Nucleon form factors from PACS10 configuration

Takeshi Yamazaki

University of Tsukuba

Center for Computational Sciences

PACS Collaboration: E. Shintani, N. Tsukamoto,

Y. Aoki, K.-I. Ishikawa, Y. Kuramashi, S. Sasaki

Refs. PRD98:7:074510(2018), PRD99:1:014510(2019)

Lattice QCD workshop @ La Posada de Santa Fe, August 26-30, 2019

Outline

- Introduction
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  - Lattice QCD status
  - Calculation method
  - Simulation parameters
- Results
  - Axialvector, Tensor, and Scalar couplings
  - Electric and magnetic form factors
  - Axialvector form factor
  - Induced pseudoscalar form factor
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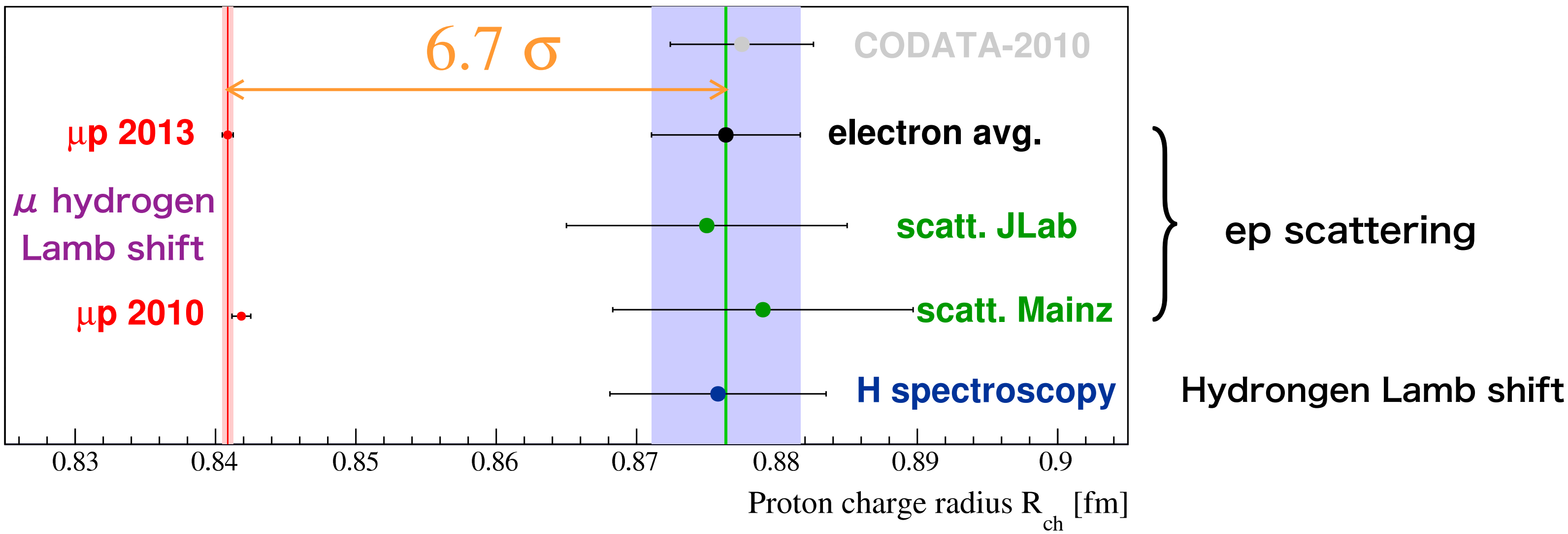
Proton size puzzle

Proton charge radius : $G_E(q^2) = 1 - \frac{q^2}{6}\langle r^2\rangle_E + \mathcal{O}(q^4)$

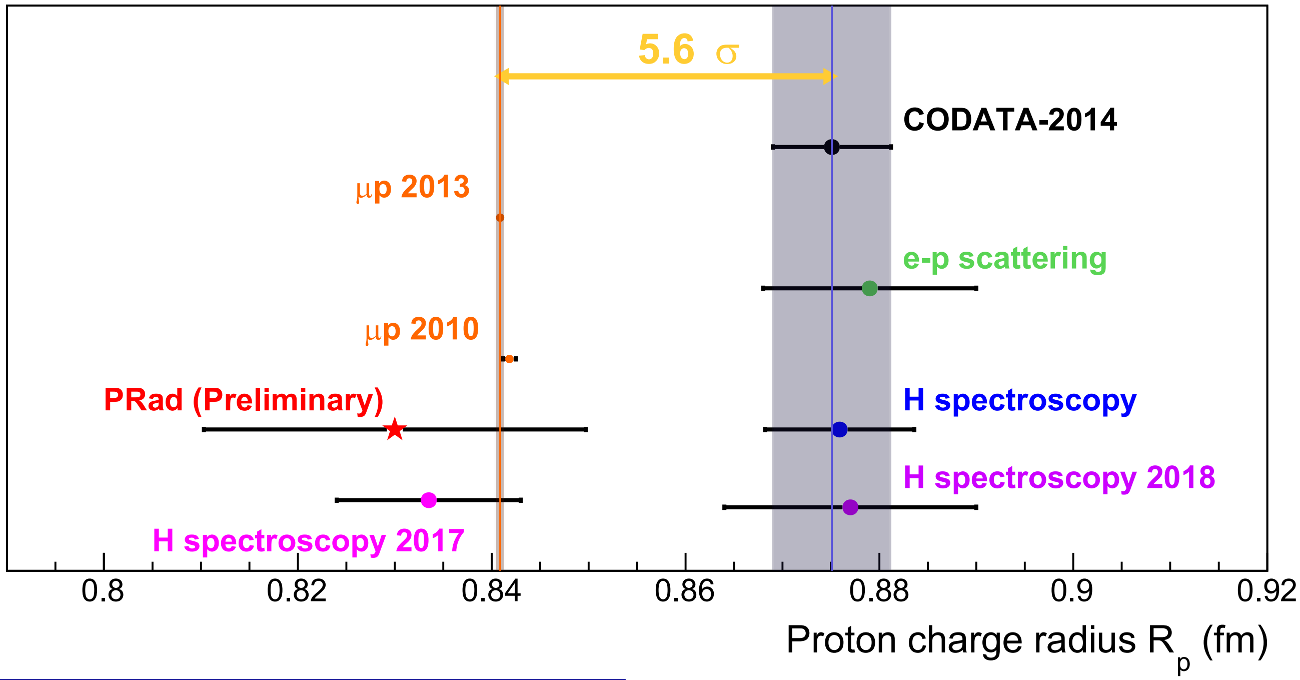


[Arrington, arXiv:1506.00873]

Slide from S. Sasaki in QCDdownunder2017

$\langle r^2\rangle_E$ differs in μ-proton and e-proton experiments.

Proton size puzzle: recent status



PRad Preliminary result:
R_p = 0.830 ± 0.008 (stat.) ± 0.018 (syst.) fm

Nilanga Liyanage 2018 DNP Meeting

NSAC Meeting NSF NP Overview NOV-2018

Slide from A. Opper, NSAC Meeting

Discrepancies in e-proton
recent e-proton agree with μ-proton $\Rightarrow$ More complicated situation

Several experiments proposed to resolve this puzzle

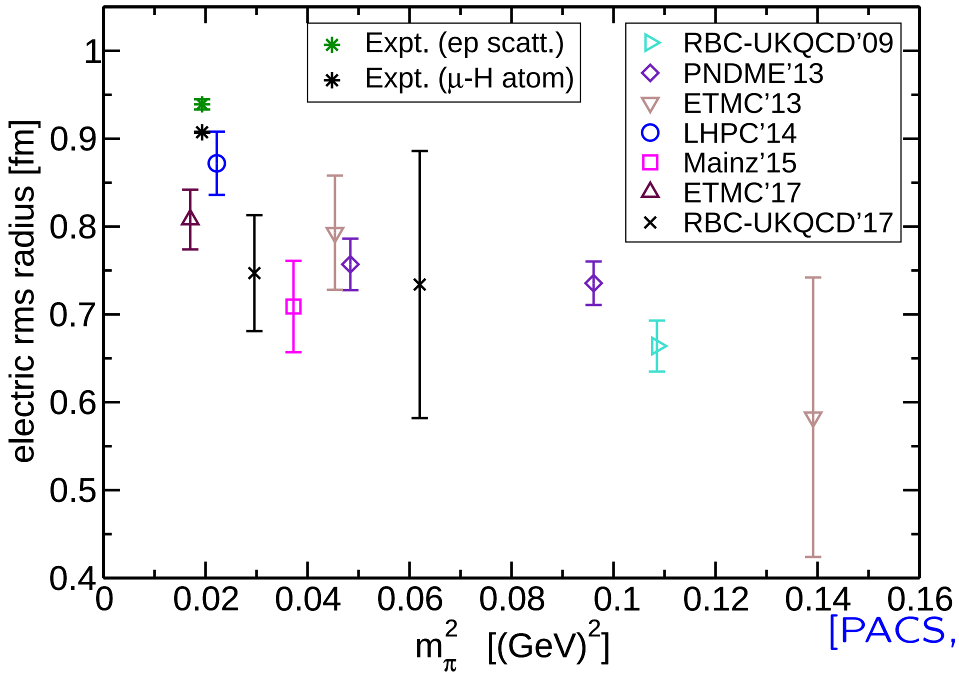
e.g. ULQ^2 Collaboration (Tohoku Univ., Japan)

Lattice QCD can calculate $\langle r_E^2 \rangle$ from the first principle.

