Multipolar Order and Multipolar Kondo Effect

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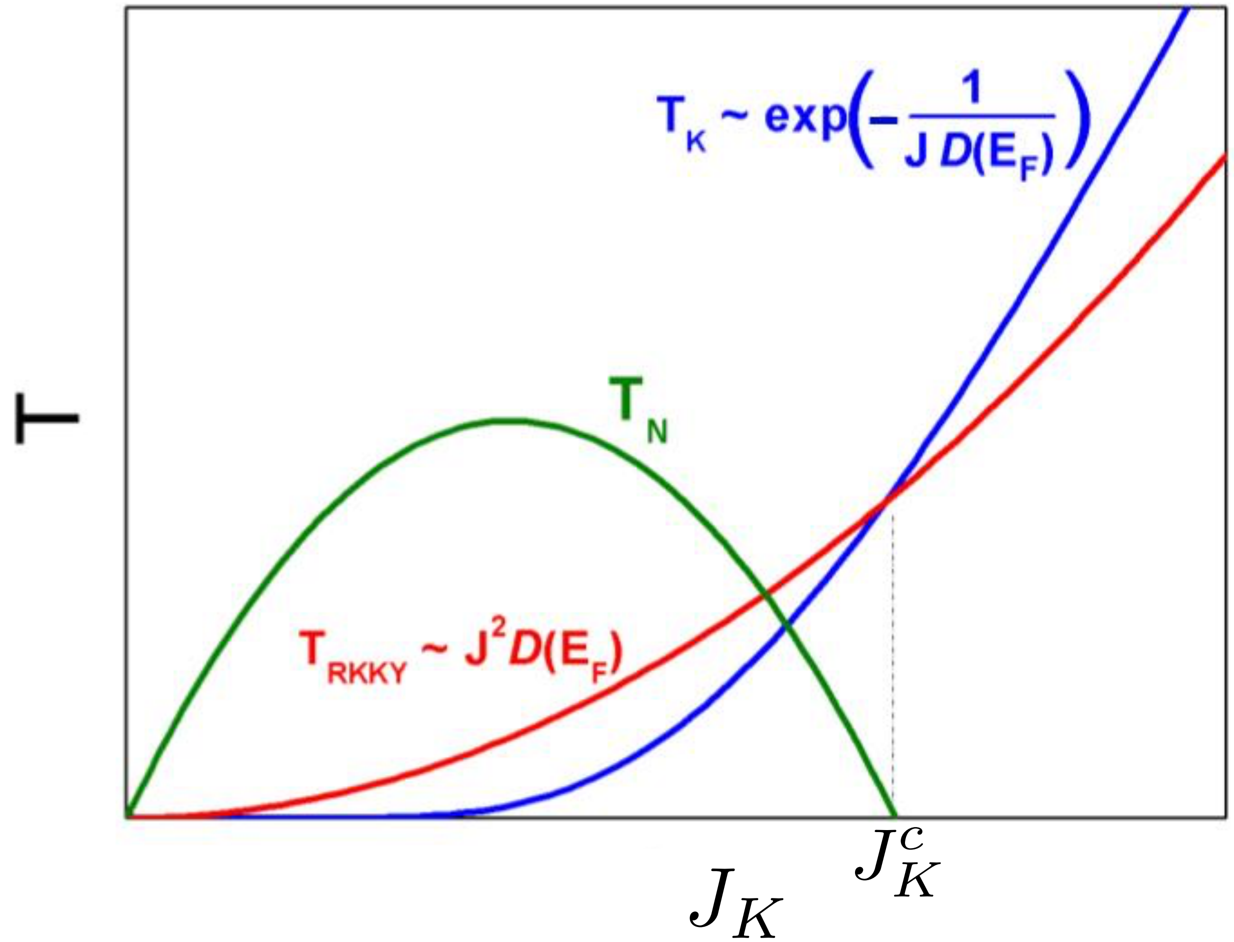
Doniach Phase Diagram

(local spin moments coupled to conduction electrons)



Magnetic order Fermi liquid

$$H = \sum_k \epsilon_k c^\dagger_{k\alpha} c_{k\alpha} + \frac{J_K}{2} \sum_r \vec{S}_r \cdot c^\dagger_{r\alpha} \vec{\sigma}_{\alpha\alpha'} c_{r\alpha'}$$

Multipolar local moments in metallic systems

Purely multipolar moments (no dipole moment)

Quadrupolar - time reversal even

Octupolar - time reversal odd

$4f^2$ Pr^{3+} ions

Unusual "Kondo" coupling between multipolar local moments and conduction electrons

RKKY induced interactions between different multipolar degrees of freedom

Reconstruction of electronic structure:

Nematic Fermi surface, Weyl metal, anomalous Hall effect

Possible presence of novel quantum critical points

Multipolar local moments in metallic systems

Classic example (~30 years old !): URu_2Si_2

"Hidden" Order" Large specific heat anomaly at 17K

Lots of theoretical proposals, but still controversial

5f orbitals are quite extended: Significant hybridization ?
Local v.s. Itinerant picture ?

What about $4f^2$ systems ? Pr^{3+} cubic systems

More localized orbitals: Clearer signatures of multipolar order ?

With pressure: Controlling the hybridization !

Outline

I. Introduction and Motivation: emergent phases with multipolar local moment in $Pr(TM)_2Al_{20}$

II. Landau theory of multipolar order

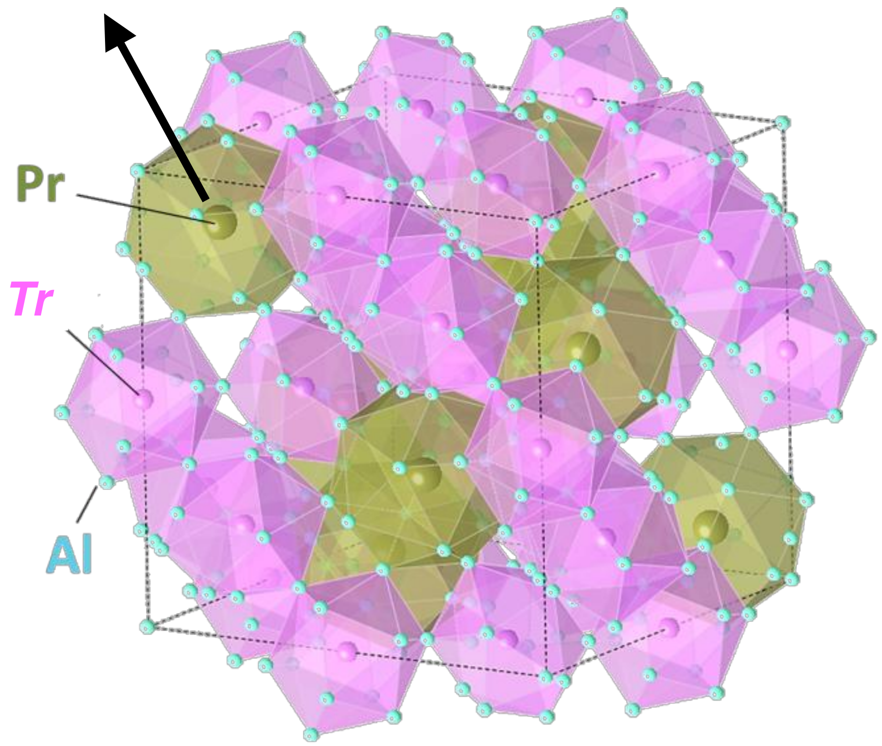
III. Magnetostriction as a tool to detect multipolar order

