39^{th} *CNLS Conference on Strongly Correlated Quantum Matter, Santa Fe, New Mexico*

Superfluid ^{3}He-A - Strongly Correlated Chiral Quantum Matter

J. A. Sauls

Department of Physics, Northwestern University

- Oleksii Shevtsov
- Joshua Wiman
- Wave Ngampruetikorn (**Poster Session**)

- Chiral Phase of Superfluid ^{3}He
- Edge Currents & Chiral Fermions
- Detecting Broken P & T
- Strong Correlation Physics in ^{3}He
- Strong Coupling: Next-to-Leading Order
- Strong Coupling: New Results

▶Supported by National Science Foundation Grant DMR-1508730

Superfluid ^{3}He

Ferromagnetic Spin-Fluctuation Mediated Pairing

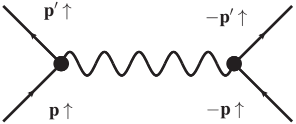
$\Downarrow$

Spin-Triplet, P-wave Condensate

Paramagnon Exchange: Ferromagnetic Spin Fluctuations $\rightsquigarrow$ Odd-Parity, Spin-Triplet Pairing for ^{3}He

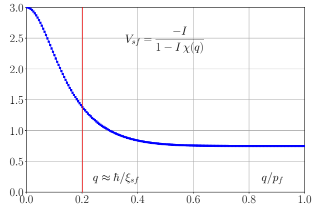
- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)

$$V_{\text{sf}}(\mathbf{q}) =$$



$$= -\frac{I}{1 - I\,\chi(\mathbf{q})}$$

$$-g_l = (2l+1)\int \frac{d\Omega_{\hat{p}}}{4\pi}\int \frac{d\Omega_{\hat{p}'}}{4\pi}\, V_{\text{sf}}(\mathbf{p}-\mathbf{p}')\, P_l(\hat{p}\cdot\hat{p}')$$

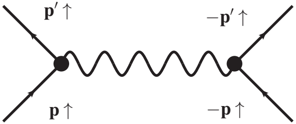


- $-g_l$ is a function of $I \approx 0.75$ & $\xi_{\text{sf}} \approx 5\,\hbar/p_f$

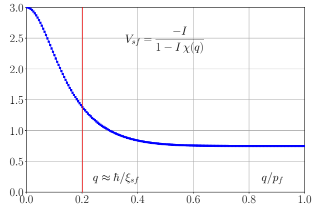
Paramagnon Exchange: Ferromagnetic Spin Fluctuations $\rightsquigarrow$ Odd-Parity, Spin-Triplet Pairing for ^{3}He

- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)

$V_{\rm sf}(\mathbf{q}) =$



$= -\dfrac{I}{1 - I\,\chi(\mathbf{q})}$



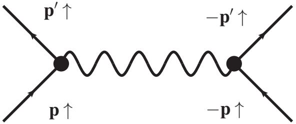
$$-g_l = (2l+1)\int \frac{d\Omega_{\hat{p}}}{4\pi}\int \frac{d\Omega_{\hat{p}'}}{4\pi}\, V_{\rm sf}(\mathbf{p}-\mathbf{p}')\, P_l(\hat{p}\cdot\hat{p}')$$

- $-g_l$ is a function of $I \approx 0.75$ & $\xi_{\rm sf} \approx 5\,\hbar/p_f$
- $l = 1$ (p-wave) is dominant pairing channel
- $S = 1$, $S_z = 0$, ± 1 Cooper Pairs:
  $|\uparrow\downarrow + \downarrow\uparrow\rangle$, $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$
  - $\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{+i\phi_{\hat{p}}} \rightsquigarrow l_z = +1$
  - $\hat{p}_z \sim \cos\theta_{\hat{p}} \rightsquigarrow l_z = 0$
  - $\hat{p}_x - i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{-i\phi_{\hat{p}}} \rightsquigarrow l_z = -1$

Paramagnon Exchange: Ferromagnetic Spin Fluctuations $\rightsquigarrow$ Odd-Parity, Spin-Triplet Pairing for ^{3}He

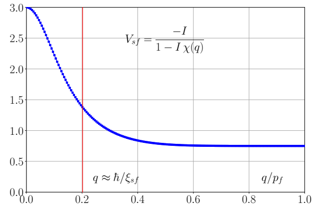
- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)

$V_{\text{sf}}(\mathbf{q}) =$



$= -\dfrac{I}{1 - I\,\chi(\mathbf{q})}$

$$-g_l = (2l+1)\int\frac{d\Omega_{\hat{p}}}{4\pi}\int\frac{d\Omega_{\hat{p}'}}{4\pi}\,V_{\text{sf}}(\mathbf{p}-\mathbf{p}')\,P_l(\hat{p}\cdot\hat{p}')$$



- $-g_l$ is a function of $I \approx 0.75$ & $\xi_{\text{sf}} \approx 5\,\hbar/p_f$
- $l = 1$ (p-wave) is dominant pairing channel
- $S = 1$, $S_z = 0$, ± 1 Cooper Pairs:
  $|\uparrow\downarrow + \downarrow\uparrow\rangle$, $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$
  - $\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{+i\phi_{\hat{p}}} \rightsquigarrow l_z = +1$
  - $\hat{p}_z \sim \cos\theta_{\hat{p}} \rightsquigarrow l_z = 0$
  - $\hat{p}_x - i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{-i\phi_{\hat{p}}} \rightsquigarrow l_z = -1$

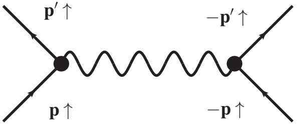
- Weak-Coupling BCS Theory based on V_{sf} leads to:
- $\rightsquigarrow$ a unique ground state for all p, T:

$$|\,B\,\rangle = \tfrac{1}{\sqrt{2}}(p_x - ip_y)|\uparrow\uparrow\rangle + \tfrac{1}{\sqrt{2}}(p_x + ip_y)|\downarrow\downarrow\rangle + p_z|\uparrow\downarrow + \downarrow\uparrow\rangle$$

Paramagnon Exchange: Ferromagnetic Spin Fluctuations $\rightsquigarrow$ Odd-Parity, Spin-Triplet Pairing for ^{3}He

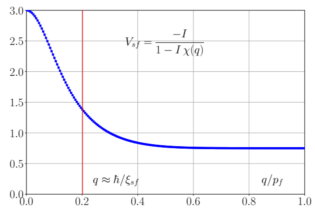
- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)

$V_{\text{sf}}(\mathbf{q}) =$



$= -\dfrac{I}{1 - I\,\chi(\mathbf{q})}$

$$-g_l = (2l+1)\int \frac{d\Omega_{\hat{p}}}{4\pi}\int \frac{d\Omega_{\hat{p}'}}{4\pi}\, V_{\text{sf}}(\mathbf{p}-\mathbf{p}')\, P_l(\hat{p}\cdot\hat{p}')$$



- $-g_l$ is a function of $I \approx 0.75$ & $\xi_{\text{sf}} \approx 5\,\hbar/p_f$
- $l = 1$ (p-wave) is dominant pairing channel
- $S = 1$, $S_z = 0$, ± 1 Cooper Pairs:
  $|\uparrow\downarrow + \downarrow\uparrow\rangle$, $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$
  - $\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{+i\phi_{\hat{p}}} \rightsquigarrow l_z = +1$
  - $\hat{p}_z \sim \cos\theta_{\hat{p}} \rightsquigarrow l_z = 0$
  - $\hat{p}_x - i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{-i\phi_{\hat{p}}} \rightsquigarrow l_z = -1$

- Weak-Coupling BCS Theory based on V_{sf} leads to:
- $\rightsquigarrow$ a unique ground state for all p, T:

$$|\,B\,\rangle = \tfrac{1}{\sqrt{2}}(p_x - ip_y)|\uparrow\uparrow\rangle + \tfrac{1}{\sqrt{2}}(p_x + ip_y)|\downarrow\downarrow\rangle + p_z|\uparrow\downarrow + \downarrow\uparrow\rangle$$

- $\rightsquigarrow$ $S = 1$, $L = 1$ and $J = 0$ ("Isotropic State")
- $\rightsquigarrow$ Fully gapped excitation spectrum:

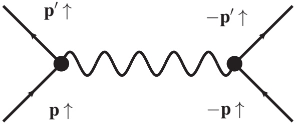
$$E_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2 + \Delta^2}$$

- R. Balian and N. Werthamer, PR 131, 1553 (1963)

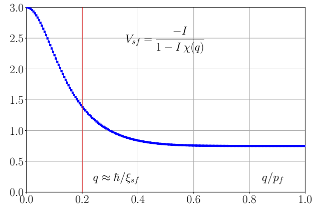
Paramagnon Exchange: Ferromagnetic Spin Fluctuations $\rightsquigarrow$ Odd-Parity, Spin-Triplet Pairing for ^{3}He

- A. Layzer and D. Fay, Int. J. Magn. 1, 135 (1971)

$V_{\mathrm{sf}}(\mathbf{q}) =$



$= -\dfrac{I}{1 - I\,\chi(\mathbf{q})}$



$$-g_l = (2l+1) \int \frac{d\Omega_{\hat{p}}}{4\pi} \int \frac{d\Omega_{\hat{p}'}}{4\pi}\, V_{\mathrm{sf}}(\mathbf{p} - \mathbf{p}')\, P_l(\hat{p}\cdot\hat{p}')$$

- $-g_l$ is a function of $I \approx 0.75$ & $\xi_{\mathrm{sf}} \approx 5\,\hbar/p_f$
- $l = 1$ (p-wave) is dominant pairing channel
- $S = 1$, $S_z = 0$, ± 1 Cooper Pairs:
  $|\uparrow\downarrow + \downarrow\uparrow\rangle$, $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$
  - $\hat{p}_x + i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{+i\phi_{\hat{p}}} \rightsquigarrow l_z = +1$
  - $\hat{p}_z \sim \cos\theta_{\hat{p}} \rightsquigarrow l_z = 0$
  - $\hat{p}_x - i\hat{p}_y \sim \sin\theta_{\hat{p}}\; e^{-i\phi_{\hat{p}}} \rightsquigarrow l_z = -1$

- Weak-Coupling BCS Theory based on V_{sf} leads to:
- $\rightsquigarrow$ a unique ground state for all p, T:

$$|\,B\,\rangle = \frac{1}{\sqrt{2}}(p_x - ip_y)|\uparrow\uparrow\rangle + \frac{1}{\sqrt{2}}(p_x + ip_y)|\downarrow\downarrow\rangle + p_z|\uparrow\downarrow + \downarrow\uparrow\rangle$$

