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Chance-Constrained AC Optimal Power Flow

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Joint work with Göran Andersson, Haoyuan Qu, Jeremias Schmidli, Spyros Chatzivasileiadis, Aldo Tobler and Dan Molzahn

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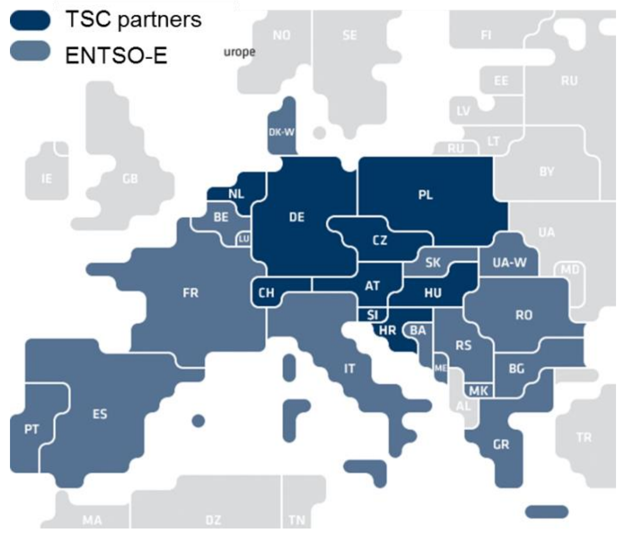
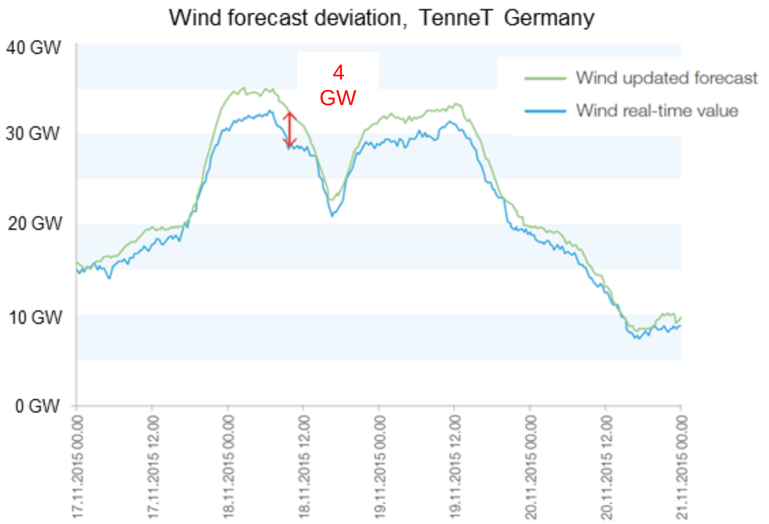
Changing Power System

Increasing uncertainty in operational planning

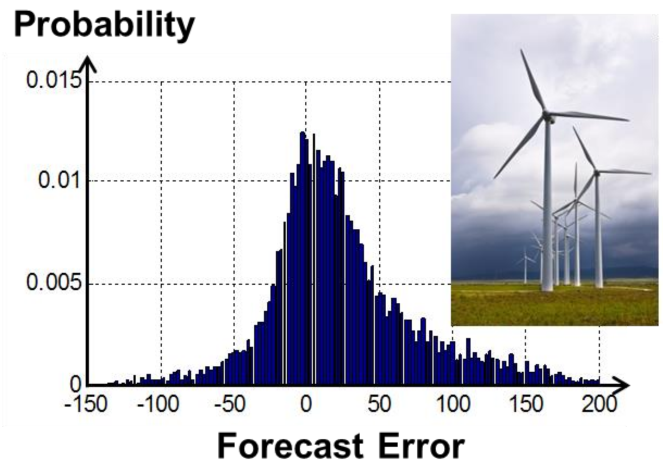
- Renewables
- Market liberalization

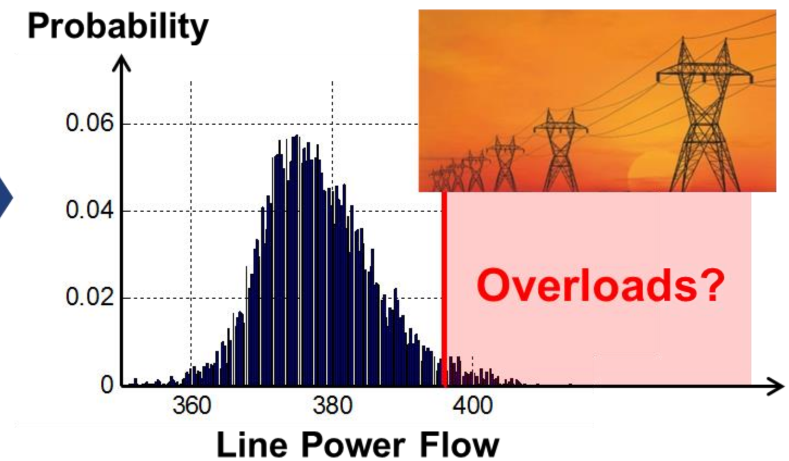
➢ Larger and more frequent deviations from schedules