VI. Multipolar Kondo Effect

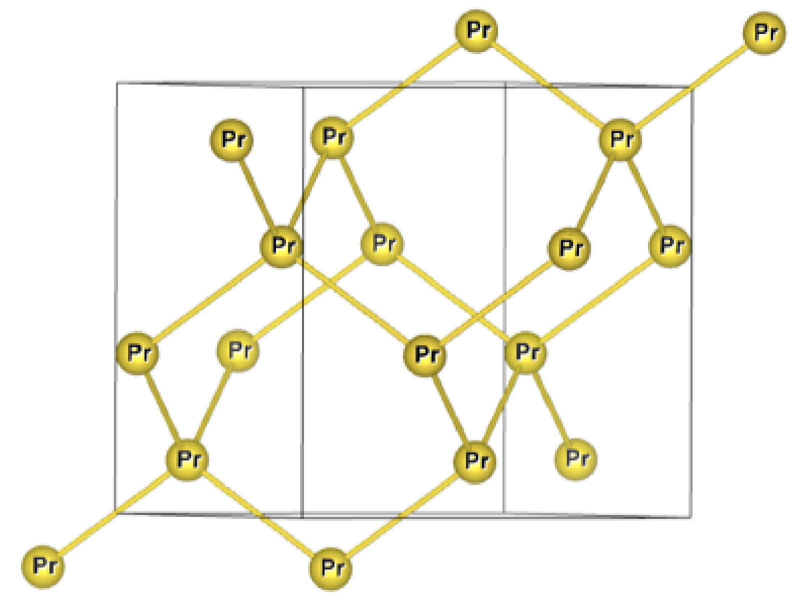
$Pr(TM)_2Al_{20}$

$\mathrm{Fd\bar{3}m}$

Frank-Kasper cage



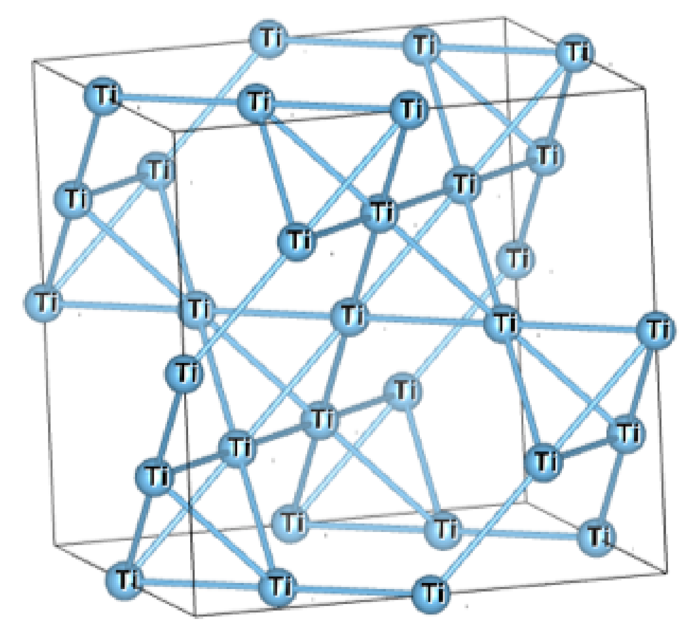
Pr: Diamond Lattice



heavy fermion
superconductivity

multipolar order
quantum critical point

TM: Pyrochlore Lattice



$PrTi_2Al_{20}$ $m^*/m_0 \sim 16$

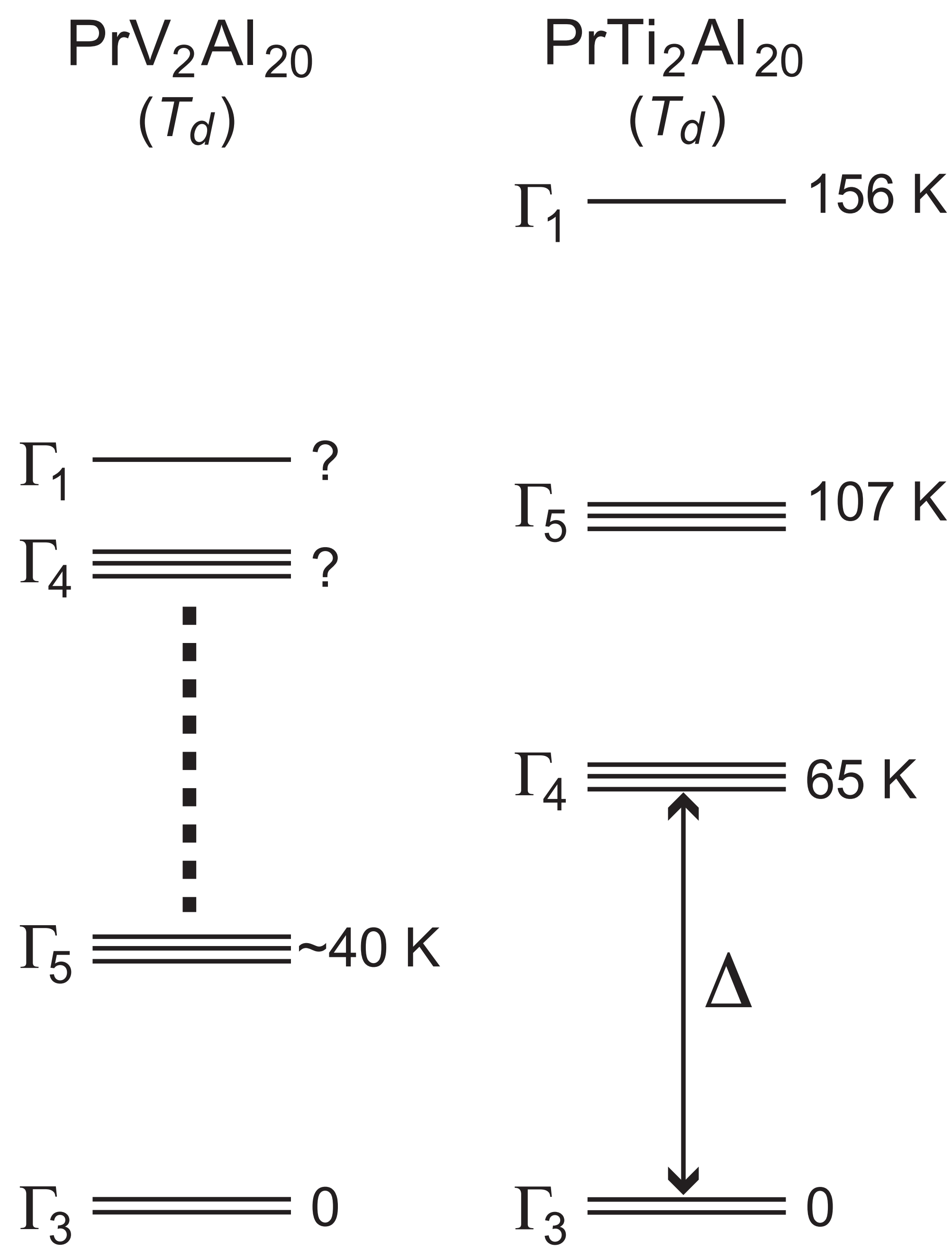
PrV_2Al_{20} $m^*/m_0 \sim 140$

S. Nakatsuji
U of Tokyo

$PrIr_2Zn_{20}$ P. Gegenwart & Hiroshima Univ

Pr^{3+} Crystal Electric Field Splitting: J=4

$4f^2$ Point Group Symmetry: T_d



Non-Kramers Doublet: Pr^{3+}

$\Gamma_3^{(1)}$	$\frac{1}{2}\sqrt{\frac{7}{6}}\|4\rangle - \frac{1}{2}\sqrt{\frac{5}{3}}\|0\rangle + \frac{1}{2}\sqrt{\frac{7}{6}}\|-4\rangle$
$\Gamma_3^{(2)}$	$\sqrt{\frac{1}{2}}\|2\rangle + \sqrt{\frac{1}{2}}\|-2\rangle$

Non-Kramers Doublet: Pr^{3+}

$$|+\rangle \equiv \Gamma_3^{(1)} \qquad |-\rangle \equiv \Gamma_3^{(2)}$$

$\langle \pm | J_\alpha | \pm \rangle = 0$ No dipole moment !

But, finite matrix elements for

$O_{22} = \frac{\sqrt{3}}{2}(J_x^2 - J_y^2)$ $\quad O_{20} = \frac{1}{2}(3J_z^2 - J^2)$ Quadrupolar

$T_{xyz} = \frac{\sqrt{15}}{6}\overline{J_x J_y J_z}$ Octupolar

Pseudo-spin basis

$$|\uparrow\rangle \equiv \frac{1}{\sqrt{2}}(|\Gamma_3^{(1)}\rangle + i\,|\Gamma_3^{(2)}\rangle)$$

$$|\downarrow\rangle \equiv \frac{1}{\sqrt{2}}(i\,|\Gamma_3^{(1)}\rangle + |\Gamma_3^{(2)}\rangle)$$

$$\tau_x = -\frac{1}{4}O_{22}$$

$$\tau_y = -\frac{1}{4}O_{20}$$

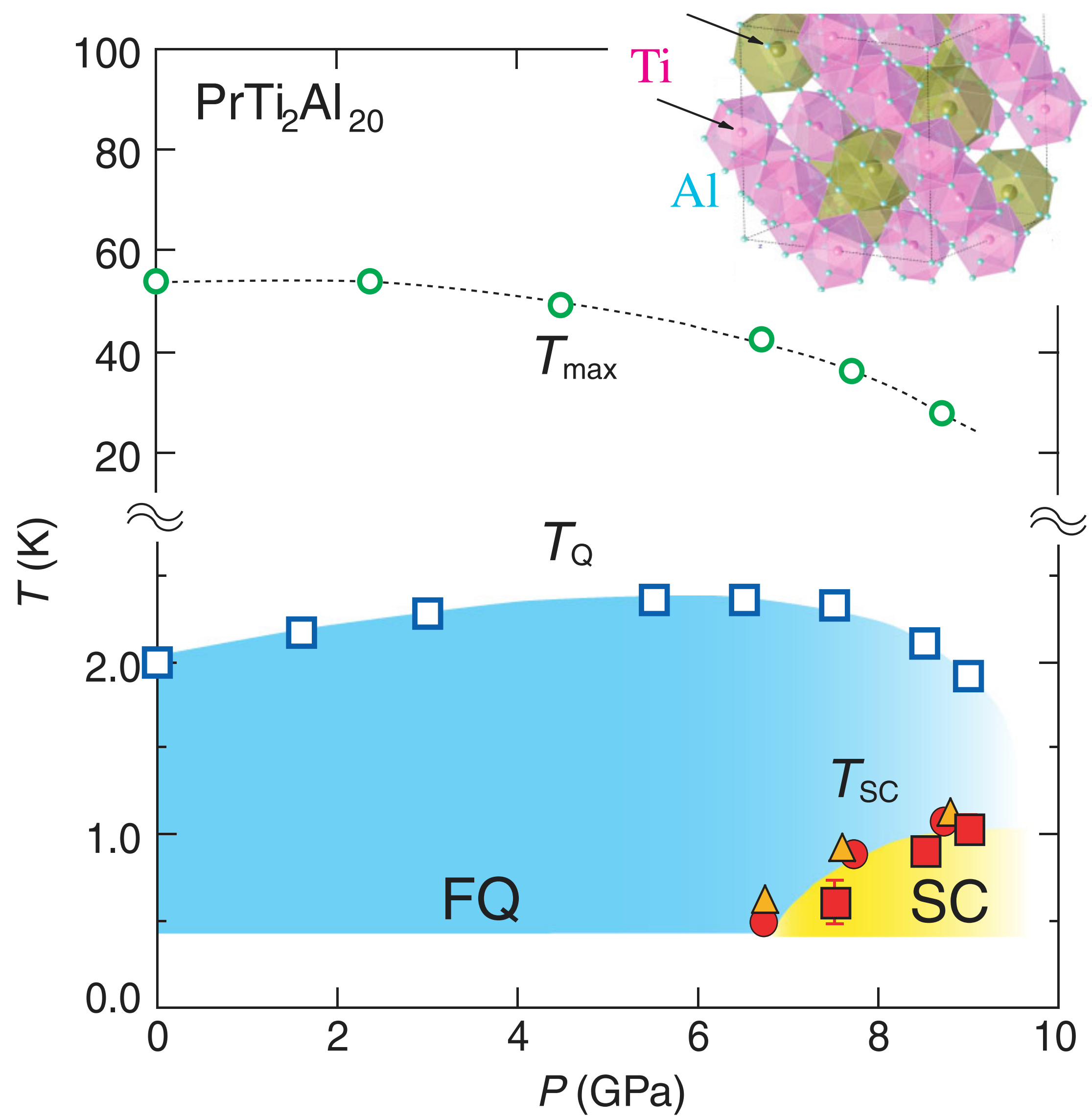
$$\tau_z = \frac{1}{3\sqrt{5}}T_{xyz}$$

$(\tau_x, \tau_y) \equiv \vec{\tau}^{\perp}$ Quadrupolar — time-reversal even

τ_z Octupolar — time-reversal odd

Pressure-induced Quantum Critical Point ?