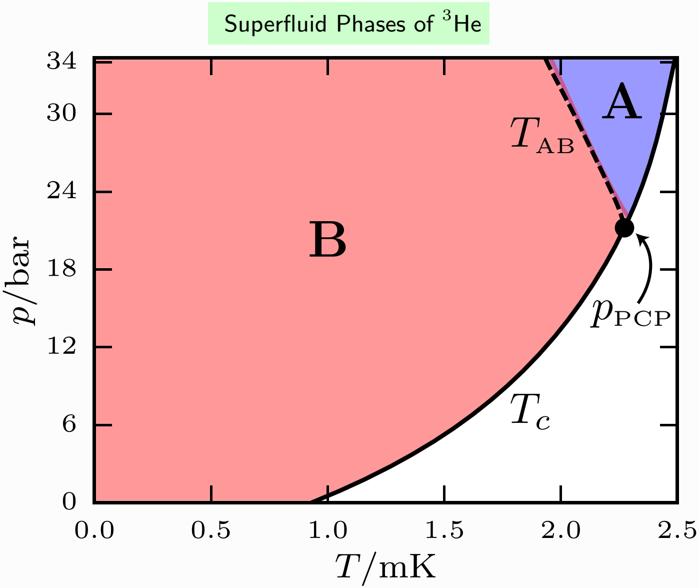
- $\rightsquigarrow$ $S = 1$, $L = 1$ and $J = 0$ ("Isotropic State")
- $\rightsquigarrow$ Fully gapped excitation spectrum:

$$E_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2 + \Delta^2}$$

- R. Balian and N. Werthamer, PR 131, 1553 (1963)

Not the Whole Story

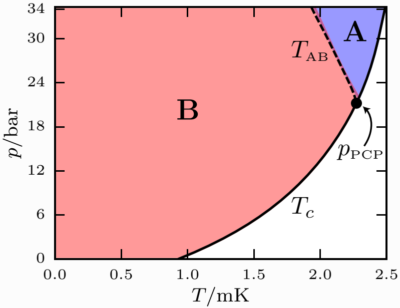
The Pressure-Temperature Phase Diagram for Liquid ^{3}He



▶J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)

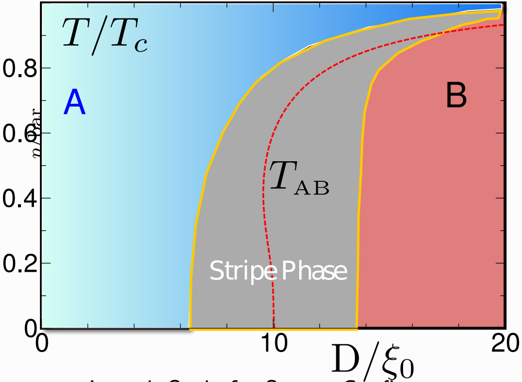
Realization of Broken Time-Reversal and Mirror Symmetry by the Vacuum State of ^{3}He Films

▶A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)



Realization of Broken Time-Reversal and Mirror Symmetry by the Vacuum State of ^{3}He Films

▶A. Vorontsov & J. A. Sauls, PRL 98, 045301 (2007)



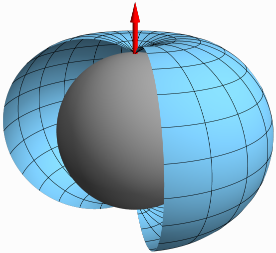
▶Length Scale for Strong Confinement:

$\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80$ nm

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{ABM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

$$\mathrm{SO(3)_S} \times \mathrm{SO(3)_L} \times \mathrm{U(1)_N} \times \mathrm{T} \times \mathrm{P}$$
$$\Downarrow$$
$$\mathrm{SO(2)_S} \times \mathrm{U(1)_{N\text{-}L_z}} \times \mathrm{Z_2}$$

Chiral ABM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

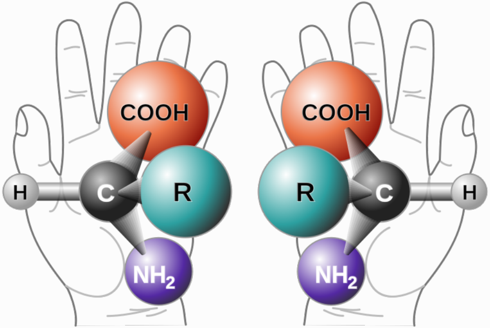
Ground-State Angular Momentum

$\langle \widehat{L}_z \rangle = \frac{N}{2}\hbar$?

Open Question

Chiral Quantum Matter

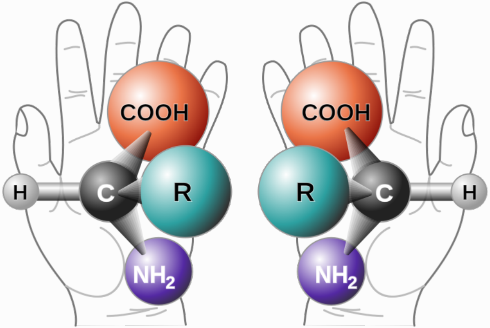
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Quantum Matter

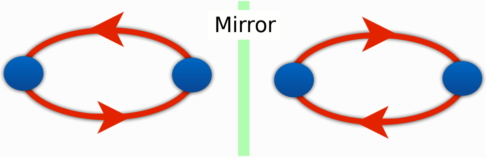
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r)\,(x + iy)$$



Broken Mirror Symmetries

$$\Pi_{zx}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

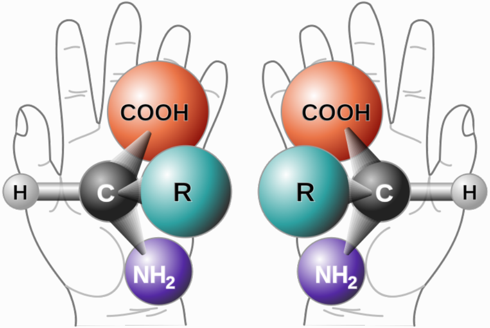
Broken Time-Reversal Symmetry

$$\mathrm{T}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Realized in Superfluid ^{3}He-A & possibly the ground states in unconventional superconductors

Chiral Quantum Matter

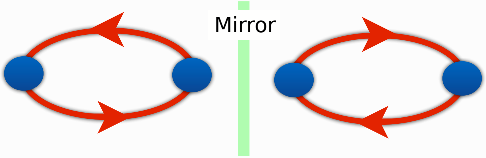
Molecular Chiral Enantiomers



Handedness: Broken Mirror Symmetry

Chiral Diatomic Molecules

$$\Psi(\mathbf{r}) = f(r)\,(x + iy)$$



Broken Mirror Symmetries

$$\Pi_{zx}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Broken Time-Reversal Symmetry

$$\mathrm{T}\,\Psi(\mathbf{r}) = f(r)\,(x - iy)$$

Realized in Superfluid ^{3}He-A & possibly the ground states in unconventional superconductors

Signatures: Chiral, Edge Fermions $\rightsquigarrow$ Anomalous Hall Transport

▶See Wave Ngampruetikorn's Poster: Anomalous Thermal Hall Effect in Sr_2RuO_4, UPt_3, etc.

Signatures of Broken T and P Symmetry in ^{3}He-A

Evidence for the Chirality of Superfluid ^{3}He-A

$\Downarrow$

Broken T and P $\rightsquigarrow$ Anomalous Hall Effect for Electrons in ^{3}He-A

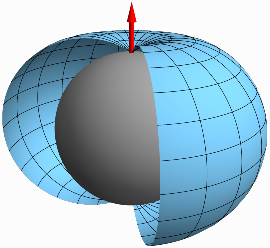
Broken Symmetries $\rightsquigarrow$ Topology of ^{3}He-A

Chirality + Topology $\rightsquigarrow$ Chiral Edge States

Momentum-Space Topology

Topology in Momentum Space

$$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$$

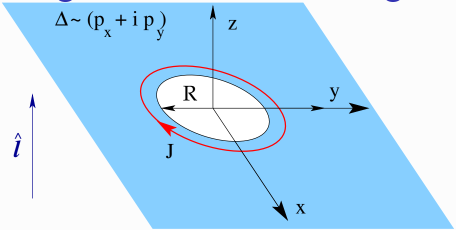


Winding Number of the Phase: $L_z = \pm 1$

$$N_{\text{2D}} = \frac{1}{2\pi} \oint d\mathbf{p} \cdot \frac{1}{|\Psi(\mathbf{p})|} \text{Im}[\boldsymbol{\nabla}_{\mathbf{p}} \Psi(\mathbf{p})] = L_z$$

- Massless Chiral Fermions
- Nodal Fermions in 3D
- Edge Fermions in 2D & 3D

Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid

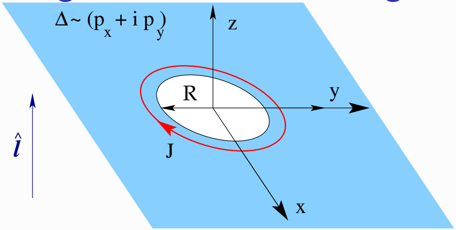


- $R \gg \xi_0 \approx 100\,\text{nm}$
- Edge Sheet Current:

$$J \equiv \int dx\, J_{\varphi}(x)$$

- Quantized Edge Current: $\frac{1}{4}\, n\, \hbar$ $(n = N/V =$ ^{3}He density)
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$

Chiral Edge Current Circulating a Hole or Defect in a Chiral Superfluid



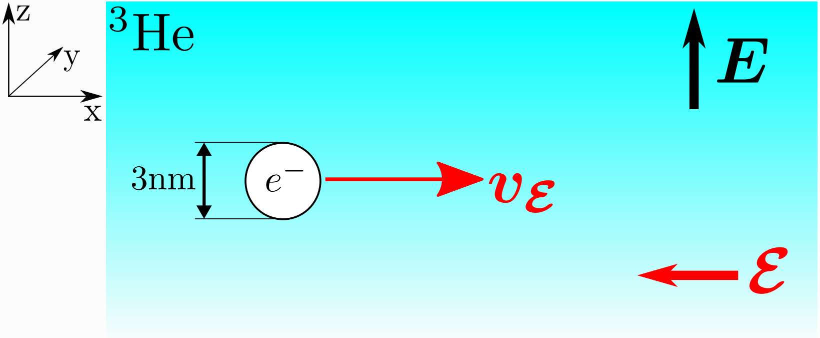
- $R \gg \xi_0 \approx 100\,\text{nm}$
- Edge Sheet Current :
  $J \equiv \int dx\, J_\varphi(x)$

- Quantized Edge Current: $\frac{1}{4}\, n\, \hbar$ ($n = N/V =$ ^{3}He density)
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$
- Angular Momentum: $L_z = 2\pi\, h\, R^2 \times (-\frac{1}{4}\, n\, \hbar) = -(N_{\text{hole}}/2)\, \hbar$

$N_{\text{hole}}/2 =$ Number of ^{3}He Cooper Pairs excluded from the Hole

- An object in ^{3}He-A *inherits* angular momentum from the Condensate of Chiral Pairs!