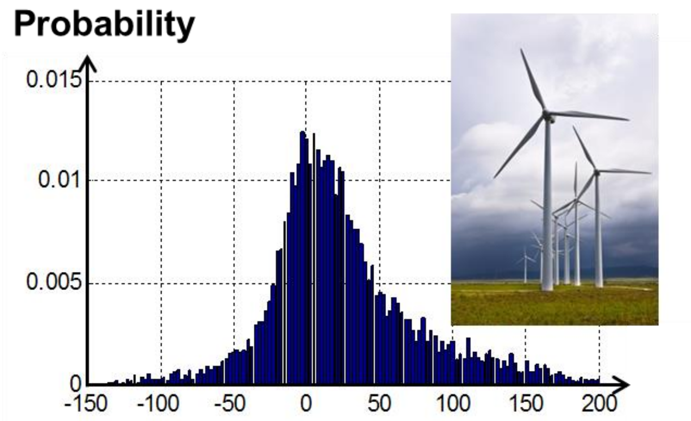
Impact of Forecast Uncertainty

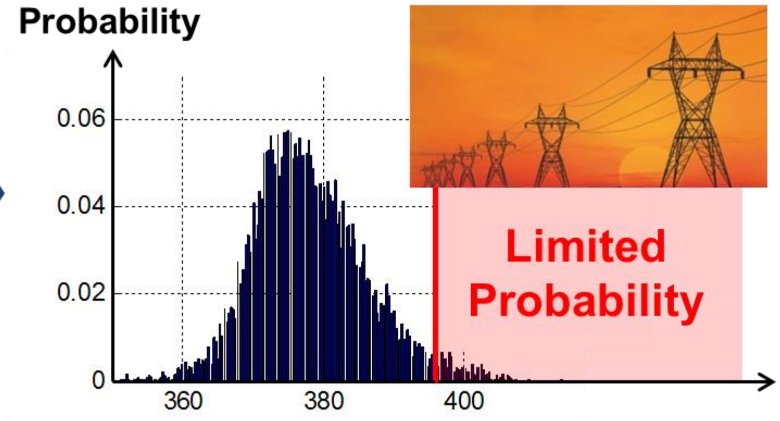




How to define and enforce **security**?

Impact of Forecast Uncertainty

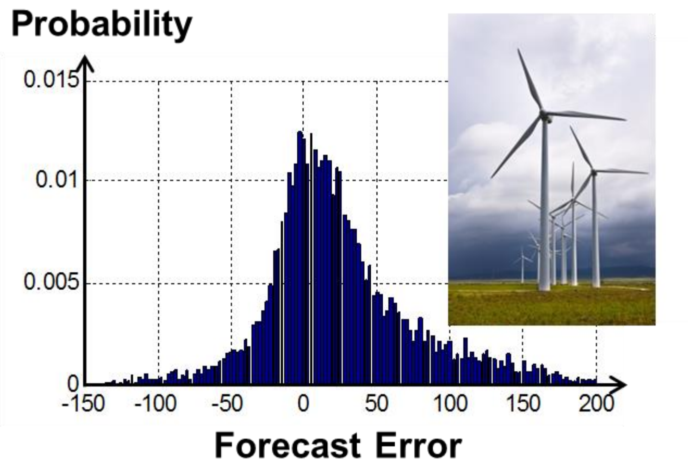


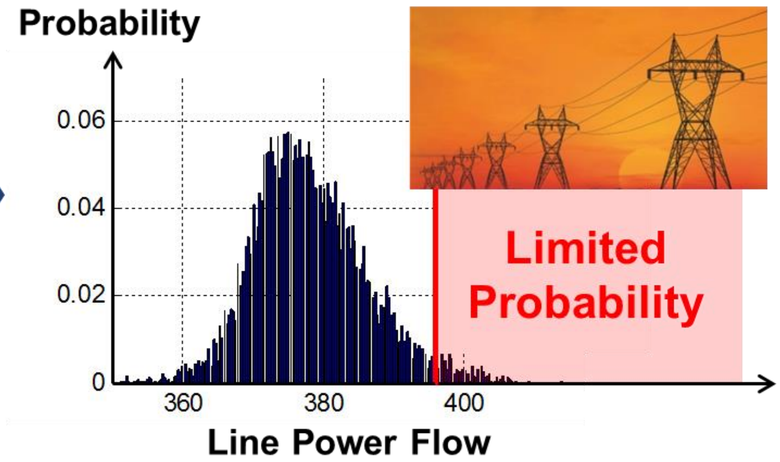


Chance-constrained Optimal Power Flow

Limit ***probability*** of constraint violation

Impact of Forecast Uncertainty





Chance-constrained **AC** Optimal Power Flow

Non-linear dependence on fluctuations!

Chance-Constrained AC Optimal Power Flow

$$\min_{p_G} \quad \sum_{i\in\mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i}\, p_{G,i} + c_{0,i}\right)$$

Minimizing generation cost

s.t.

