Dimension transcendence and anomalous charge transport in magnets with *moving* multiple-Q spin textures

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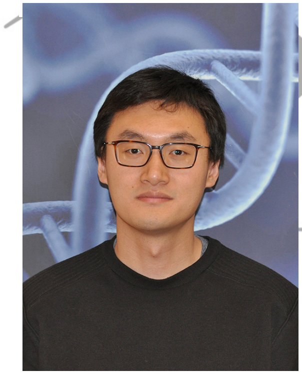
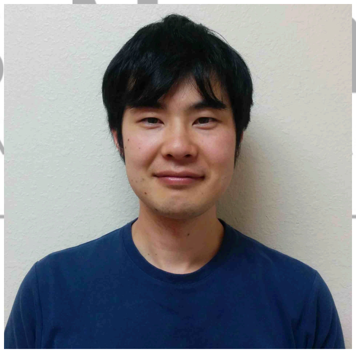
May 2, 2019@Santa Fe

Collaborators



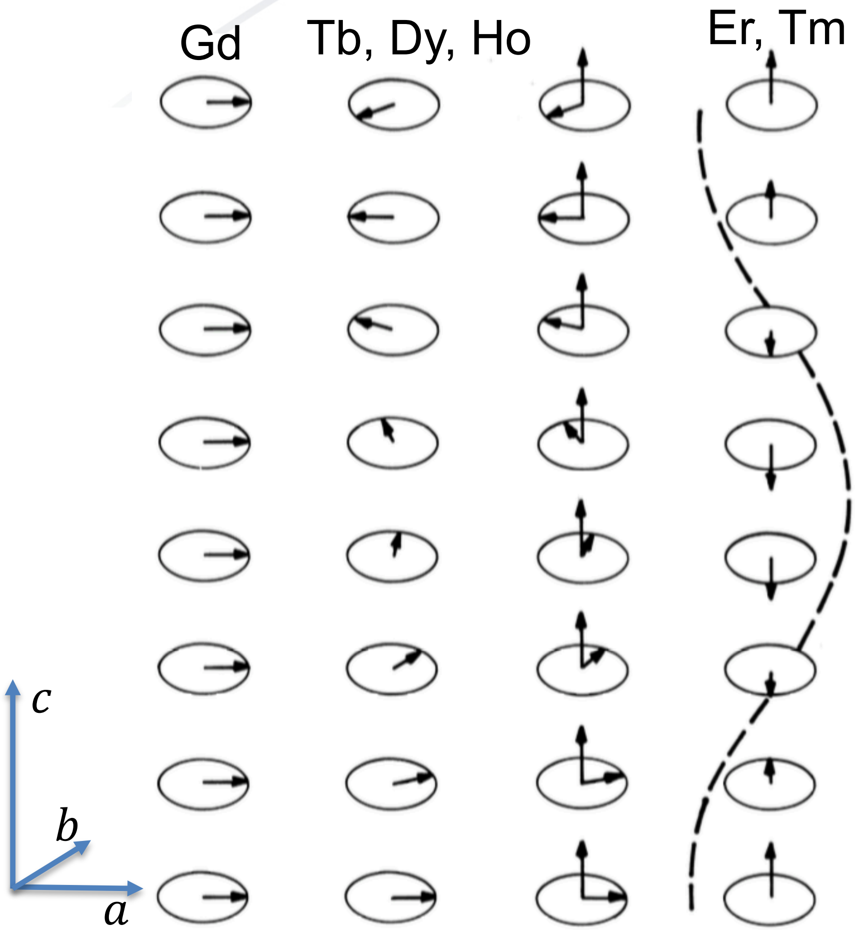
Ying Su



Satoru Hayami

Single-Q magnetic helix/spiral

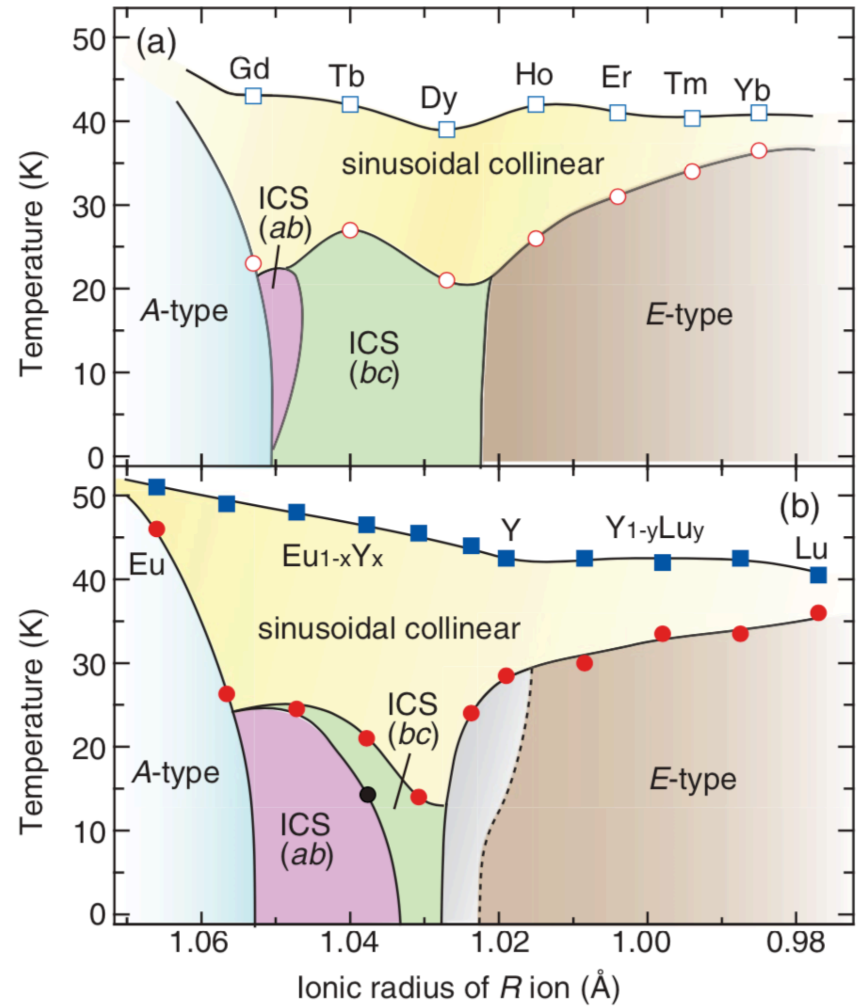
Rare earth magnets: Magnetic structure near T_N



At low temperature   Er, Ho

Jensen and Makintosh, <<Rare Earth Magnetism: Structures and Excitations>>

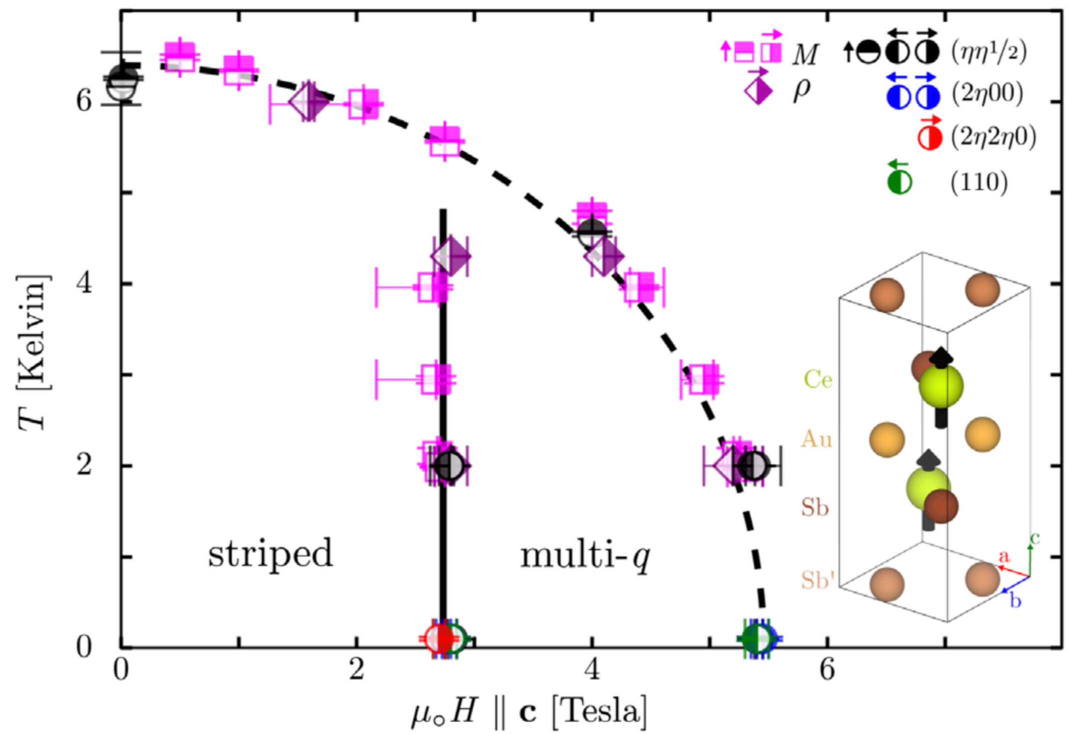
manganese perovskites RMnO$_3$

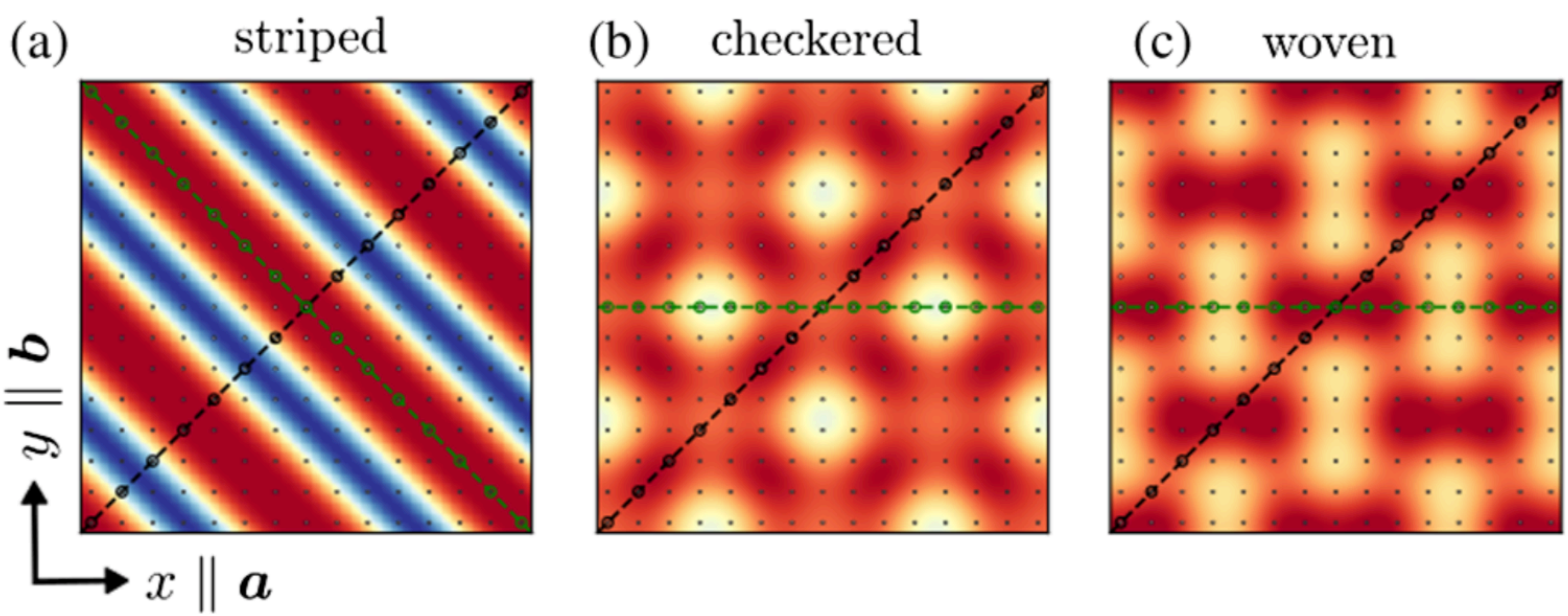


Mochizuki et al., PRB (2011)