Status of Lattice QCD at ∼2017

Isovector charge radius

$$G_E^v(q^2) = G_E^p(q^2) - G_E^n(q^2) = 1 - \frac{1}{6}q^2\langle r_E^2\rangle_v + \cdots$$



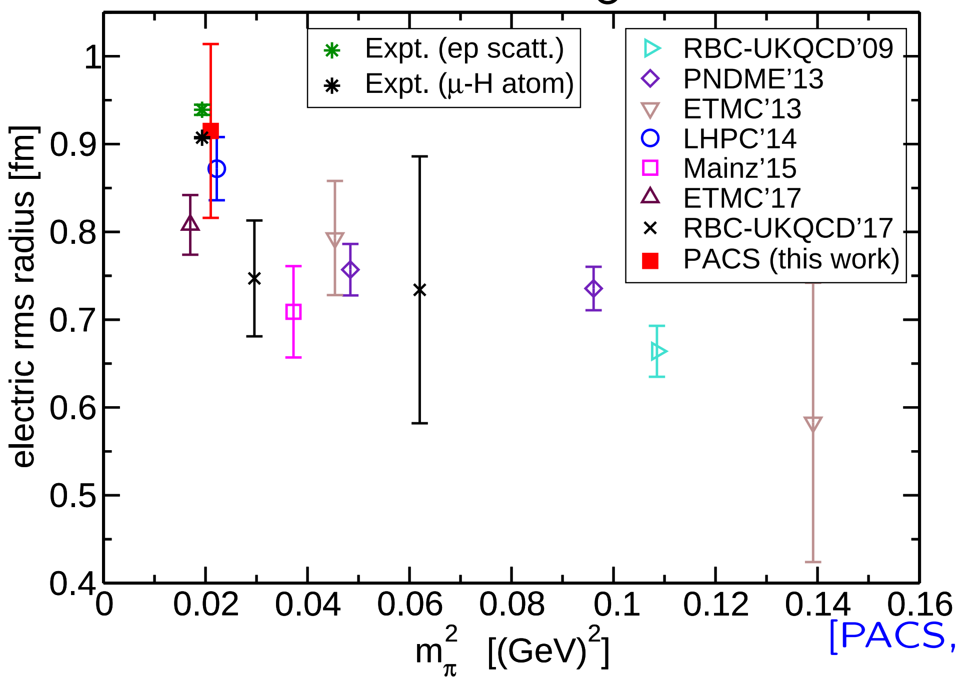
[PACS, PRD98:7:074510(2018)]

Our goal of nucleon form factor calculation

Status of Lattice QCD at ∼2017

Isovector charge radius

$$G_E^v(q^2) = G_E^p(q^2) - G_E^n(q^2) = 1 - \frac{1}{6}q^2\langle r_E^2\rangle_v + \cdots$$



[PACS, PRD98:7:074510(2018)]

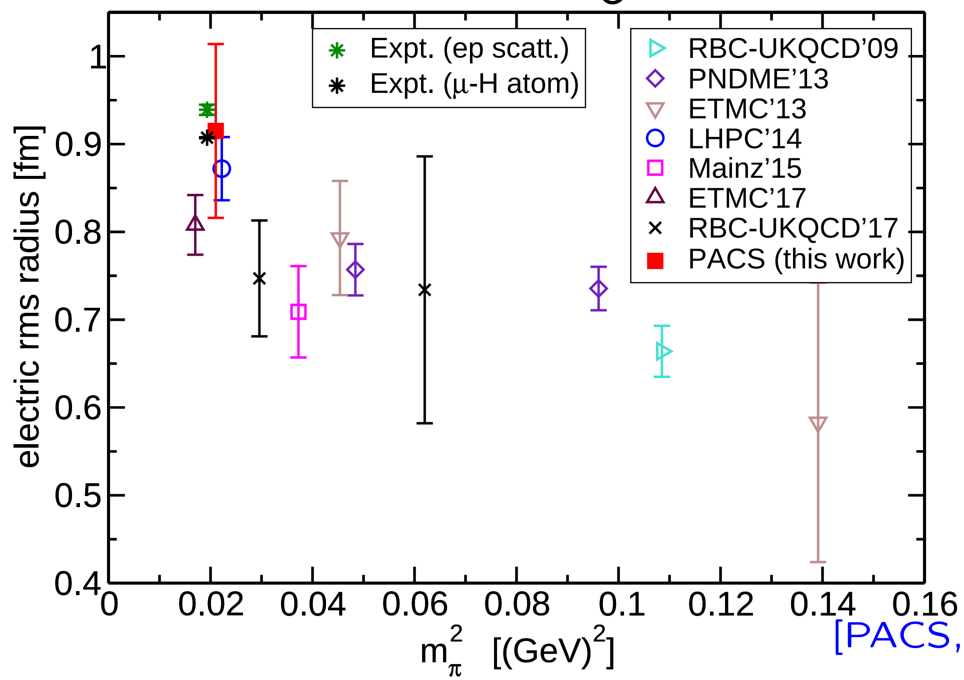
Our goal of nucleon form factor calculation

near $m_\pi^{\text{phys}} = 0.135$ GeV

Status of Lattice QCD at ~2017

Isovector charge radius

$$G_E^v(q^2) = G_E^p(q^2) - G_E^n(q^2) = 1 - \frac{1}{6}q^2\langle r_E^2\rangle_v + \cdots$$



[PACS, PRD98:7:074510(2018)]

Our goal of nucleon form factor calculation

near $m_\pi^{\text{phys}} = 0.135$ GeV $\rightarrow$ reduce error $\lesssim$ 1% at m_π^{phys}

$\langle r_E^2\rangle$: judge two experiments

g_A: calculation with 3-pt; cf) 2-pt calculation [CalLat, Nature:558:91(2018)]

Calculation method

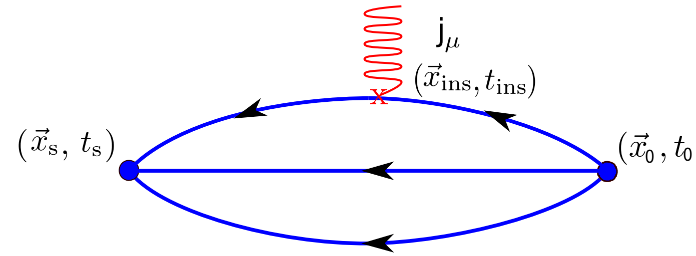


Figure [ETM, arXiv:1812.10311]

Nucleon 3-point function

$$C^3_{j_\mu}(t,q) = \langle 0|N(\vec{0},t_s)j_\mu(\vec{q},t)\overline{N}(-\vec{q},t_0)|0\rangle \xrightarrow[t_s-t\gg 1]{t-t_0\gg 1} \langle N|j_\mu|N\rangle$$

$$j_\mu = \begin{cases} j^{\rm EM}_\mu & ; \quad G_E(q^2), G_M(q^2) \\ A_\mu & ; \quad g_A, F_A(q^2), F_P(q^2) \\ T_{\mu\nu} & ; \quad g_T \\ S & ; \quad g_S \end{cases}$$

- isovector form factors ($m_u = m_d$) w/o disconnected diagrams
- large $t_{\rm sep} = t_s - t_0$ or investigation of $t_{\rm sep}$ dependence

  tune smearing parameters for early plateau

  using wider smearing than other groups

Simulation parameters

$N_f = 2+1$ Iwasaki gauge + stout smeared Wilson clover quarks

$\beta = 1.82$ corresponding to $a^{-1} = 2.3$ GeV

L^3T	96^4	128^4					
La[fm]	8.1	10.8					
m_π[MeV]	146	135					
Smear	Exp	Exp				Gauss	
t_{sep}	15	10	12	14	16	13	16
N_{meas}	12800	2560	5120	6400	10240	2560	8960

Preliminary result

96^4 lattice [PRD98:7:074510(2018)]

generated by K computer (HPCI Strategic Program Field 5) [PoS(LATTICE2015):075(2016)]

128^4 lattice (PACS10 configuration) [PRD99:1:014510(2019)]

parameters of configuration [PRD99:1:014504(2019)]

PACS10 configuration: $La > 10$ fm at m_π^{phys}

$L^3T = 128^4, 160^4, 256^4$ at $a = 0.08, 0.06, 0.04$ fm, respectively

remove main systematic uncertainties in lattice QCD calculation

Simulation parameters

$N_f = 2+1$ Iwasaki gauge + stout smeared Wilson clover quarks

$\beta = 1.82$ corresponding to $a^{-1} = 2.3$ GeV

L^3T	96^4	128^4					
La[fm]	8.1	10.8					
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Preliminary result

96^4 lattice [PRD98:7:074510(2018)] represented by PACS'18

Exponential smearing: parameters tuned for smear-local 2pt.

128^4 lattice (PACS10 configuration) [PRD99:1:014510(2019)]

Cost reduction comparing 96^4 calculation

Exponential smearing: parameters tuned for 3pt. ($\sim \times 2$)

all-mode-averaging method ($\sim \times 3$) [Blum *et al.*, PRD88:094503(2013);$\cdots$]

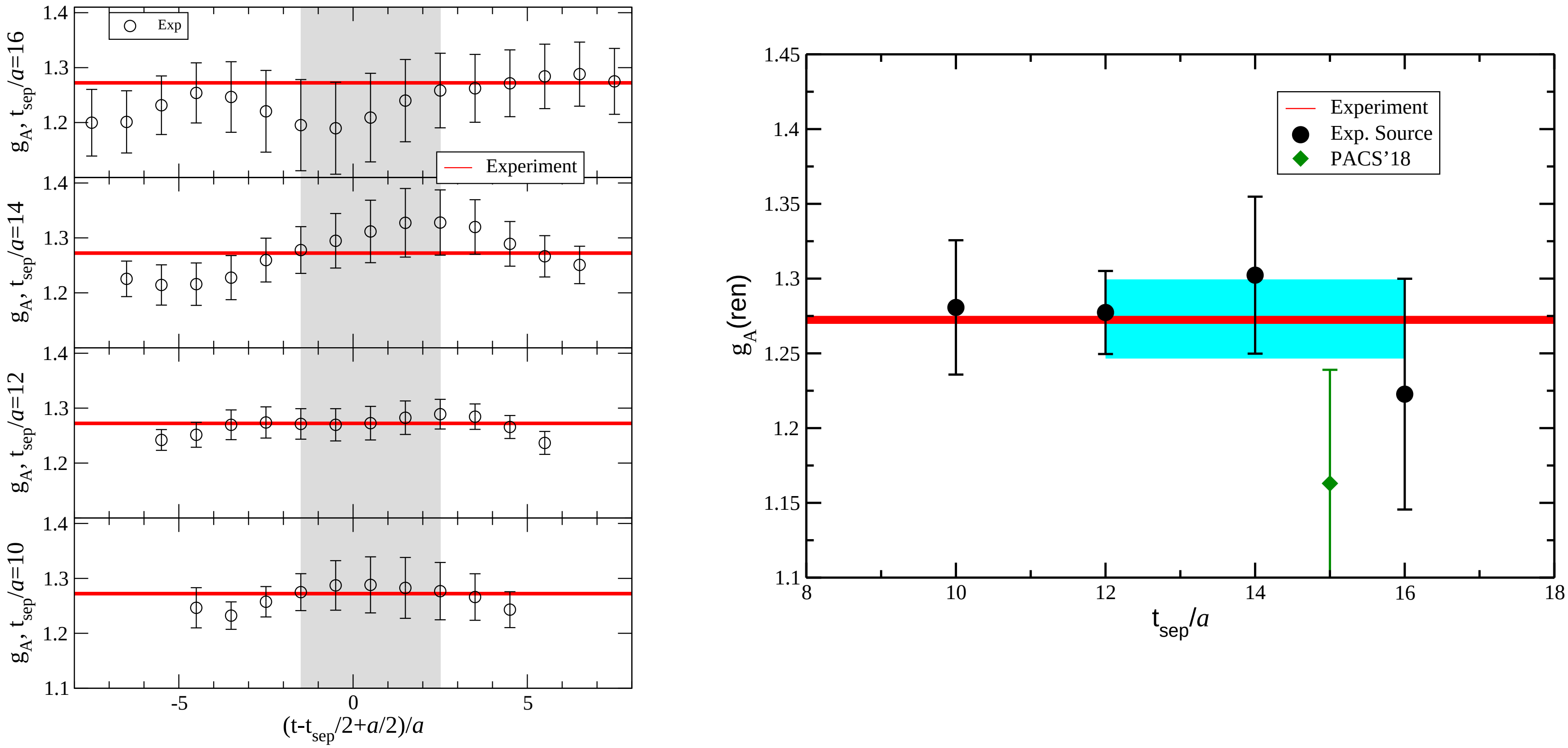
deflated low mode ($\sim \times 10$) [Lüscher, JHEP0707:81(2007)]