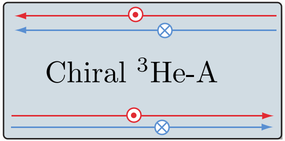
Electron bubbles in the Normal Fermi liquid phase of ^{3}He



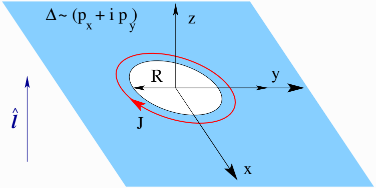
- Bubble with $R \simeq 1.5$ nm, $0.1\,\text{nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80\,\text{nm}$
- Effective mass $M \simeq 100 m_3$ (m_3 – atomic mass of ^{3}He)
- QPs mean free path $l \gg R$
- Mobility of ^{3}He is *independent of* T for $T_c < T < 50$ mK

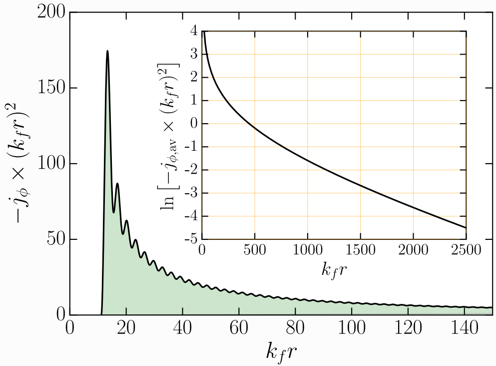
B. Josephson and J. Leckner, PRL 23, 111 (1969)

Current bound to an electron bubble ($k_f R = 11.17$)

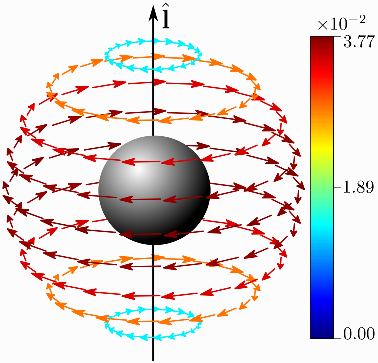


$\Longrightarrow$





$\Longrightarrow$

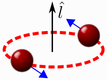


$$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi$$

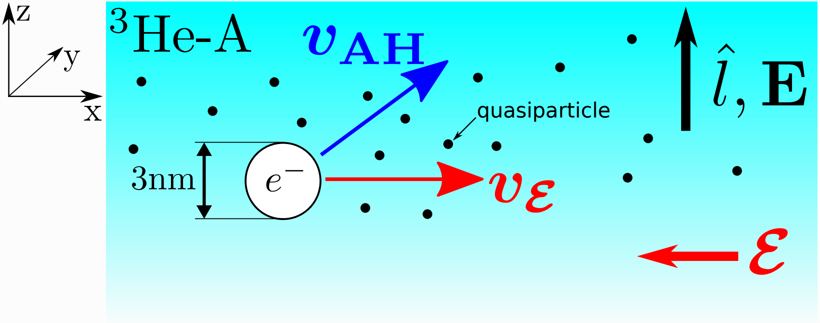
$$\mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}}/2\,\hat{\mathbf{l}} \approx -100\,\hbar\,\hat{\mathbf{l}}$$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Electron bubbles in chiral superfluid ^{3}He-A



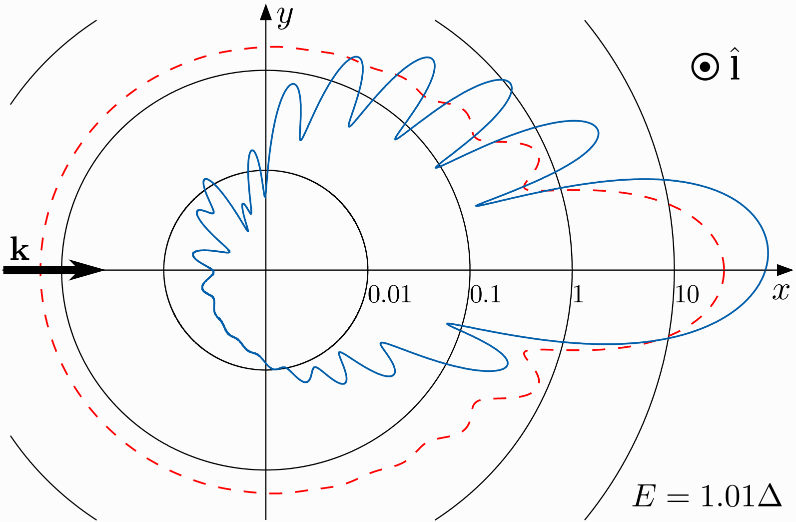
$\Delta(\hat{k}) = \Delta(\hat{k}_x + i\hat{k}_y) = \Delta\, e^{i\phi_{\mathbf{k}}}$



- Current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\text{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\text{AH}}}$
- Hall ratio: $\tan\alpha = v_{\text{AH}}/v_{\mathcal{E}} = |\mu_{\text{AH}}/\mu_\perp|$

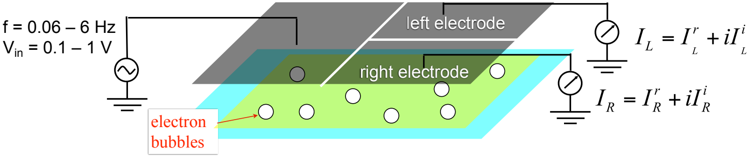
▶R. Salmelin, M. Salomaa & V. Mineev, PRL **63**, 868 (1989) ▶O. Shevtsov and J.A. Sauls, Phys. Rev. B 96, 064511 (2016)

Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$



▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

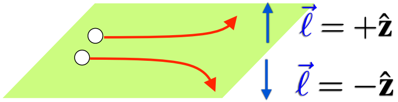
Measurement of the Transverse e$^-$ mobility in Superfluid ^{3}He Films



Transverse Force from ***Skew* Scattering**

$$\rightsquigarrow \quad \Delta I = I_R - I_L \neq 0$$

$$\vec{v} = \left[\mu_\perp \vec{E} + \mu_{xy}\, \hat{\ell} \times \vec{E} \right]$$

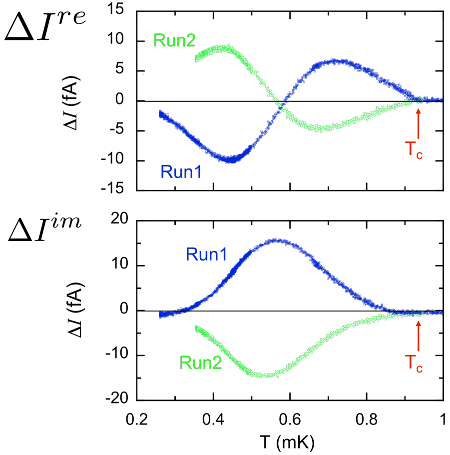


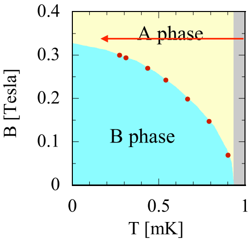
H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Detection of Broken Time-Reversal & Mirror Symmetry in ^{3}He-A

Transverse e$^-$ bubble current in ^{3}He-A

$$\Delta I = I_R - I_L$$





Single Domains:

Run 1 $\vec{\ell} = +\hat{\mathbf{z}}$

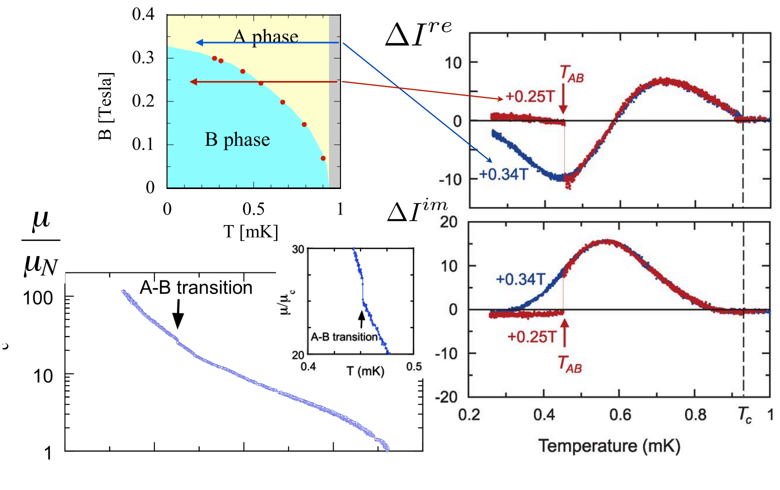
Run 2 $\vec{\ell} = -\hat{\mathbf{z}}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\,\%$$

H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

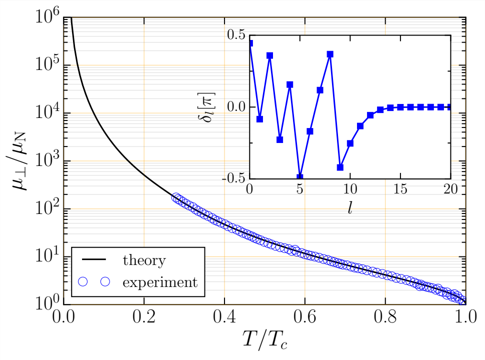
Detection of Broken Time-Reversal & Mirror Symmetry in ^{3}He-A

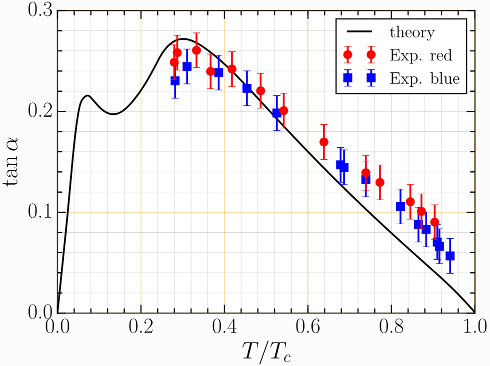
Zero **Transverse e⁻ current in ^{3}He-B (*T - symmetric phase*)**



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Comparison between Theory and Experiment for the Drag and Transverse Forces