K. Matsubayashi et al. (2012)



$m^*/m_0 \sim 16$ $m^*/m_0 \sim 110$

$PrTi_2Al_{20}$

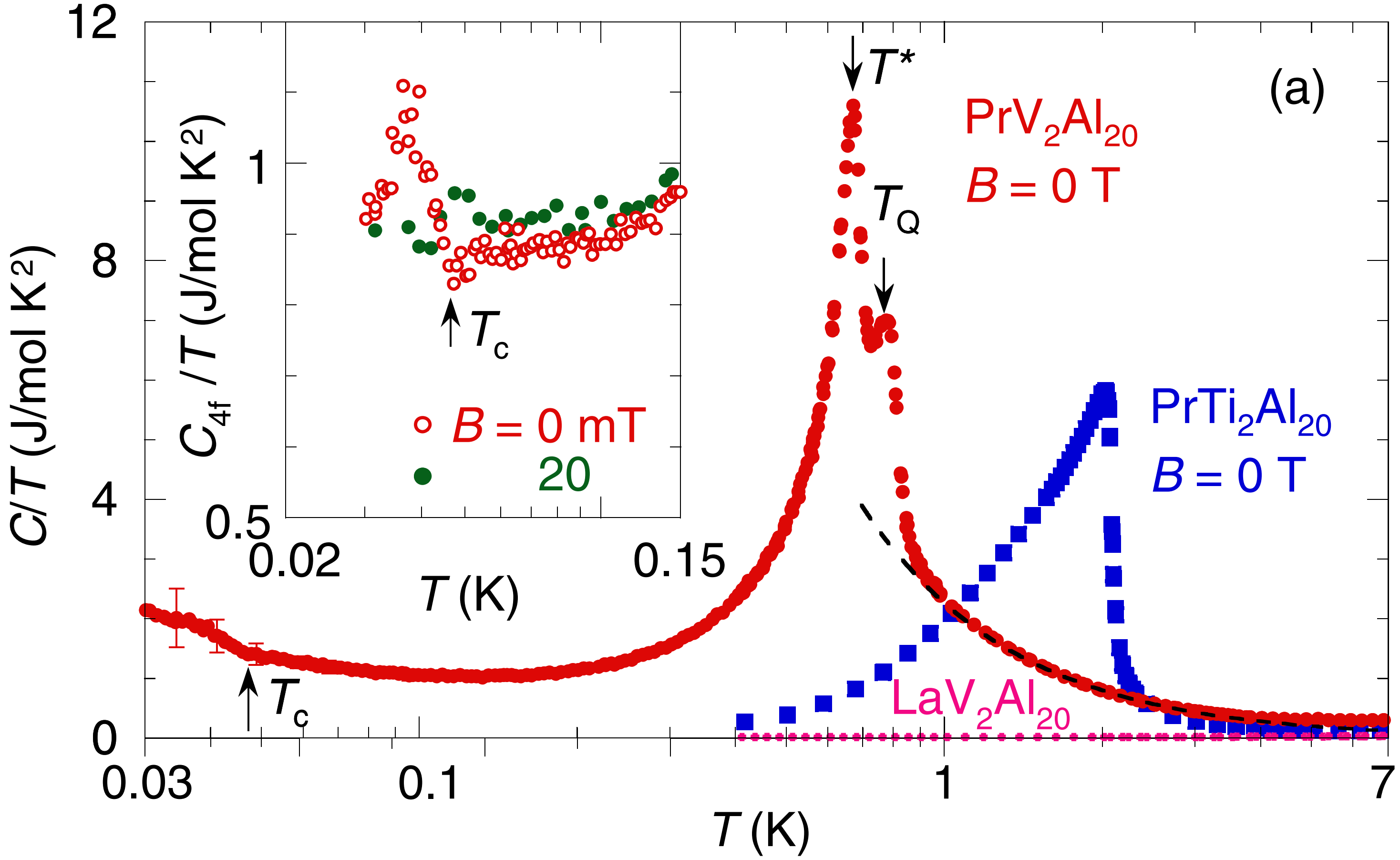
Ferro-Qudrupolar (FQ) Order
(field-induced dipole moment, elastic constant, NMR)

Pressure-induced suppression of FQ

enhanced effective mass

enhancement of SC

PrV_2Al_{20} Antiferro-Quadrupolar (AFQ) Order



elastic constant,
sharp transition
in magnetic field …

Non-Fermi liquid
behavior

Stronger
hybridization ?

Specific heat:
thermal transitions at
0.7K and 0.8K

Presence of
Octupolar order ?

M.Tsujimoto, Y.Matsumoto, S.Nakatsuji (2015)

Landau-Ginzburg Theory

Arun Paramekanti

SungBin Lee

Simon Trebst

arXiv:1806.02842

Landau-Ginzburg Theory

Order Parameters $\langle \tau_\mu^\pm \rangle = \langle \tau_\mu^x \rangle \pm i \langle \tau_\mu^y \rangle$

F-Q $\phi_u \equiv \langle \tau_A^+ \rangle + \langle \tau_B^+ \rangle$

Ferro-Quadrupole

$\phi_s \equiv \langle \tau_A^+ \rangle - \langle \tau_B^+ \rangle$ AF-Q

Antifero-Quadrupole

F-O $m_u \equiv \langle \tau_A^z \rangle + \langle \tau_B^z \rangle$

Ferro-Octupole

$m_s \equiv \langle \tau_A^z \rangle - \langle \tau_B^z \rangle$ AF-O

Antifero-Octupole

$\tau_x = -\frac{1}{4} O_{22}$ $\tau_y = -\frac{1}{4} O_{20}$ $\tau_z = \frac{1}{3\sqrt{5}} T_{xyz}$

$(\tau_x, \tau_y) \equiv \vec{\tau}^\perp$ Quadrupolar

time-reversal even

τ_z Octupolar

time-reversal odd

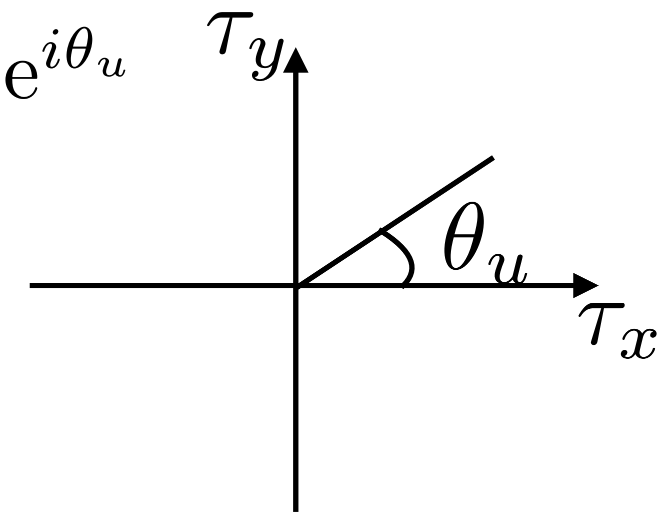
Landau-Ginzburg Theory

F-Q $\quad \mathcal{F}_{\phi u} = r_{u\phi}|\phi_u|^2 + \boxed{iv(\phi_u^3 - \phi_u^{*3})} + g_{u\phi}|\phi_u|^4 + \ldots$

Z3 clock anisotropy $\quad$ first order in clean system

Hattori+Tsunetsugu

$\phi_u \equiv |\phi_u| e^{i\theta_u}$



Cubic anisotropy

locks the phase θ_u

$\theta_u = \pi/6 + 2n\pi/3 \qquad v > 0$

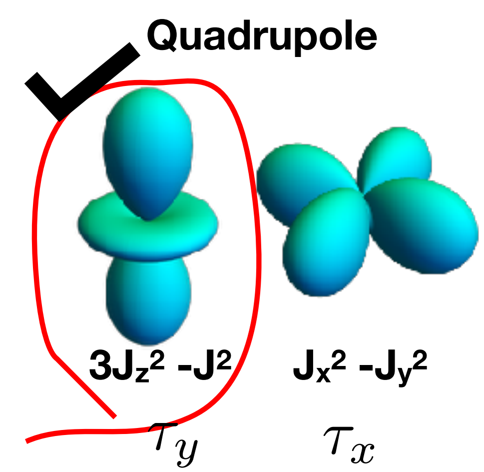
$\theta_u = \pi/6 + (2n+1)\pi/3 \qquad v < 0$

favors

$O_{20} = \frac{1}{2}(3J_z^2 - J^2)$

(consistent with NMR, Takigawa)

PrTi2Al20



Considering Only Quadrupolar Order Parameters

F-Q $\quad \mathcal{F}_{\phi u} = r_{u\phi}|\phi_u|^2 + \boxed{iv(\phi_u^3 - \phi_u^{*3})} + g_{u\phi}|\phi_u|^4 + \dots$

AF-Q $\quad \mathcal{F}_{\phi s} = r_{s\phi}|\phi_s|^2 + g_{s\phi}|\phi_s|^4 + \boxed{w(\phi_s^6 + \phi_s^{*6})}$

Interaction

$$\mathcal{F}_{int}^{(4)} = c_1|\phi_u|^2|\phi_s|^2$$

In general, symmetry also allows

$$\mathcal{F}_{\rm int}^{(3)} = i\lambda(\phi_s^2\phi_u - \phi_s^{*2}\phi_u^*) \qquad \phi_u \sim \phi_s^{*2}$$

Parasitic F-Q order in AF-Q state !