Gaussian smearing (preliminary) $\omega = 8, N = 110$ w/ APE step

Results

Axialvector coupling [PACS, PRD99:1:014510(2019)]

$Z_A C_A^3(t,0)/C_N(t_s)$, $C_N(t)$: 2-pt function, Z_A calculated in SF scheme



averaging data in shaded region

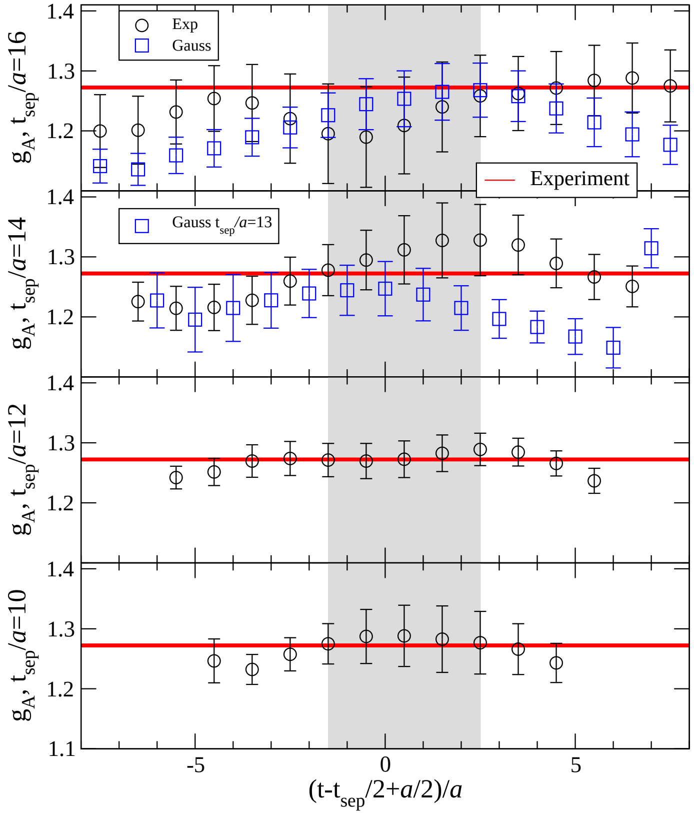
Small $t_{\rm sep}$ and t dependence

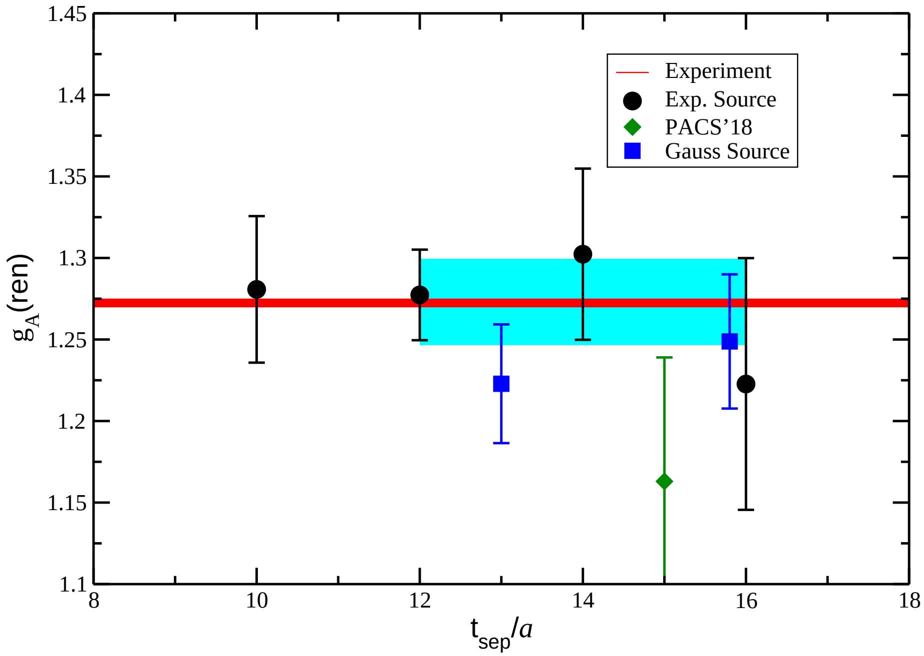
Consistent with experimental value

Central value: Exp source averaging $t_{\rm sep}/a = 12, 14, 16$

Axialvector coupling [PACS, PRD99:1:014510(2019)]

