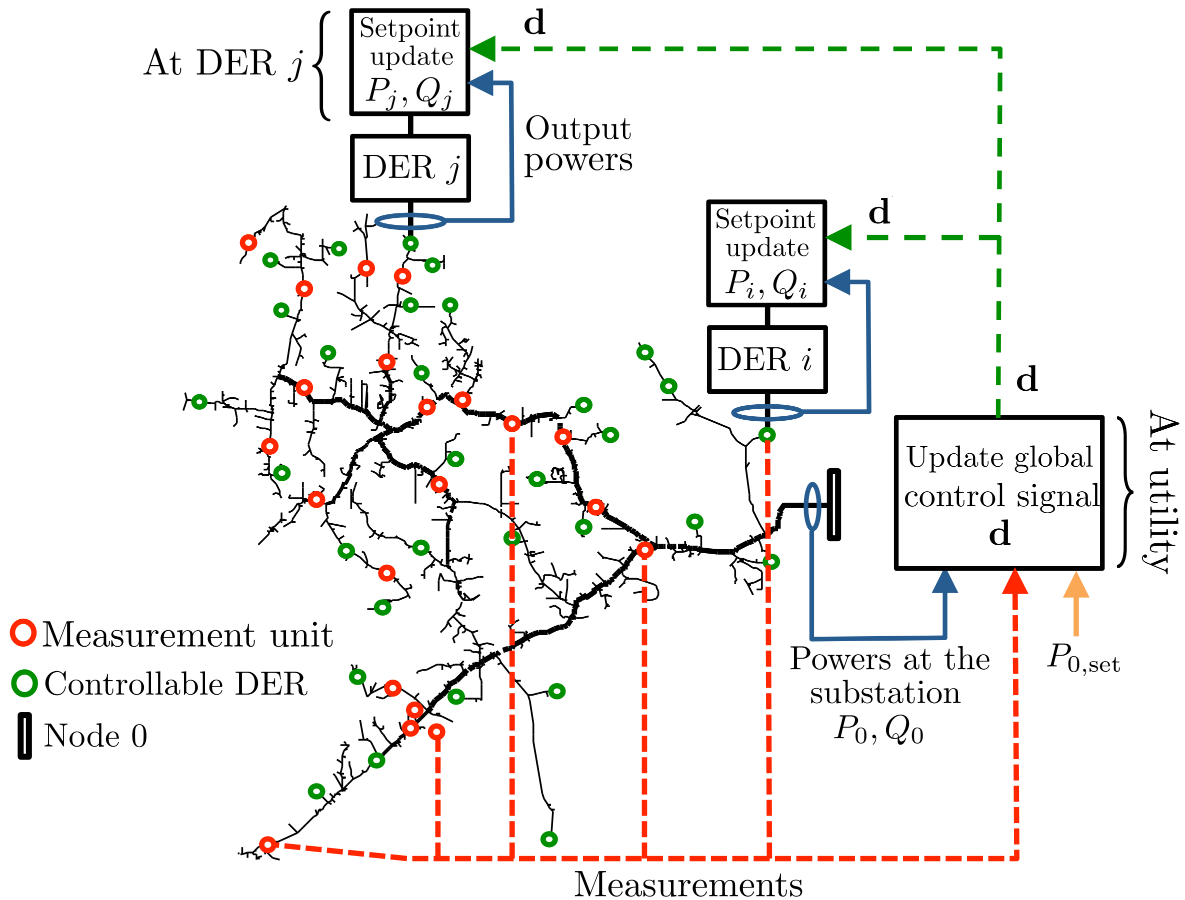
$$\lambda^{t_{k+1}} = \mathrm{proj}_{\mathbb{R}_+} \left\{ \lambda^{t_k} + \alpha(\hat{P}_0^{t_k} - P_{0,\mathrm{set}}^{t_k} - E^{t_k} - \epsilon\lambda^{t_k}) \right\}$$

$$\zeta^{t_{k+1}} = \mathrm{proj}_{\mathbb{R}_+} \left\{ \zeta^{t_k} + \alpha(P_{0,\mathrm{set}}^{t_k} - \hat{P}_0^{t_k} - E^{t_k} - \epsilon\zeta^{t_k}) \right\}$$