Including Octupolar Order Parameters

F-Q $\quad \mathcal{F}_{\phi u} = r_{u\phi}|\phi_u|^2 + \boxed{iv(\phi_u^3 - \phi_u^{*3})} + g_{u\phi}|\phi_u|^4 + \ldots$

AF-Q $\quad \mathcal{F}_{\phi s} = r_{s\phi}|\phi_s|^2 + g_{s\phi}|\phi_s|^4 + \boxed{w(\phi_s^6 + \phi_s^{*6})}$

F-O $\quad \mathcal{F}_{mu} = r_{um} m_u^2 + g_{um} m_u^4 + \ldots$

Interaction

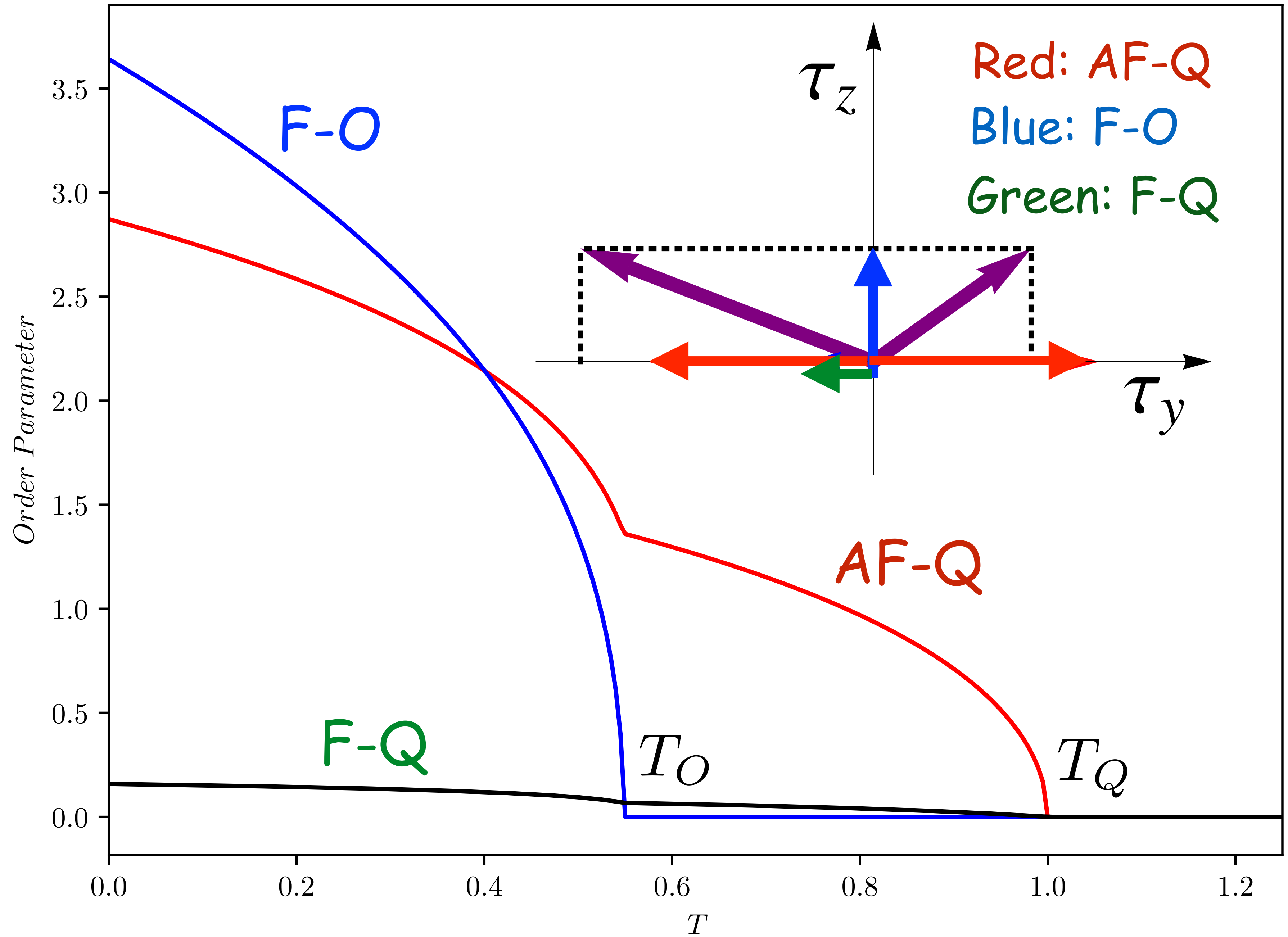
$$\mathcal{F}_{int}^{(4)} = c_1|\phi_u|^2|\phi_s|^2 + c_2|\phi_u|^2 m_u^2 + c_3|\phi_s|^2 m_u^2$$

In general, symmetry also allows

$$\mathcal{F}_{\text{int}}^{(3)} = i\lambda(\phi_s^2\phi_u - \phi_s^{*2}\phi_u^*) \qquad \phi_u \sim \phi_s^{*2}$$

Parasitic F-Q order in AF-Q state !

Thermal transitions at B=0



$$\tau_y = -\frac{1}{4} O_{20}$$

$$\tau_z = \frac{1}{3\sqrt{5}} T_{xyz}$$

$$O_{20} = \frac{1}{2}\left(3J_z^2 - J^2\right)$$

$$T_{xyz} = \frac{\sqrt{15}}{6} \overline{J_x J_y J_z}$$

Magnetostrsiction: Detection of Multipolar Order

Adarsh Patri

Akito Sakai

SungBin Lee

Arun Paramekanti

Satoru Nakatsuji

arXiv:1901.00012

Parellel Work: Piers Coleman and Premi Chandra

Elastic Energy of a Cubic Crystal

$$
\begin{aligned}
F_{\text{lattice}} &= \frac{c_{11}}{2}\left(\epsilon_{xx}^2+\epsilon_{yy}^2+\epsilon_{zz}^2\right)+\frac{c_{44}}{2}\left(\epsilon_{xy}^2+\epsilon_{yz}^2+\epsilon_{xz}^2\right) \\
&\quad + c_{12}\left(\epsilon_{xx}\epsilon_{yy}+\epsilon_{yy}\epsilon_{zz}+\epsilon_{zz}\epsilon_{xx}\right) \ , \\
&= \frac{c_B}{2}\left(\epsilon_B^2\right)+\frac{c_{11}-c_{12}}{2}\left(\epsilon_\mu^2+\epsilon_\nu^2\right) \\
&\quad + \frac{c_{44}}{2}\left(\epsilon_{xy}^2+\epsilon_{yz}^2+\epsilon_{xz}^2\right) \ ,
\end{aligned}
$$

$\epsilon_B \equiv \epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz}$ volume expansion

$$
\left.
\begin{aligned}
\epsilon_\nu &\equiv (2\epsilon_{zz}-\epsilon_{xx}-\epsilon_{yy})/\sqrt{3} \\
\epsilon_\mu &\equiv (\epsilon_{xx}-\epsilon_{yy})
\end{aligned}
\right\} \ e_g \text{ symmetry distortion}
$$

Coupling to Quadrupolar Order Parameter

$$F^{(1)}_{\rm strain}[h=0] = \frac{c_{11}-c_{12}}{2}\left(\epsilon_\mu^2+\epsilon_\nu^2\right)$$

$$-\, g_{\mathcal{Q}}\epsilon_\mu\left[\langle\tau_A^x\rangle+\langle\tau_B^x\rangle\right] - g_{\mathcal{Q}}\epsilon_\nu\left[\langle\tau_A^y\rangle+\langle\tau_B^y\rangle\right]$$

coupled to e_g symmetry distortion

$$O_{22} = \tfrac{\sqrt{3}}{2}(J_x^2 - J_y^2) \qquad O_{20} = \tfrac{1}{2}(3J_z^2 - J^2)$$

Quadrupolar

$$\tau_x = -\tfrac{1}{4}O_{22} \qquad \tau_y = -\tfrac{1}{4}O_{20}$$

time-reversal even

$$\left.\begin{aligned} \epsilon_\mu &\equiv (\epsilon_{xx}-\epsilon_{yy}) \\ \epsilon_\nu &\equiv (2\epsilon_{zz}-\epsilon_{xx}-\epsilon_{yy})/\sqrt{3} \end{aligned}\right\}$$

e_g symmetry distortion

Coupling to Quadrupolar Order Parameter

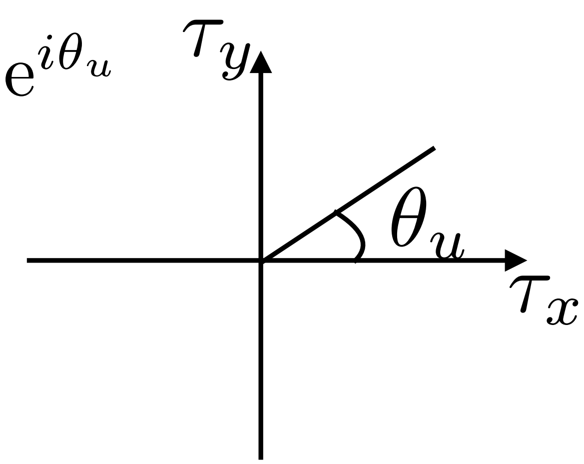
$$F^{(1)}_{\text{strain}}[h=0] = \frac{c_{11}-c_{12}}{2}\left(\epsilon_\mu^2+\epsilon_\nu^2\right)$$

$$-\, g_Q \epsilon_\mu \left[\langle \tau_A^x\rangle + \langle \tau_B^x\rangle\right] - g_Q \epsilon_\nu \left[\langle \tau_A^y\rangle + \langle \tau_B^y\rangle\right]$$

coupled to e_g symmetry distortion

$$\boxed{\begin{array}{ll} \phi_u \equiv \langle \tau_A^+\rangle + \langle \tau_B^+\rangle & \text{F-Q} \\ \phi_s \equiv \langle \tau_A^+\rangle - \langle \tau_B^+\rangle & \text{AF-Q} \end{array}}$$

$$\phi_u \equiv |\phi_u| \mathrm{e}^{i\theta_u}$$



$$\boxed{\begin{aligned} \epsilon_\mu &= \frac{g_Q}{c_{11}-c_{22}} |\phi_u| \cos\theta_u \\ \epsilon_\nu &= \frac{g_Q}{c_{11}-c_{22}} |\phi_u| \sin\theta_u \end{aligned}}$$