$Z_A C_A^3(t,0)/C_N(t_s)$, $C_N(t)$: 2-pt function, Z_A calculated in SF scheme





averaging data in shaded region

Gauss source preliminary

Smaller error than exp source at $t_{\rm sep}/a = 16$

Small $t_{\rm sep}$ and t dependence

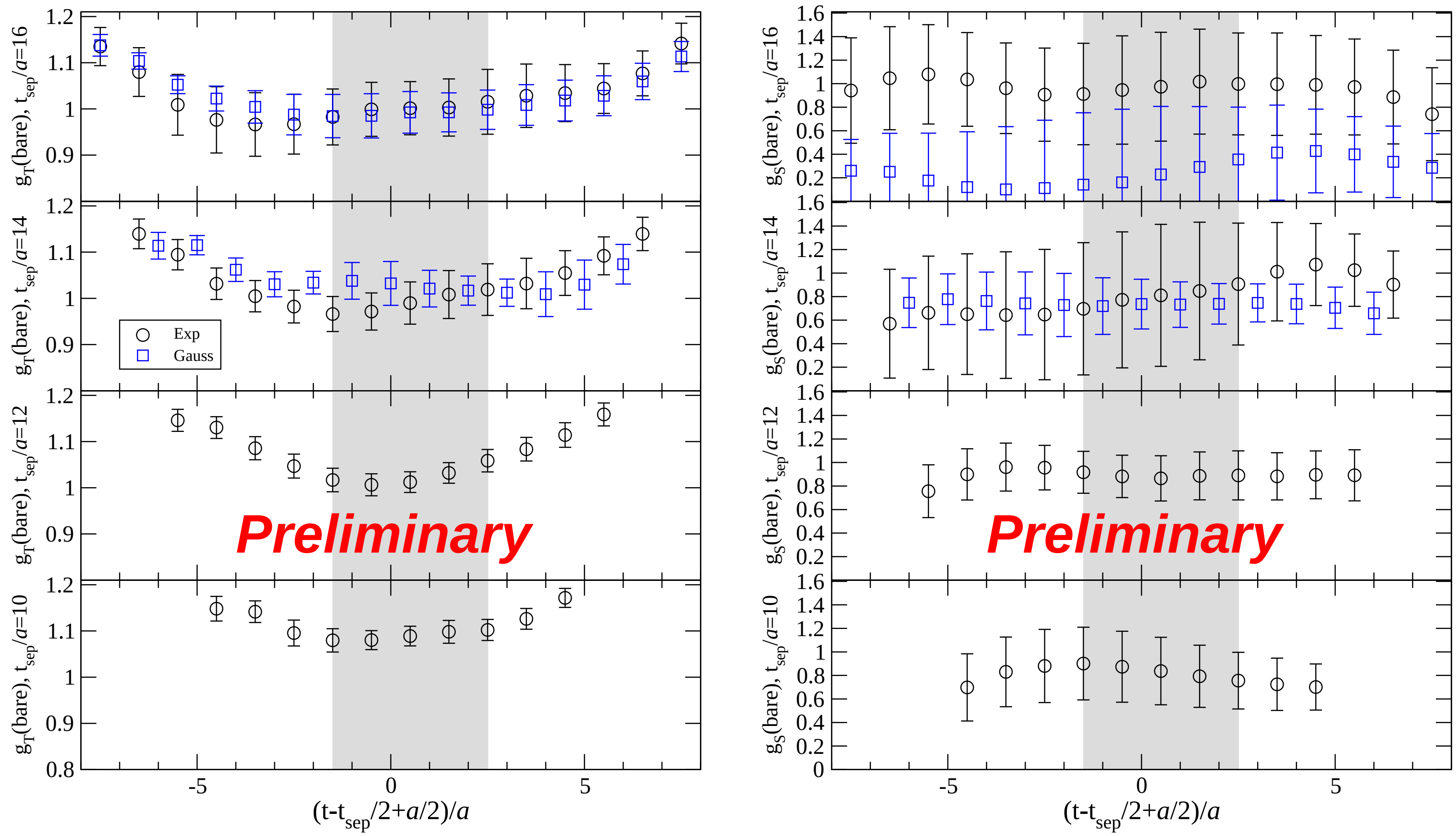
Consistent with experimental value

Central value: Exp source averaging $t_{\rm sep}/a = 12, 14, 16$

Bare Tensor and Scalar couplings

Tensor coupling $C_T^3(t,0)/C_N(t_s)$

Scalar coupling $C_S^3(t,0)/C_N(t_s)$



averaging data in shaded region

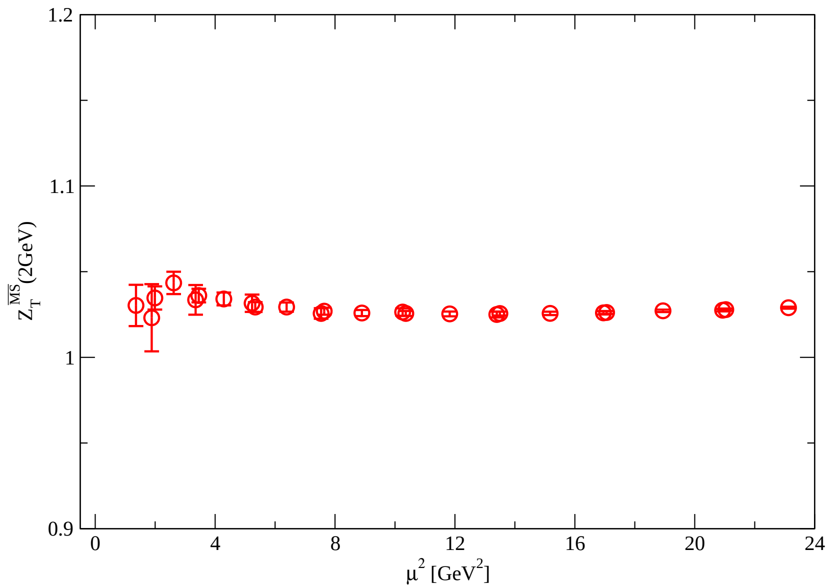
g_T : Little t dependence seen

g_S : Large error at large $t_{\rm sep}$

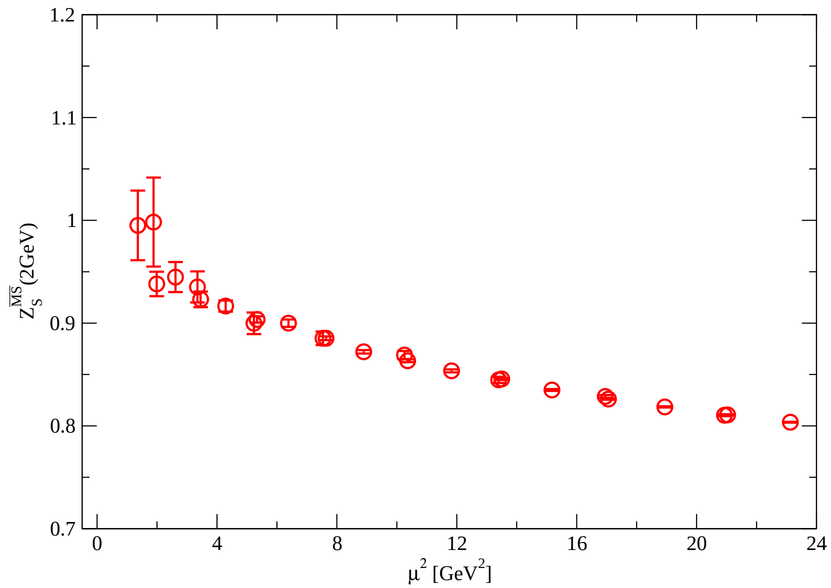
$Z_T^{\overline{\mathsf{MS}}}(2\mathrm{GeV})$ and $Z_S^{\overline{\mathsf{MS}}}(2\mathrm{GeV})$ calculated by N. Tsukamoto

RI/SMOM scheme in $\mu = 1$–5 [GeV] $\rightarrow$ convert to $\overline{\mathsf{MS}}$ scheme at 2 GeV

$Z_T^{\overline{\mathsf{MS}}}(2\mathrm{GeV})$



$Z_S^{\overline{\mathsf{MS}}}(2\mathrm{GeV})$



μ dependence is seen.

- lattice artifact at large μ
- non-perturbative effect at small μ

Remove μ dependence by fit with $c_0 + c_1\mu^2 + c_2\mu^4 + c_{-1}/\mu^2$

$Z_T^{\overline{\text{MS}}}(\text{2GeV})$ and $Z_S^{\overline{\text{MS}}}(\text{2GeV})$ calculated by N. Tsukamoto

RI/SMOM scheme in $\mu = 1$–5 [GeV] $\rightarrow$ convert to $\overline{\text{MS}}$ scheme at 2 GeV

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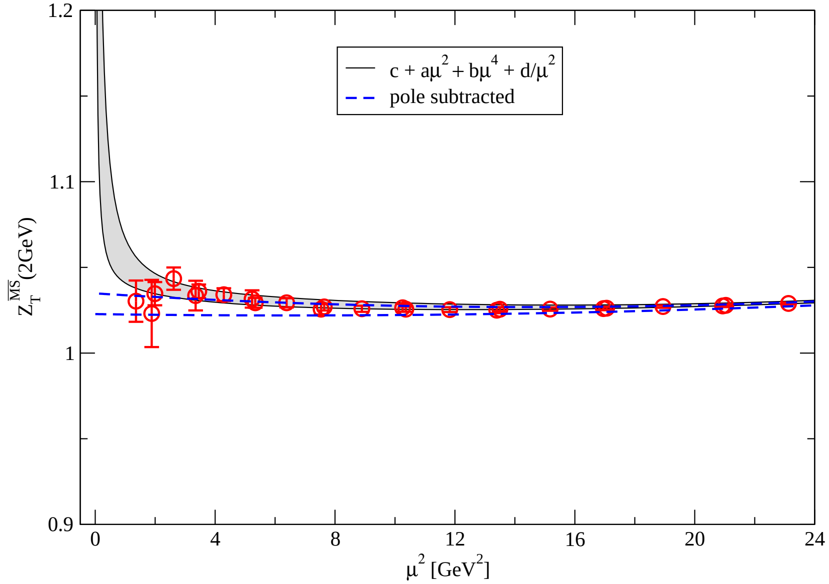
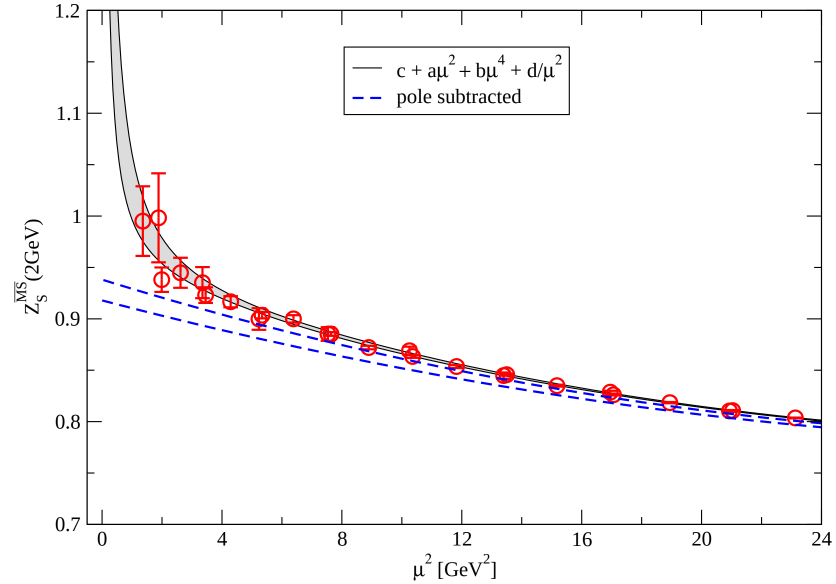


$Z_S^{\overline{\text{MS}}}(\text{2GeV})$



Remove μ dependence by fit with $c_0 + c_1\mu^2 + c_2\mu^4 + c_{-1}/\mu^2$

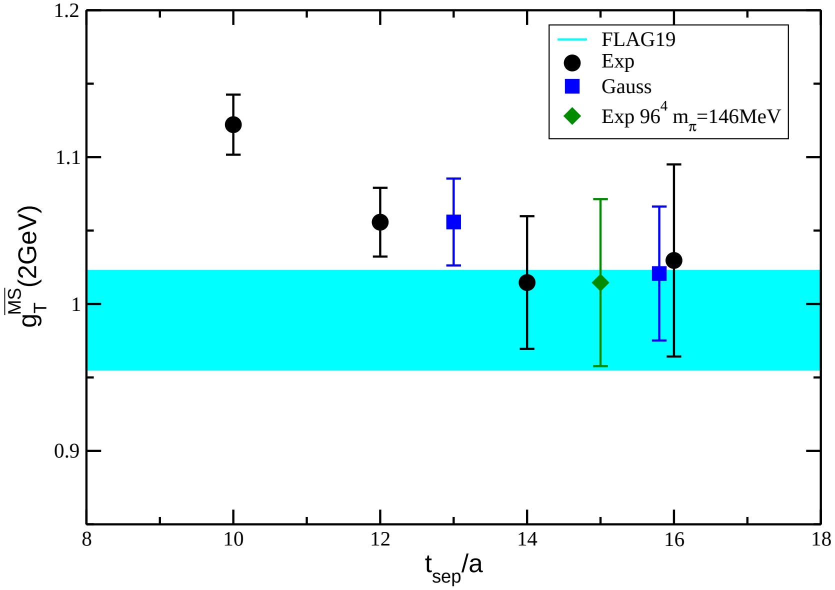
Preliminary results

$$
\begin{aligned}
Z_T^{\overline{\text{MS}}}(\text{2GeV}) &= 1.030(5)_{\text{stat}}(X)_{\text{sys}} \\
Z_S^{\overline{\text{MS}}}(\text{2GeV}) &= 0.933(7)_{\text{stat}}(Y)_{\text{sys}}
\end{aligned}
$$