- $\mu_\perp = e\,\dfrac{\eta_\perp}{\eta_\perp^2 + \eta_{\mathsf{AH}}^2}$
- $\mu_{\mathsf{AH}} = -e\,\dfrac{\eta_{\mathsf{AH}}}{\eta_\perp^2 + \eta_{\mathsf{AH}}^2}$

- $\tan\alpha = \left|\dfrac{\mu_{\mathsf{AH}}}{\mu_\perp}\right| = \dfrac{\eta_{\mathsf{AH}}}{\eta_\perp}$
- Ionic Radius of the e^- Bubble: $k_f R = 11.17$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

▶O. Shevtsov and JAS, JLTP 187, 340353 (2017)

Summary I

- Electrons in ^{3}He-A are "dressed" by a spectrum of Chiral Fermions
- Electrons are "Left handed" in a Right-handed Chiral Vacuum $\rightsquigarrow L_z = -100\,\hbar$

- Experiment: RIKEN mobility experiments $\rightsquigarrow$ Observation an AHE in ^{3}He-A

- Origin: Broken Mirror & Time-Reversal Symmetry + Branch-Conversion Scattering

- Theory: Scattering of Bogoliubov QPs by the dressed Ion $\rightsquigarrow$
  - Drag Force $(-\eta_{\perp}\mathbf{v})$
  - Transverse Force $(\frac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}})$

- *Anomalous Hall Field:* $\mathbf{B}_{\text{eff}} \approx \frac{\Phi_0}{3\pi^2}\,k_f^2\,(k_f R)^2\left(\frac{\eta_{\text{AH}}}{\eta_{\text{N}}}\right)\mathbf{l} \simeq 10^3 - 10^4\,\text{T}\,\mathbf{l}$

Strong Correlation Physics $\rightsquigarrow$ the Stability of the Chiral Phase of ^{3}He

Beyond Weak-coupling BCS pairing

$$\Downarrow$$

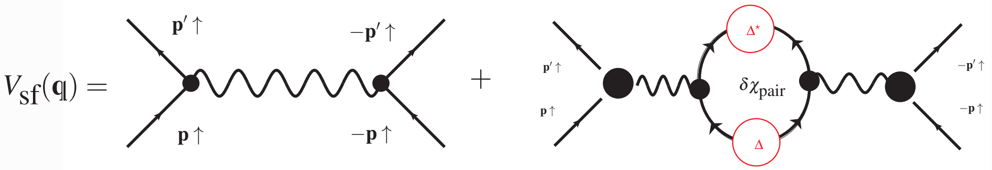
Strong-Coupling Theory of Superfluid ^{3}He

$$\Downarrow$$

Nearly Ferromagnetic or Nearly Localized?

Spin Fluctuation Exchange: Feedback Effect $\rightsquigarrow$ Stabilization of ^{3}He-A

Spin-Triplet Pairing Fluctuations *modify* the Spin-Fluctuation Pairing Interaction



- $S = 1$ pairing fluctuations modify V_{sf}:

$$\delta V_{sf} \propto \delta\chi_{\text{pair}} \propto -\chi_N \left(\Delta\Delta^\dagger\right)$$

$$|\,A\,\rangle \sim (\hat{p}_x + i\hat{p}_y)(\,|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\,) \rightsquigarrow \delta\chi^{\text{A}}_{\text{pair}} = 0$$

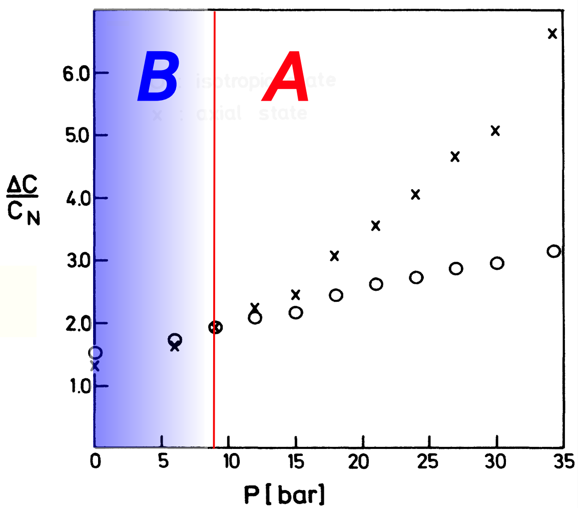
$$|\,B\,\rangle \sim (\hat{p}_x + i\hat{p}_y)|\downarrow\downarrow\rangle + (\hat{p}_x + i\hat{p}_y)|\uparrow\uparrow\rangle + \hat{p}_z\,|\uparrow\downarrow + \downarrow\uparrow\rangle\,) \rightsquigarrow \delta\chi^{\text{B}}_{\text{pair}} \sim -\chi_N \left(|\Delta|/\pi T_c\right)^2$$

"Feedback" Stabilization of ^{3}He-A

- P. W. Anderson and W. Brinkman, PRL 30, 1108 (1973)
- W. Brinkman, J. Serene, and P. Anderson, PRA 10, 2386 (1974)

Over Stabilization of the A-phase by Ferromagnetic Spin-Fluctuations

$$\Delta F = -\frac{\pi^2}{3} N(0) \left(\frac{\Delta C}{C_N} \right) (T - T_c)^2$$

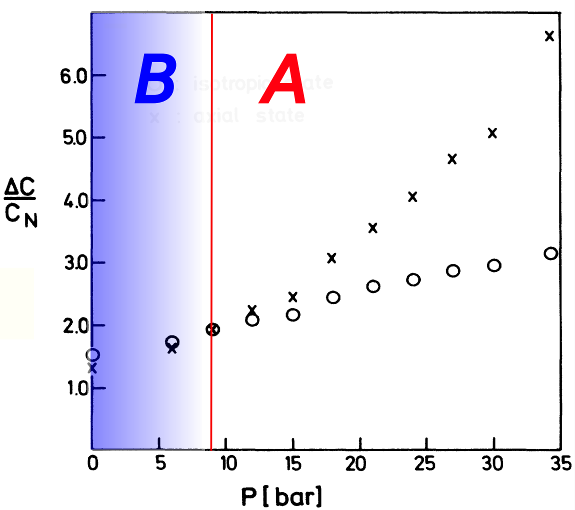


- D. Rainer & J.W. Serene, PRB 13, 4745 (1976)

- Feedback Corrections to FM $(q \to 0)$ Spin-Fluctuation Exchange
- Ferromagnetic Exchange with $F_0^a \in \{-0.67, -0.75\}$
- B phase is strongly suppressed
- A phase is *over stabilized*

Over Stabilization of the A-phase by Ferromagnetic Spin-Fluctuations

$$\Delta F = -\frac{\pi^2}{3} N(0) \left(\frac{\Delta C}{C_N} \right) (T - T_c)^2$$



- D. Rainer & J.W. Serene, PRB 13, 4745 (1976)

- Feedback Corrections to FM ($q \to 0$) Spin-Fluctuation Exchange
- Ferromagnetic Exchange with $F_0^a \in \{-0.67, -0.75\}$
- B phase is strongly suppressed
- A phase is *over stabilized*

What is missing?

- Retardation (conventional strong-coupling)
- Full spin- and density-fluctuation spectrum

Beyond Weak-Coupling BCS Theory

Asymptotic Expansion in **small** $= \; k_B T_c/E_f, \quad \Delta/E_f, \quad \hbar/p_f\xi \; \ldots$

$$\Downarrow$$

Thermodynamic Potential for a Strongly Correlated Fermionic Superfluids

$$\Downarrow$$

$$\Omega(p,T) = \Omega_{\text{N}} + \Omega_{\text{wc}} + \Omega_{\text{sc}_1} + \ldots$$

$$\Downarrow$$

- $\Omega_{\text{N}} \sim \textbf{small}^0 \leftarrow$ Normal Fermi-Liquid Ground-State Energy
- $\Omega_{\text{wc}} \sim \Omega_{\text{N}} \times \textbf{small}^2 \leftarrow$ weak-Coupling BCS theory
- $\Omega_{\text{sc}_1} \sim \Omega_{\text{N}} \times \textbf{small}^3 \leftarrow$ leading order Strong-Coupling Theory
- $\Omega_{\text{sc}_2} \sim \Omega_{\text{N}} \times \textbf{small}^4 \log \textbf{small} \leftarrow$ next-to-leading order

- D. Rainer & J. Serene, Phys. Rev. B 13, 4745 (1976)
- JAS & J. Serene, Phys. Rev. B 24, 181 (1981)

Strong-Coupling Free-Energy Functional

- Expansion of the Luttinger-Ward functional in $\frac{k_B T}{E_f}, \frac{\hbar}{p_f \xi_0}, \ldots$

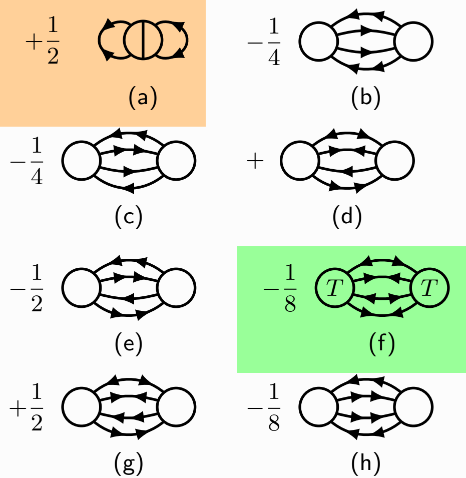
$$\Omega = -\frac{T}{2} \sum_{\epsilon_n} \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}_4 \Big\{ \Delta\widehat{\Sigma}\, \widehat{G} + \ln[-\widehat{G}_N^{-1} + \Delta\widehat{\Sigma}] - \ln[-\widehat{G}_N^{-1})] \Big\} + \Delta\Phi[\Delta\widehat{G}]$$

$$\widehat{G}(\vec{p},\, \epsilon_n) = \begin{pmatrix} \hat{G}(\vec{p},\, \epsilon_n) & \hat{F}(\vec{p},\, \epsilon_n) \\ \hat{F}^{\dagger}(\vec{p},\, -\epsilon_n) & -\hat{G}^{\mathrm{tr}}(-\vec{p},\, -\epsilon_n) \end{pmatrix}$$

$$\widehat{\Sigma}(\vec{p},\, \epsilon_n) = \begin{pmatrix} \hat{\Sigma}(\vec{p},\, \epsilon_n) & \hat{\Delta}(\vec{p},\, \epsilon_n) \\ \hat{\Delta}^{\dagger}(\vec{p},\, -\epsilon_n) & -\hat{\Sigma}^{\mathrm{tr}}(-\vec{p},\, -\epsilon_n) \end{pmatrix}$$