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Minimizing generation cost

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0, \quad \forall \omega \in \mathcal{U}$$

Full AC power flow equations for all uncertainty realizations

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$$\mathbb{P}\left(p_{G,g}(\omega) \le p_{G,g}^{max}\right) \ge 1-\epsilon, \qquad g \in \mathcal{G}$$

$$\mathbb{P}\left(p_{G,g}(\omega) \ge p_{G,g}^{min}\right) \ge 1-\epsilon, \qquad g \in \mathcal{G}$$

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Minimizing generation cost

Full AC power flow equations for all uncertainty realizations

Chance constraints on power generation

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Full AC power flow equations for all uncertainty realizations

Chance constraints on power generation

Chance constraints on voltage and current magnitudes

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Chance constraints on power generation

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$$\mathbb{P}\left(i_{L,j}(\omega) \le i_{L,j}^{max}\right) \ge 1-\epsilon, \quad j \in \mathcal{L}$$

Chance constraints on voltage and current magnitudes

Automatic generation control (AGC) for active power balancing: $\Delta p_G = \alpha \sum \omega$

Local voltage control at PV buses for reactive power balancing: $v(\omega) = v,\ q(\omega)$ is varying

Chance Constraint Reformulation

$$\mathbb{P}(i(x,\omega) \leq i^{max}) \geq 1 - \epsilon$$

acceptable violation probability

Chance Constraint Reformulation

$$\mathbb{P}(i(\boldsymbol{x}, \boldsymbol{\omega}) \leq i^{max}) \geq 1 - \epsilon$$

Non-linear dependence on **decision variables** and **uncertainty**

acceptable violation probability

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$$\mathbb{P}(i(\boldsymbol{x}, \boldsymbol{\omega}) \leq i^{max}) \geq 1 - \epsilon$$

Non-linear dependence on **decision variables** and **uncertainty**

acceptable violation probability

- Linearization – similar methods as for DC power flow
  - Needs **good linearization** point!

(Zhang and Li, 2011)
(Dall'Anese, Baker and Summers 2016)

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  - Needs **good linearization** point!
- Sample-based – based on convex relaxation (Vrakopoulou et al, 2013)
  - Ensure **tight relaxation** & **scalability**

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(Zhang and Li, 2011)
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- Sample-based – based on convex relaxation
  - Ensure **tight relaxation** & **scalability**

(Vrakopoulou et al, 2013)

- Partial linearization + analytical reformulation
  - Still an approximation...

(Qu, Roald and Andersson, 2015)
(Schmidli et al, 2016)
(Roald, 2016)

Chance Constraint Reformulation

(H. Qu, L. Roald, G. Andersson, 2015)
(Roald 2016)

$$\mathbb{P}(\boldsymbol{i}(\boldsymbol{x},\boldsymbol{\omega}) \leq i^{max}) \geq 1-\epsilon$$

non-linear dependence

Chance Constraint Reformulation

(H. Qu, L. Roald, G. Andersson, 2015) (Roald 2016)

$$\mathbb{P}(\boldsymbol{i}(\boldsymbol{x}, \boldsymbol{\omega}) \leq i^{max}) \geq 1 - \epsilon$$

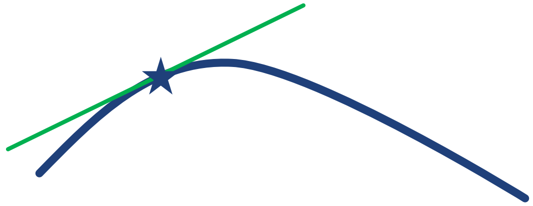
non-linear dependence

1. Partial linearization

$$\mathbb{P}(\boldsymbol{i}(\boldsymbol{x}, \mathbf{0}) + \boldsymbol{h}(\boldsymbol{x}) \cdot \boldsymbol{\omega} \leq i^{max}) \geq 1 - \epsilon$$

Expected operating point
Full AC power flow

Linearization around expected point
Accurate when uncertainty is small



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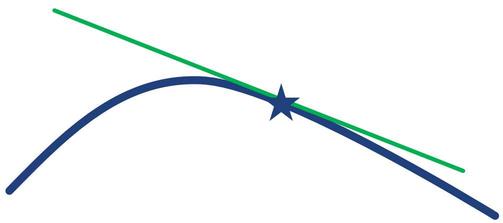

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non-linear dependence

1. **Partial linearization**

$$\mathbb{P}(\boldsymbol{i}(\boldsymbol{x},\boldsymbol{0}) + \boldsymbol{h}(\boldsymbol{x}) \cdot \boldsymbol{\omega} \leq i^{max}) \geq 1-\epsilon$$

linear dependence on $\boldsymbol{\omega}$

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(H. Qu, L. Roald, G. Andersson, 2015) (Roald 2016)