Double-Q state (magnetic bubble lattice) in $CeAuSb_2$

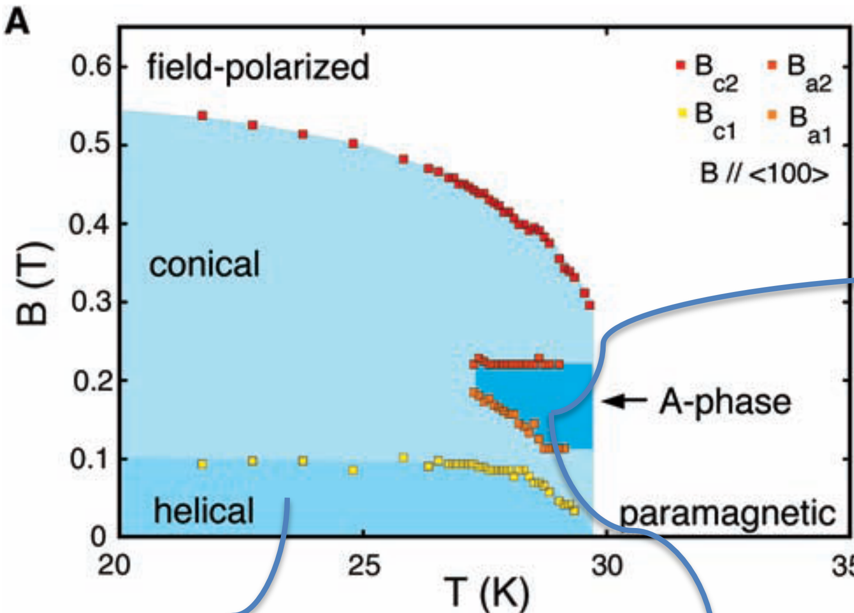
Marcus et al. PRL (2018)