Real-time control strategy

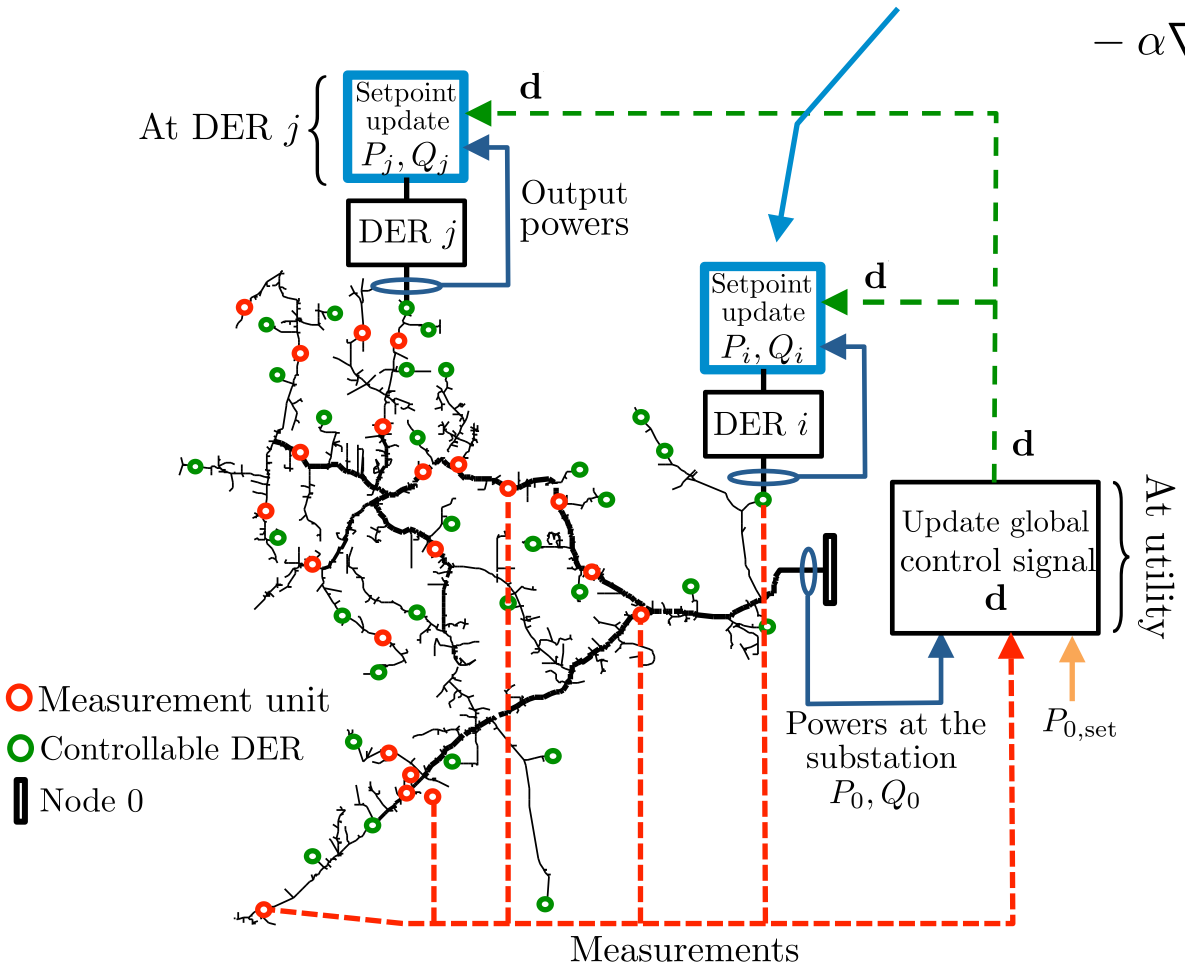
[S2b] Broadcast dual variables



Real-time control strategy

[S3] Compute and command setpoints

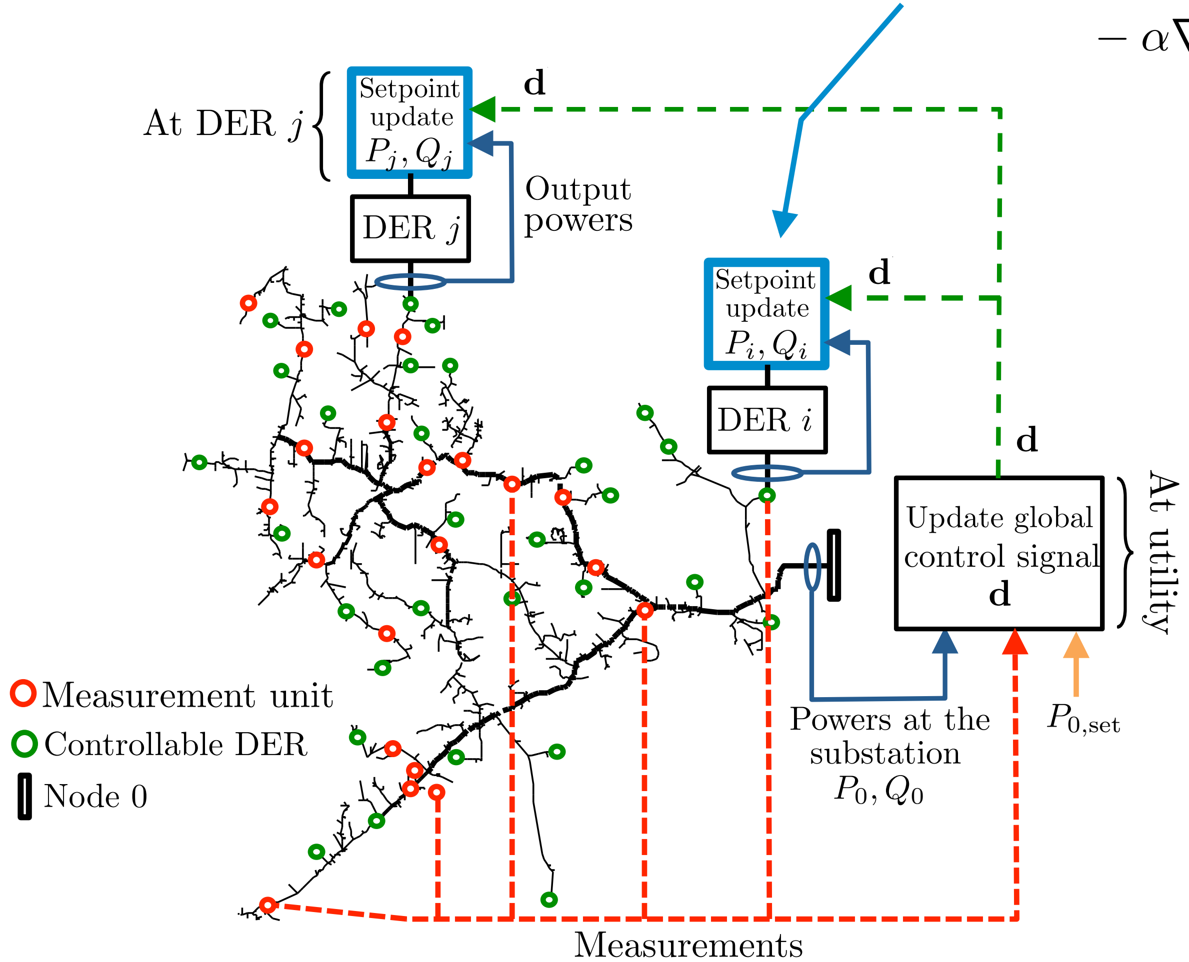
$$\begin{bmatrix} P_i^{t_{k+1}} \\ Q_i^{t_{k+1}} \end{bmatrix} = \mathrm{proj}_{\mathcal{Y}_i^{t_k}} \left\{ \begin{bmatrix} P_i^{t_k} \\ Q_i^{t_k} \end{bmatrix} - \alpha \nabla_{[P_i, Q_i]} \mathcal{L}^{t_k}(\mathbf{p}, \mathbf{q}, \mathbf{d})\big|_{\hat{P}_i^{t_k}, \hat{Q}_i^{t_k}, \mathbf{d}^{t_k}} \right\}$$