Coupling to Octupolar Order Parameter

$$F^{(2)}_{\mathrm{strain}}[h \neq 0] = \frac{c_{44}}{2}\left(\epsilon^2_{xy} + \epsilon^2_{yz} + \epsilon^2_{xz}\right) - g_{\mathcal{O}} m \left[h_x \epsilon_{yz} + h_y \epsilon_{xz} + h_z \epsilon_{xy}\right]$$

$$- \gamma_c \left[h_x h_y \epsilon_{xy} + h_x h_z \epsilon_{xz} + h_y h_z \epsilon_{yz}\right]$$

$$\boxed{m = \langle \tau_z \rangle}$$

$$T_{xyz} = \frac{\sqrt{15}}{6}\overline{J_x J_y J_z}$$

Octupolar

$$\tau_z = \frac{1}{3\sqrt{5}} T_{xyz}$$

time-reversal odd

Coupling to Octupolar Order Parameter

$$F^{(2)}_{\text{strain}}[h \neq 0] = \frac{c_{44}}{2}\left(\epsilon_{xy}^2 + \epsilon_{yz}^2 + \epsilon_{xz}^2\right) - g_{\mathcal{O}} m \left[h_x \epsilon_{yz} + h_y \epsilon_{xz} + h_z \epsilon_{xy}\right]$$
$$- \gamma_c \left[h_x h_y \epsilon_{xy} + h_x h_z \epsilon_{xz} + h_y h_z \epsilon_{yz}\right]$$

$$\boxed{m = \langle \tau_z \rangle}$$

$$\rightarrow \quad \boxed{\begin{aligned} \epsilon_{xy} &= \mathcal{M} h_z + \gamma h_x h_y \\ \epsilon_{zx} &= \mathcal{M} h_y + \gamma h_z h_x \\ \epsilon_{yz} &= \mathcal{M} h_x + \gamma h_y h_z \end{aligned}}$$

$$\mathcal{M} = g_{\mathcal{O}} m / c_{44}$$
$$\gamma = \gamma_c / c_{44}$$

Measurement of elastic tensor	⟷	Measurement of multipolar order

Relative Length Change

$$\left(\frac{\Delta L}{L}\right)_{\vec{l}} = \sum_{i,j=1}^{3} \epsilon_{ij}\hat{l}_i\hat{l}_j \qquad \epsilon_{ij} \equiv \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\hat{l}_i \equiv i^{th} \text{ component of unit vector } \hat{l}.$$

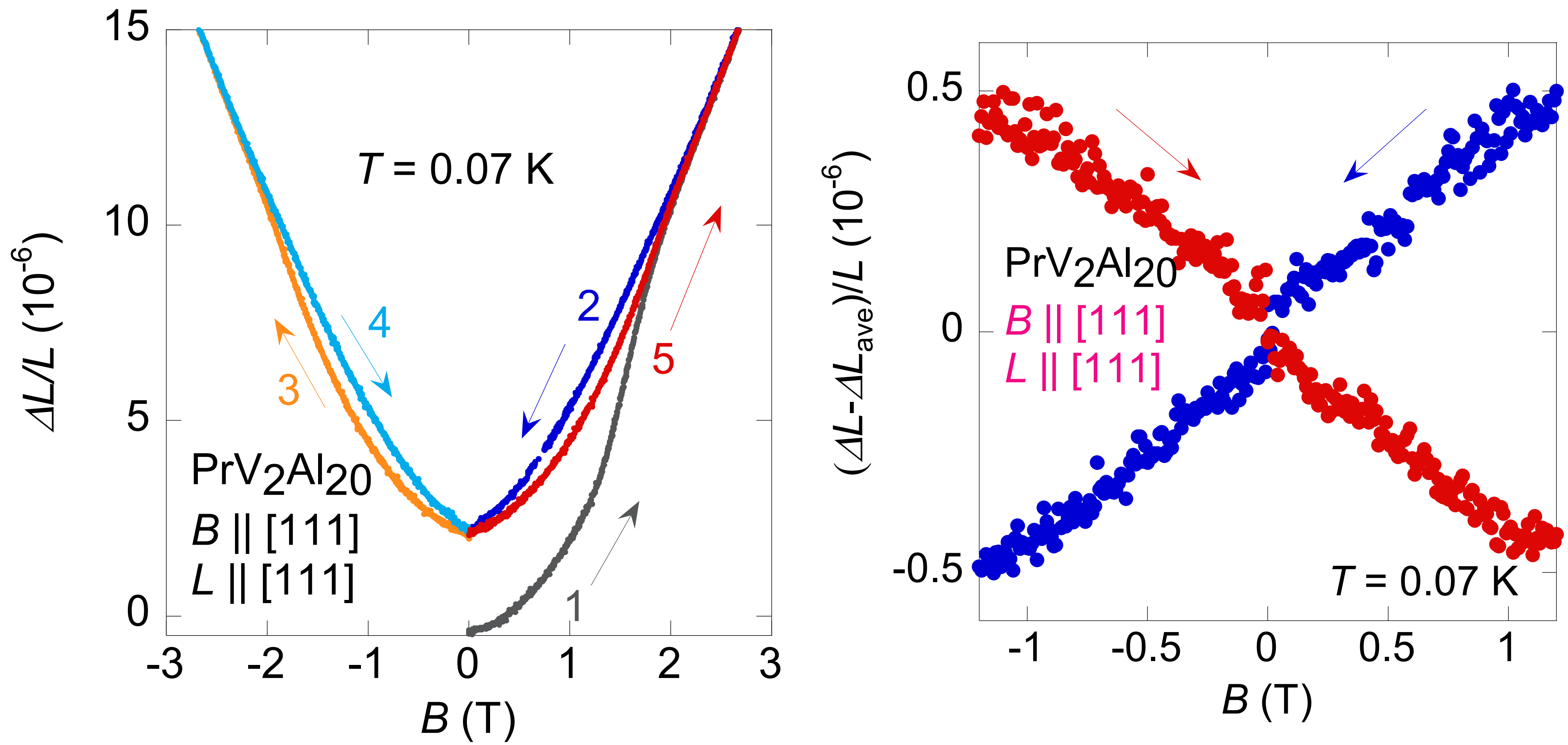
Example:

$$\left(\frac{\Delta L}{L}\right)_{(1,\pm1,1)} = \frac{\epsilon_B}{3} + \frac{2\left(\pm\epsilon_{xy} \pm \epsilon_{yz} + \epsilon_{xz}\right)}{3}$$

$$= \frac{1}{3}\epsilon_B + \frac{2}{3}\mathcal{M}\left[\pm h_z \pm h_x + h_y\right] + \frac{2}{3}\gamma\left[\pm h_x h_y \pm h_y h_z + h_z h_x\right]$$

Magnetostrsiction measures Octupolar order !

experimental results (A. Sakai, S. Nakatsuji)



- Existence of odd function $f(B) = -f(-B)$ in L.
- Hysteresis below 2 T.

Predictions for different magnetic field directions + different directions of length changes

Magnetic field $\vec{h} = h\ \hat{n}$	$\vec{\ell}$	$\Delta L / L_{\vec{\ell}}$ scaling
$\hat{n} = \frac{1}{\sqrt{3}}[111]$	$\vec{\ell} = (1,1,1)$	$\frac{2}{\sqrt{3}}\mathcal{M}h + \frac{2}{3}\gamma h^2$
	$\vec{\ell} = (1,-1,0)$ $\vec{\ell} = (1,1,-2)$	$\Phi_2 - \frac{1}{\sqrt{3}}\mathcal{M}h - \frac{1}{3}\gamma h^2 + \frac{1}{3}\kappa_2 h^2$

Sign change !

Predictions for different magnetic field directions + different directions of length changes

Magnetic field $\vec{h} = h\,\hat{n}$	$\vec{\ell}$	$\Delta L/L_{\vec{\ell}}$ scaling
$\hat{n} = \frac{1}{\sqrt{3}}[111]$	$\vec{\ell} = (1,1,1)$	$\frac{2}{\sqrt{3}}\mathcal{M}h + \frac{2}{3}\gamma h^2$
	$\vec{\ell} = (1,-1,0)$ $\vec{\ell} = (1,1,-2)$	$\Phi_2 - \frac{1}{\sqrt{3}}\mathcal{M}h - \frac{1}{3}\gamma h^2 + \frac{1}{3}\kappa_2 h^2$
$\hat{n} = [100]$	$\vec{\ell} = (1,0,0)$	$\Phi_1 + \kappa_1 h^2$
	$\vec{\ell} = (0,1,\pm 1)$	$\Phi_1 \pm \mathcal{M}h + \kappa_1 h^2$
$\hat{n} = \frac{1}{\sqrt{2}}[110]$	$\vec{\ell} = (1,1,0)$	$\Phi_2 + \frac{1}{2}\gamma h^2 + \frac{1}{2}\kappa_2 h^2$
	$\vec{\ell} = (1,-1,1)$	$-\frac{1}{3}\gamma h^2$
	$\vec{\ell} = (-1,1,2)$	$\Phi_2 - \frac{1}{6}\gamma h^2 + \frac{1}{2}\kappa_2 h^2$

Multipolar Kondo Problem
single impurity

Adarsh
Patri

arXiv:1904.02717

How to model conduction electrons ?