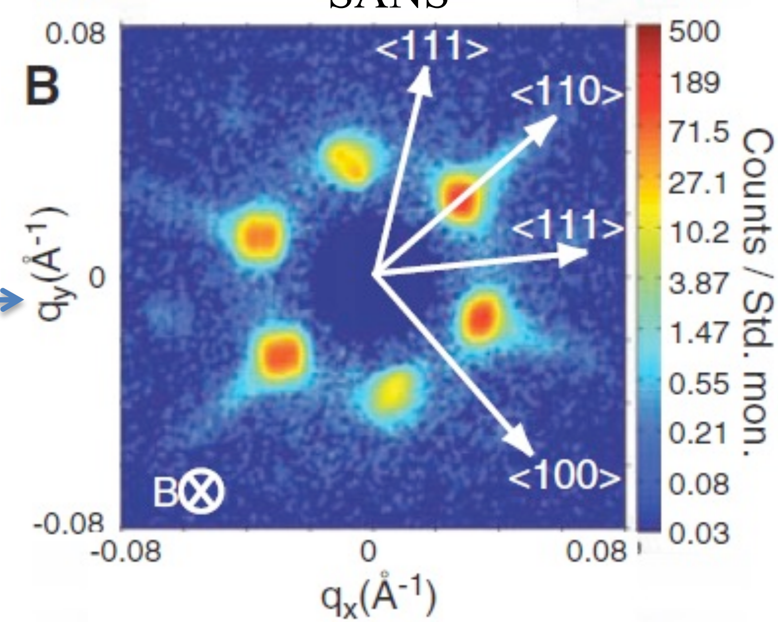


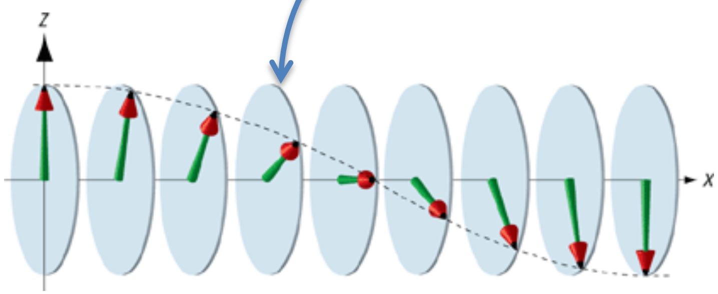


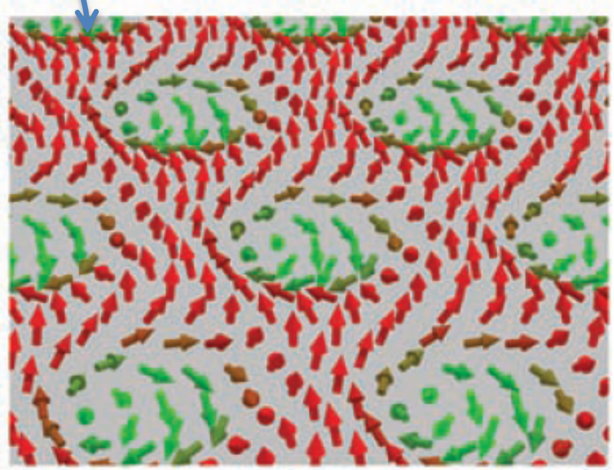
Helix and skyrmions in bulk materials

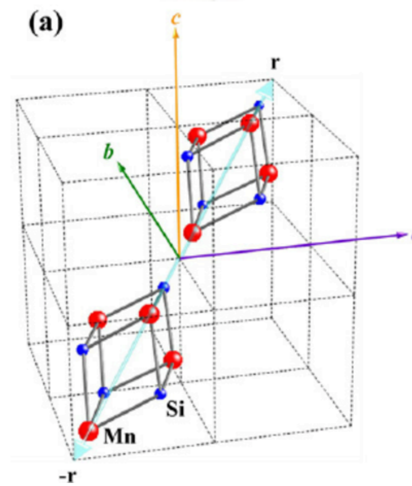
SANS







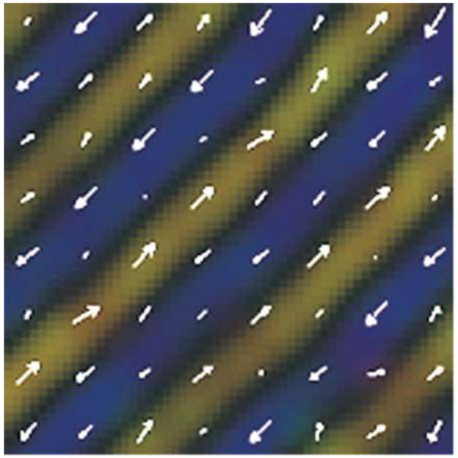




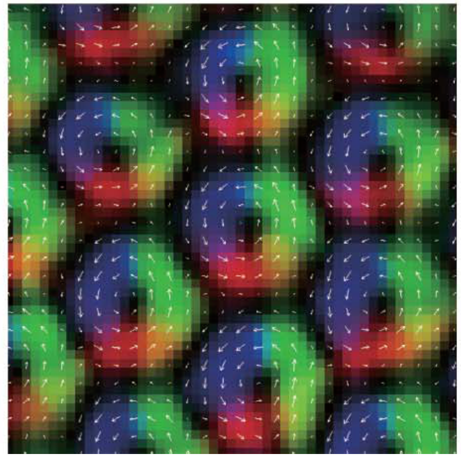
Bulk chiral magnet: MnSi

S. Mühlbauer, et al. Science 323, 915 (2009).

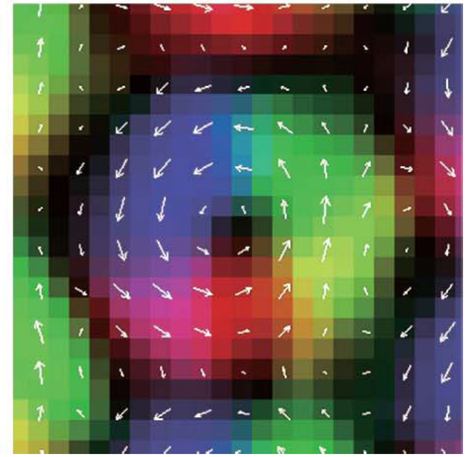
Helix and skyrmions in thin films



Helical



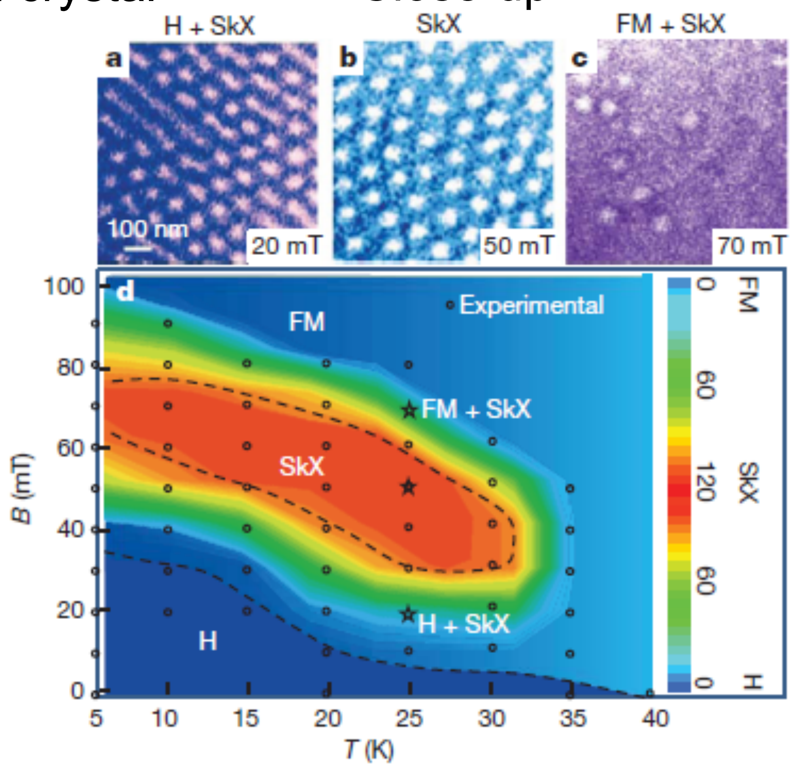
Skyrmion crystal



Close-up

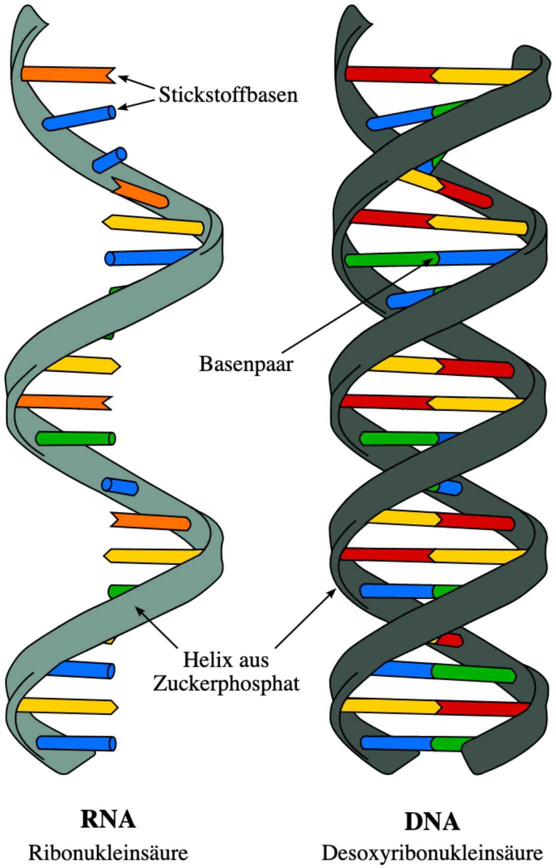
- Lorentz transmission electron microscopy; measure the in-plane component of spin.
- Materials: $Fe_{0.5}Co_{0.5}Si$

X. Z. Yu et al. Nature 465, 901 (2010).

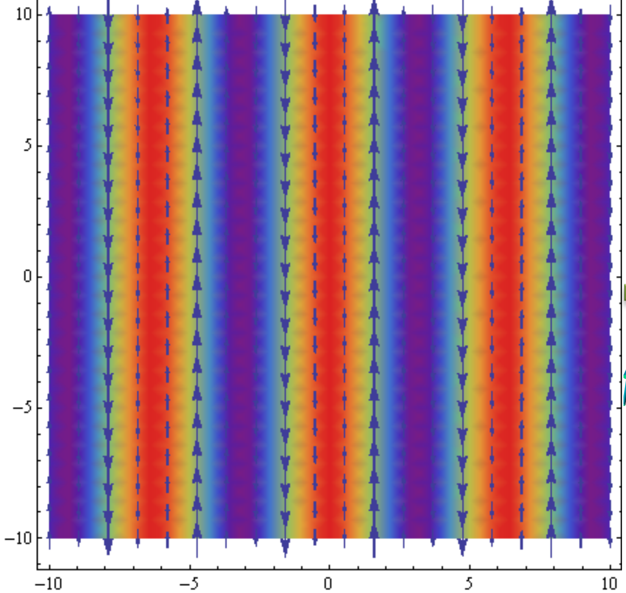


Multiple-Q spin texture

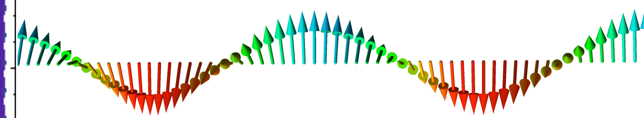
Building block of life

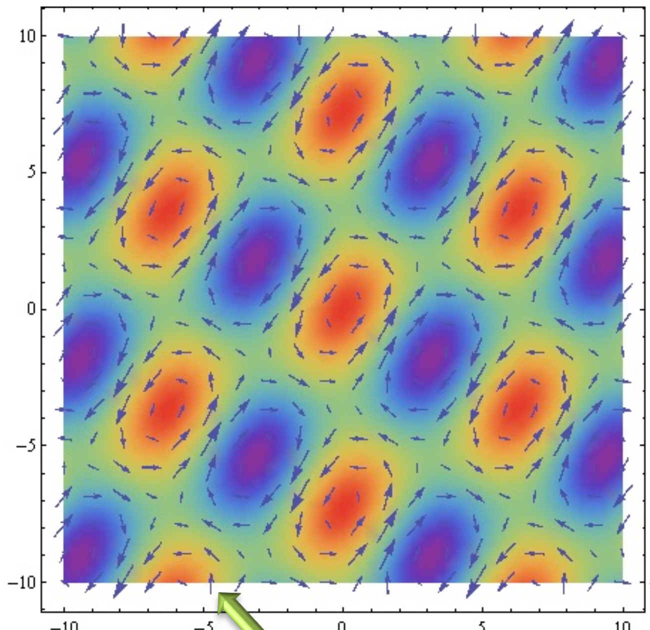


Single-Q helix vs double-Q helix

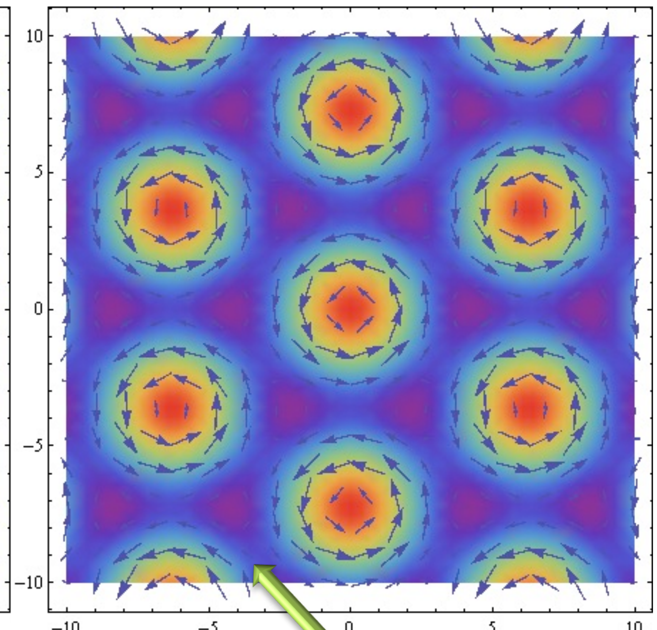


1Q state (helix or spiral)



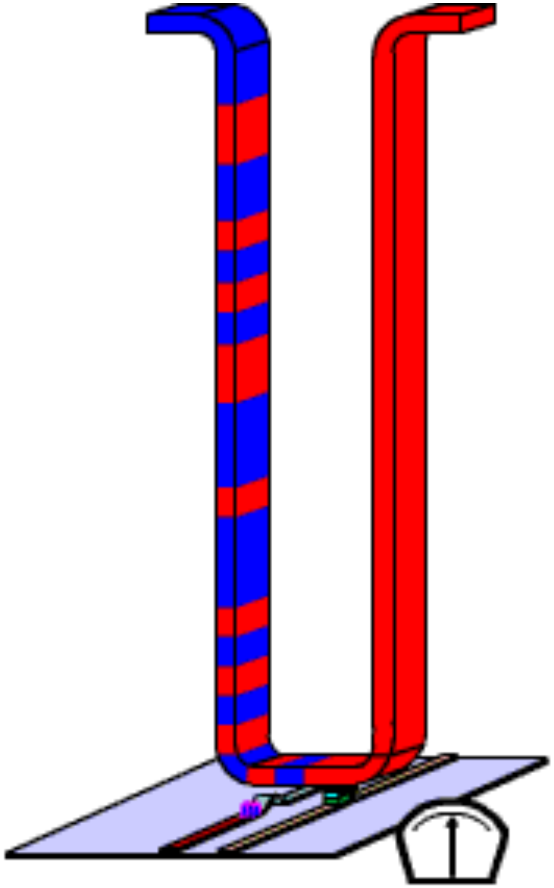


2Q state (vortex lattice)

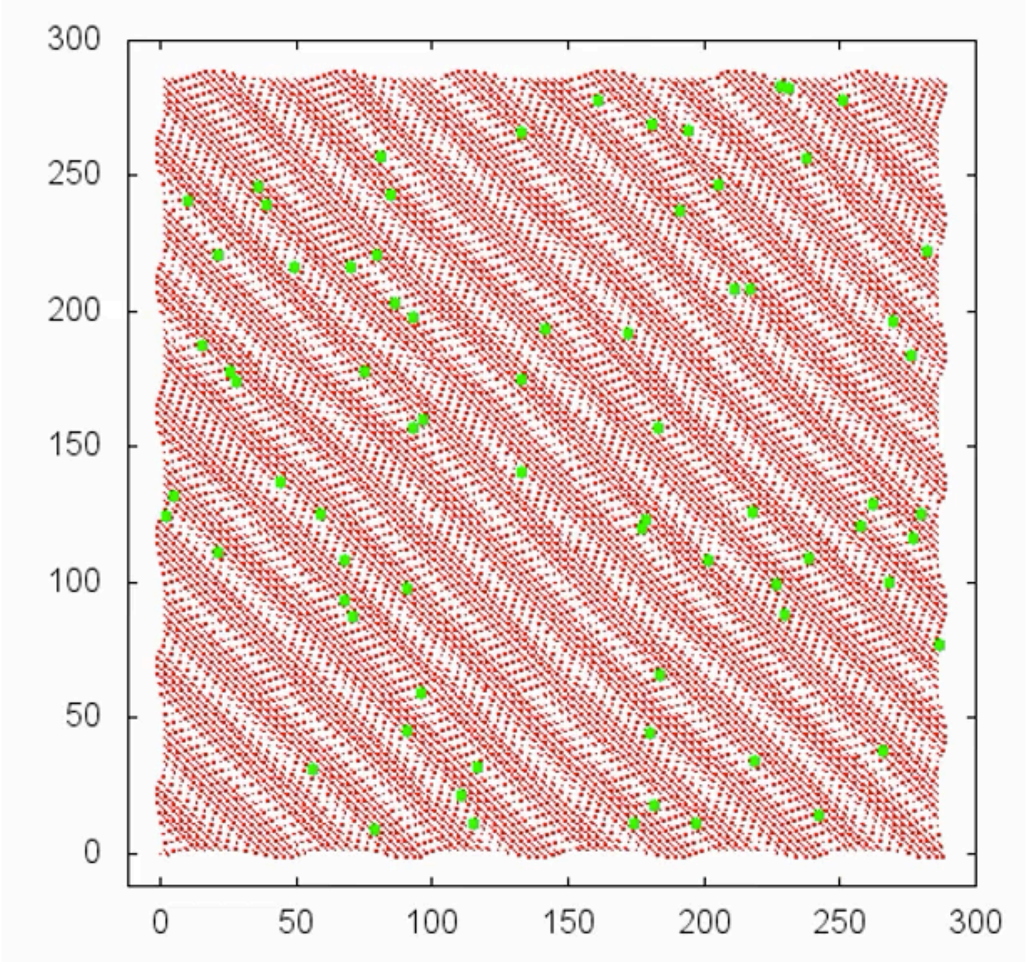


3Q state (skyrmion lattice)