Real-time control strategy

[S3] Compute and command setpoints

$$\begin{bmatrix} P_i^{t_{k+1}} \\ Q_i^{t_{k+1}} \end{bmatrix} = \mathrm{proj}_{\mathcal{Y}_i^{t_k}} \left\{ \begin{bmatrix} P_i^{t_k} \\ Q_i^{t_k} \end{bmatrix} - \alpha \nabla_{[P_i, Q_i]} \mathcal{L}^{t_k}(\mathbf{p}, \mathbf{q}, \mathbf{d}) \big|_{\hat{P}_i^{t_k}, \hat{Q}_i^{t_k}, \mathbf{d}^{t_k}} \right\}$$



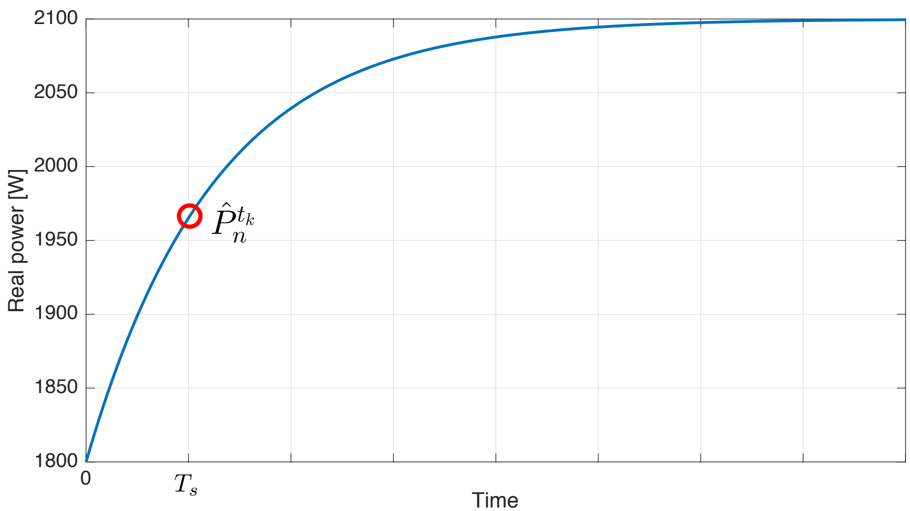
Note: Can handle node-level DER aggregations

Convergence

- $$\mu_n^{t_{k+1}} = \mathrm{proj}_{\mathbb{R}_+}\left\{\mu_n^{t_k} + \alpha \underbrace{\left(|\hat{V}_n^{t_k}| - V^{\max} - \epsilon\mu_n^{t_k}\right)}_{\neq \nabla_{\mu_n}\bar{\mathcal{L}}^{t_k}|_{\mathbf{p}^{t_k},\mathbf{q}^{t_k},\mathbf{d}^{t_k}}}\right\}$$

- *(Ass. 4)* Measurement and model errors for voltages and powers at the feeder head are bounded.

- *(Ass. 5)* There exists $0 \leq e_{\mathrm{out}} < +\infty$ such that: $\left\| \begin{bmatrix} \mathbf{p}^{t_k} \\ \mathbf{q}^{t_k} \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{p}}^{t_k} \\ \hat{\mathbf{q}}^{t_k} \end{bmatrix} \right\|_2 \leq e_{\mathrm{out}}$



Convergence

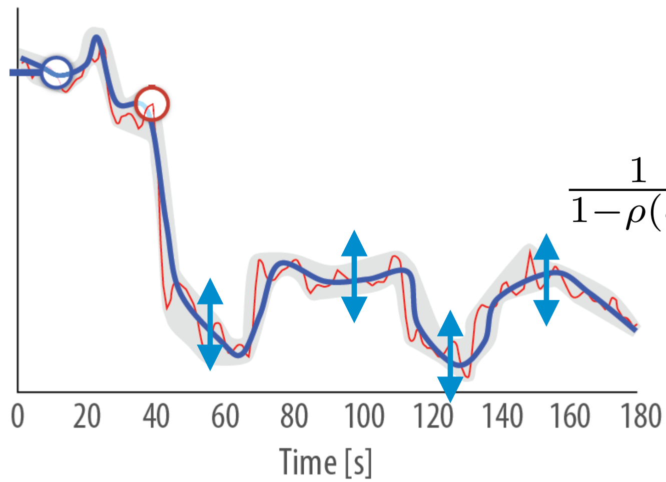
Theorem [Dall'Anese et al'16]. Under current modeling assumptions, if the stepsize is chosen such that:

$$\rho(\alpha) := \sqrt{1 - 2\alpha \min\{\nu, \epsilon\} + \alpha^2 B} < 1$$

then the following holds for the closed-loop system above:

$$\limsup_{t_k \to \infty} \|\mathbf{z}^{t_k} - \mathbf{z}^{*,t_k}\|_2 = \frac{1}{1-\rho(\alpha)} \left[\alpha e + \sigma\right]$$