$\delta\Omega[\widehat{\Sigma}]/\delta\widehat{\Sigma}^{\mathrm{tr}}(\vec{p},\, \epsilon_n) = 0$ and $\delta\Omega[\widehat{\Sigma}]/\delta\widehat{G}^{\mathrm{tr}}(\vec{p},\, \epsilon_n) = 0$

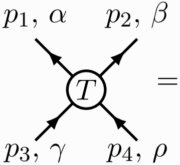
$$\widehat{\Sigma} = \widehat{\Sigma}_{\mathrm{skel}} = 2\,\delta\Phi[\widehat{G}]/\delta\widehat{G}^{\mathrm{tr}}$$



- D. Rainer & J. Serene, Phys. Rev. B 13, 4745 (1976)
- JAS & J. Serene, Phys. Rev. B 24, 181 (1981)
- J. Wiman & JAS, unpublished (2019)

Quasiparticle Scattering Amplitude - T-Matrix on the Fermi Surface

Effective Quasiparticle-Quasiparticle Potentials in the spirit of *Pines' Polarization Potentials*



$$= \delta_{\alpha\gamma}\delta_{\beta\rho}\, v(q_3) + \vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\rho}\, j(q_3) - \delta_{\alpha\rho}\delta_{\beta\gamma}\, v(q_4) - \vec{\sigma}_{\alpha\rho}\cdot\vec{\sigma}_{\beta\gamma}\, j(q_4)$$

- $v(q) =$ spin-independent potential
- $j(q) =$ spin-exchange potential

- Singlet/Triplet Amplitudes: $T_s(\theta,\phi) = W_s(q_3) + W_s(q_4)$ & $T_t(\theta,\phi) = W_t(q_3) - W_t(q_4)$

$$W_s(q) = v(q) - 3j(q) \qquad W_t(q) = v(q) + j(q)$$

- J.A. Sauls & J.W. Serene, Phys. Rev. B 24, 181 (1981)

Determination of the Quasiparticle Interaction Potentials $\forall$ Pressures

Thermodynamic Properties

- $C_v/T \propto m^*/m$
- $\chi/\chi_{\text{Pauli}} = (m^*/m)/(1+F_0^a)$
- First Sound: c_1/v_f
- Zero Sound: $(c_0 - c_1)/c_1$

Transport Properties

- QP Lifetime: $\tau T^2 \propto \langle \mathcal{W}(\theta,\phi) \rangle_{FS}$
- Heat: $\kappa T \propto \tau_\kappa T^2 \leftarrow \langle \mathcal{W}(\theta,\phi)(1+2\cos\theta) \rangle_{FS}$
- Spin: $D_S T^2 \propto \tau_S T^2 \leftarrow \langle \mathcal{W}_{\uparrow\downarrow}(\theta,\phi)(\sin^2\frac{\theta}{2}\sin^2\frac{\phi}{2} \rangle_{FS}$
- Viscosity: $\eta T^2 \propto \tau_\eta T^2 \leftarrow \langle \mathcal{W}(\theta,\phi)(1-3\sin^4\frac{\theta}{2}\sin^2\phi) \rangle_{FS}$

- Heat Capacity Jumps at $T_c(p)$: $\Delta C_{\text{B}}/T_c$, $\Delta C_{\text{A}}/T_c \leftarrow \langle \mathcal{W}(\theta,\phi) X_{\text{B,A}}(\theta,\phi) \rangle_{FS}$

- QP Scattering Rates Determine <u>All</u> Thermodynamic & Transport Coefficients of Normal ^{3}He

$$\mathcal{W}(\theta,\phi) = |T_s(\theta,\phi)|^2 + 3|T_t(\theta,\phi)|^2 + 2T_t(\theta,\phi)T_s(\theta,\phi)$$
$$\mathcal{W}_{\uparrow\downarrow}(\theta,\phi) = |T_s(\theta,\phi)|^2 + |T_t(\theta,\phi)|^2 + 2T_t(\theta,\phi)T_s(\theta,\phi)$$

- J.A. Sauls & J.W. Serene, Phys. Rev. B 24, 181 (1981)
- J.J. Wiman & J. A. Sauls, unpublished (2019)

Determining the Quaiparticle T-Matrix Amplitudes & Effective Potentials

- $V_i[\{W_l^{s,t}\}] \quad \forall i = 1, \ldots N$ are N Theortical Functions for Each Observable
- $X_i(p) \quad \forall i = 1, \ldots N$ are N Experimental Constraints at a fixed Pressure

Minimize: $E[\{W_l^{s,t}\}] = \sum_i^N \Big(V_i[\{W_l^{s,t}\}] - X_i(p) \Big)^2$

$$\Downarrow$$

$v(q)$ and $j(q)$

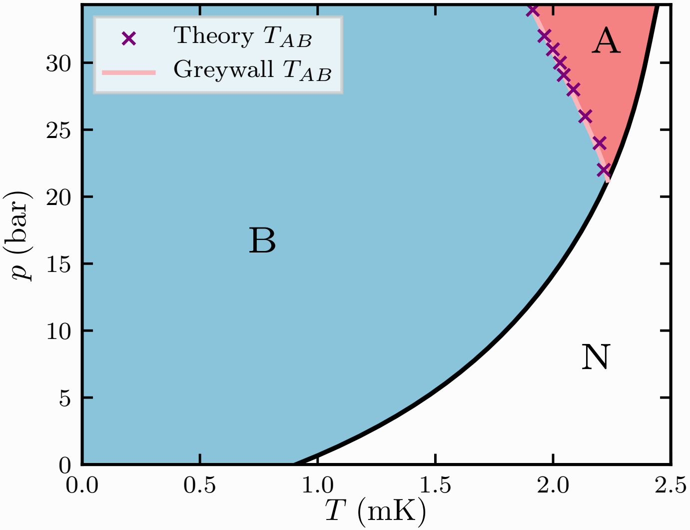
$$\Downarrow$$

$$\Omega(p,T) = \Delta\Omega[\Delta\widehat{G}_*, \Delta\widehat{\Sigma}_*]$$

$$\Downarrow$$

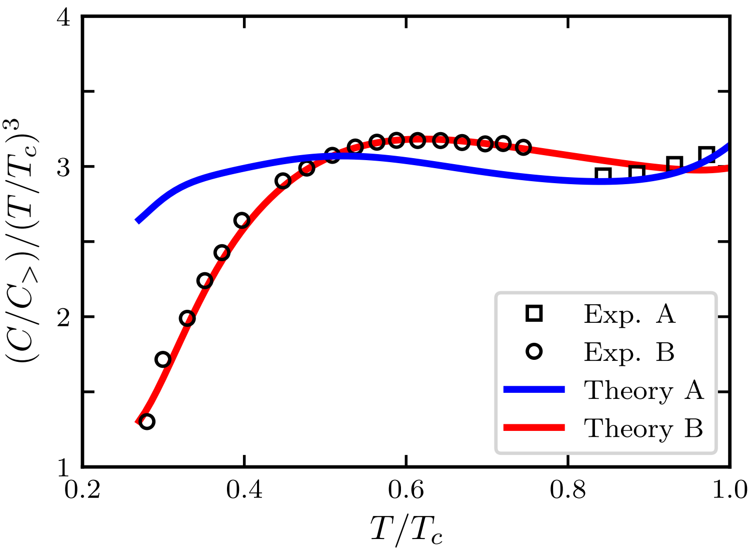
$$\Omega_{\text{A,B}}(p,T)\,, \quad S_{\text{A,B}}(p,T) = -\frac{\partial \Omega_{\text{A,B}}}{\partial T}\,, \quad C_{\text{A,B}}(p,T) = T\frac{\partial S_{\text{A,B}}}{\partial T}$$

Strong-Coupling Results - The A-B Transition Line: $\Omega_A(p, T_{AB}) = \Omega_B(p, T_{AB})$



- Data: D. S. Greywall. Phys. Rev. B, 33(11):7520, 1986
- ▶J.J. Wiman & J. A. Sauls, unpublished (2019)

Strong-Coupling Results - Heat Capacity: $C(T) = -T\partial^2\Omega(p,T)/\partial T^2$



• Data: D. S. Greywall. Phys. Rev. B, 33(11):7520, 1986 ▶J.J. Wiman & J. A. Sauls, unpublished (2019)

^{3}He: Nearly Ferromagnetic vs. Almost Localized

Paramagnon Theory (Levin and Valls, Phys. Rep. 1 1983):

- Spin Susceptibility in Paramagnon Theory: $\chi/\chi_{\mathrm{P}} = \dfrac{1}{1-I} \gg 1$

$$\Downarrow$$

^{3}He is near to a ferromagnetic instability

$$\Downarrow$$

finite, but long-lived FM spin fluctuations.

- Effective Mass: $m^*/m - 1 = \ln(1/(1-I))$

^{3}He: Nearly Ferromagnetic vs. Almost Localized

Paramagnon Theory (Levin and Valls, Phys. Rep. 1 1983):

- Spin Susceptibility in Paramagnon Theory: $\chi/\chi_{\mathsf{P}} = \dfrac{1}{1-I} \gg 1$

$$\Downarrow$$

^{3}He is near to a ferromagnetic instability

$$\Downarrow$$

finite, but long-lived FM spin fluctuations.

- Effective Mass: $m^*/m - 1 = \ln(1/(1-I))$

- Fermi Liquid Theory: $\chi/\chi_{\mathsf{P}} = \dfrac{m^*/m}{1+F_0^a} \gg 1$
- Exchange Interaction: $F_0^a = -0.70$ to -0.75 is nearly constant
- $\therefore \chi/\chi_{\mathsf{P}}$ increases with pressure mainly due m^*/m

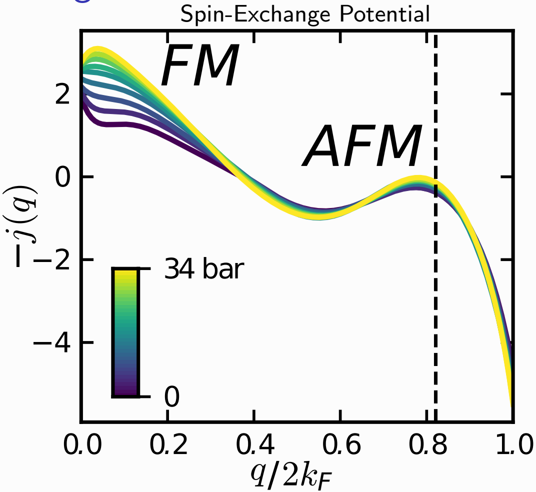
$$\Downarrow$$

^{3}He is nearly localized (à la Mott) due to short-range repulsive interactions
P. W. Anderson, W. Brinkman, Scottish Summer School, St. Andrews (1975).