Current driven domain wall motion



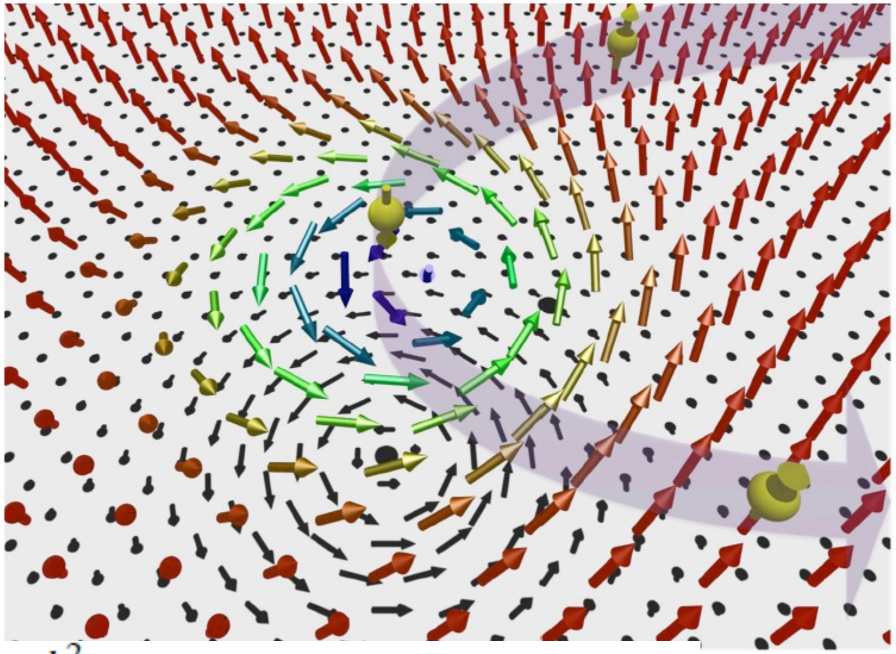
Concept of racetrack memory



Current driven motion of helix

Iwasaki et al., Nature Communications (2013)

Effect of *static* (non-coplanar) spin textures on electronic state



- Band folding
- Open a gap
- Berry phase due to the non-coplanar spin texture

  - Conduction electrons Hamiltonian

$$S_{\mathrm{el}} = \int dt\, dx^3 [i\hbar \psi^\dagger \dot{\psi} - \mathcal{H}] \text{ with } \mathcal{H} = \frac{\hbar^2}{2m} \nabla\psi^\dagger \nabla\psi - J_H S \psi^\dagger \boldsymbol{\sigma}\cdot\mathbf{n}\,\psi$$

  - Large Hund's coupling (adiabatic limit) $J_H >> E_F$

$$S_{el} = \int \mathrm{d}r^3 \mathrm{d}t [i\hbar\chi^\dagger \dot{\chi} + eA_0 - \frac{1}{2m}[(i\hbar\nabla - \frac{e}{c}\mathbf{A}^*)\chi^\dagger][(-i\hbar\nabla - \frac{e}{c}\mathbf{A})\chi] - \frac{\hbar^2}{8m}(\nabla\mathbf{n})^2 + J_H S$$

  - Emergent electromagnetic fields

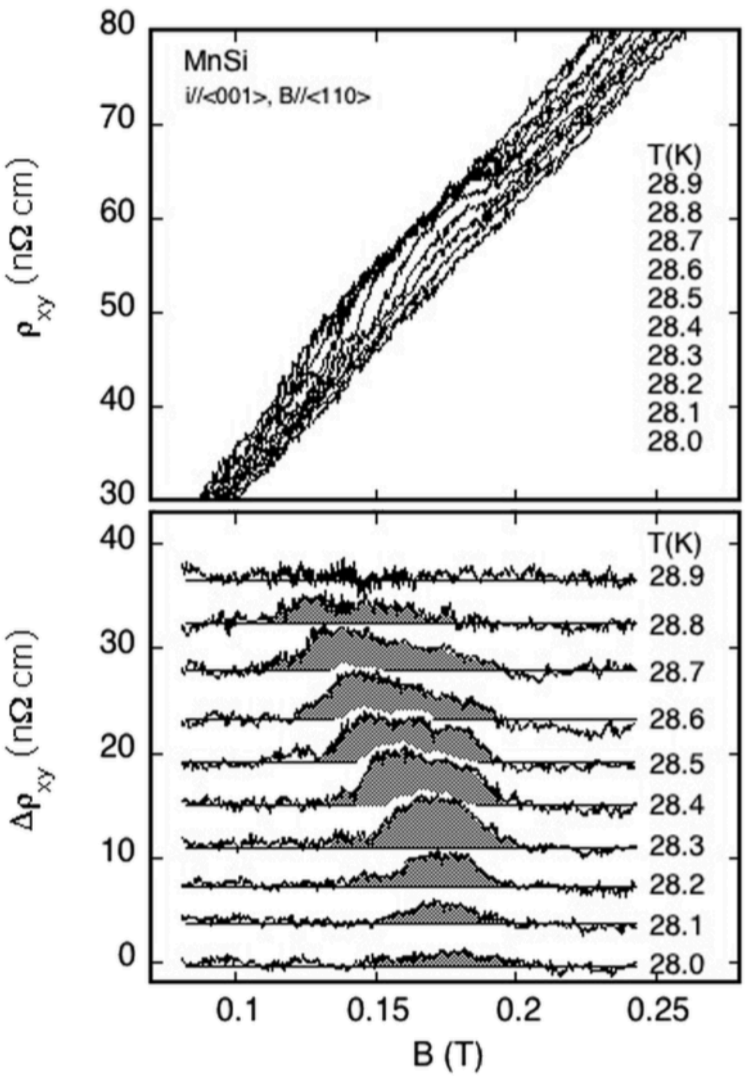
$$(\nabla\times\mathbf{A})_z = B_z = \frac{\hbar c}{2e}[\mathbf{n}\cdot(\partial_x \mathbf{n}\times\partial_y \mathbf{n})],\ \nabla A_0 = \mathbf{E} = \frac{\hbar}{2e}[\mathbf{n}\cdot(\nabla\mathbf{n}\times\partial_t \mathbf{n})]$$

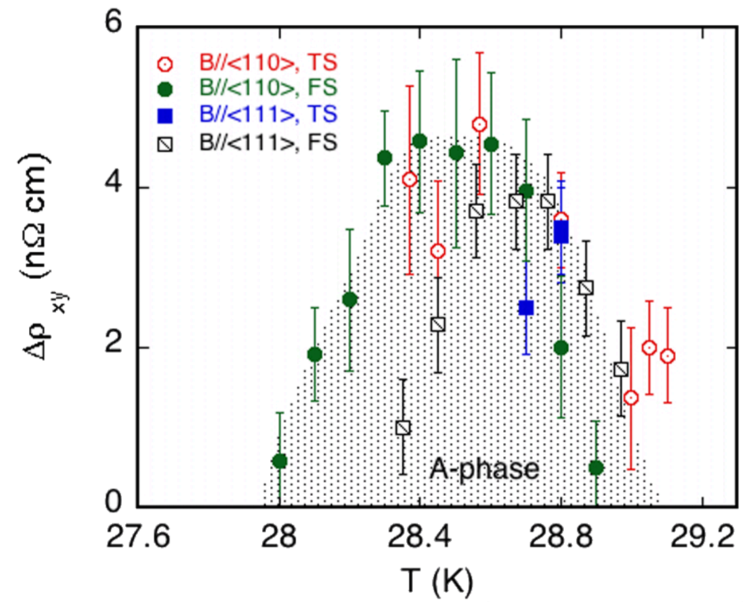
  - For skyrmion size 10 nm, B ~ 100 T !!!
  - Longitudinal and topological Hall conductivity

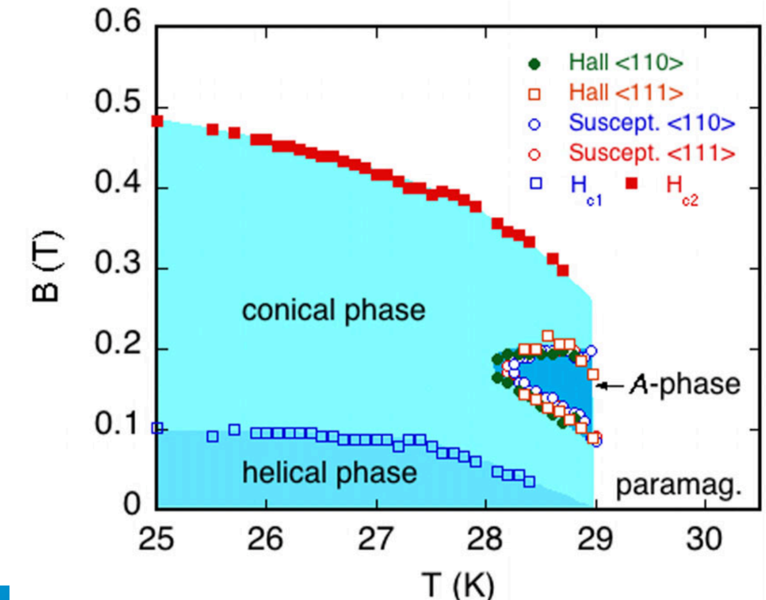
$$\sigma_{\parallel} = \frac{e^2\rho_n\tau}{m}\frac{1}{1+(\omega_c\tau)^2},\ \sigma_{\perp} = \frac{e^2\rho_n\tau}{m}\frac{\omega_c\tau}{1+(\omega_c\tau)^2} \text{ with } \omega_c = eB_z/mc$$

Topological Hall effect induced by spin texture

Neubauer et al., PRL 102, 186602 (2009)







Question: What is the effect of *moving* spin textures on the electronic wave functions?