Assume conduction electrons are mostly from 16 Al cage

Local symmetry around Pr ion is T_d

Molecular orbitals can be classified in terms of irreducible representation of T_d

A_1	$xyz, \quad x^4+y^4+z^4, \quad x^2y^2z^2$
A_2	$x^4(y^2-z^2)+y^4(z^2-x^2)+z^4(x^2-y^2)$
E	$\{x^2-y^2, \; 2z^2-x^2-y^2\}$
T_1	$\{x(z^2-y^2), \; y(z^2-x^2), \; z(x^2-y^2)\}$
T_2	$\{x, \; y, \; z\}, \quad \{xy, \; xz, \; yz\},$

For simplicity and symmetry reasons, we work with

E (e_g molecular orbitals) and T_2 (p molecular orbitals)

Kondo coupling in E-representation (e_g-orbital)

$$H_Q^{e_g} = J_Q \left\{ S_i^x \left(c^\dagger_{i,x^2-y^2,\alpha} c_{i,3z^2-r^2,\beta} + c^\dagger_{i,3z^2-r^2,\alpha} c_{i,x^2-y^2,\beta} \right) \delta_{\alpha\beta} + S_i^y \left(c^\dagger_{i,x^2-y^2,\alpha} c_{i,x^2-y^2,\beta} - c^\dagger_{i,3z^2-r^2,\alpha} c_{i,3z^2-r^2,\beta} \right) \delta_{\alpha\beta} \right\}$$

$$H_O^{e_g} = -iJ_O \; S_i^z \left(c^\dagger_{i,x^2-y^2,\alpha} c_{i,3z^2-r^2,\beta} - c^\dagger_{i,3z^2-r^2,\alpha} c_{i,x^2-y^2,\beta} \right) \delta_{\alpha\beta}$$

"Famous" two-channel Kondo model (Dan Cox)

Orbital fluctuations   Spectator Spin: two channels

Kondo coupling in E-representation (e_g-orbital)

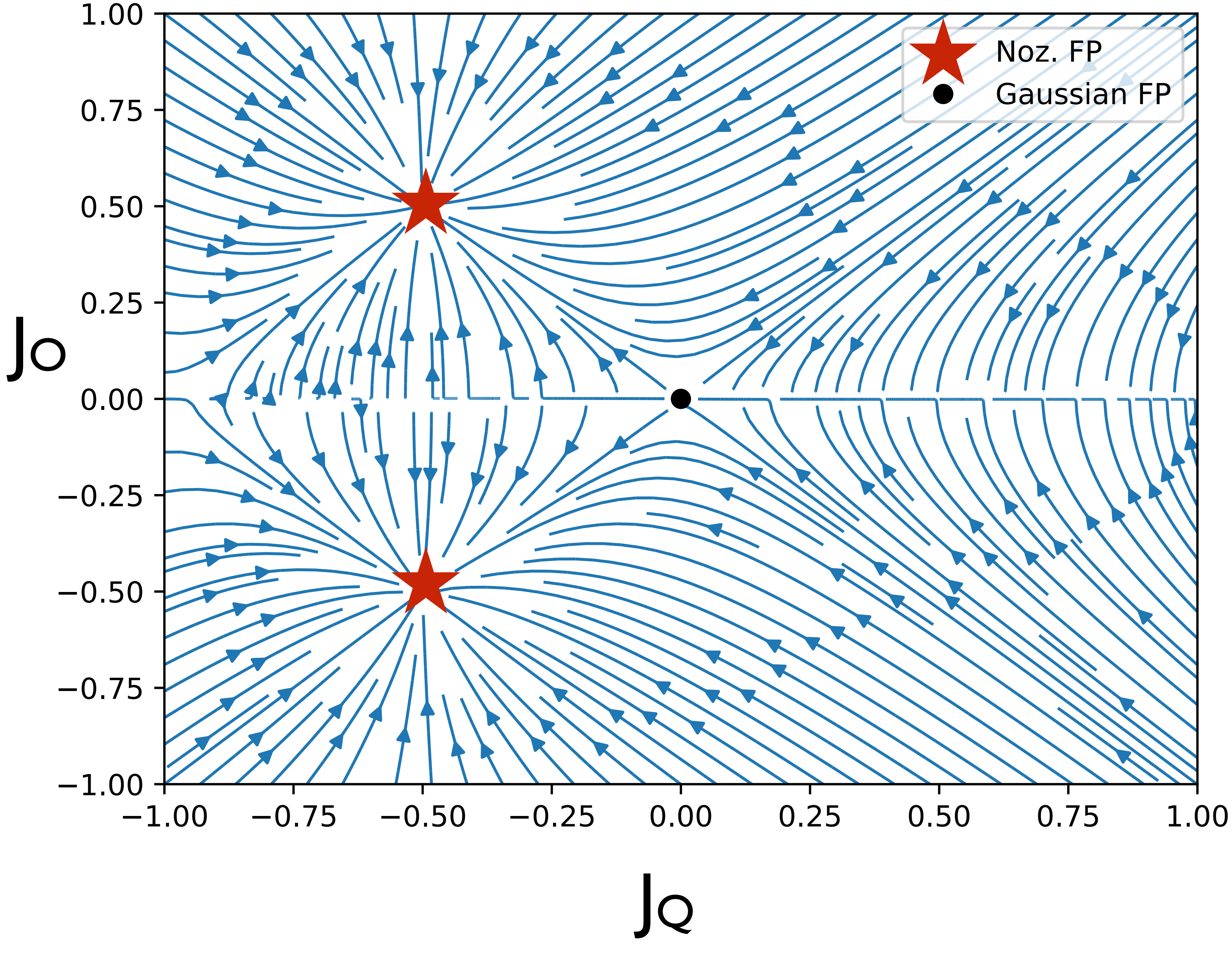
$$H_Q^{e_g} = J_Q \qquad c^{\dagger}_{j,a,\alpha}\left[S^x_j \sigma^0_{\alpha\beta} \otimes \tau^x_{ab} - S^y_j \sigma^0_{\alpha\beta} \otimes \tau^z_{ab}\right] c_{j,b,\beta} \ ,$$

$$H_O^{e_g} = -J_O \qquad S^z_j c^{\dagger}_{j,a,\alpha}\left[\sigma^0_{\alpha\beta} \otimes \tau^y_{ab}\right] c_{j,b,\beta},$$

"Famous" two-channel Kondo model (Dan Cox)

Orbital fluctuations   Spectator Spin: two channels

$$\frac{dJ_Q}{d\ln D} = 2J_Q J_O + 2J_Q\left(J_Q^2 + J_O^2\right),$$
$$\frac{dJ_O}{d\ln D} = 2J_Q^2 + 4J_O J_Q^2,$$



stable fixed points

$$(J_Q, J_O) = (1/2, -1/2)$$
$$(J_Q, J_O) = (-1/2, -1/2)$$

Kondo coupling in T_2-representation (p-orbital)

$$H^p_{Q1} = K_{Q1} \quad c^\dagger_{j,a,\alpha}\left[S^x_j \sigma^0_{\alpha\beta} \otimes \lambda^{x^2-y^2}_{ab} - \frac{S^y_j}{\sqrt{3}} \sigma^0_{\alpha\beta} \otimes \lambda^{2z^2-x^2-y^2}_{ab}\right] c_{j,b,\beta},$$

$$H^p_{Q2} = K_{Q2} \quad c^\dagger_{j,a,\alpha}\left[\left(\sqrt{3}S^x_j - S^y_j\right)\sigma^x_{\alpha\beta} \otimes \lambda^{yz,i}_{ab} + \left(\sqrt{3}S^x_j + S^y_j\right)\sigma^y_{\alpha\beta} \otimes \lambda^{xz,i}_{ab}\right.$$

$$\left. + 2S^y_j \sigma^z_{\alpha\beta} \otimes \lambda^{xy,i}_{ab}\right] c_{j,b,\beta},$$

$$H^p_O = K_O \quad S^z_j c^\dagger_{j,a,\alpha}\left[\sigma^x_{\alpha\beta} \otimes \lambda^{yz,r}_{ab} + \sigma^y_{\alpha\beta} \otimes \lambda^{xz,r}_{ab} + \sigma^z_{\alpha\beta} \otimes \lambda^{xy,r}_{ab}\right] c_{j,b,\beta},$$

Entangled fluctuations of BOTH orbital and spin !