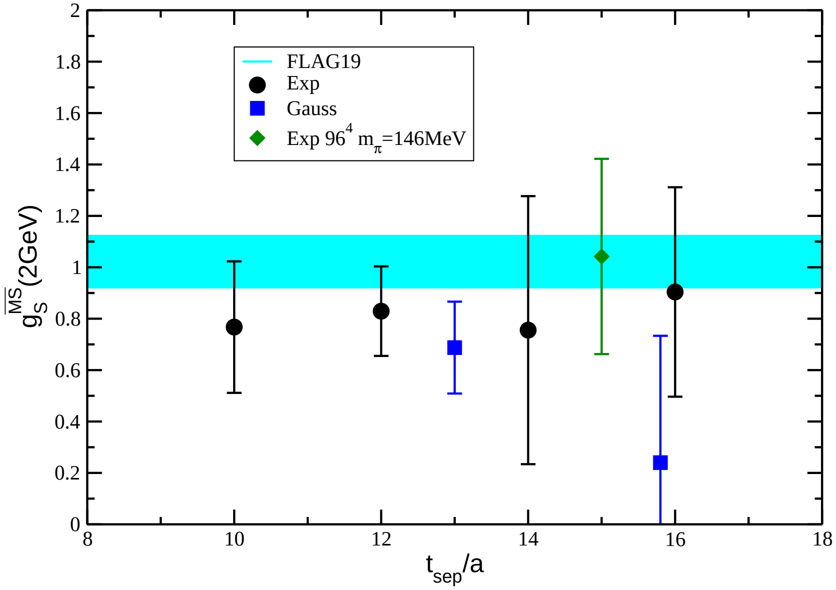
$g_T^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$ and $g_S^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$ calculated by N. Tsukamoto

Preliminary results

$g_T^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$



$g_S^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$



g_T : t_{sep} dependence seen; $t_{\mathrm{sep}}/a \gtrsim 12$ is safe from excited state

g_S : small t_{sep} dependence; large statistical error

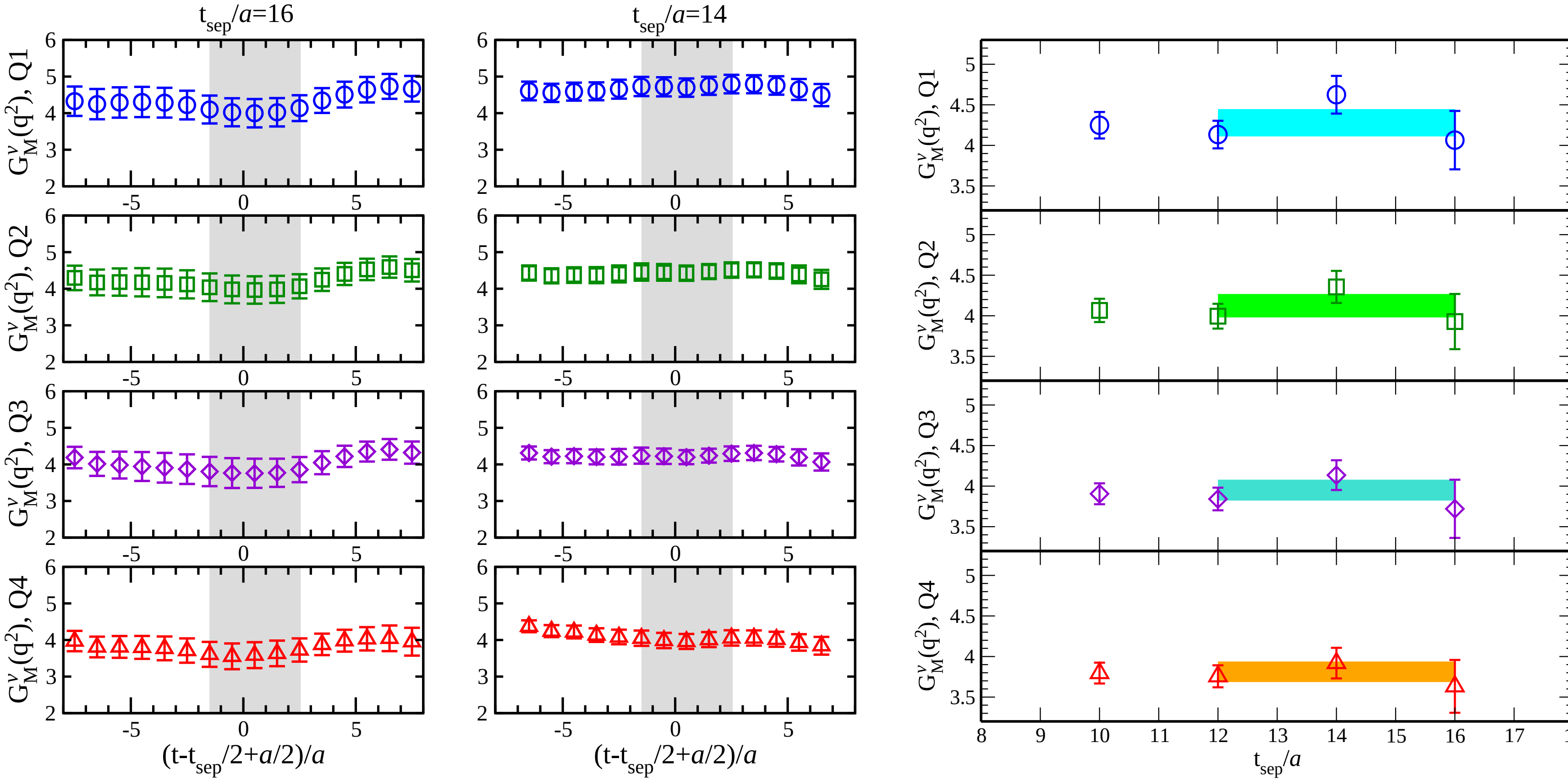
Consistent with PACS'18 [PRD98:7:074510(2018)]

Consistent with FLAG19 average in g_T and g_S

$t_{\rm sep}$ dependence of $G_M^v(q^2)$ [PACS, PRD99:1:014510(2019)]

Exponential source, Z_V calculated in SF scheme

Ratio of 3-pt to 2-pt functions



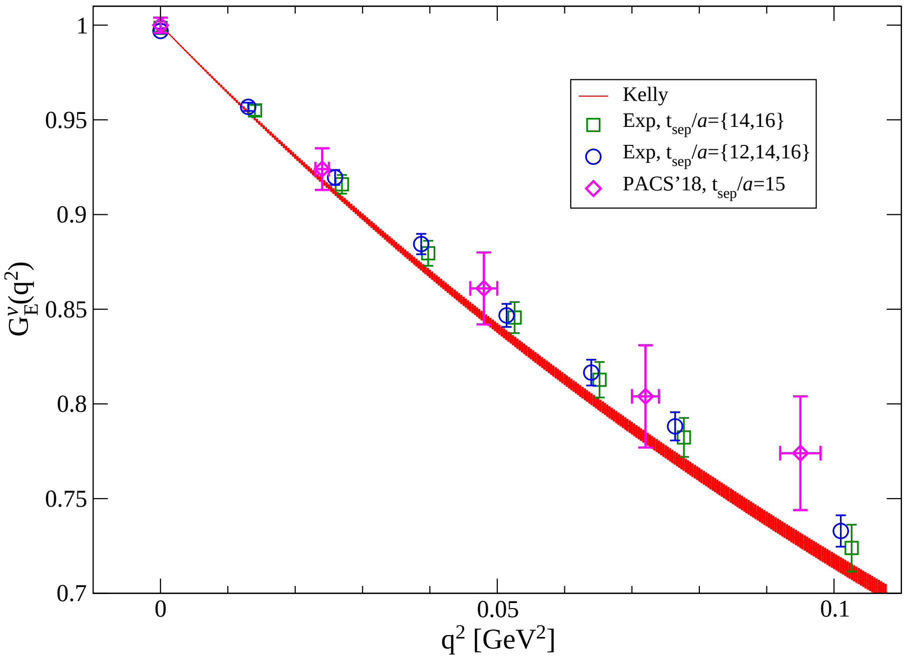
averaging data in shaded region

Small $t_{\rm sep}$ and t dependence in all q^2

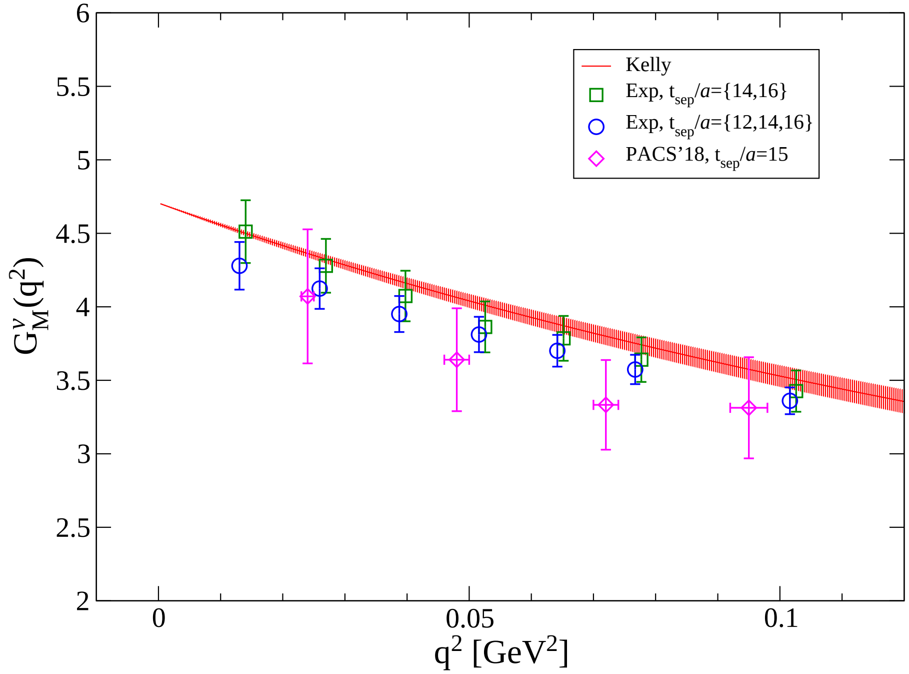
Central value: averaging $t_{\rm sep}/a = 12, 14, 16$

Isovector EM form factors [PACS, PRD99:1:014510(2019)]