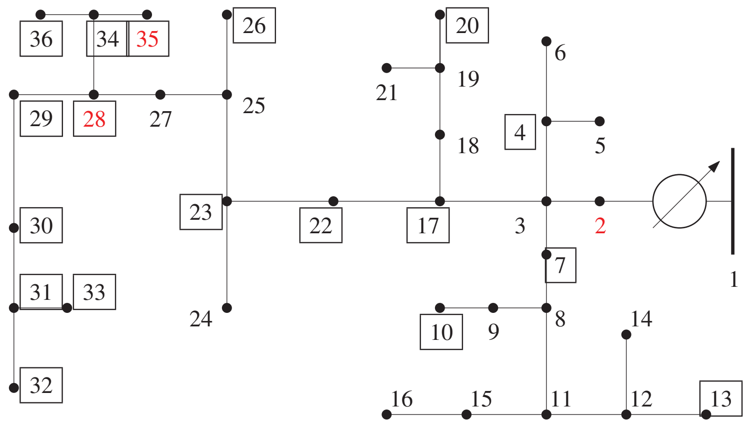
Where $e = \sqrt{(L+\nu)^2 e_{\text{out}}^2 + 2e_v^2 + 2e_0^2}$ and $\sigma := \sup \|\mathbf{z}^{*,t_{k+1}} - \mathbf{z}^{*,t_k}\|$.