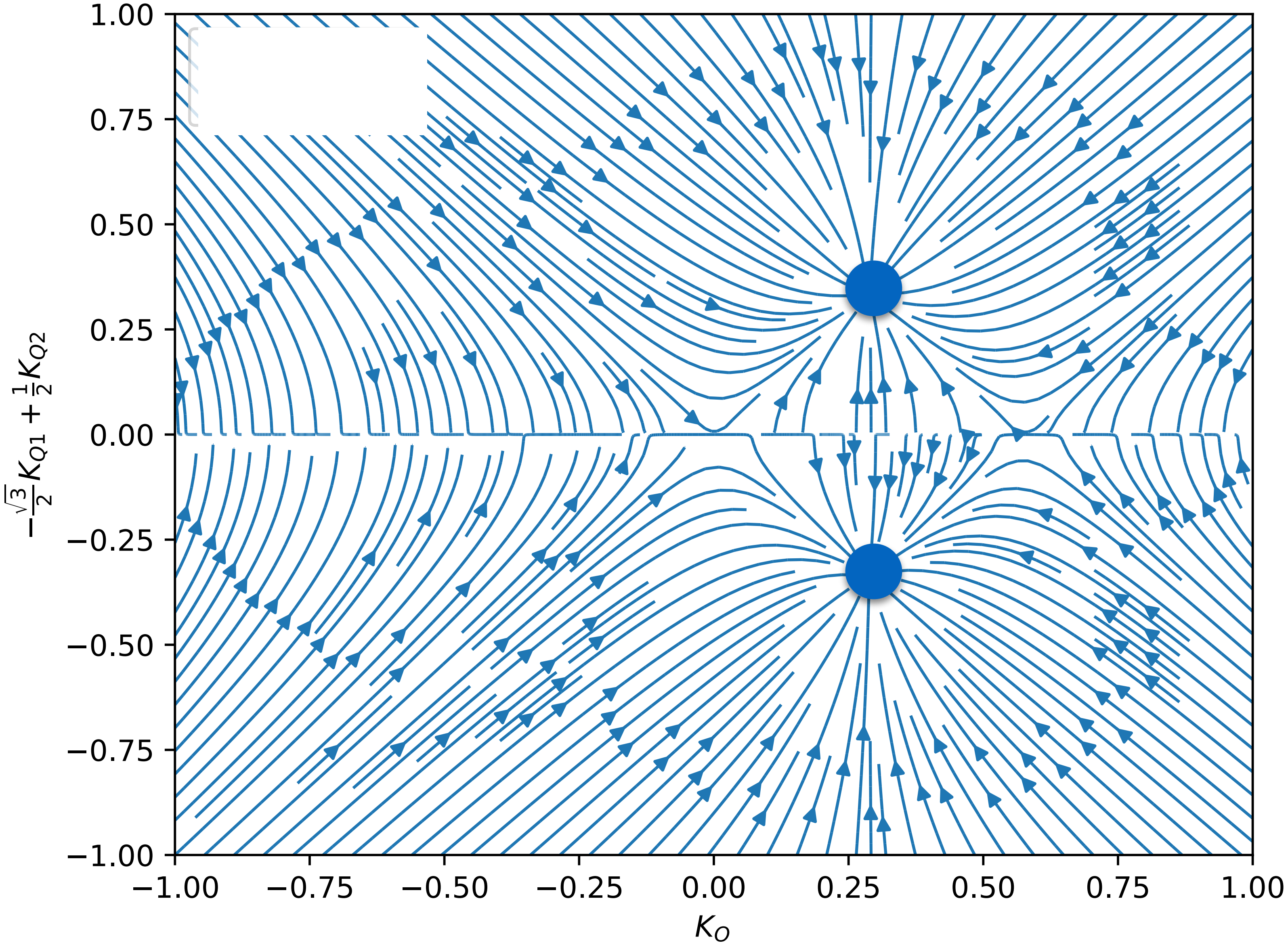
$$\frac{dK_{Q1}}{d\ln D} = 6K_{Q2}K_O + K_{Q1}\left(12K_{Q2}^2 + 6K_O^2 + 2K_{Q1}^2\right)$$

$$\frac{dK_{Q2}}{d\ln D} = K_O\left(K_{Q1} - \sqrt{3}K_{Q2}\right) + K_{Q2}\left(12K_{Q2}^2 + 6K_O^2 + 2K_{Q1}^2\right)$$

$$\frac{dK_O}{d\ln D} = 4K_{Q1}K_{Q2} - 2\sqrt{3}K_{Q2}^2 + K_O\left(24K_{Q2}^2 + 4K_{Q1}^2\right)$$

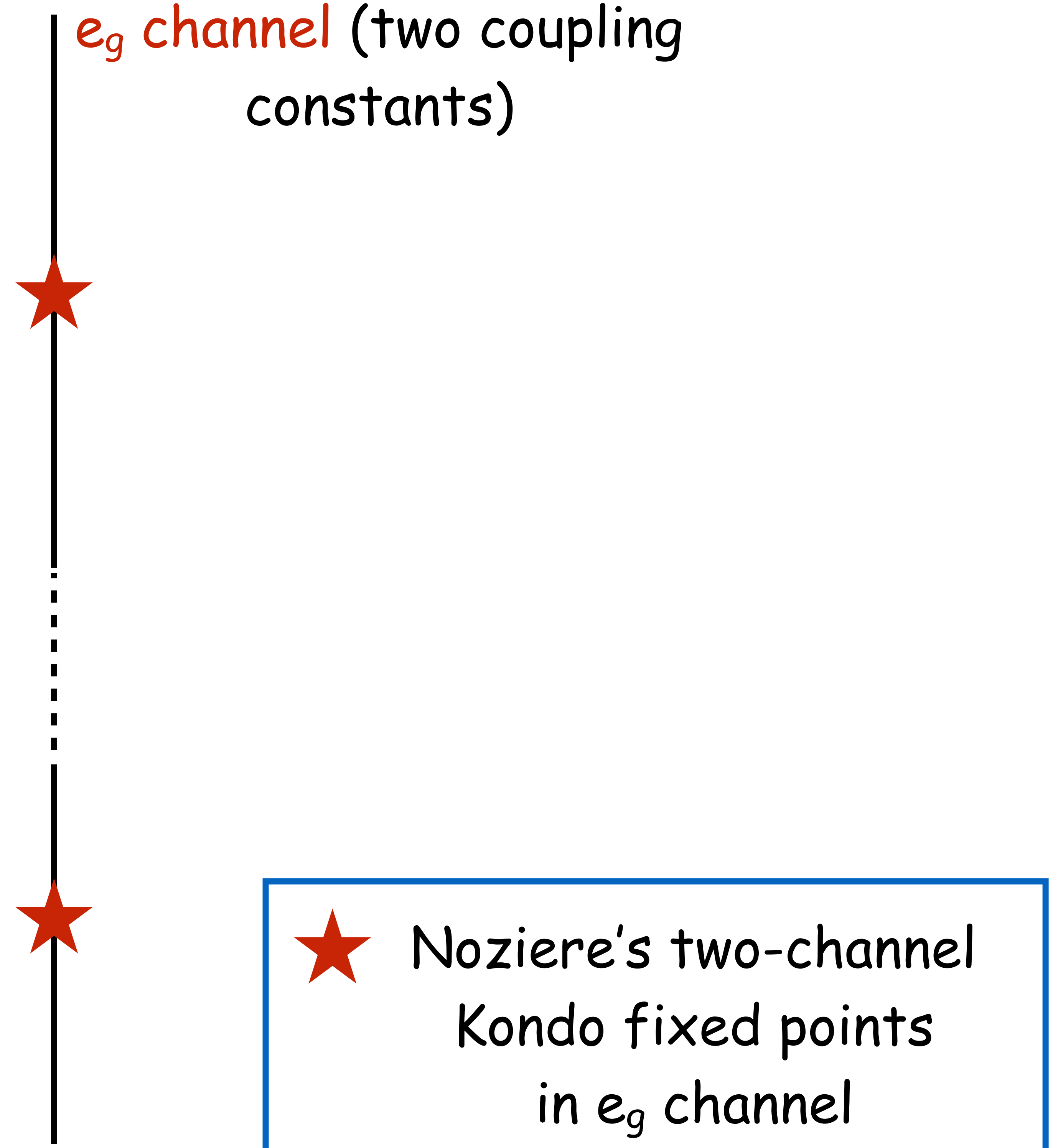
Four fixed points !