Electric form factor



Magnetic form factor

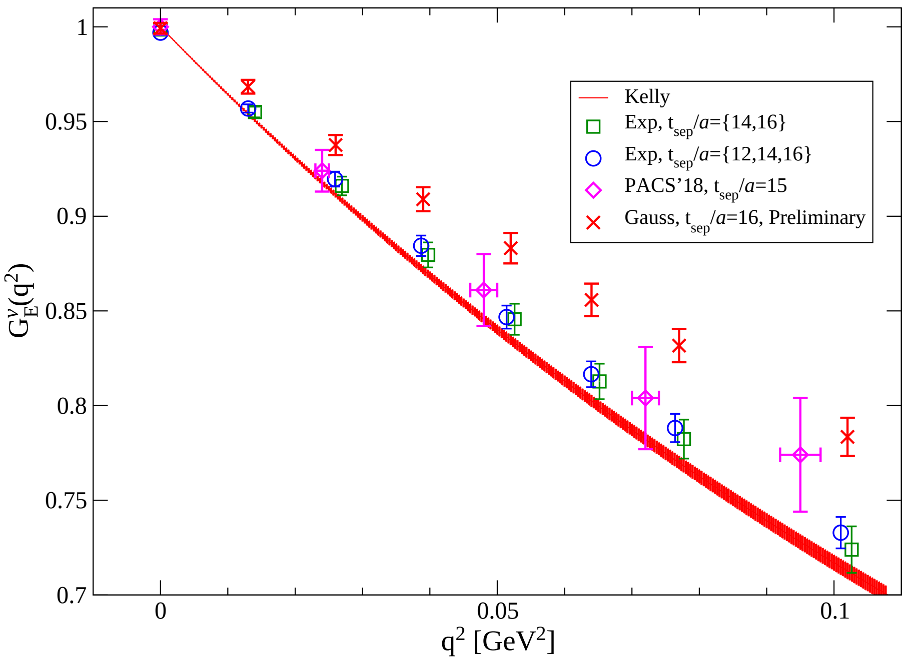


Small $t_{\rm sep}$ dependence

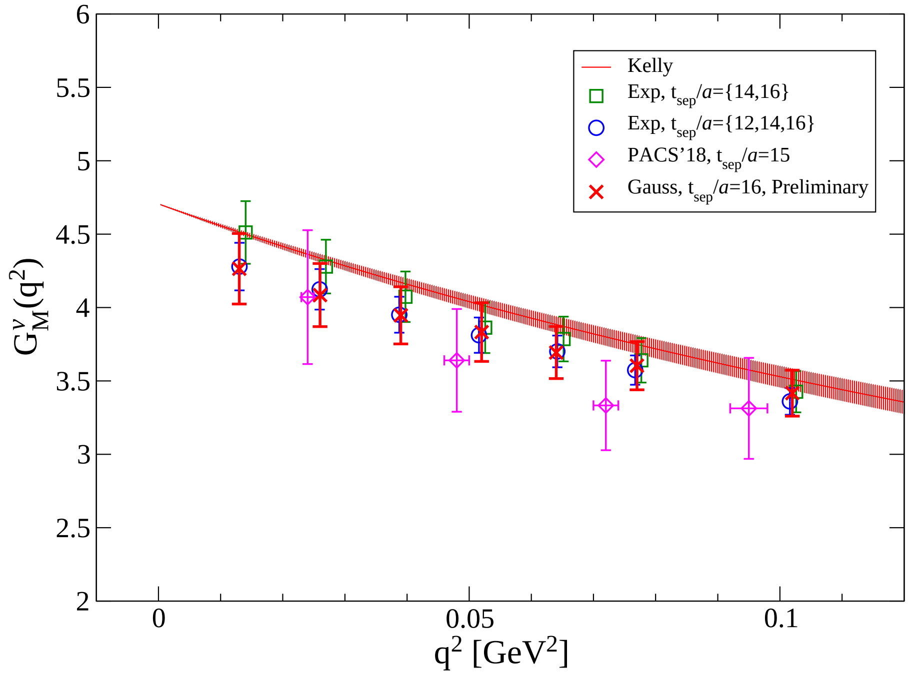
Consistent with PACS'18 [PRD98:7:074510(2018)] with much smaller error

Isovector EM form factors [PACS, PRD99:1:014510(2019)]

Electric form factor



Magnetic form factor



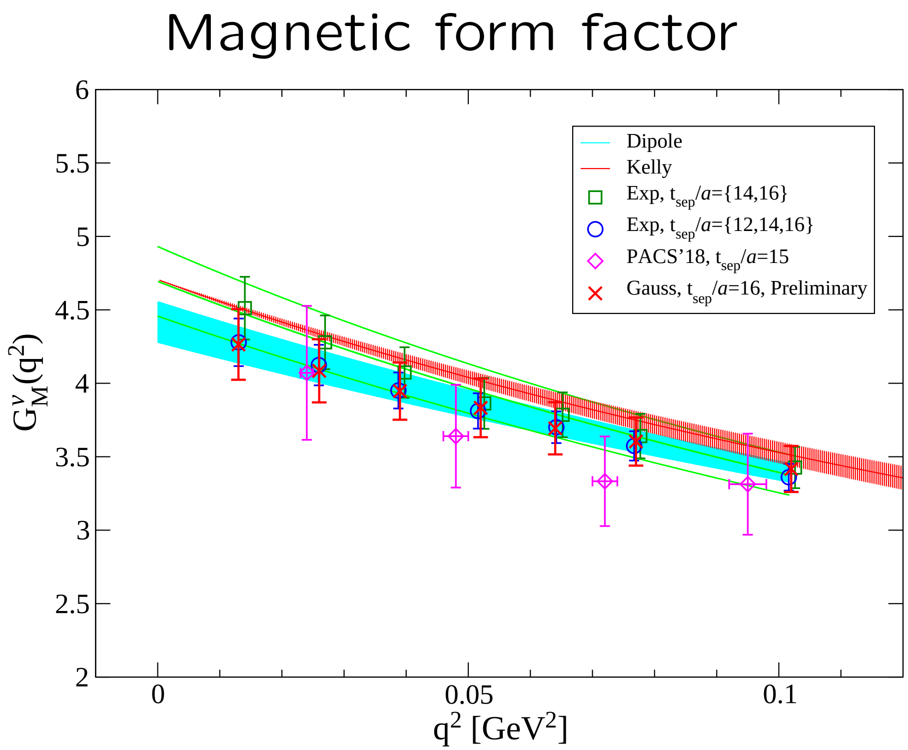
Small t_{sep} dependence

Consistent with PACS'18 [PRD98:7:074510(2018)] with much smaller error

Discrepancy in G_E with Gauss source

$\rightarrow$ under investigation, will not discuss G_E in the following

Isovector EM form factors [PACS, PRD99:1:014510(2019)]



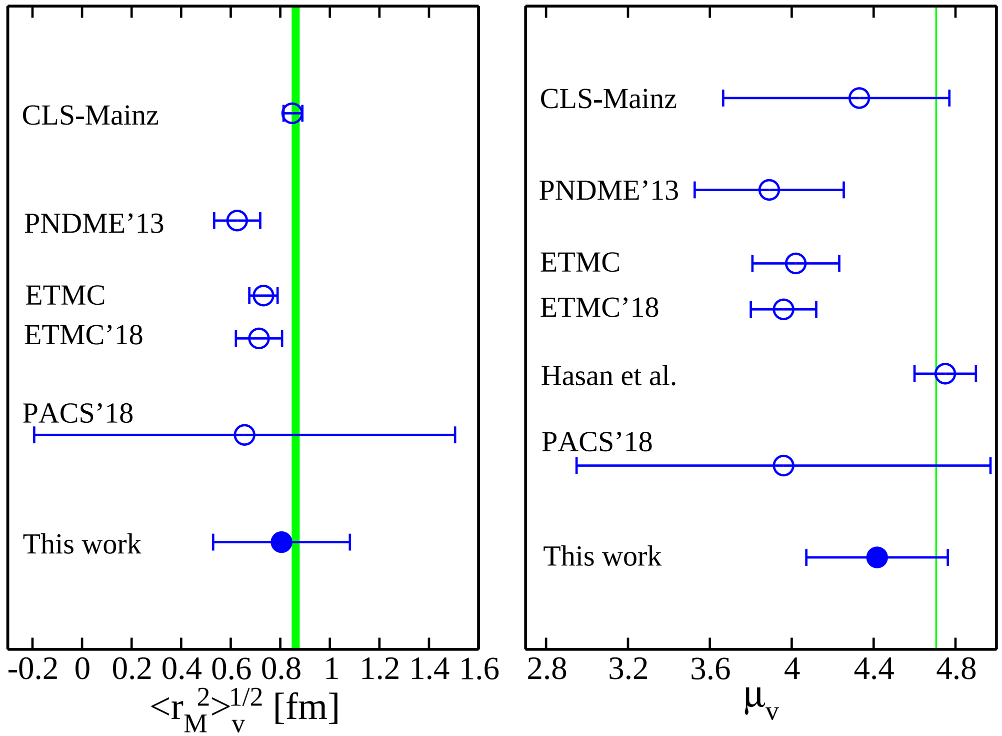
Small t_{sep} dependence

Consistent with PACS'18 [PRD98:7:074510(2018)] with much smaller error

$G_M^v(q^2)$ agrees with experimental curve.