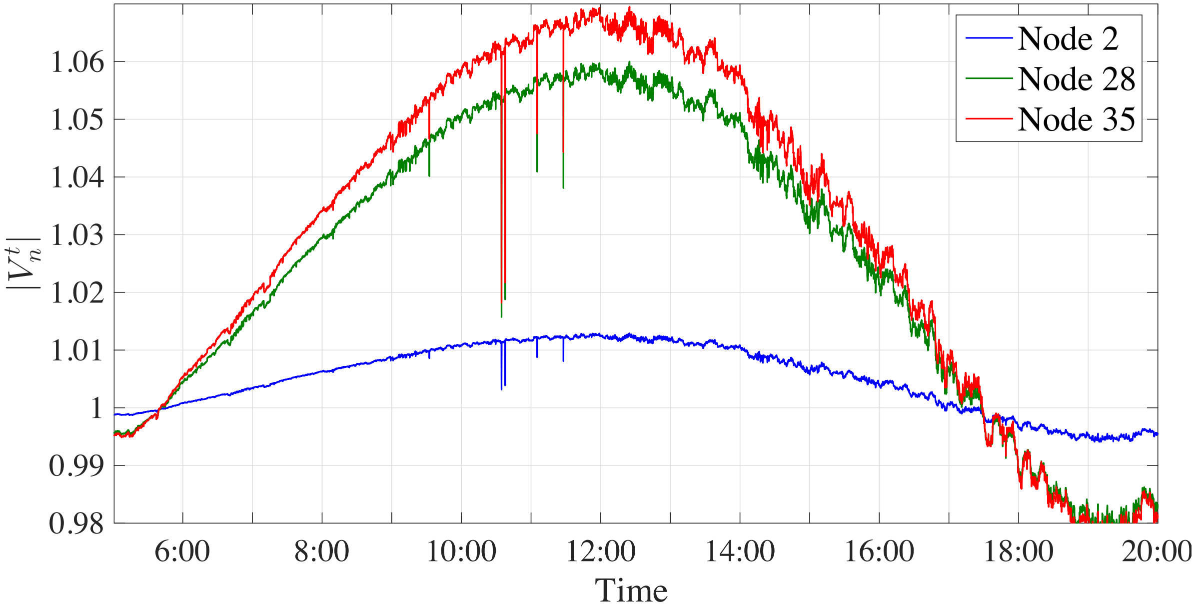


$$\frac{1}{1-\rho(\alpha)} \left[\alpha e + \sigma\right]$$

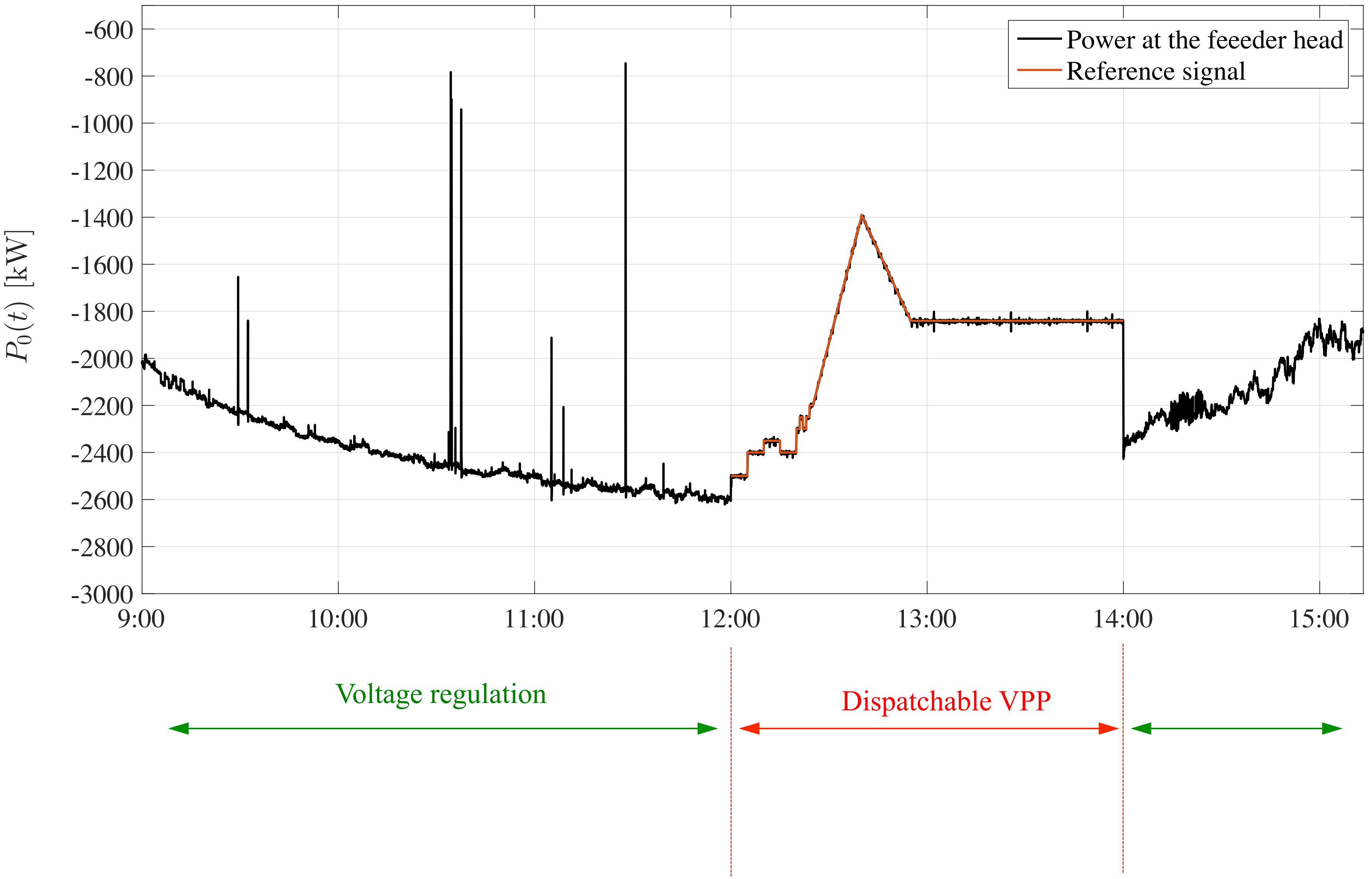
Representative results

- IEEE 37-node test feeder
- Real load and solar irradiance data from Anatolia, CA
- PQ updated every 300ms
- $\text{cost} = \sum_i c_p P_{i,\text{curtailed}}^2 + c_q Q_i^2$
- PV inverter mimics first-order system