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non-linear dependence

1. **Partial linearization**

$$\mathbb{P}(\boldsymbol{i}(\boldsymbol{x},\boldsymbol{0}) + \boldsymbol{h}(\boldsymbol{x})\cdot\boldsymbol{\omega} \leq i^{max}) \geq 1-\epsilon$$

linear dependence on $\boldsymbol{\omega}$

2. **Analytical reformulation**

$$\boldsymbol{i}(\boldsymbol{x},\boldsymbol{0}) \leq i^{max} - \boldsymbol{\Phi}^{-1}(\boldsymbol{1}-\boldsymbol{\epsilon})\sqrt{\boldsymbol{h}(\boldsymbol{x})\,\Sigma_{\text{cov}}\,\boldsymbol{h}(\boldsymbol{x})}$$

Full AC power flow **«uncertainty margin»**

Chance Constraint Reformulation

(H. Qu, L. Roald, G. Andersson, 2015)
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$$\mathbb{P}(\boldsymbol{i}(\boldsymbol{x},\boldsymbol{\omega}) \leq i^{max}) \geq 1-\epsilon$$

non-linear dependence

1. **Partial linearization**

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linear dependence on $\boldsymbol{\omega}$

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$$\boldsymbol{i}(\boldsymbol{x},\mathbf{0}) \leq i^{max} - \boldsymbol{\Phi}^{-1}(\mathbf{1}-\boldsymbol{\epsilon})\sqrt{\boldsymbol{h}(\boldsymbol{x})\,\Sigma_{\text{cov}}\,\boldsymbol{h}(\boldsymbol{x})}$$

+ Known or partially known distributions
+ Scalable and transparent

Chance-Constrained AC Optimal Power Flow

$$\min_{P_G} \quad \sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i}\, p_{G,i} + c_{0,i} \right)$$

Quadratic objective

s.t.

$$f(\theta, v, p, q) = 0,$$

Full AC power flow equations for forecasted operating point

$$p_G \leq p_G^{max} - \sqrt{h_P(x) \Sigma_{cov} h_P(x)^T}$$

$$p_G \geq p_G^{min} + \sqrt{h_P(x) \Sigma_{cov} h_P(x)^T}$$

$$q_G \leq q_G^{max} - \sqrt{h_Q(x) \Sigma_{cov} h_Q(x)}$$

$$q_G \geq q_G^{min} + \sqrt{h_Q(x) \Sigma_{cov} h_Q(x)}$$

$$v \leq v^{max} - \sqrt{h_V(x) \Sigma_{cov} h_V(x)^T}$$

$$v \geq v^{min} + \sqrt{h_V(x) \Sigma_{cov} h_V(x)^T}$$

$$i \leq i^{max} - \sqrt{h_I(x) \Sigma_{cov} h_I(x)^T}$$

Reformulated chance constraints

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Chance-Constrained AC Optimal Power Flow

$$\min_{P_G} \quad \sum_{i \in \mathcal{G}} \left(c_{2,i} p_{G,i}^2 + c_{1,i}\, p_{G,i} + c_{0,i} \right)$$

s.t.

$$f(\theta, v, p, q) = 0,$$

$$p_G \leq p_G^{max} - \sqrt{h_P(x) \Sigma_{cov} h_P(x)^T}$$

$$p_G \geq p_G^{min} + \sqrt{h_P(x) \Sigma_{cov} h_P(x)^T}$$

$$q_G \leq q_G^{max} - \sqrt{h_Q(x) \Sigma_{cov} h_Q(x)}$$

$$q_G \geq q_G^{min} + \sqrt{h_Q(x) \Sigma_{cov} h_Q(x)}$$

$$v \leq v^{max} - \sqrt{h_V(x) \Sigma_{cov} h_V(x)^T}$$

$$v \geq v^{min} + \sqrt{h_V(x) \Sigma_{cov} h_V(x)^T}$$

$$i \leq i^{max} - \sqrt{h_I(x) \Sigma_{cov} h_I(x)^T}$$

Solution approaches

One-shot optimization problem

+ Optimize AGC response

Chance-Constrained AC Optimal Power Flow

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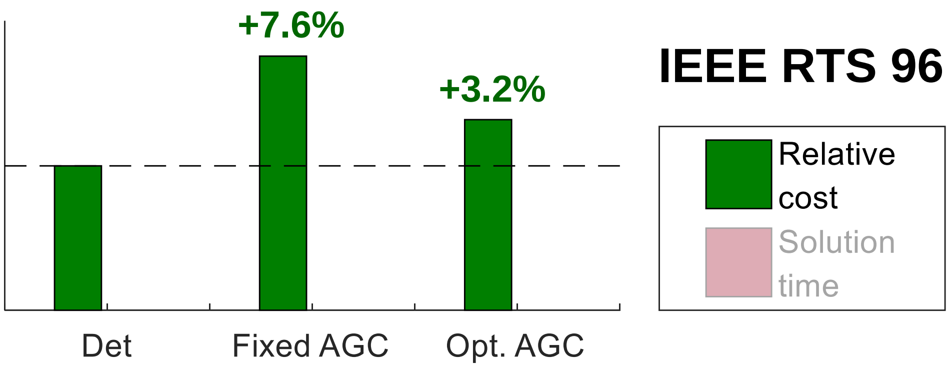
LOWER COST OF HANDLING UNCERTAINTY