- The motion of the spin texture is described by dynamics of the phason mode $\phi(r,t)$ of the spin texture lattice.
- Sine-Gordon equation

$$\partial_x^2\phi - \partial_t^2\phi - \sin\phi = F_d$$

- The dynamics of motion of spin texture is slow compared to the dynamics of electrons. For instance, spin texture with lattice parameter $a = 1\ \mathrm{nm}$ moving at a velocity $1\ \mathrm{m/s}$, the frequency $\omega = 1\ \mathrm{GHz}$. Therefore we can use adiabatic approximation.

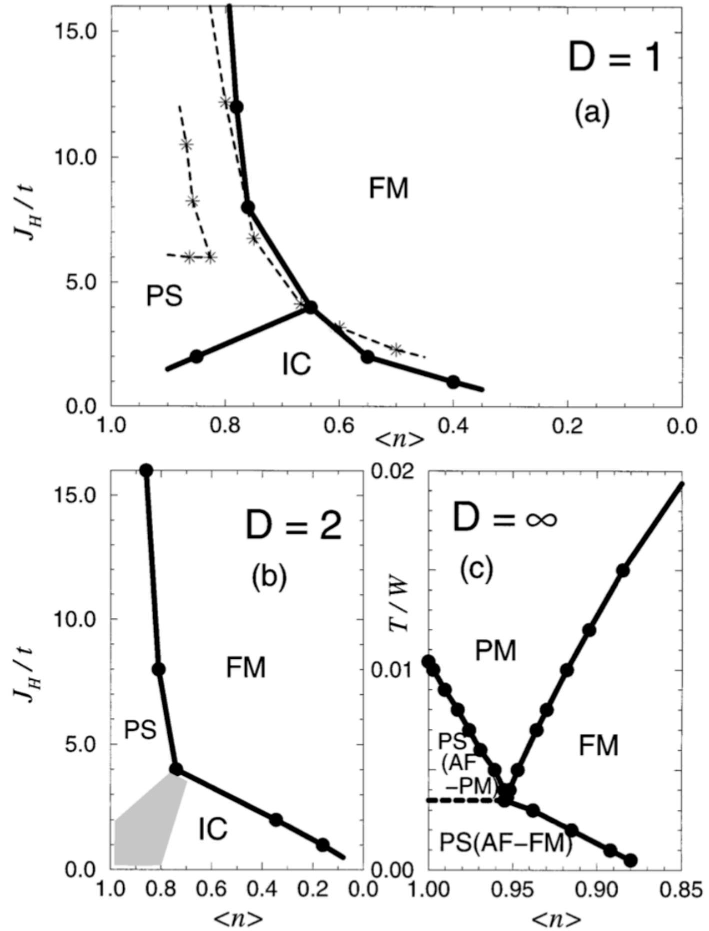
The ferromagnetic Kondo lattice/double exchange model

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j - J \sum_i c_i^\dagger \mathbf{S}_i \cdot \boldsymbol{\sigma} c_i,$$

- The magnetic order S_i can be derived for a given filling.
- Here we assume S_i are determined by other magnetic interactions specified by $\mathcal{F}(S_i)$.
- We consider the classical limit $|S_i|>>1$ and one dimension system.
- For ease of discussion, we parametrize the spin texture by

$$\mathbf{S} = (0, b\sin(Q\, i + \phi), \cos(Q\, i + \phi))$$

Elliptical helix

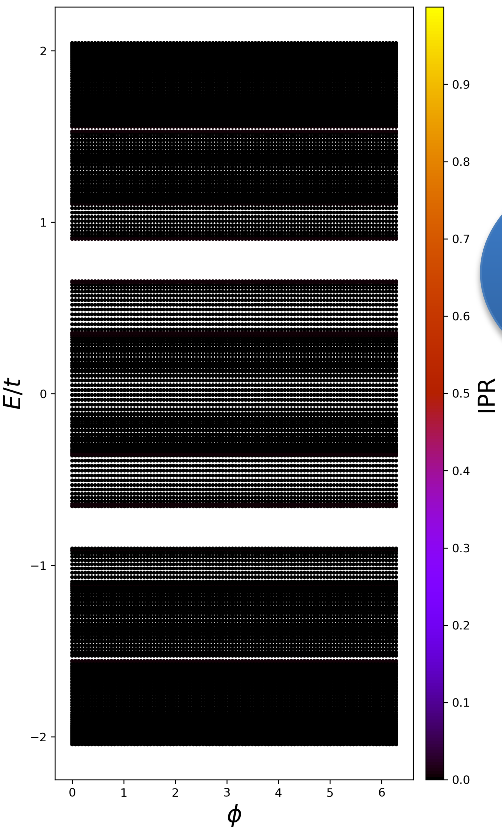


Phase diagram from self-consistent calculations

Yunoki et al., PRL (1998)

1D model

- **For incommensurate helix, irrational Q, ϕ is a Goldstone mode corresponding to the translation of the helix.**
- **How does the spectrum depend on ϕ?**



PBC

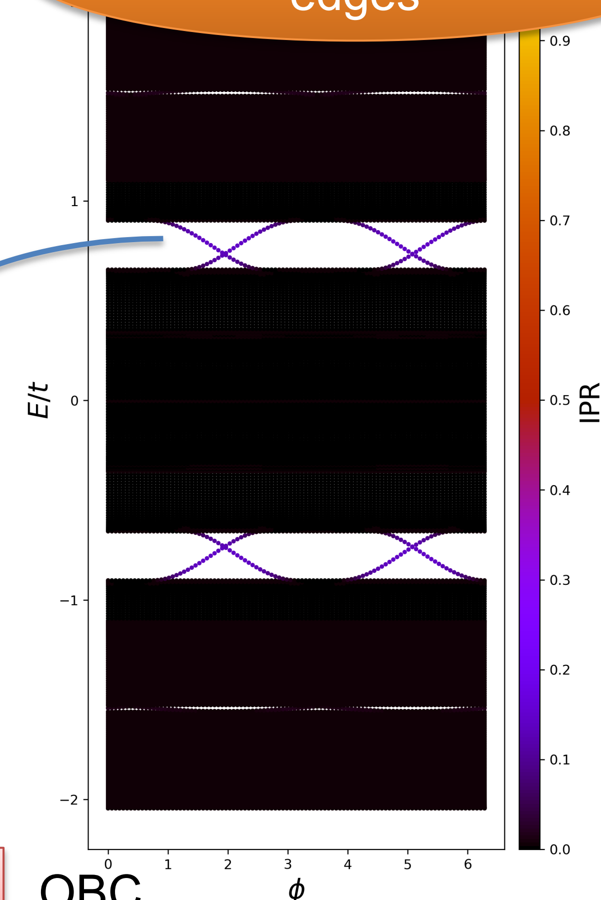
The spectra do not change

Nevertheless, it is true only for the bulk spectra

Spectrum depends on ϕ, $E(k,\phi)$

The spectra with edges

non-trivial topology



OBC

Connection to the integer Quantum Hall effect

When b=0, the KLM is the same as the Hofstadter model (with two fold degeneracy), which describes the IQHE.

Landau gauge $A = (0, B\ i\ a\ , 0)$

$$\mathcal{H} = t c^{+}_{i+1,j} c_{i,j} + t \exp(\frac{ie}{\hbar c} B\ i\ a^2\ ) c^{+}_{i,j+1} c_{i,j} + H.C.$$

Fourier transform in the y direction, $c_{i,j} \rightarrow c_{i,k_y} \exp(i\ k_y j a)$

$$\mathcal{H} = t c^{+}_{i+1,k_y} c_{i,k_y} + 2t \cos(Qi + k_y a) c^{+}_{i,k_y} c_{i,k_y} + H.C.$$

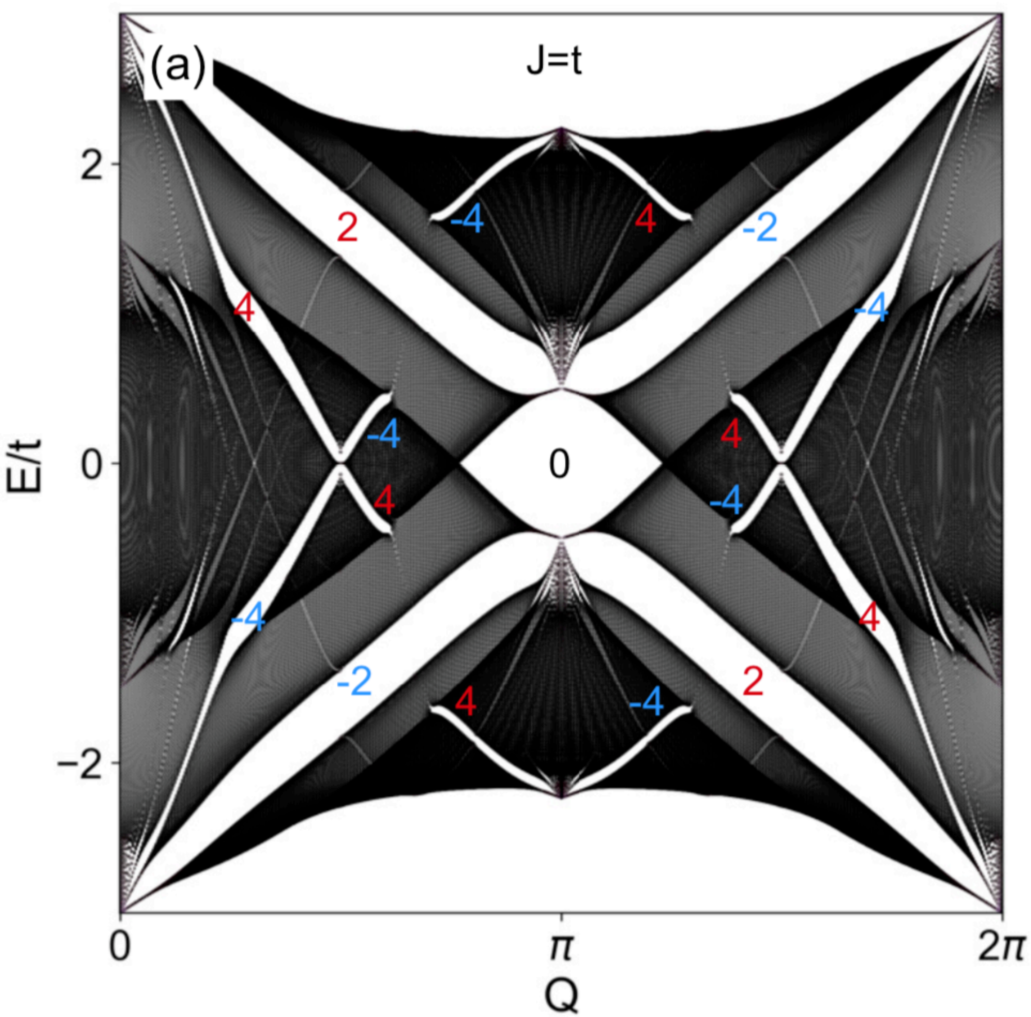
$$Q = 2\pi B a^2 e / hc$$

Corresponding to one-half of the KLM model with $\mathbf{S} = (0, 0, \cos(Q\ i + \phi))$

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} c_i^{\dagger} c_j - J \sum_i c_i^{\dagger} \mathbf{S}_i \cdot \boldsymbol{\sigma} c_i,$$

$J = 2t$ and $\phi = k_y a$

2Z Chern insulator in the k_x and ϕ space



Chern number

$$C = \frac{1}{2\pi i}\int_0^{2\pi} dk \int_0^{2\pi} d\phi \mathrm{Tr}\left(\mathcal{U}\left[\partial_k \mathcal{U}, \partial_\phi \mathcal{U}\right]\right),$$

with $\mathcal{U}(k,\phi) = \sum_{E_n<E_F} |\psi_n(k,\phi)\rangle\langle\psi_n(k,\phi)|$

Features:

(1) The spectrum is invariant when ϕ is shifted by π.