RG flow

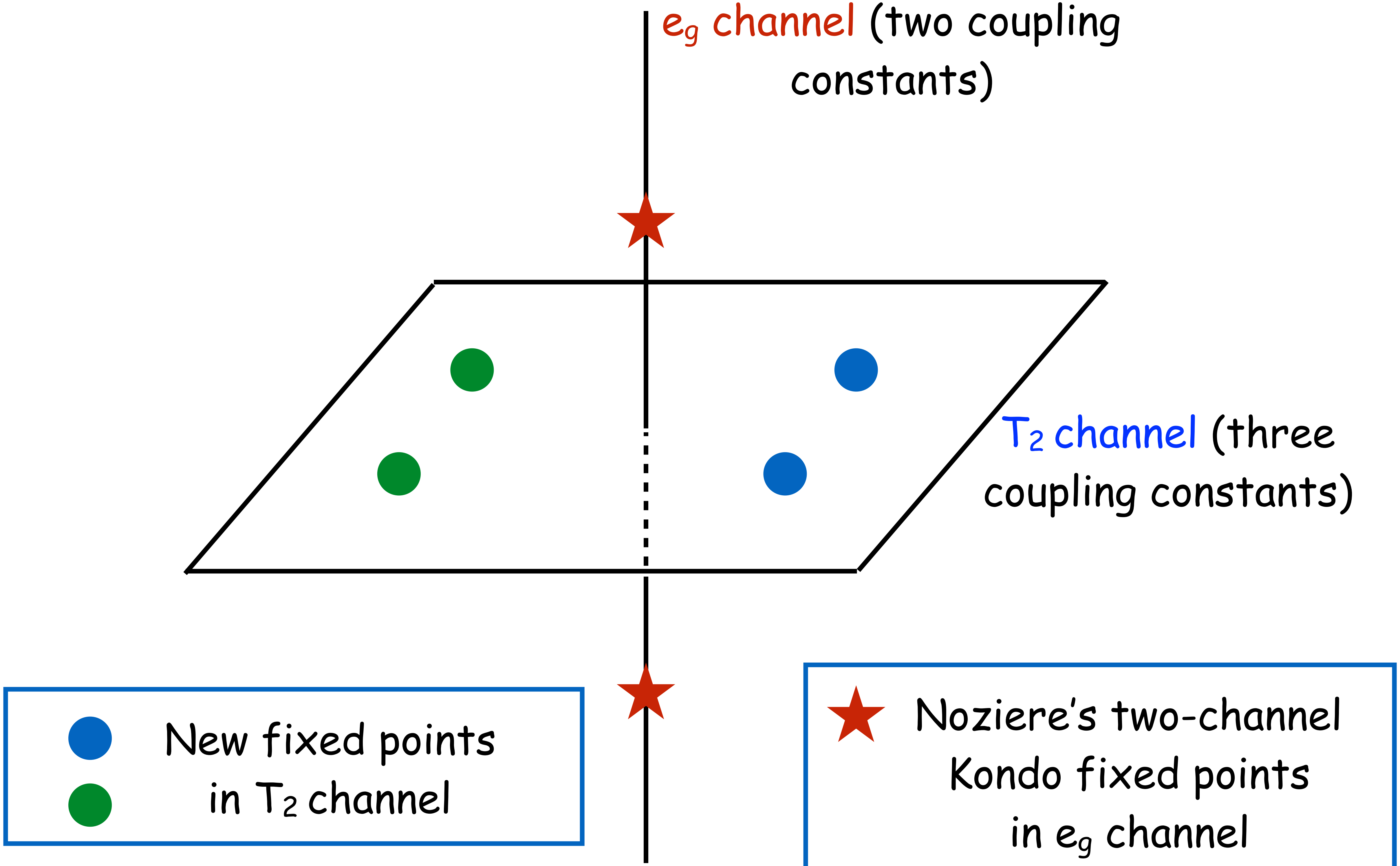


Only two are shown

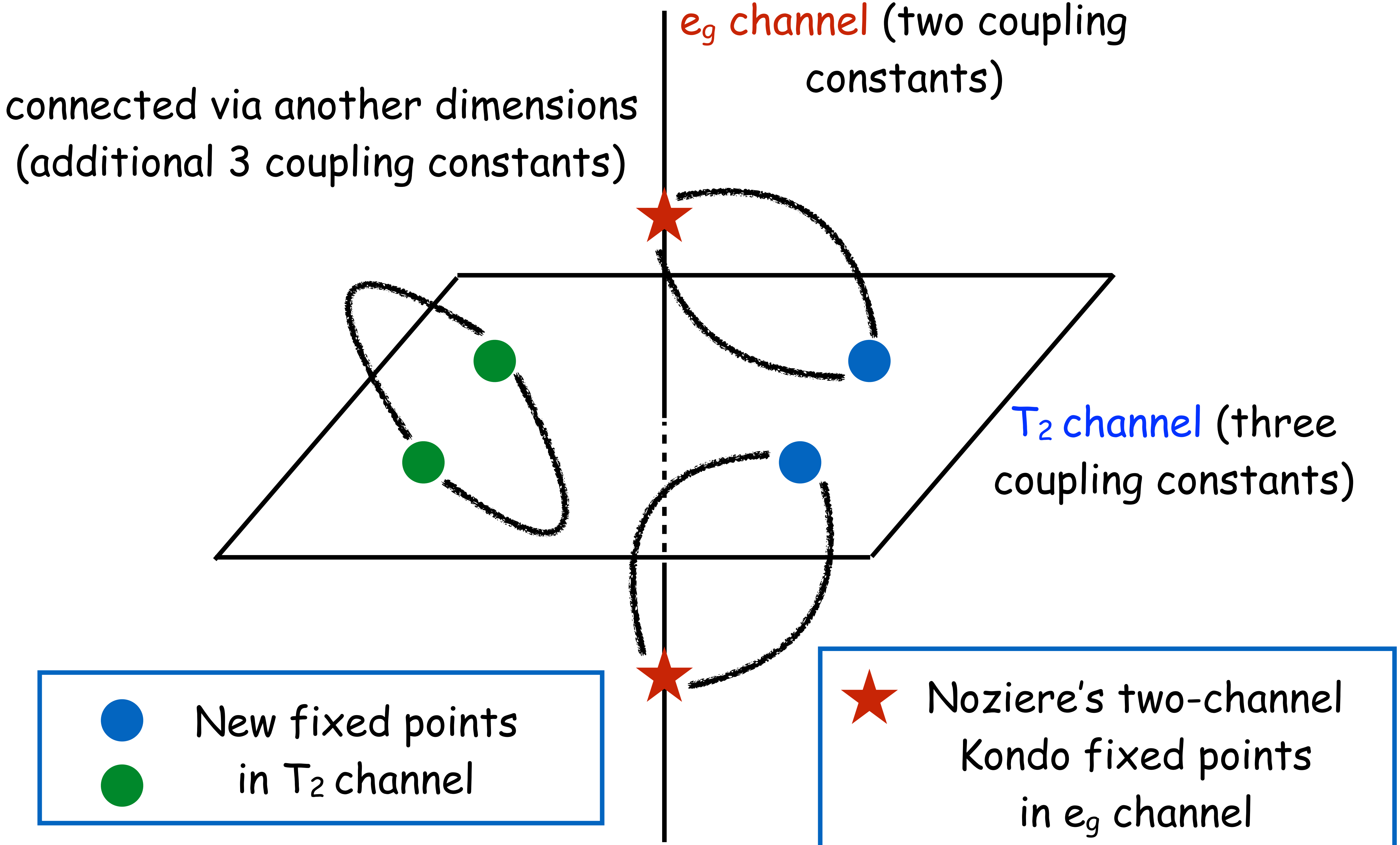
Fixed points in RG (up to third order)



Fixed points in RG (up to third order)



Fixed points in RG (up to third order)



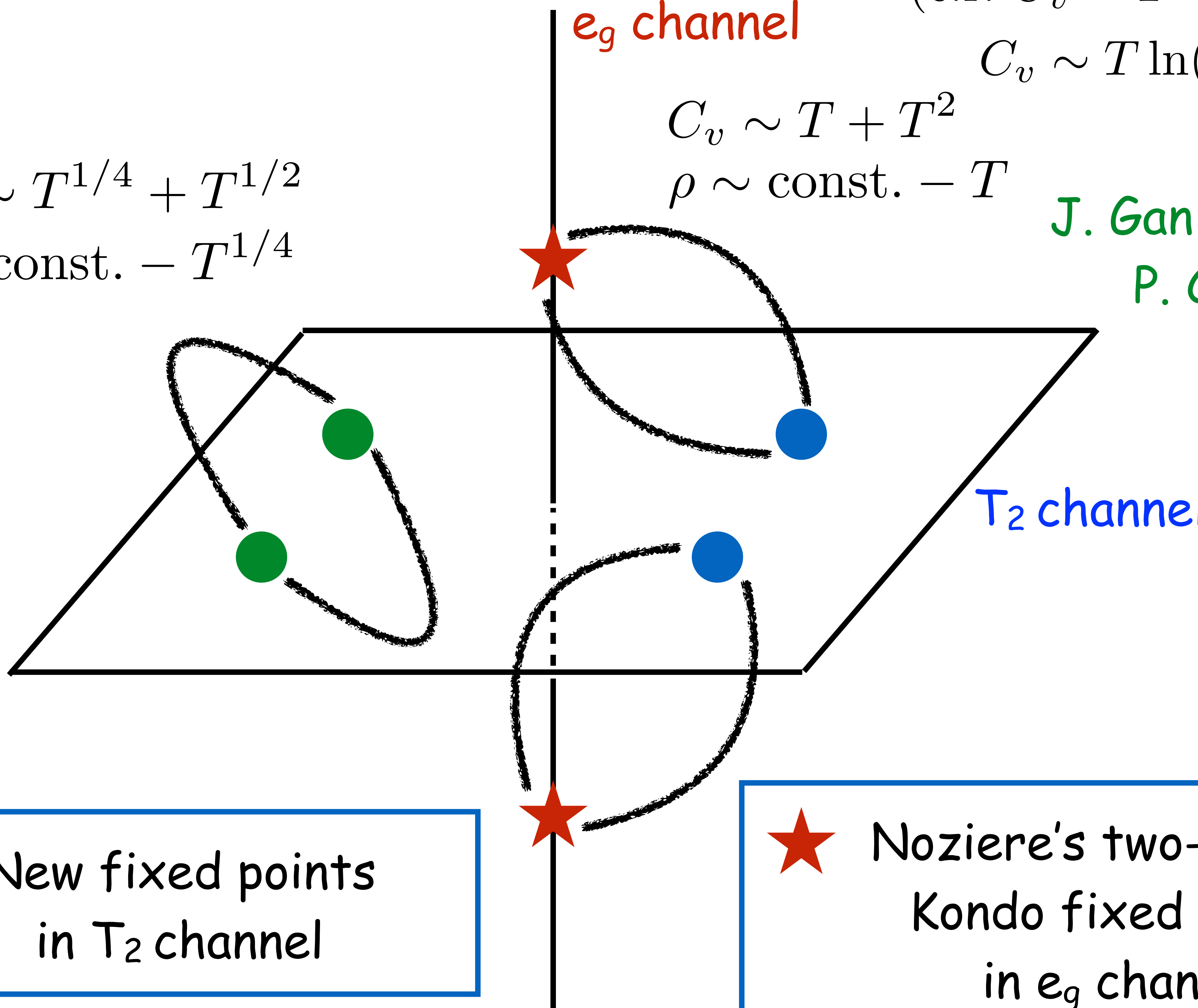
Fixed points in RG (up to third order)

(c.f. $C_v \sim T^{4/(k+2)}\ k > 2,$
$C_v \sim T\ln(1/T)\ k = 2)$

$C_v \sim T + T^2$
$\rho \sim \text{const.} - T$

$C_v \sim T^{1/4} + T^{1/2}$
$\rho \sim \text{const.} - T^{1/4}$

J. Gan, N. Andrei.
P. Coleman



New fixed points
in T_2 channel

Noziere's two-channel
Kondo fixed points
in e_g channel

Summary

Pr^{2+} local moments - non-Kramers doublet

XY - Quadrupolar   Ising - Octupolar $Pr(TM)_2X_{20}$

Landau-Ginzburg Theory

Coexisting Quadrupolar and Octupolar orders at low temperature, Two thermal transitions

Magnetostrsiction

Very useful way to detect multipolar order !

Multipolar “Kondo" Effect

Beyond two-channel Kondo effect