- ^{3}He is very incompressible: $F_0^s \approx 10$ to 100 at $p = 34\,$bar

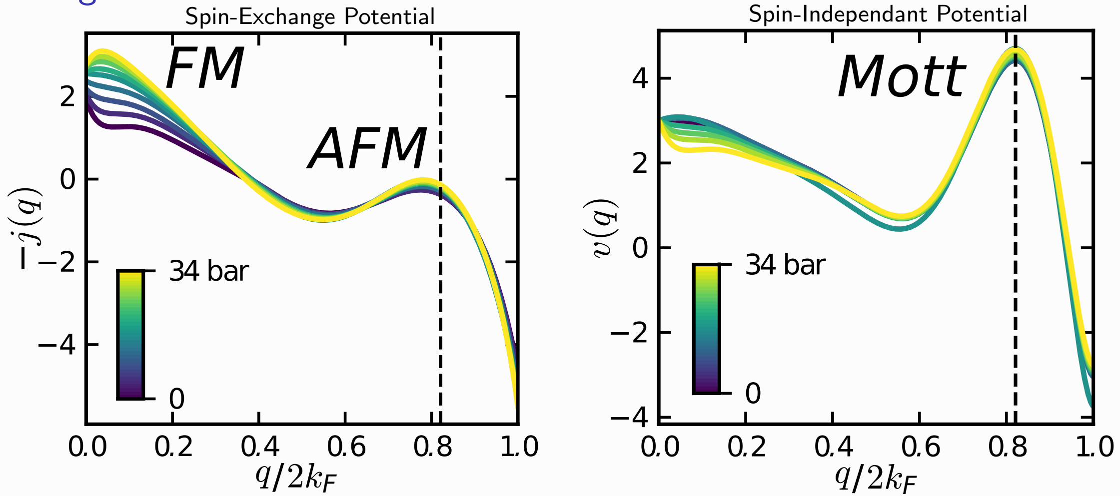
▶D. Vollhardt, RMP 56, 101 (1984) ▶Mott transition in 2D 3He Films, J. Saunders et al., PRL

Strong-Correlations in ^{3}He: Effective Interactions



- FM Spin-Fluctuation resonance at $q \approx 0$
- AFM Spin-Fluctuation resonance at $q/2k_f \approx 0.82$
- Exchange distributed over multiple length scales

Strong-Correlations in ^{3}He: Effective Interactions



- FM Spin-Fluctuation resonance at $q \approx 0$
- AFM Spin-Fluctuation resonance at $q/2k_f \approx 0.82$
- Exchange distributed over multiple length scales
- Density-Fluctuation peak at $q/2k_f \approx 0.82$
- Solid ^{3}He @ $p = 34$ bar: $a = 4.32\,\mathring{A}$
- $\rightsquigarrow q_a = 2\pi/a \approx 1.64\,k_f$

▶J.J. Wiman & J. A. Sauls, unpublished (2019)

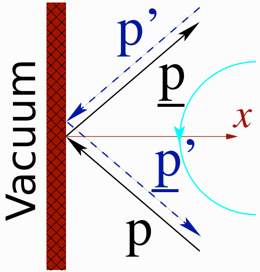
Summary II

- Quasiclassical reduction of the Luttinger-Ward Functional - with effective interactions obtained from the normal Fermi liquid - provides a quantitative account of the thermodynamics of the superfluid ^{3}He-A and ^{3}He-B at all T, p.
- The stability of ^{3}He-A at high pressure (beyond weak-coupling BCS) is derived from effective interactions for a nearly localized Fermi liquid with both FM and AFM correlations.
- Interactions Stabilizing the Equal-Spin Pairing of ^{3}He-A ($|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$) appear to also drive localization and the UUDD AFM phase of Solid ^{3}He.

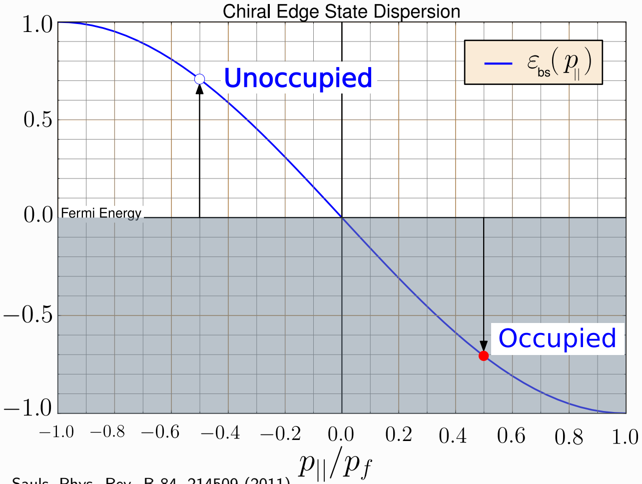
Massless Chiral Fermions in the 2D ^{3}He-A Films

Edge Fermions: $G^{\text{R}}_{\text{edge}}(\mathbf{p},\varepsilon;x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\text{bs}}(\mathbf{p}_{||})}\, e^{-x/\xi_\Delta}$ $\qquad \xi_\Delta = \hbar v_f/2\Delta \approx 10^2\,\AA \gg \hbar/p_f$

- $\varepsilon_{\text{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$



- Broken P & T $\rightsquigarrow$ Edge Current



- M. Stone, R. Roy, PRB 69, 184511 (2004)
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

- Structure of Electrons in Superfluid ^{3}He-A
- Forces of Moving Electrons in Superfluid ^{3}He-A

$\Downarrow$

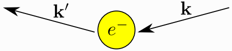
- Scattering Theory of ^{3}He Quasiparticles by Electron Bubbles

Forces on the Electron bubble in ^{3}He-A:

- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\rm QP}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\rm AH} & 0 \\ -\eta_{\rm AH} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for broken PT symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$
- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \frac{e}{c}\mathbf{v}\times\mathbf{B}_{\rm eff}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\rm eff} = -\frac{c}{e}\eta_{\rm AH}\hat{\mathbf{l}}$ $\quad B_{\rm eff} \simeq 10^3 - 10^4$ T !!!
- Mobility: $\frac{d\mathbf{v}}{dt} = 0 \rightsquigarrow \mathbf{v} = \overleftrightarrow{\mu}\,\boldsymbol{\mathcal{E}}$, where $\overleftrightarrow{\mu} = e\,\overleftrightarrow{\eta}^{-1}$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

T-matrix description of Quasiparticle-Ion scattering



- Lippmann-Schwinger equation for the T-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

$$\hat{T}^R_S(\mathbf{k}',\mathbf{k},E) = \hat{T}^R_N(\mathbf{k}',\mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3} \hat{T}^R_N(\mathbf{k}',\mathbf{k}'') \Big[\hat{G}^R_S(\mathbf{k}'',E) - \hat{G}^R_N(\mathbf{k}'',E) \Big] \hat{T}^R_S(\mathbf{k}'',\mathbf{k},E)$$

$$\hat{G}^R_S(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E^2_{\mathbf{k}}} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

- Normal-state T-matrix:

$$\hat{T}^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \begin{pmatrix} t^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) & 0 \\ 0 & -[t^R_N(-\hat{\mathbf{k}}',-\hat{\mathbf{k}})]^\dagger \end{pmatrix}$$ in p-h (Nambu) space , where

$$t^R_N(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = -\frac{1}{\pi N_f} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}),$$ $P_l(x)$ – Legendre function

- Hard-sphere potential $\rightsquigarrow \tan\delta_l = j_l(k_f R)/n_l(k_f R)$ – spherical Bessel functions

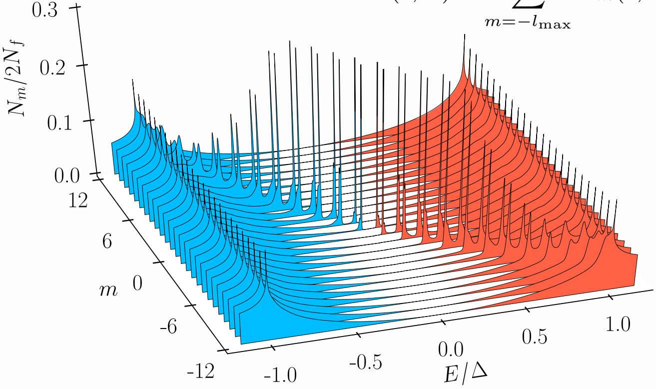
  - $k_f R$ – determined by the Normal-State Mobility $\rightsquigarrow$ $k_f R = 11.17\ (R = 1.42\,\text{nm})$

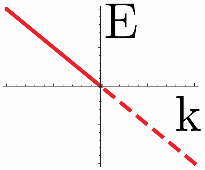
Weyl Fermion Spectrum bound to the Electron Bubble

$$\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma_{\mathsf{N}}^{\mathsf{tr}}} \quad \Leftarrow \quad \mu_{\mathsf{N}}^{\mathsf{exp}} = 1.7 \times 10^{-6}\, \frac{m^2}{V\, s}$$

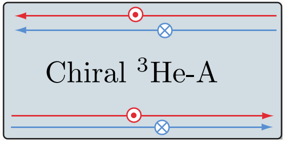
$$\tan\delta_l = j_l(k_f R)/n_l(k_f R) \;\Rightarrow\; \sigma_{\mathsf{N}}^{\mathsf{tr}} = \frac{4\pi}{k_f^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1}-\delta_l) \;\rightsquigarrow\; k_f R = 11.17$$

$$N(\mathbf{r},E) = \sum_{m=-l_{\max}}^{l_{\max}} N_m(\mathbf{r},E), \quad l_{\max} \simeq k_f R$$

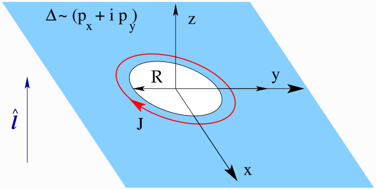


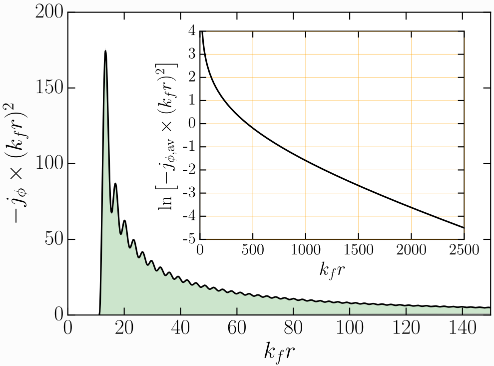


Current bound to an electron bubble ($k_f R = 11.17$)