(2) All the Chern numbers are even.

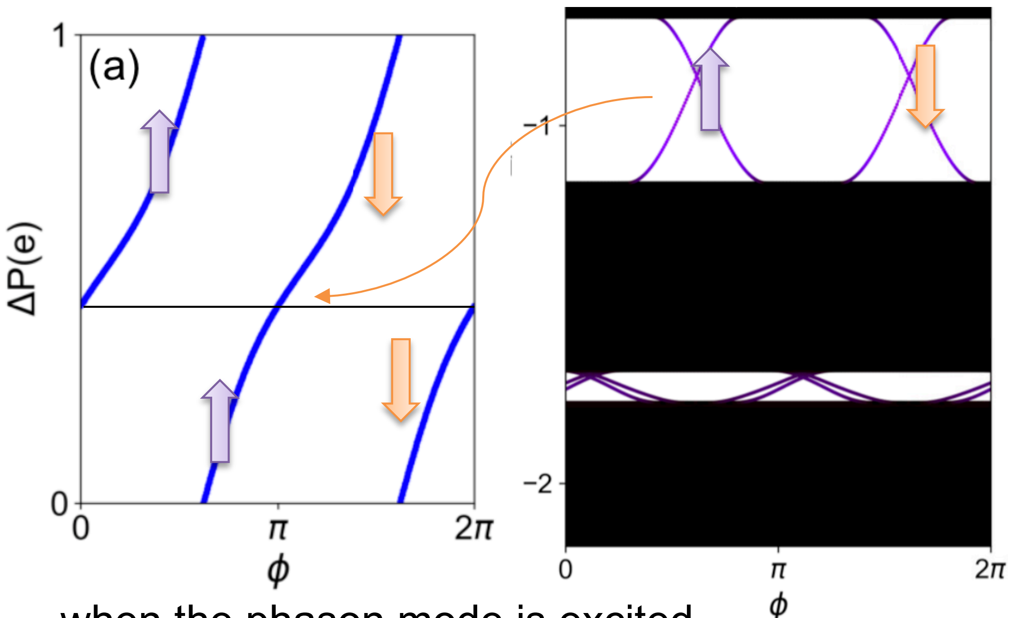
(3) The spectrum is symmetric at zero energy and $Q = \pi$.

(4) The Chern number changes sign with respect to $E = 0$ and $Q = \pi$.

Charge transport

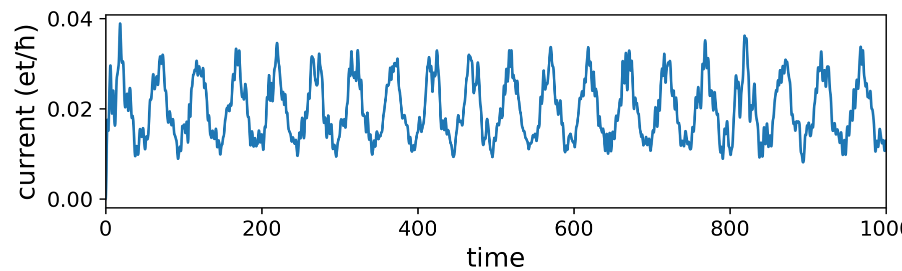
$$P = \langle w(x)|x|w(x)\rangle = \frac{i}{2\pi}\int_{BZ} dk^3 \langle u_k|\partial_k u_k\rangle$$

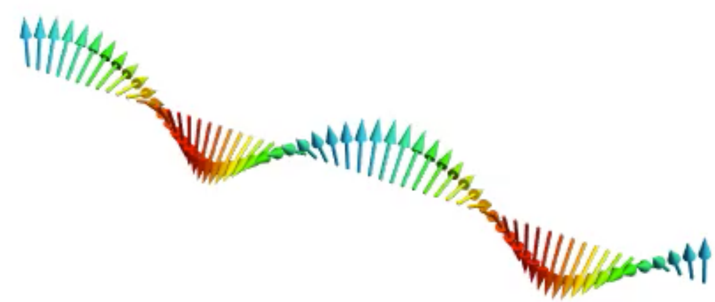
$$\Delta P(\phi) = P(\phi) - \min(P).$$



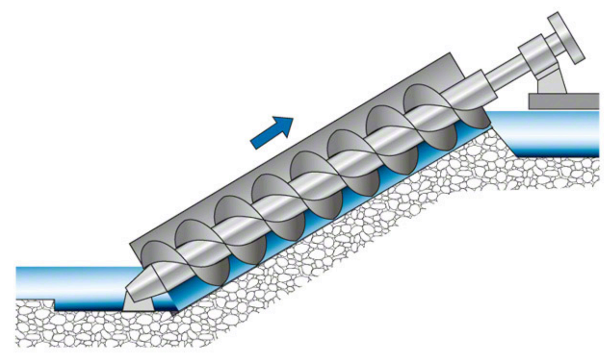
when the phason mode is excited.

Explicitly calculated current when $\phi = \omega t$





Magnetic version of Archimedes pump



The number of pump charge is just the Chern number

$$N = \int_0^{2\pi} d\phi \Delta P(\phi) = \frac{1}{2\pi}\oint \mathbf{A}\cdot dk = C$$

Thouless, PRB 27, 6083 (1983).

Dimension of the momentum space is increased

- In the presence of moving multiple-Q spin texture

$$\boldsymbol{S}_i = \sum_{\nu} A_{\nu}^{\mu}(\boldsymbol{Q}_{\nu} \cdot \boldsymbol{r}_i + \phi_{\nu})\hat{\boldsymbol{e}}_{\mu} \qquad \phi_{\nu} \rightarrow \phi_{\nu} + Q_{\nu} \cdot v_i t$$

- The electronic spectrum depends on crystal momentum $\boldsymbol{k}$ and ϕ_n, $E(\boldsymbol{k}, \phi_n) = E(k, \phi_n + 2\pi)$. The dimension of momentum space is increased.
- Semi-classical dynamics of electron wave packet with momentum $\tilde{k} = \boldsymbol{k} \oplus \phi_n$ and position $\boldsymbol{r}$

$$\dot{r}^{\mu}\left(\tilde{\boldsymbol{k}}\right) = \frac{\partial E_n\left(\tilde{\boldsymbol{k}}\right)}{\hbar \partial k_{\mu}} - \dot{\tilde{k}}_{\nu} \Omega_n^{\mu\nu}\left(\tilde{\boldsymbol{k}}\right),$$

where $\Omega^{\mu\nu}(\tilde{k})$ is the Berry curvature in the hybrid momentum space. Generally $\Omega^{\mu\nu}(\tilde{k}) \neq 0$ in the presence of spin textures

In the absence of physical EM fields

$$\dot{k} = 0$$

$$\dot{\phi}_{\nu} = \omega_{\nu}$$

Can be regarded as an effective electric field in the hybrid momentum space.

Electric current

- The current induced by the motion of the spin texture is

$$j_0^{\mu} = e\sum_n \int \frac{d^2k}{4\pi^2} f\left(E_n - E_F\right)\left(\frac{\partial E_n}{\hbar \partial k_\mu} - \omega_\nu \Omega_n^{\mu\nu}\right),$$

- When the Fermi energy is in the spectra gap, the transported charge

$$q_0^{\mu} = -\frac{eN^{\mu}}{|\omega_\nu|}\sum_{E_n \le E_F}\int_0^{2\pi} d\phi_\nu \int \frac{d^2k}{4\pi^2}\omega_\nu \Omega_n^{\mu\nu} = \frac{eN^{\mu}\omega_\nu C_1^{\mu\nu}}{|\omega_\nu|},$$

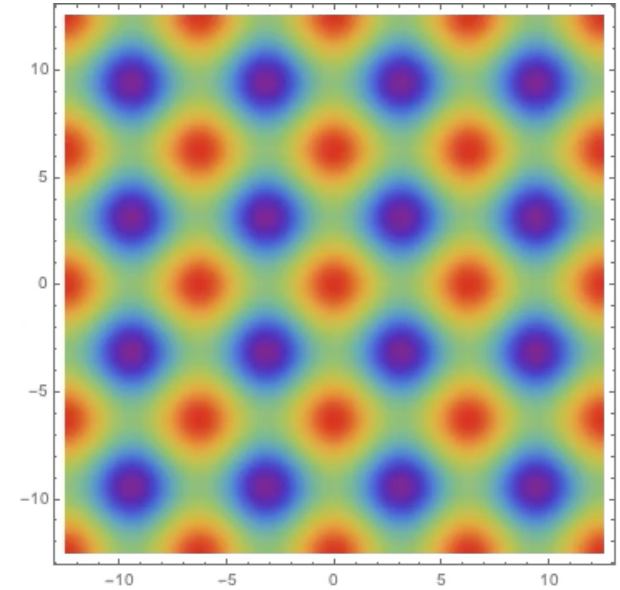
The first Chern number $C_1^{\mu\nu} = -\frac{1}{2\pi}\sum_{E_n \le E_F}\int dk_\mu d\phi_\nu \Omega_n^{\mu\nu},$

The translational motion of multiple-*Q* spin texture transports electric charge.

The dimension of hybrid momentum space $d = D + d_\phi$ can be $d \ge 4$. This allows for the higher topological order defined by the second Chern number. How?

A: We need to couple the dynamics of ϕ_ν to $\dot{r}$

Shearing the spin texture



- Consider double-Q spin texture (square lattice of magnetic bubble) and apply shear to the crystal, such that $Q_\nu \rightarrow Q_\nu + Q'_\nu$

$$\dot{\phi}_\nu = \omega_\nu + \boldsymbol{Q}'_\nu \cdot \dot{\boldsymbol{r}} \equiv \omega_\nu + \tilde{B}_{\nu\mu}\dot{r}^\mu,$$

- Shear strain generates an effective magnetic field $\tilde{B}_{\nu\mu}$ normal to the $\nu\mu$ plane.