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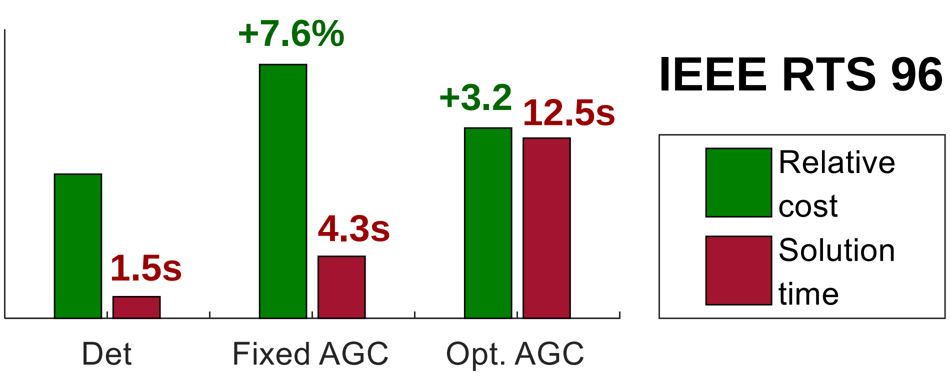
– Complex problem

➡ **LONG SOLUTION TIMES**



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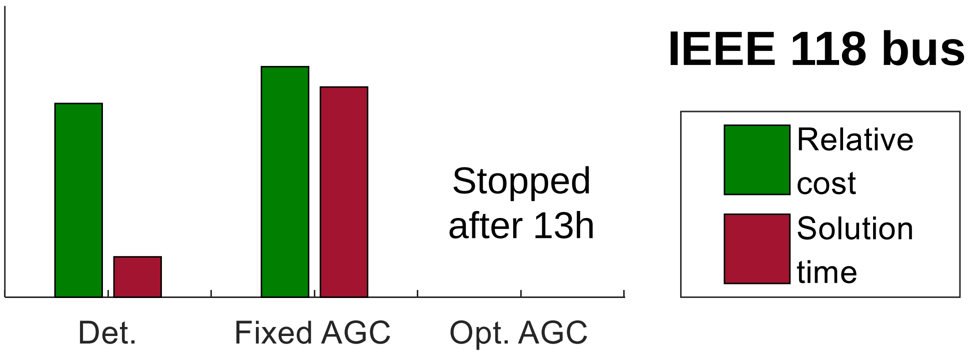
➡ **LONG SOLUTION TIMES**

IEEE 118 bus



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Impact of uncertainty is inside the uncertainty margins

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➢ Iterative solution algorithm

(J. Schmidli, L. Roald, S. Chatzivasileiadis, G. Andersson, 2016), (Roald 2016)

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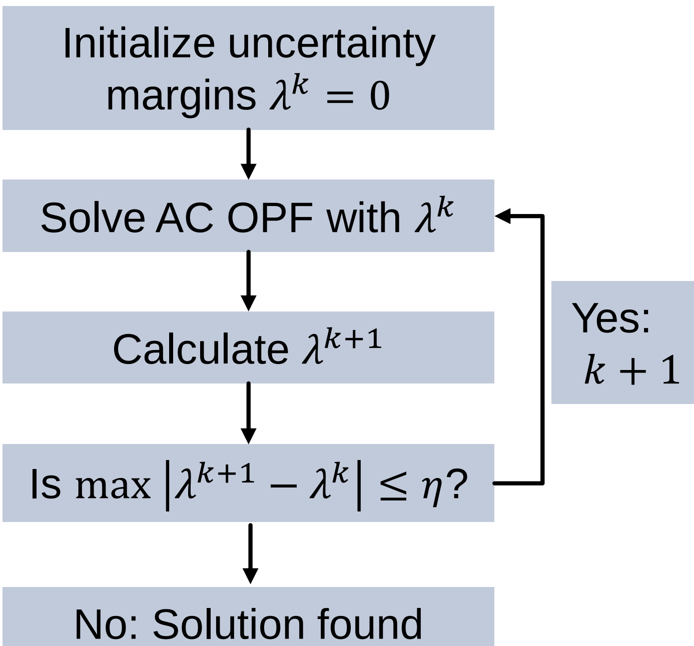
Iterative Solution Scheme

- Outer loop on existing AC OPF



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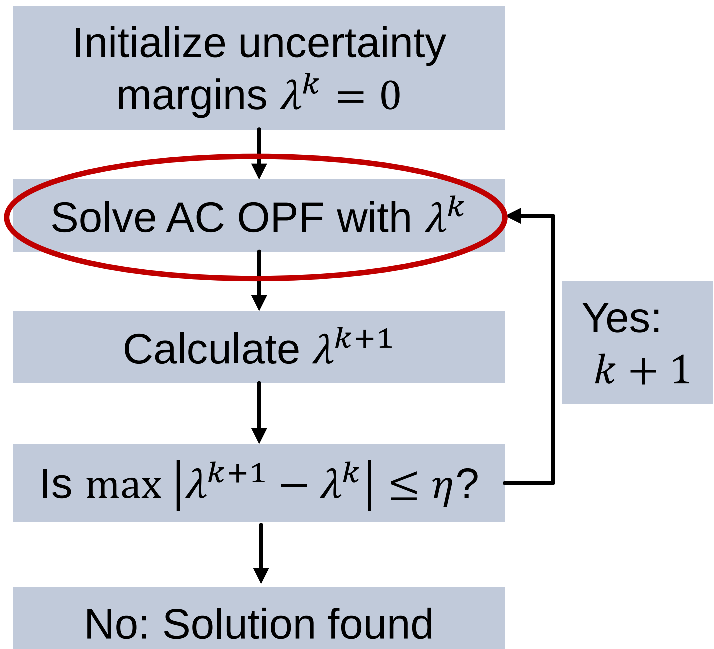

Use your favourite AC OPF solver!