Dipole form fit works. dipole form: $G_M^v(q^2) = \dfrac{G_M^v(0)}{(1 + \langle r_M^2 \rangle_v q^2/[12 G_M^v(0)])^2}$

Comparison of $\langle r_M^2 \rangle_v$ and μ_v [PACS, PRD99:1:014510(2019)]



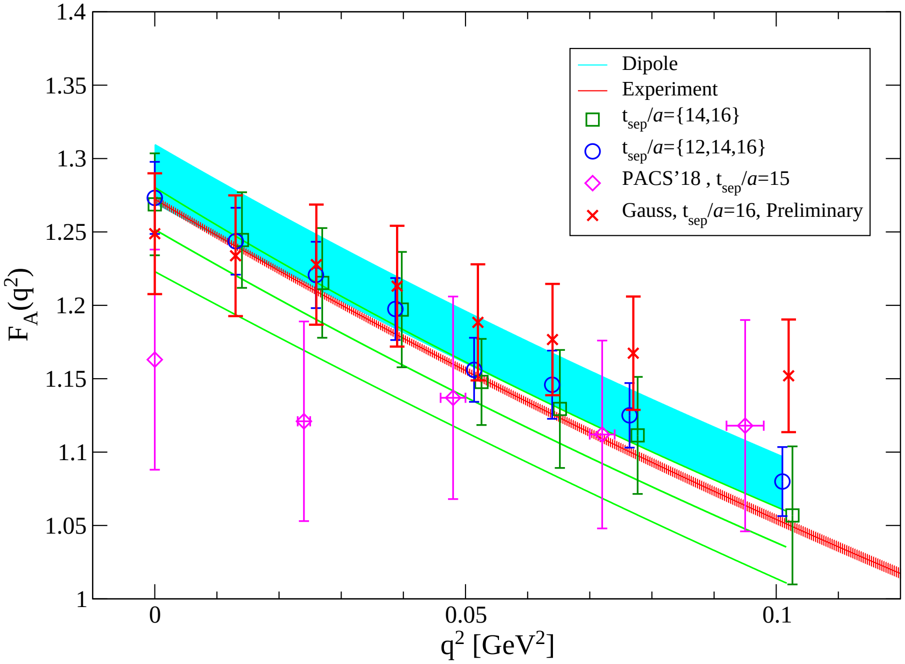
statistical and systematic errors (fit form and t_{sep} dependences) added in quadrature

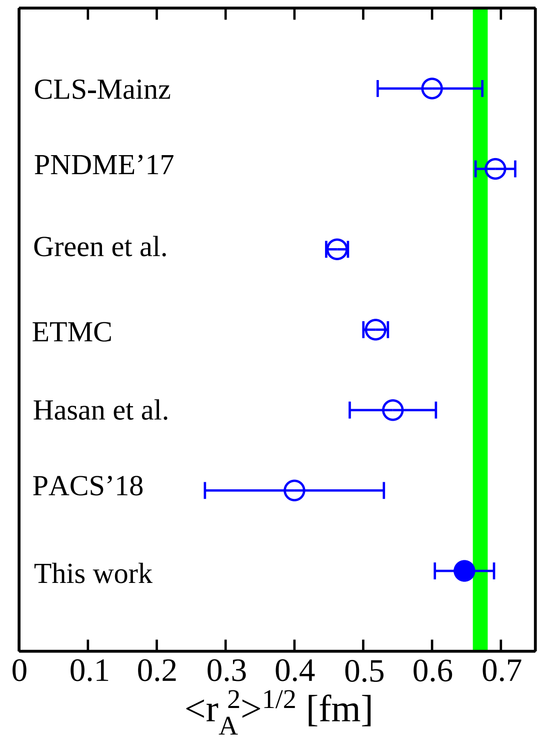
Much smaller error than PACS'18

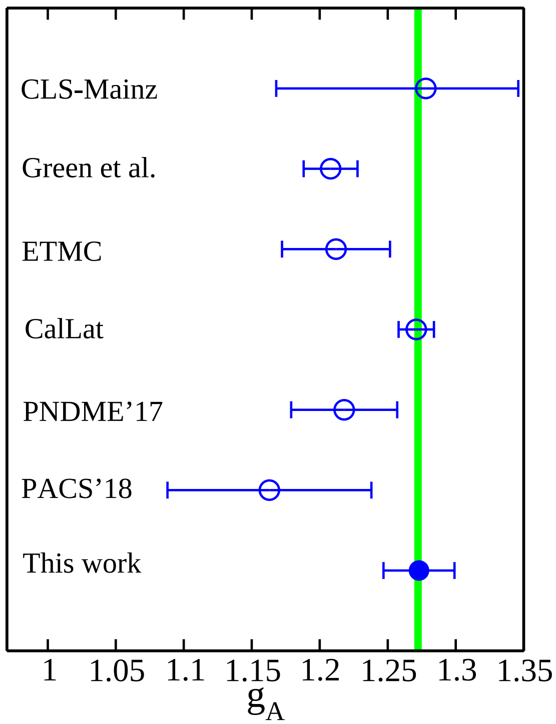
Consistent with experiment and recent lattice results

Axialvector form factor $F_A(q^2)$ [PACS, PRD99:1:014510(2019)]

Z_A calculated in SF scheme







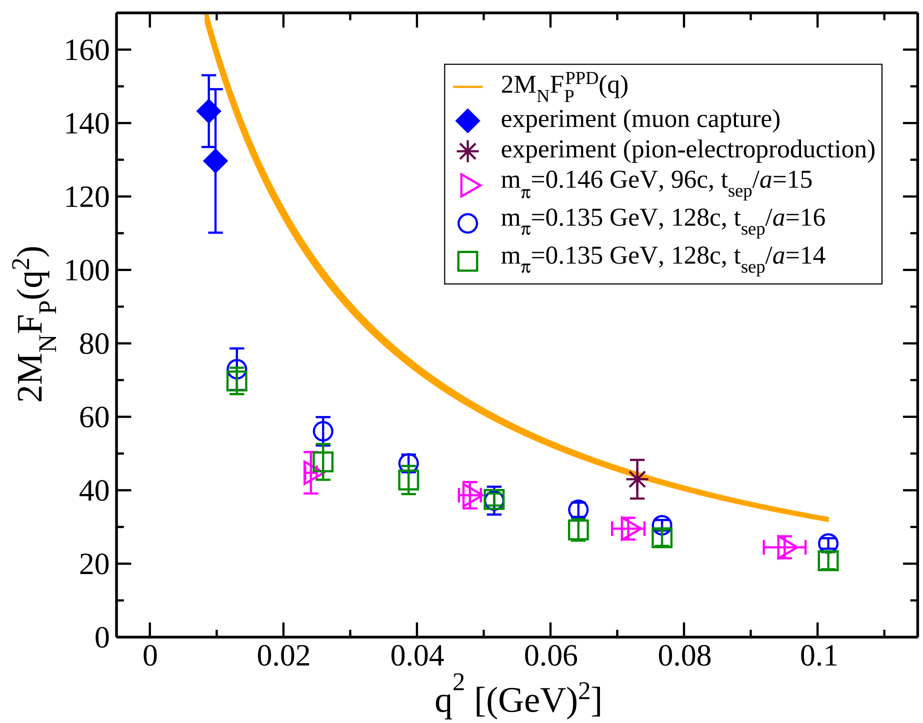
Experiment $\sqrt{\langle r_A^2\rangle} = 0.67(1)$ fm [Bernard *et al.*, JPG28:R1(2002)]

small t_{sep} dependence; Consistent with Gauss source

Consistent with PACS'18 with much smaller error

g_A and $\sqrt{\langle r_A^2\rangle}$ agree with experiment

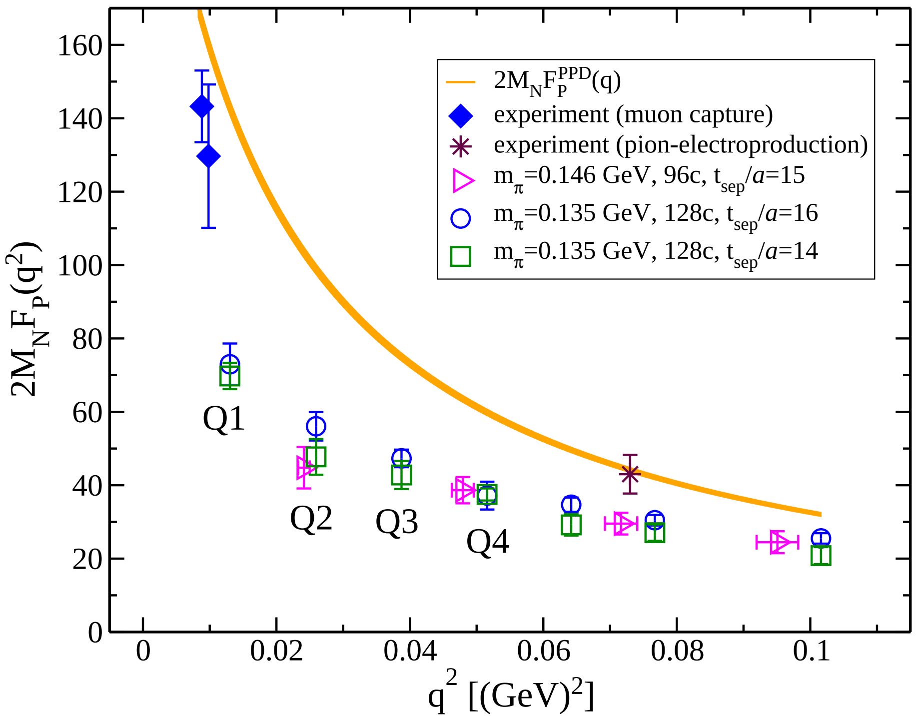
Induced pseudoscalar form factor $F_P(q^2)$ [PACS, PRD99:1:014510(2019)]

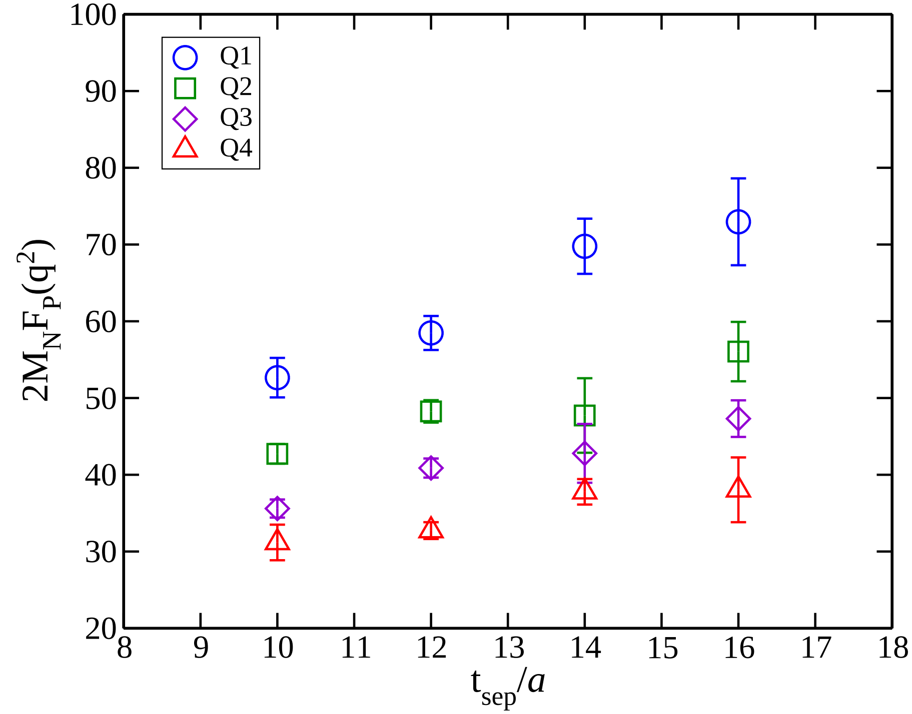


significantly smaller than experiments and PPD

Pion pole dominance (PPD): $2M_N F_P(q^2) = \dfrac{4M_N^2 F_A(q^2)}{m_\pi^2 + q^2}$