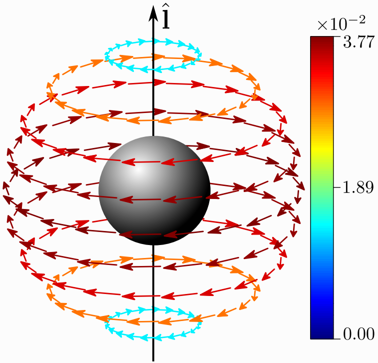


$\Longrightarrow$





$\Longrightarrow$



$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

$\mathbf{L}(T \to 0) \approx -\hbar N_{\mathrm{bubble}}/2\,\hat{\mathbf{l}} \approx -100\,\hbar\,\hat{\mathbf{l}}$

Determination of the Stokes Tensor from the QP-Ion T-matrix

(i) Fermi's golden rule and the QP scattering rate:

$$\Gamma(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}',\hat{\mathbf{k}})\delta(E_{\mathbf{k}'} - E_{\mathbf{k}}), \qquad W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = \frac{1}{2}\sum_{\tau'\sigma';\tau\sigma} |\overbrace{\langle \mathbf{k}',\sigma',\tau' |}^{\text{outgoing}} \hat{T}_S \overbrace{| \mathbf{k},\sigma,\tau \rangle}^{\text{incoming}}|^2$$

(ii) Drag force from QP-ion collisions (linear in $\mathbf{v}$): ▶Baym et al. PRL **22**, 20 (1969)

$$\mathbf{F}_{\mathsf{QP}} = -\sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}' - \mathbf{k}) \left[\hbar\mathbf{k}'\mathbf{v} f_{\mathbf{k}} \left(-\frac{\partial f_{\mathbf{k}'}}{\partial E}\right) - \hbar\mathbf{k}\mathbf{v}(1 - f_{\mathbf{k}'}) \left(-\frac{\partial f_{\mathbf{k}}}{\partial E}\right)\right] \Gamma(\mathbf{k}',\mathbf{k})$$

(iii) Microscopic reversibility condition: $W(\hat{\mathbf{k}}',\hat{\mathbf{k}} : +\mathbf{l}) = W(\hat{\mathbf{k}},\hat{\mathbf{k}}' : -\mathbf{l})$

Broken T and mirror symmetries in ^{3}He-A $\Rightarrow$ fixed $\hat{\mathbf{l}}$ $\rightsquigarrow$ $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}},\hat{\mathbf{k}}')$

(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\mathsf{QP}} = -\overleftrightarrow{\eta} \cdot \mathbf{v} \quad \rightsquigarrow \quad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E), \quad \overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\mathsf{AH}} & 0 \\ -\eta_{\mathsf{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\| \end{pmatrix}$$

$n_3 = \frac{k_f^3}{3\pi^2}$ – ^{3}He particle density, $\sigma_{ij}(E)$ – transport scattering cross section,

$f(E) = [\exp(E/k_B T) + 1]^{-1}$ – Fermi Distribution

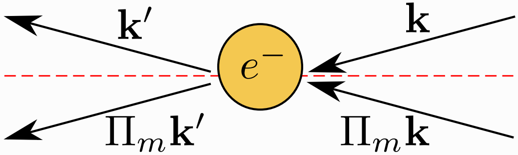
Mirror-symmetric scattering ⇒ longitudinal drag force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[(\hat{\mathbf{k}}'_i-\hat{\mathbf{k}}_i)(\hat{\mathbf{k}}'_j-\hat{\mathbf{k}}_j)\right]\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)$$

Mirror-symmetric cross section: $W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}}W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

$\rightsquigarrow$ Stokes Drag $\eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_\perp,\ \eta_{zz}^{(+)} \equiv \eta_\parallel$, No transverse force $\left[\eta_{ij}^{(+)}\right]_{i\neq j} = 0$

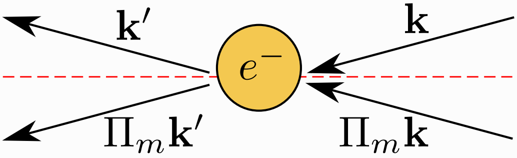
Mirror-antisymmetric scattering $\Rightarrow$ transverse force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(-)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\geq|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[\epsilon_{ijk}(\hat{\mathbf{k}}'\times\hat{\mathbf{k}})_k\right]\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)\left[f(E)-\frac{1}{2}\right]$$

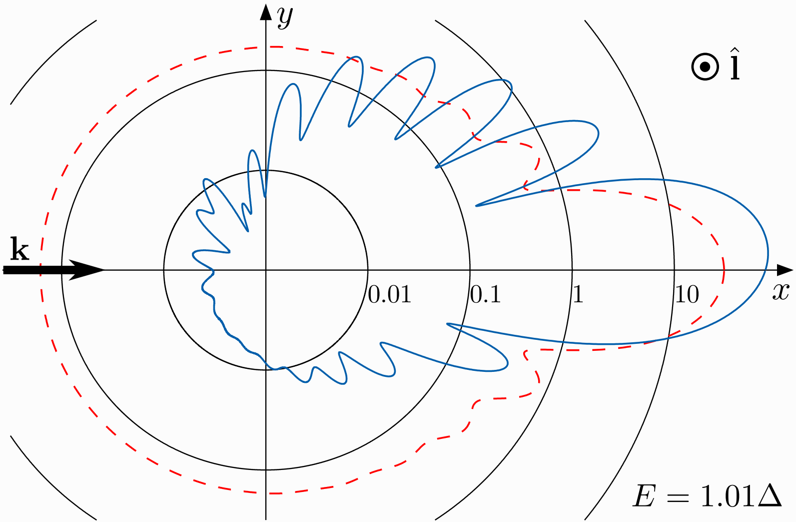
Mirror-antisymmetric cross section: $W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) - W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}}')|^2}}W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}})\frac{E}{\sqrt{E^2-|\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force $\quad \eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\text{AH}} \quad \Rightarrow \quad$ anomalous Hall effect

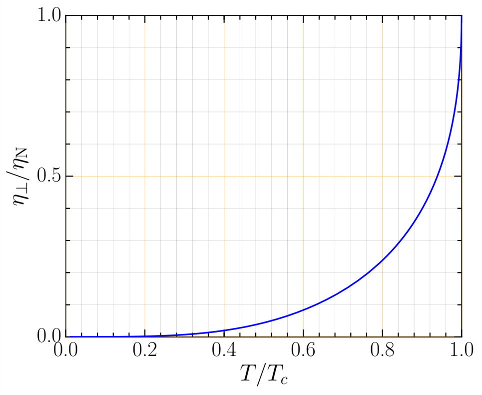
▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

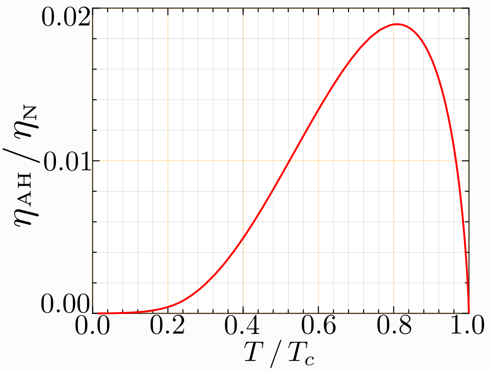
Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$



▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Theoretical Results for the Drag and Transverse Forces





- $\Delta p_x \approx p_f \qquad \sigma_{xx}^{\mathrm{tr}} \approx \sigma_{\mathrm{N}}^{\mathrm{tr}} \approx \pi R^2$
- $F_x \approx n\, v_x\, \Delta p_x\, \sigma_{xx}^{\mathrm{tr}}$
  $\approx n\, v_x\, p_f\, \sigma_{\mathrm{N}}^{\mathrm{tr}}$

- $\Delta p_y \approx \hbar/R \quad \sigma_{xy}^{\mathrm{tr}} \approx (\Delta(T)/k_{\mathrm{B}}T_c)^2 \sigma_{\mathrm{N}}^{\mathrm{tr}}$
- $F_y \approx n\, v_x\, \Delta p_y\, \sigma_{xy}^{\mathrm{tr}}$
  $\approx n\, v_x \; (\hbar/R)\, \sigma_{\mathrm{N}}^{\mathrm{tr}} (\Delta(T)/k_{\mathrm{B}}T_c)^2$

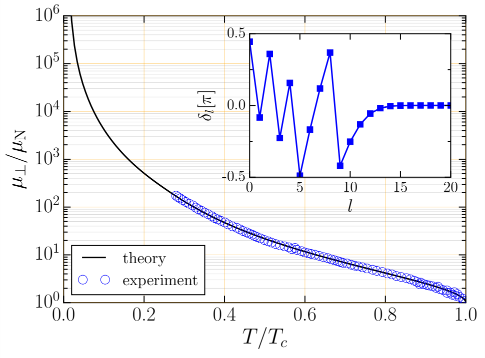
$$|F_y/F_x| \approx \frac{\hbar}{p_f R}\,(\Delta(T)/k_B T_c)^2$$

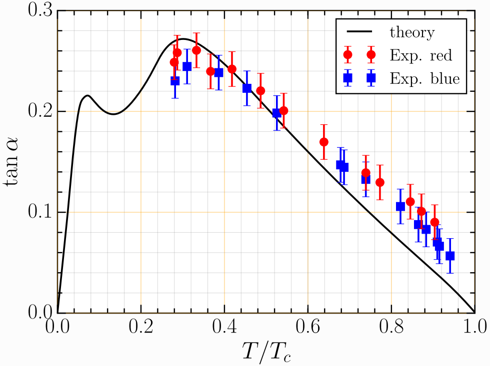
$k_f R = 11.17$

Branch Conversion Scattering in a Chiral Condensate

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Comparison between Theory and Experiment for the Drag and Transverse Forces





- $\mu_\perp = e\,\dfrac{\eta_\perp}{\eta_\perp^2 + \eta_{\text{AH}}^2}$
- $\mu_{\text{AH}} = -e\,\dfrac{\eta_{\text{AH}}}{\eta_\perp^2 + \eta_{\text{AH}}^2}$

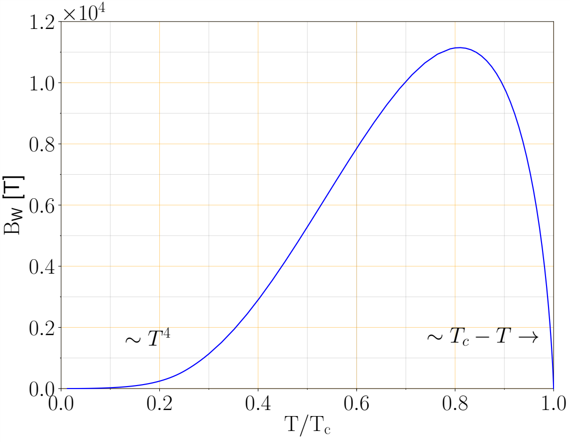
- $\tan\alpha = \left|\dfrac{\mu_{\text{AH}}}{\mu_\perp}\right| = \dfrac{\eta_{\text{AH}}}{\eta_\perp}$
- Ionic Radius of the e^- Bubble: $k_f R = 11.17$