Iterative Solution Scheme

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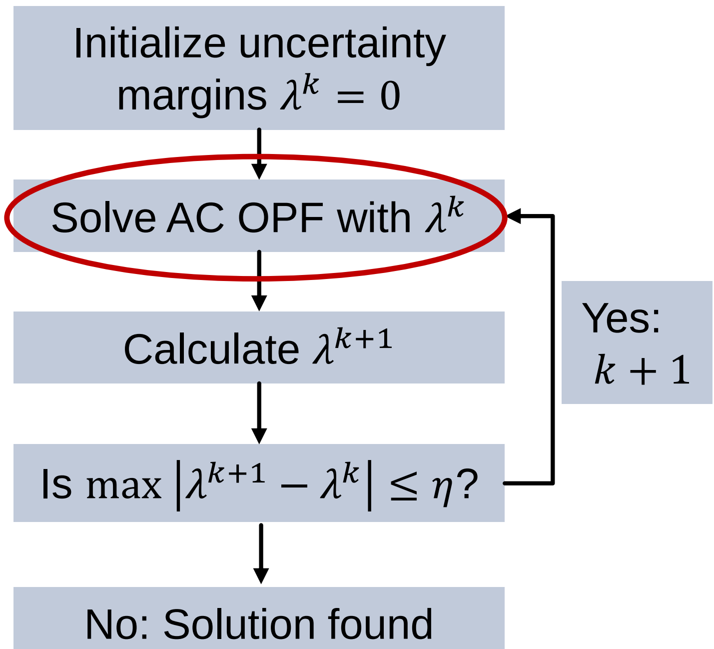


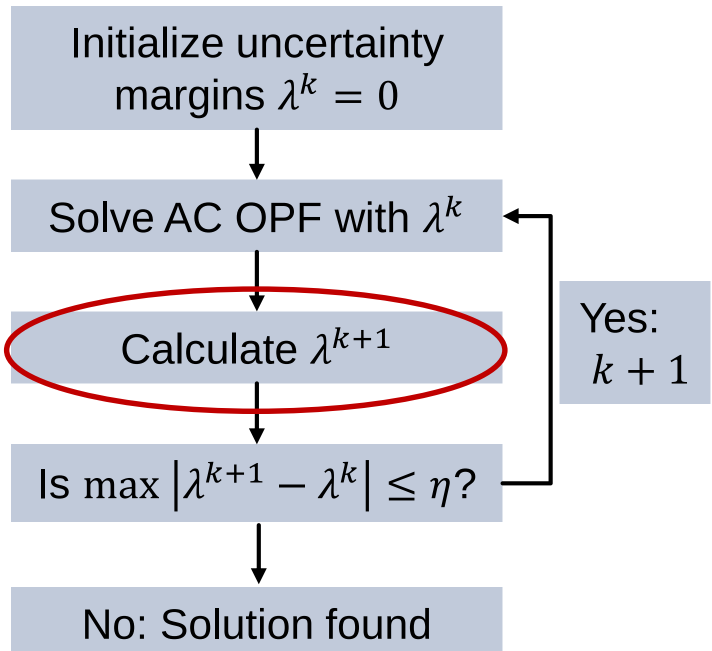
Use your favourite AC OPF solver!

Scalability

	RTS 96 24 bus	**IEEE 300 Bus**	**Polish 2383 bus**
Random loads	19	131	**941**
Time	0.54s	3.37s	**31.89s**
Iterations	5	5	4
Cost	40 127	17 143	802 238

Iterative Solution Scheme

- Outer loop on existing AC OPF



Not restricted to analytical chance constraint

Monte Carlo
Scenario Approach
...