Representative results

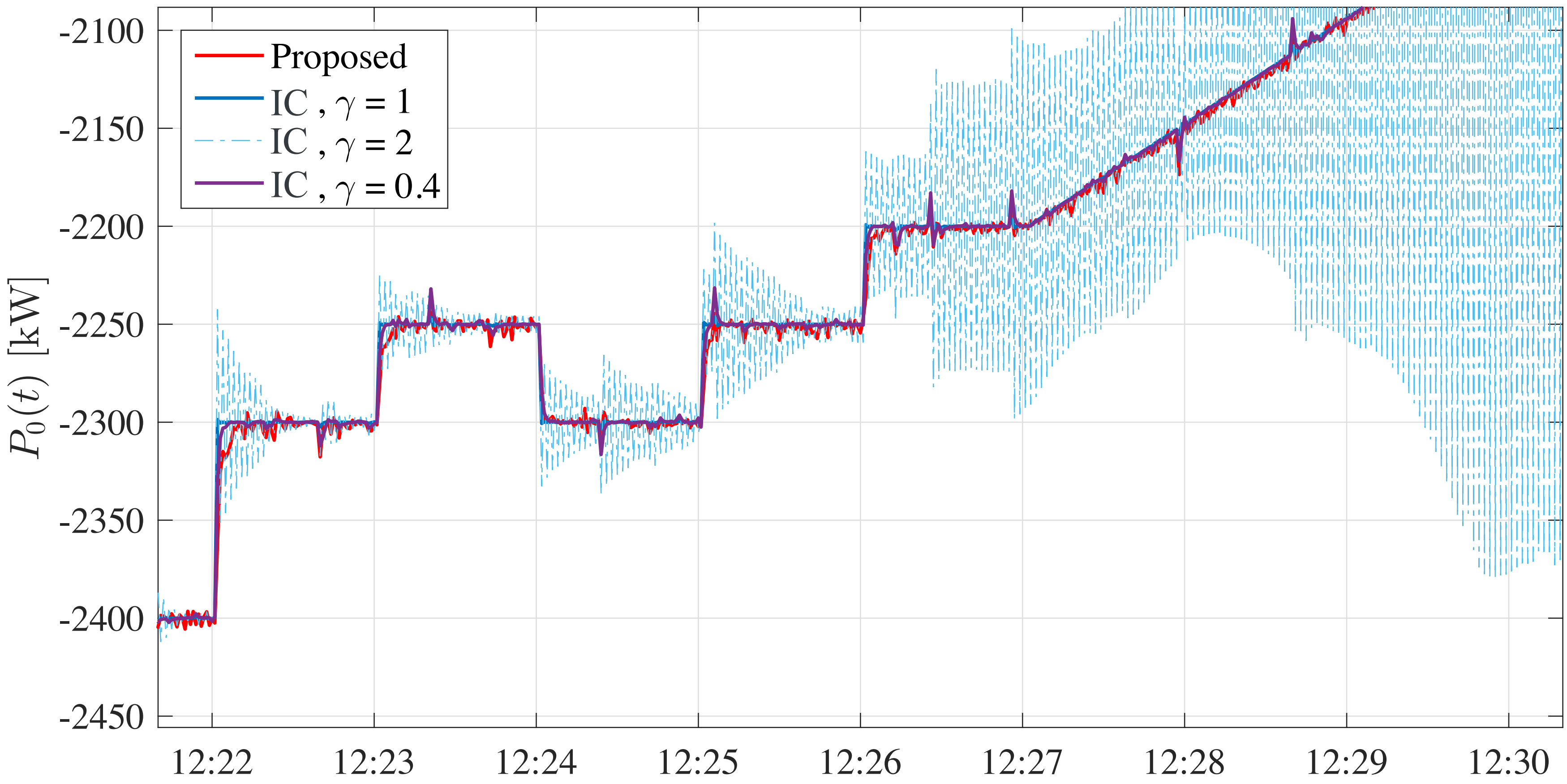


Representative results

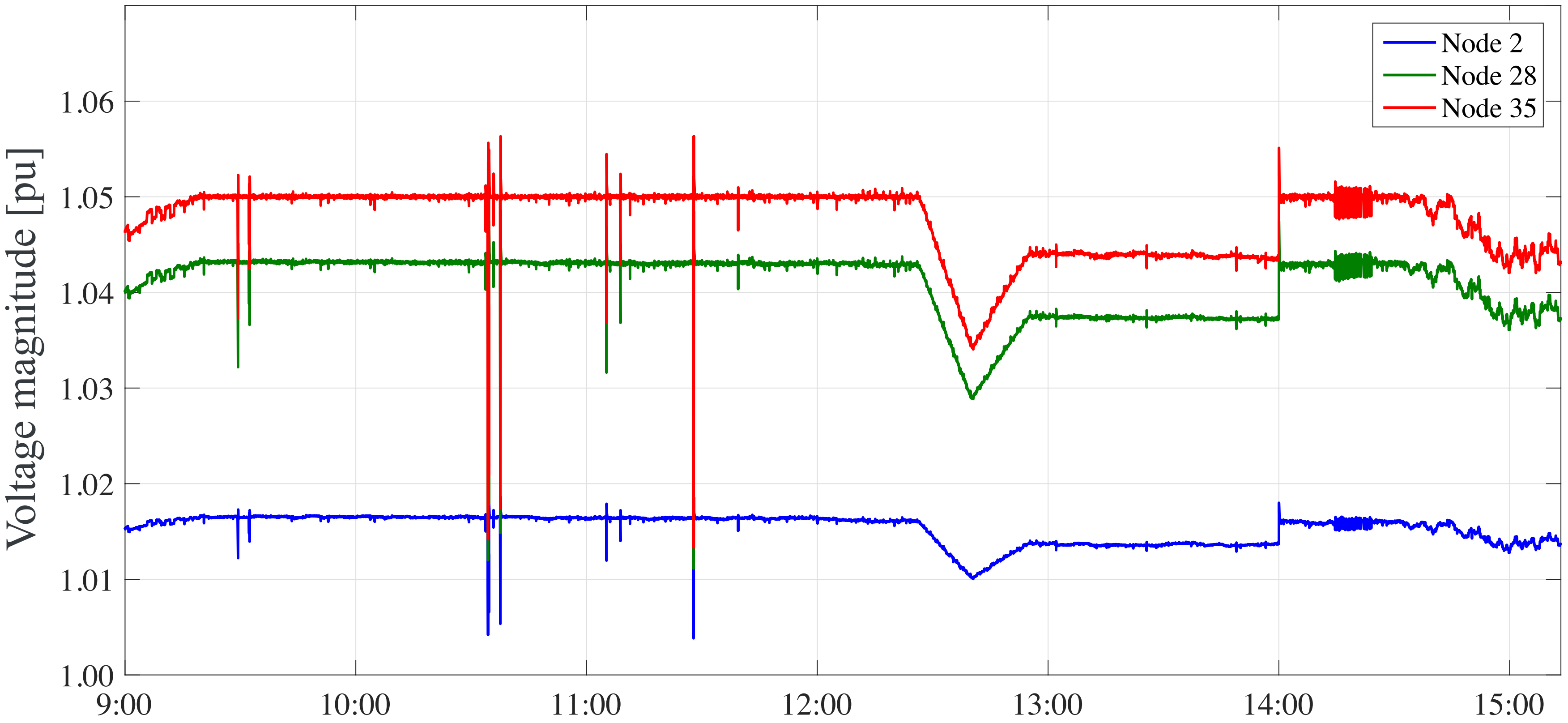
❑ The value of communication

Representative results

- Comparison: $P_n^{t_{k+1}} = \mathrm{proj}_{\mathcal{Y}_n} \left\{ P_n^{t_{k+1}} - \gamma_n (P_{0,\mathrm{set}}^{t_k} - \hat{P}_0^{t_k}) \right\}$ **+ Volt/VAr**



Representative results



Concluding remarks

- Feeder as virtual power plants to provide services to the main grid
  - Address voltage regulation and pursue optimization objectives
  - Can be extended to control currents
- Value of communications to enable coordination

- Future efforts towards
  - Convergence to solution of nonconvex problems
  - Application of “closed-loop optimization” to other problems
  - Optimization layer to dispatch HVAC systems, EVs, and other DERs
  - Validation and implementation

Thank you!

Comments?