▶O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

▶O. Shevtsov and JAS, JLTP 187, 340353 (2017)

Vanishing of the Effective Magnetic Field for $T \to 0$

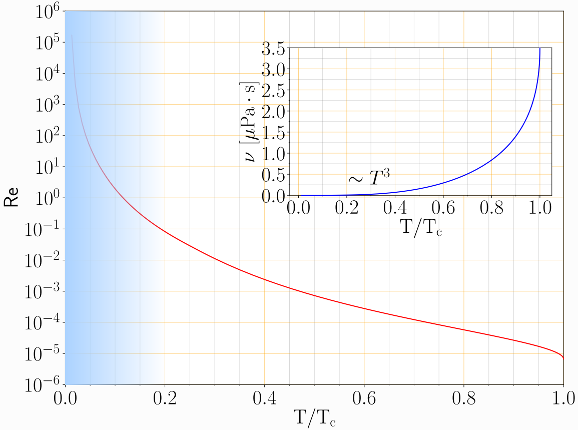
$$B_{\mathsf{W}} = 5.9 \times 10^5\ \mathsf{T} \left(\frac{\eta_{xy}}{\eta_{\mathsf{N}}}\right)$$



$$\eta_{xy}/\eta_{\mathsf{N}}|_{T=0.8\,T_c} \approx \frac{\hbar}{p_f R}$$

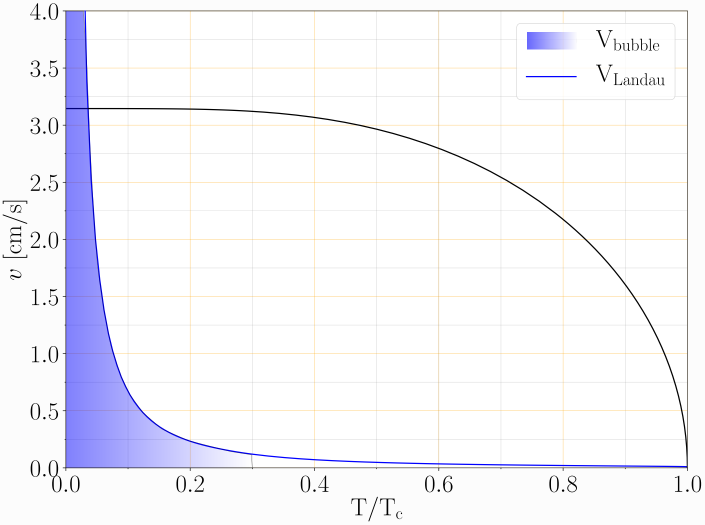
Breakdown of Laminar Flow

$$\mathsf{Re} = \mathsf{Re}_{\mathsf{N}} \left(\frac{\eta_{\mathsf{N}}}{\eta}\right)^{3/2} \xrightarrow[T\to 0]{} \sim \left(\frac{T_c}{T}\right)^{9/2}$$



$$\mathsf{Re}_{\mathsf{N}} = 6.7 \times 10^{-6}$$

Breakdown of Scattering Theory for $T \to 0$



Electron Bubble Velocity

- $V_{\mathsf{N}} = \mu_{\mathsf{N}} E_{\mathsf{N}} = 1.01 \times 10^{-4}$ m/s
- $V = \mu_{\mathsf{N}} E_{\mathsf{N}} \sqrt{\frac{\eta_{\mathsf{N}}}{\eta}}$

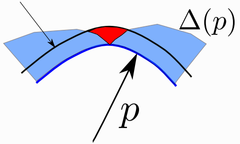
Maximum Landau critical velocity

- $V_c^{\mathsf{max}} \approx 155 \times 10^{-4}$ m/s $\frac{\Delta_{\mathsf{A}}(T)}{k_b T_c}$

Nodal Superfluids:

- $V_c = \Delta(p)/p_f \to 0$ for $p \to p_{\mathsf{node}}$

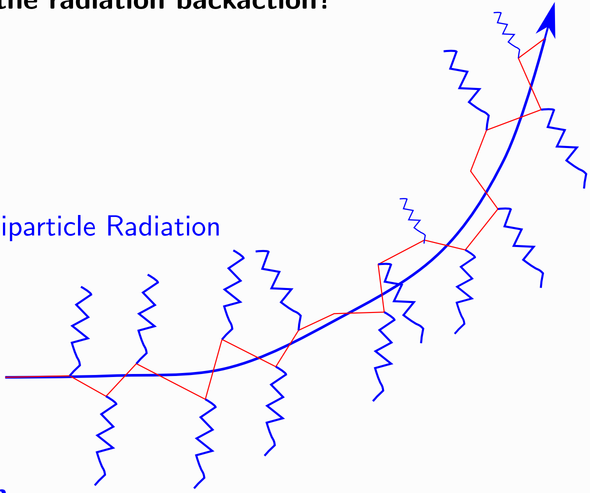
$$\varepsilon_{\mathrm{Doppler}} = p_f V$$



- Radiation Dominated Damping for $T \lesssim 0.1 T_c$

Radiation Damping - Pair-Breaking at $T \to 0$

Is their a transverse component of the radiation backaction?

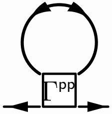


Fluctuations of the Chiral Vacuum

- Mesoscopic Ion coupled and driven through a Chiral "Bath"

Leading Order Pairing Self-Energy

$\Delta_{\alpha\beta}(\mathbf{p}, \varepsilon_n) =$ $+\mathbf{p}, \varepsilon_n, \alpha$



$-\mathbf{p}, -\varepsilon_n, \beta$ $= -T \sum_{\varepsilon'_n} \int \frac{d^3p'}{(2\pi)^3} \Gamma^{\mathsf{pp}}_{\alpha\beta;\gamma\rho}(\mathbf{p}, \mathbf{p}'; \varepsilon_n - \varepsilon'_n) \, F_{\gamma\rho}(\mathbf{p}', \varepsilon'_n)$

$$\Gamma^{\mathsf{pp}}_{\alpha\beta;\gamma\rho}(p, p') = \overbrace{\Gamma^{(0)}(p, p')(i\sigma_y)_{\alpha\beta}(i\sigma_y)_{\gamma\rho}}^{\text{Even Parity, Spin Singlet}} + \overbrace{\Gamma^{(1)}(p, p') \, (i\vec{\sigma}\sigma_y)_{\alpha\beta} \cdot (i\sigma_y\vec{\sigma})_{\gamma\rho}}^{\text{Odd Parity, Spin Triplet}} \, ,$$

▶ Dominant Pairing Channel: $S = 1$ and $L = 1$:

$$2N(0)\Gamma^{(1)}(p, p') \rightsquigarrow -g_1(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}') \, \chi(\varepsilon_n - \varepsilon'_n)$$

Retarded Spin-Fluctuation Mediated Interaction in the Cooper Channel

- Spin-fluctuation-mediated interaction

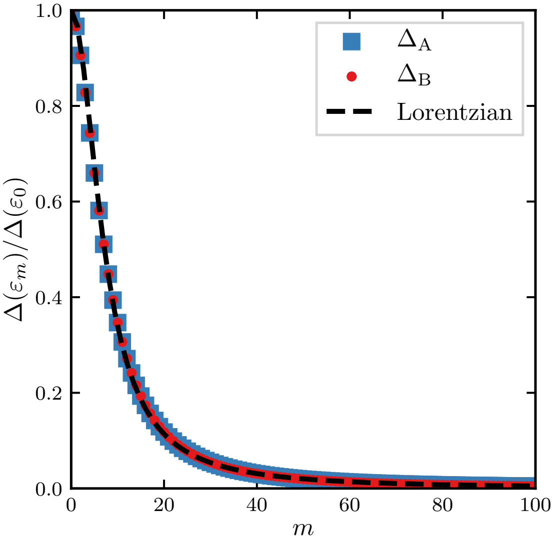
$$\Gamma_{pp} = -g_1\,(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\,\chi(\varepsilon_n - \varepsilon_{n'})$$
$$\chi = \frac{\omega_{\mathsf{sf}}^2}{[(\varepsilon_n - \varepsilon_{n'})^2 + \omega_{\mathsf{sf}}^2]} \qquad (1)$$

- Self-Consistent Pairing Self Energy

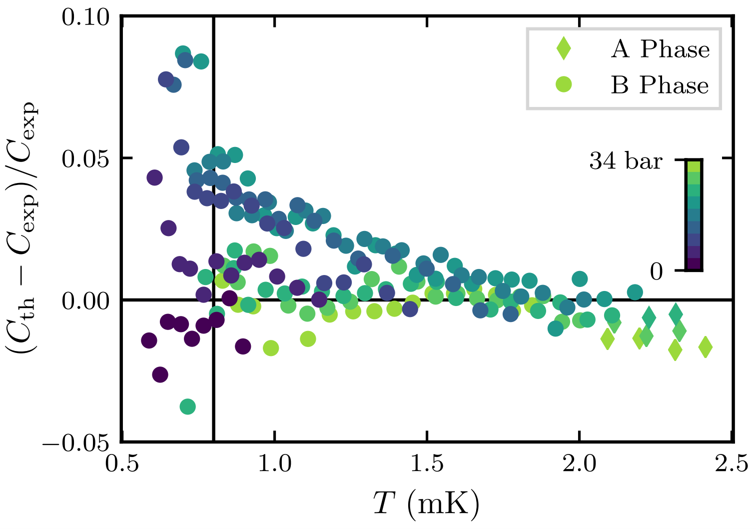
$$\Delta_{\alpha\beta}(\hat{\mathbf{p}};\varepsilon_m) \approx \Delta_{\alpha\beta}(\hat{\mathbf{p}}) \times \chi(\varepsilon_m)$$

- $\omega_{\mathsf{sf}} = x_{\mathsf{sf}}\,(1 + F_0^a(p))\,E_F(p)$
  $\rightsquigarrow$ pairing bandwidth

- Fit to $T_c(p) \rightsquigarrow x_{\mathsf{sf}} \approx 0.4\ \forall$ pressures



Strong-Coupling Results - Heat Capacity: $C(T) = -T\partial^2\Omega(p,T)/\partial T^2$



- Data: D. S. Greywall. Phys. Rev. B, 33(11):7520, 1986