Iterative Solution Scheme

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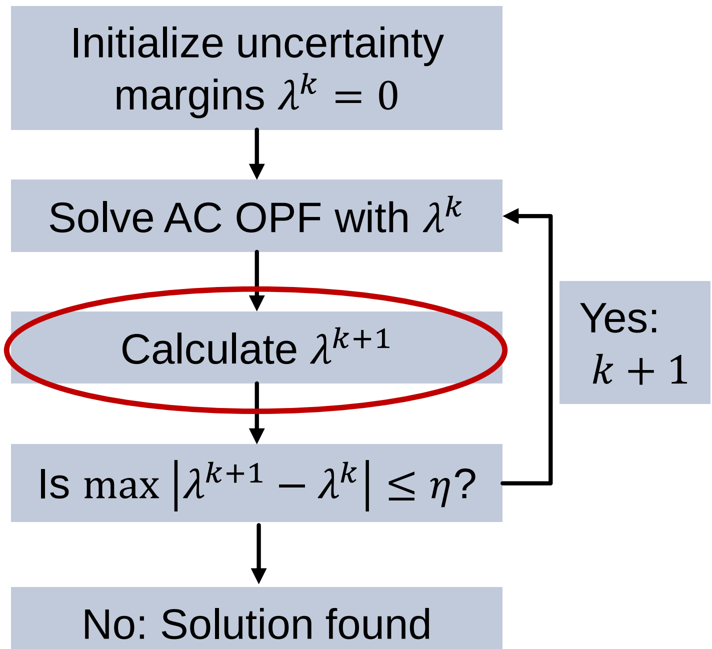

Not restricted to analytical chance constraint

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➢ **Benchmarking!**

Comparison with Sample-Based Reformulations

Analytical Reformulation

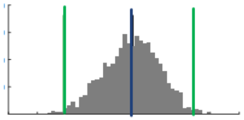
Analytic expression

$p(x, 0) \leq p^{max} - f^{-1}\sqrt{h(x)\, \Sigma_{cov}\, h(x)^T}$

Violation probability $\epsilon \leq 1\%$

Monte Carlo Simulation

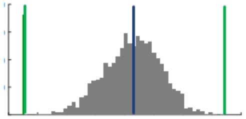
Empirical evaluation



Violation probability $\epsilon \leq 1\%$

Scenario Approach

Worst-case realization



Joint violation probability $\epsilon_J \leq 10\%$

Comparison with Sample-Based Reformulations

Analytical Reformulation

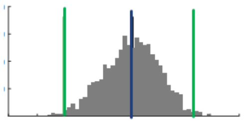
Analytic expression

$$\boldsymbol{p}(\boldsymbol{x}, \boldsymbol{0}) \leq p^{max} - f^{-1}\sqrt{\boldsymbol{h}(\boldsymbol{x})\, \Sigma_{\text{cov}}\, \boldsymbol{h}(\boldsymbol{x})^T}$$

Violation probability $\epsilon \leq 1\%$

Monte Carlo Simulation

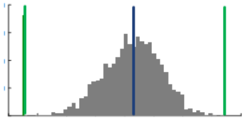
Empirical evaluation



Violation probability $\epsilon \leq 1\%$

Scenario Approach

Worst-case realization



Joint violation probability $\epsilon_J \leq 10\%$

+ No linearization error
+ No explicit assumption on distribution

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Comparison with Sample-Based Reformulations

Analytical Reformulation

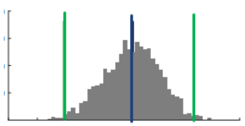
Analytic expression

$$p(x,0) \leq p^{max} - f^{-1}\sqrt{h(x)\,\Sigma_{\text{cov}}\,h(x)^T}$$

Violation probability $\epsilon \leq 1\%$

Monte Carlo Simulation

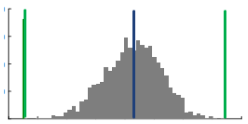
Empirical evaluation



Violation probability $\epsilon \leq 1\%$

Scenario Approach

Worst-case realization



Joint violation probability $\epsilon_J \leq 10\%$

- RTS 96 (19 random loads)
- Historical data from Austrian Power Grid (APG)

NOT NORMALLY DISTRIBUTED

\+ No linearization error

\+ No explicit assumption on distribution

Comparison with Sample-Based Reformulations

Analytical Reformulation

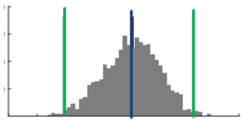
Analytic expression

$$\boldsymbol{p}(\boldsymbol{x},\boldsymbol{0}) \leq p^{max} - f^{-1}\sqrt{\boldsymbol{h}(\boldsymbol{x})\ \Sigma_{\text{cov}}\ \boldsymbol{h}(\boldsymbol{x})^T}$$

Violation probability $\epsilon \leq 1\%$

Monte Carlo Simulation

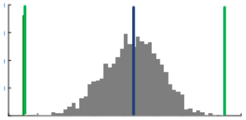
Empirical evaluation



Violation probability $\epsilon \leq 1\%$

Scenario Approach

Worst-case realization



Joint violation probability $\epsilon_J \leq 10\%$

Cost increase (from deterministic)

Max. violation probability $\leq 1\%$

➡ Dependent on samples

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Comparison with Sample-Based Reformulations