- Equation of motion up to the second order in ω_ν and $\tilde{B}_{\nu\mu}$

$$\dot{r}^\mu = \frac{\partial E_n}{\hbar\partial k_\mu} - \omega_\nu\Omega_n^{\mu\nu} - \left(\frac{\partial E_n}{\hbar\partial k_\gamma} - \omega_\delta\Omega_n^{\gamma\delta} - \frac{\partial E_n}{\hbar\partial k_\eta}Q'_{\delta\eta}\Omega_n^{\gamma\delta}\right)Q'_{\nu\gamma}\Omega_n^{\mu\nu},$$

- The transported charge in one period

$$q^\mu = q_0^\mu + \frac{eL^\mu}{|\omega_\nu|}\int_0^{2\pi} d\phi_\nu \sum_{E_n\le E_F}\int \frac{d^2k}{4\pi^2}F_n^{\mu\nu\gamma\delta}\omega_\nu Q'_{\gamma\delta}$$

$$\approx q_0^\mu + \frac{eL^\mu C_2^{\mu\nu\gamma\delta}\omega_\nu Q'_{\gamma\delta}}{2\pi|\omega_\nu|}$$

second Chern number

$$C_2^{\mu\nu\gamma\delta} = \frac{1}{4\pi^2}\sum_{E_n\le E_F}\int dk_\mu d\phi_\nu dk_\gamma d\phi_\delta F_n^{\mu\nu\gamma\delta},$$

Mapping to the high-dimensional Hofstadter model

- For the collinear spin texture in physical 2D described by the ansatz

$$\boldsymbol{S}_i = \left(0, 0, \sum_{n=1}^{N} \cos(\boldsymbol{Q}_n \cdot \boldsymbol{r}_i + \phi_n)\right).$$

Double-Q state

$$Q_1 = \left(\frac{2\pi}{a}, 0\right), Q_2 = (0, \frac{2\pi}{a}),$$

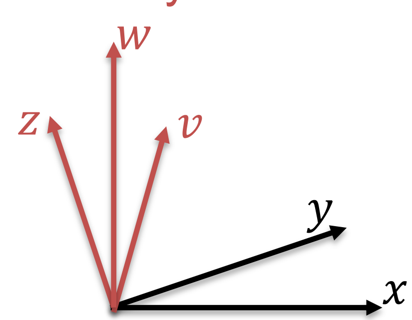
Triple-Q state

$$Q_1 = \left(-\frac{\pi}{a}, \frac{\sqrt{3}\pi}{a}\right), Q_2 = \left(-\frac{\pi}{a}, -\frac{\sqrt{3}\pi}{a}\right), Q_3 = \left(\frac{2\pi}{a}, 0\right)$$

The double-exchange model can be mapped to higher dimensional Hofstadter model

$$\mathcal{H}' = -t \sum_{\substack{x,y,k_z,\\ k_w,k_v}} \left(c^{\dagger}_{\substack{x+1,y,k_z,\\ k_w,k_v}} c_{\substack{x,y,k_z,\\ k_w,k_v}} + c^{\dagger}_{\substack{x,y+1,k_z,\\ k_w,k_v}} c_{\substack{x,y,k_z,\\ k_w,k_v}} + \text{H.c.} \right)$$

$$-2t \sum_{\substack{x,y,k_z,\\ k_w,k_v}} \begin{bmatrix} \cos(B_{xz}x + B_{yz}y + k_z) \\ + \cos(B_{xw}x + B_{yw}y + k_w) \\ + \cos(B_{xv}x + B_{yv}y + k_v) \end{bmatrix} c^{\dagger}_{\substack{x,y,k_z,\\ k_w,k_v}} c_{\substack{x,y,k_z,\\ k_w,k_v}},$$



with the correspondence

$$\tilde{B}_{xz} = Q_{1x}, \quad \tilde{B}_{yz} = Q_{1y}, \quad \tilde{B}_{xw} = Q_{2x},$$
$$\tilde{B}_{yw} = Q_{2y}, \quad \tilde{B}_{xv} = Q_{3x}, \quad \tilde{B}_{yv} = Q_{3y},$$

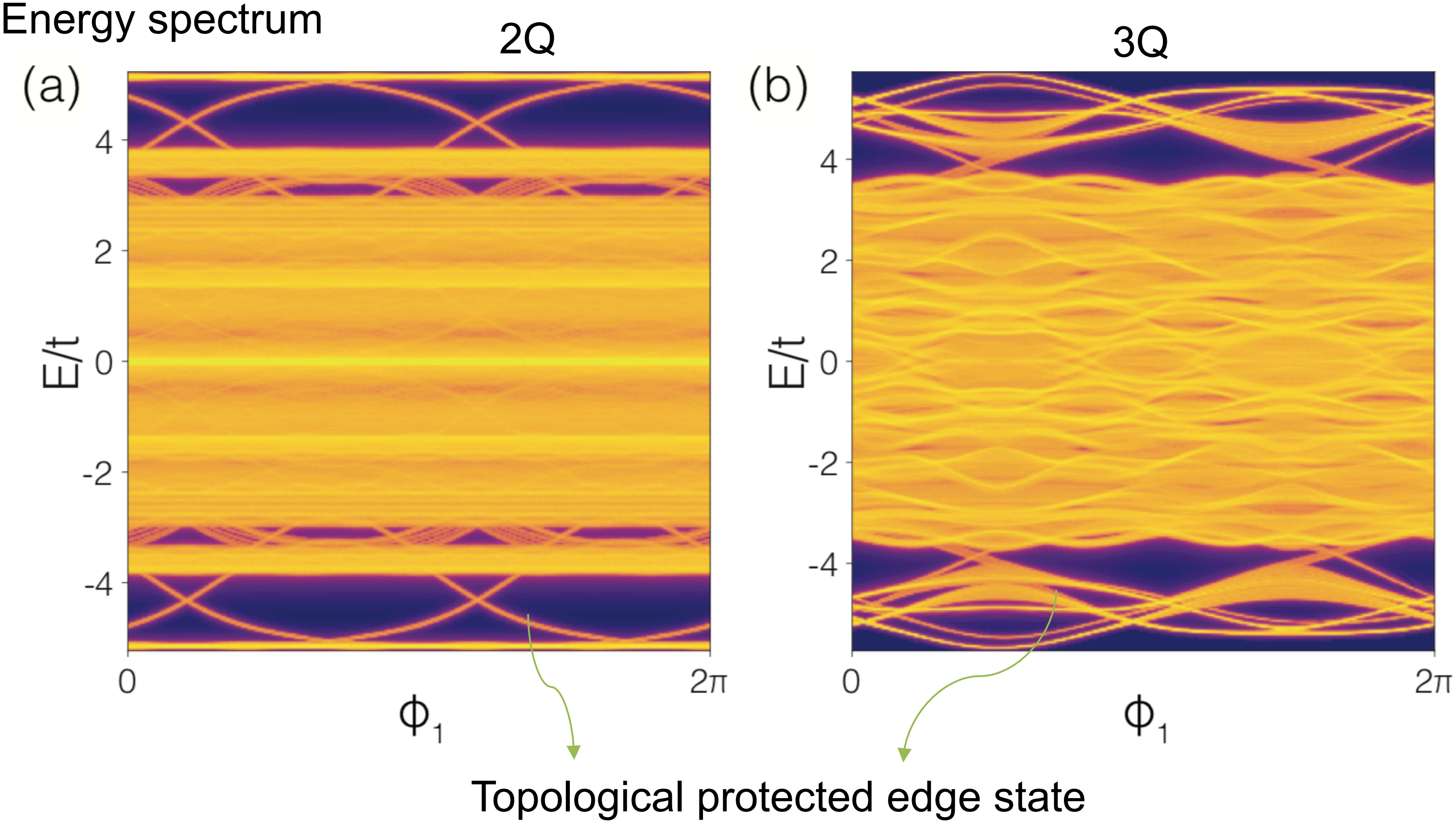
the effective crystal momenta

$$\tilde{k}_z = \phi_1, \quad \tilde{k}_w = \phi_2, \quad \tilde{k}_v = \phi_3,$$

Their time derivatives become effective electric field.

This exact mapping shows explicitly the dimension of the momentum space is increased.

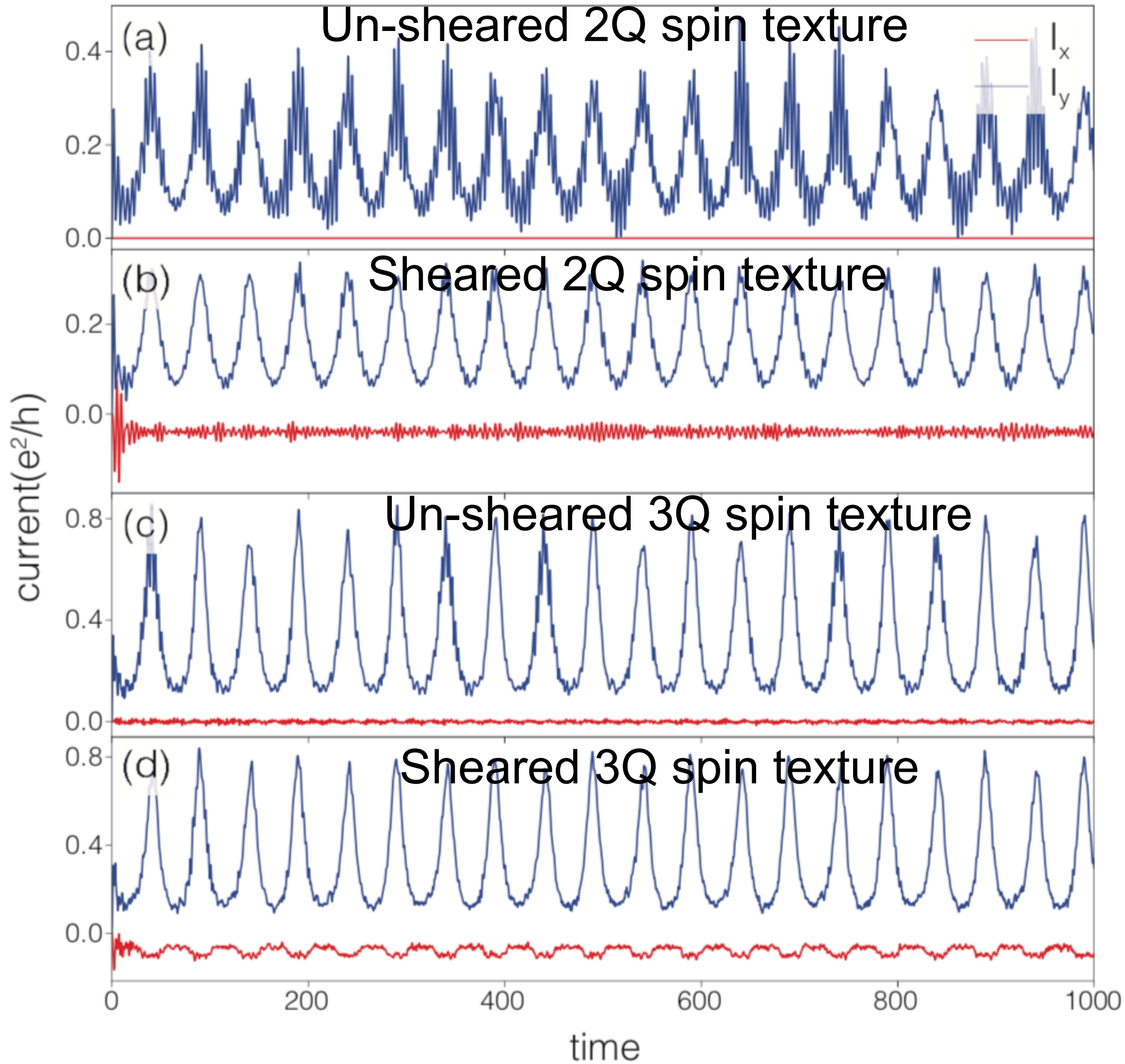
- Double-Q state maps to 4D and triple-Q maps to 5D systems, $\tilde{r} = (x, y, z, w, v)$.
- Energy spectrum