Analytical Reformulation

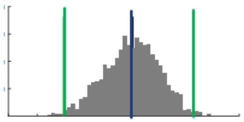
Analytic expression

$p(x,0) \leq p^{max} - f^{-1}\sqrt{h(x)\, \Sigma_{cov}\, h(x)^T}$

Violation probability $\epsilon \leq 1\%$

Monte Carlo Simulation

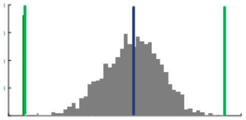
Empirical evaluation



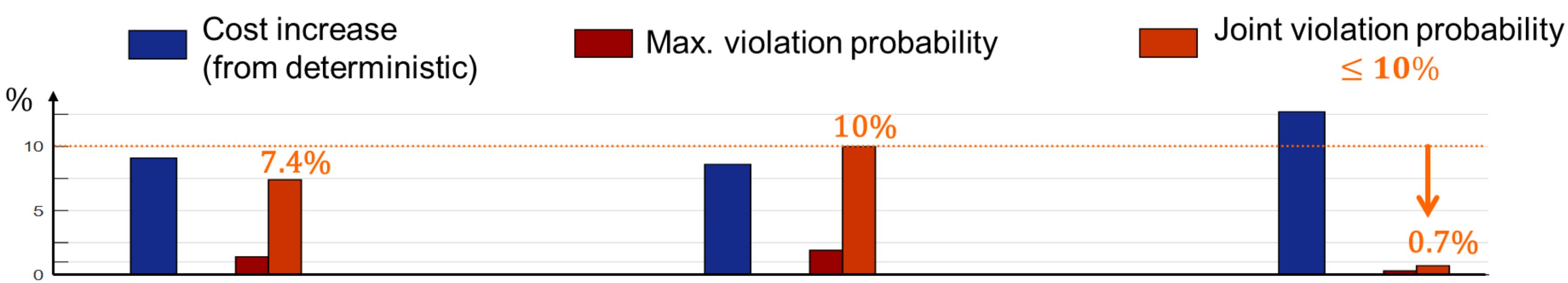
Violation probability $\epsilon \leq 1\%$

Scenario Approach

Worst-case realization



Joint violation probability $\epsilon_J \leq 10\%$



➡ Dependent on samples

➡ Very conservative

Comparison with Sample-Based Reformulations

Analytical Reformulation

Analytic expression

$\boldsymbol{p}(\boldsymbol{x},\boldsymbol{0}) \leq p^{max} - f^{-1}\sqrt{\boldsymbol{h}(\boldsymbol{x})\,\Sigma_{\text{cov}}\,\boldsymbol{h}(\boldsymbol{x})^T}$

Violation probability $\epsilon \leq 1\%$

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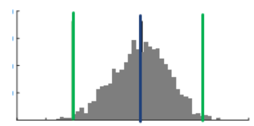
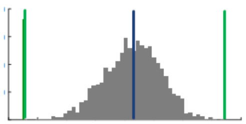
Empirical evaluation



Violation probability $\epsilon \leq 1\%$

Scenario Approach

Worst-case realization



Joint violation probability $\epsilon_J \leq 10\%$

Three conclusions:

1. Analytical reformulation is faster

Comparison with Sample-Based Reformulations

Analytical Reformulation

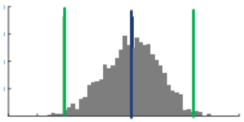
Analytic expression

$p(x, 0) \leq p^{max} - f^{-1}\sqrt{h(x)\ \Sigma_{\text{cov}}\ h(x)^T}$

Violation probability $\epsilon \leq 1\%$

Monte Carlo Simulation

Empirical evaluation



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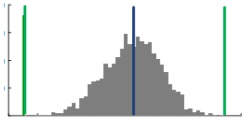
Scenario Approach

Worst-case realization



Joint violation probability $\epsilon_J \leq 10\%$

Three conclusions:

1. Analytical reformulation is faster
2. Similar accuracy as Monte Carlo

Line flows are close to normal distribution, even when injections are not!

«Central limit theorem»

Comparison with Sample-Based Reformulations

Analytical Reformulation

Analytic expression

$$\boldsymbol{p}(\boldsymbol{x},\boldsymbol{0}) \leq p^{max} - f^{-1}\sqrt{\boldsymbol{h}(\boldsymbol{x})\, \Sigma_{\text{cov}}\, \boldsymbol{h}(\boldsymbol{x})^T}$$

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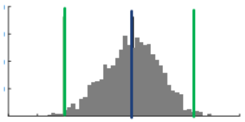
Monte Carlo Simulation

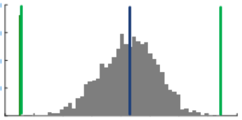
Empirical evaluation



Violation probability $\epsilon \leq 1\%$

Scenario Approach

Worst-case realization



Joint violation probability $\epsilon_J \leq 10\%$

Three conclusions:

1. Analytical reformulation is faster
2. Similar accuracy as Monte Carlo
3. Scenario Approach is very conservative

Conclusions

- Chance-constrained AC Optimal Power Flow
  - Partial linearization → Accurate for «small» fluctuations
  - Analytical reformulation → Normal distribution is not that bad
  - Performs well when compared with sample-based approaches

- Iterative solution approach
  - Decouples uncertainty and AC OPF
  - Utilizes scalability and robustness of existing implementations → Polish test case, 2000+ buses, 900+ random loads in 30s

ETH zürich

Thank you!

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Assessment of iterative algorithm

From Aldo Tobler’s semester thesis 2016 (co-supervision with Dan Molzahn)

Cut and Branch Approach