2Q state: $C_1^{xz} = C_1^{yw} = 2$, $C_2^{xyzw} = -2$ and others are zero.

3Q state: $C_1^{xz} = C_1^{xw} = -C_1^{yz} = C_1^{yw} = -2$, $C_2^{xyzw} = -2$ and others are zero.

Numerically calculated current induced by the motion of spin texture



Current response

$$j^{\mu} = -\frac{E_{\nu}e^2}{(2\pi)^2\hbar}\sum_{E_n\leq E_F}\int \Omega_n^{\mu\nu} d^2k + \frac{e^2}{(2\pi)^2\hbar} C_2^{\mu\nu\gamma\delta} E_{\nu} B'_{\gamma\delta},$$

The transported charge is consistent with the theory using the Chern number calculated from the Bloch wave function.

Realistic spin texture from spin Hamiltonian

Spin-Hamiltonian supporting multiple-Q state

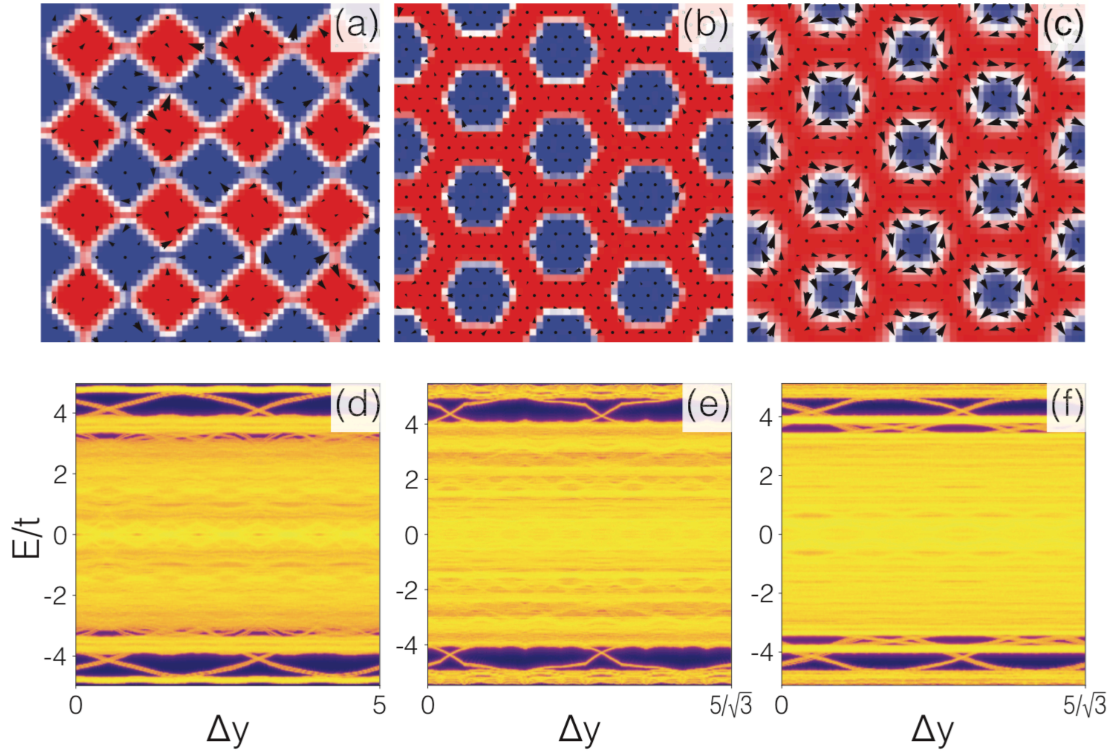
Hayami, Ying and SZL (2019)

Model 1

$$\mathcal{H} = 2\sum_{\nu}\Big[-\tilde{J}\left\{\alpha(S^x_{Q_\nu}S^x_{-Q_\nu} + S^y_{Q_\nu}S^y_{-Q_\nu}) + S^z_{Q_\nu}S^z_{-Q_\nu}\right\} + \tilde{K}\left\{\alpha(S^x_{Q_\nu}S^x_{-Q_\nu} + S^y_{Q_\nu}S^y_{-Q_\nu}) + S^z_{Q_\nu}S^z_{-Q_\nu}\right\}^2\Big] - A\sum_i (S^z_i)^2,$$

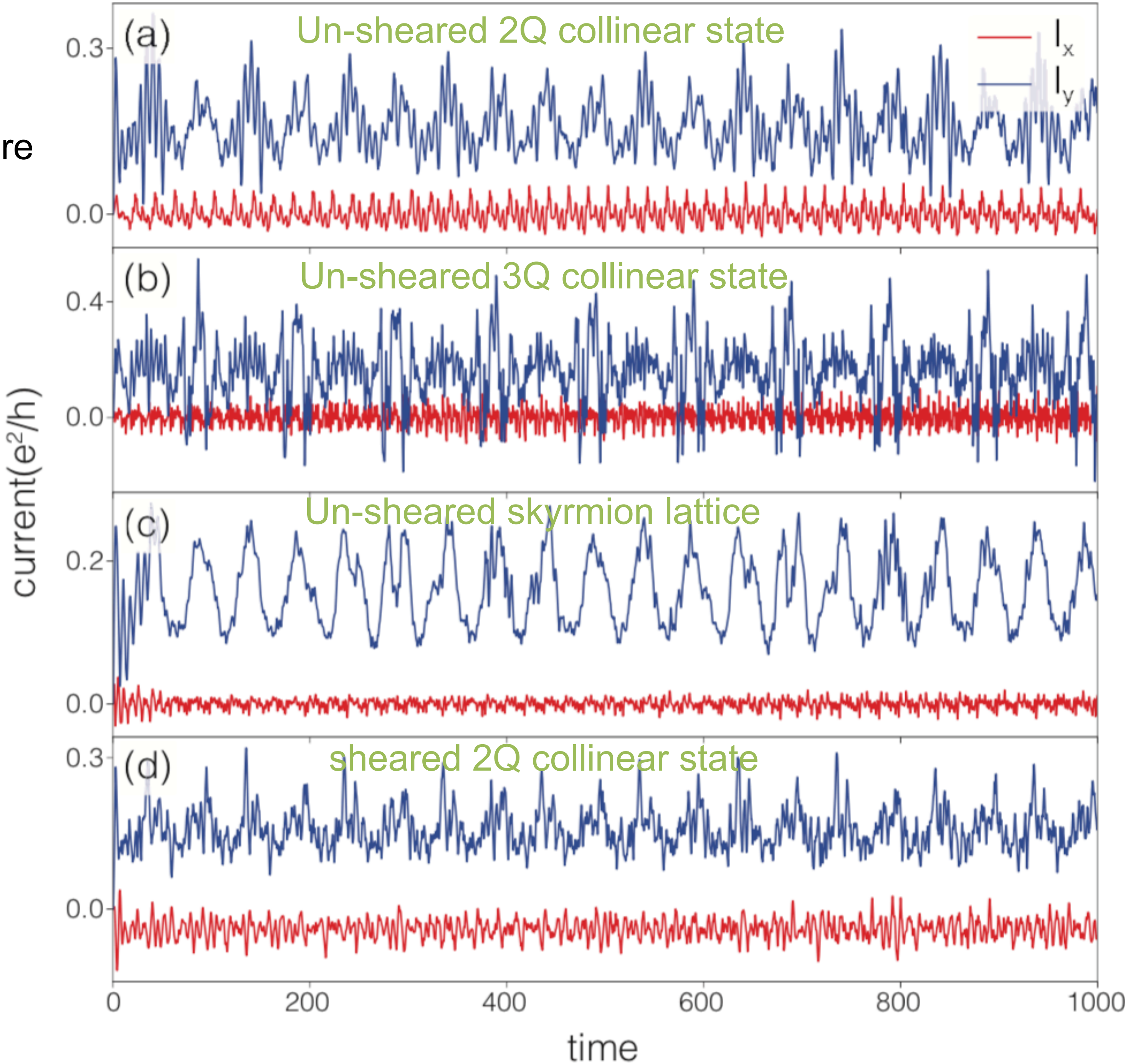
Model 2

$$\mathcal{H} = \sum_{\langle i,j\rangle} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j - H\sum_i S^z_i - A\sum_i (S^z_i)^2,$$



Spin textures obtained by Monte Carlo simulation of spin Hamiltonian

Current induced by
motion of spin texture



Discussions and conclusions

- **It is necessary to increase the dimension of the momentum space in the presence of moving spin texture.**
- **In the higher dimensional hybrid momentum space, the electronic wave functions are topological nontrivial, which is generated dynamically.**
- **As a consequence, the motion of spin textures induces electric charge transport in magnetic insulators.**

- **The magnets with multiple-Q spin texture provide a platform to explore higher dimensional topological physics.**
  - Physical 2D magnets with double-Q spin texture ➔ 4D quantum Hall systems
- **The dynamically generated topology does not require noncoplanar spin texture, cf. real Berry phase.**

- **The high dimensional topological index can be accessed by measurement of current in low dimensional systems.**
- **The Berry curvature in the hybrid momentum space also affects the electronic properties, i.e. thermoelectric coefficient and conductivity, in metallic magnets.**

S. Ying and SZL, PRB 98, 235116 (2018)
S. Ying, S. Hayami and SZL, arXiv:1904.05473