Induced pseudoscalar form factor $F_P(q^2)$ [PACS, PRD99:1:014510(2019)]





clear t_{sep} dependence → large excited state contribution

Several discussions of excited state contribution

quark mass shift in Axial Ward-Takahashi identity

[Sasaki and TY, PRD78:014510(2008); PACS, PRD98:7:074510(2018)]

πN scattering contribution in HBChPT [Bär, PRD99:5:054506(2018)]

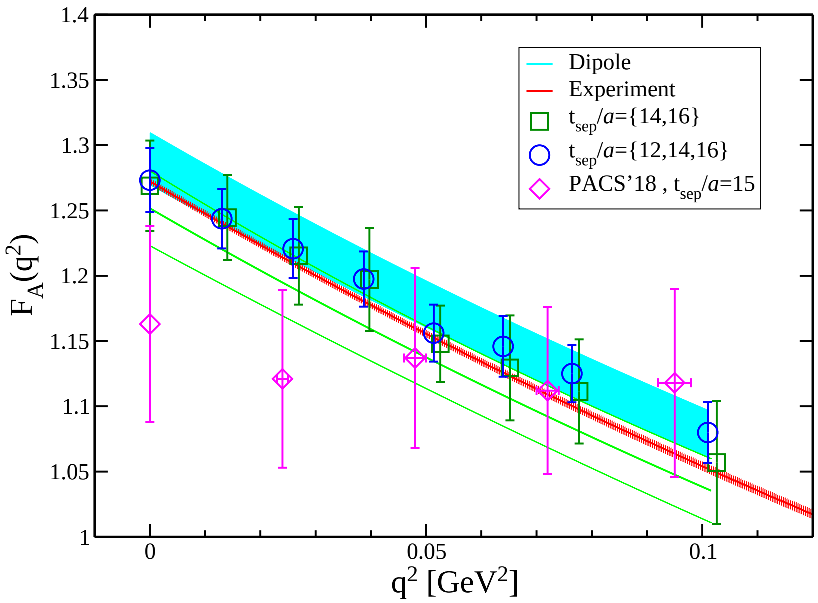
projection using $\langle N|A_i|N\rangle$ and $\langle N|A_4|N\rangle$ [Bali *et al.*, PLB789:666(2019)]

multi-exponential fits of 3-pt function [PNDME, Lattice2019]

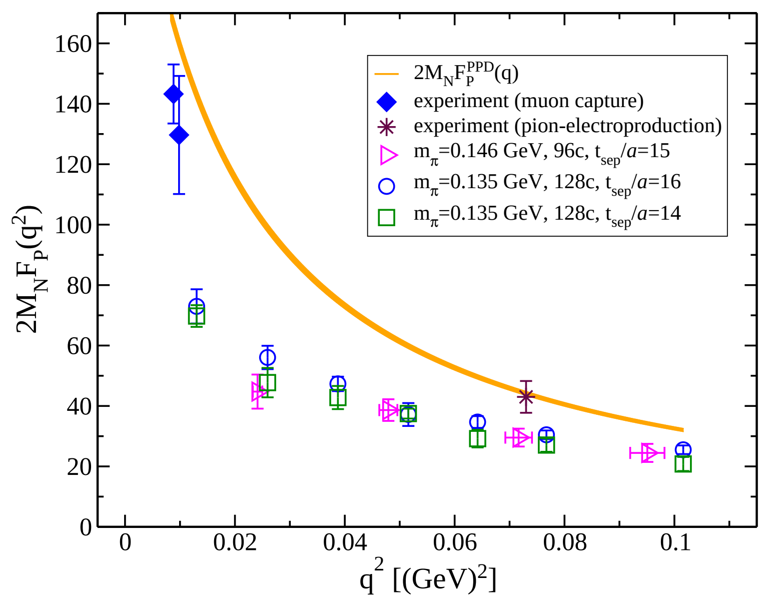
Excited state contamination in A_i matrix elements

$$R_{A_i} = \frac{C_{A_i}(t,q)}{e^{-M_N(t-t_{\mathsf{src}})}e^{-E_N(t_{\mathsf{sink}}-t)}} \propto F_A(q^2)\delta_{i3} - \frac{q_i q_3}{E_N+M_N}F_P(q^2)$$

$$C_{A_\mu}(t,q) = \mathrm{Tr}\left[\mathcal{P}_{53}\langle 0|N(t_{\mathsf{sink}},0)A_\mu(t,q)\overline{N}(t_{\mathsf{src}},-q)|0\rangle\right],\ \mathcal{P}_{53} = \frac{1+\gamma_4}{2}\gamma_5\gamma_3$$



not significant contamination



significant contamination

Significant excited state contamination is observed only in $F_P(q^2)$.

Excited state contamination in A_i matrix elements

$$R_{A_i} = \frac{C_{A_i}(t,q)}{e^{-M_N(t-t_{\text{src}})}e^{-E_N(t_{\text{sink}}-t)}} \propto F_A(q^2)\delta_{i3} - \frac{q_i q_3}{E_N+M_N}F_P(q^2)$$

$$C_{A_\mu}(t,q) = \text{Tr}\left[\mathcal{P}_{53}\langle 0|N(t_{\text{sink}},0)A_\mu(t,q)\overline{N}(t_{\text{src}},-q)|0\rangle\right], \; \mathcal{P}_{53} = \frac{1+\gamma_4}{2}\gamma_5\gamma_3$$

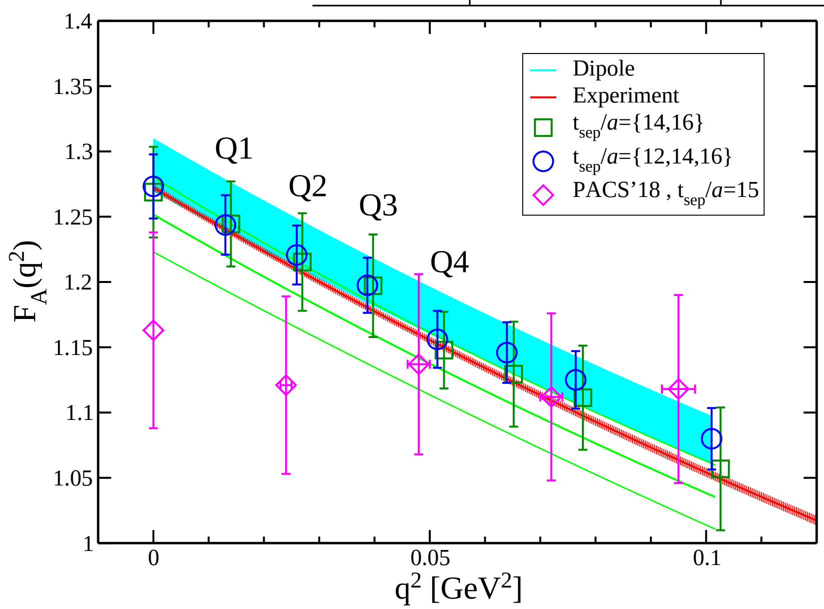
$$\begin{aligned} R^0_{A_3} &\propto F_A(q^2) && (q_3 = 0) \\ R^{NZ}_{A_3} &\propto F_A(q^2) - \frac{q_3^2}{E_N+M_N}F_P(q^2) && (q_3 \neq 0) \\ R_{A_j} &\propto -\frac{q_3 q_j}{E_N+M_N}F_P(q^2) && (q_3, q_j \neq 0, j = 1,2) \end{aligned}$$

Determination of $F_A(q^2)$ and $F_P(q^2)$

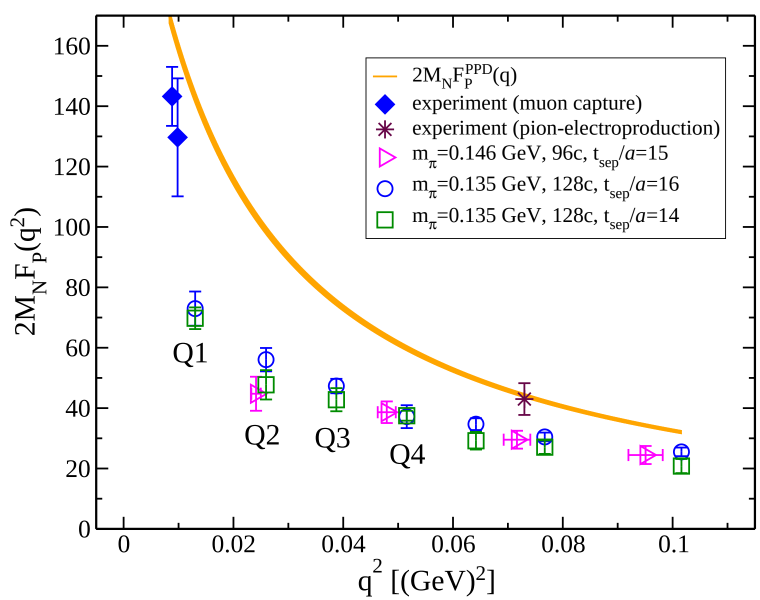
$(\vec{q}L/2\pi)^2$	$\vec{q}L/2\pi$ example	$F_A(q^2)$	$F_P(q^2)$
1	(1,0,0)	$R^0_{A_3}$	$R^{NZ}_{A_3}$, $R^0_{A_3}$
2	(1,1,0)	$R^0_{A_3}$	R_{A_j}
3	(1,1,1)	$R^{NZ}_{A_3}$, R_{A_j}	R_{A_j}
4	(2,0,0)	$R^0_{A_3}$	$R^{NZ}_{A_3}$, $R^0_{A_3}$

Excited state contamination in A_i matrix elements

label	$(\vec{q}L/2\pi)^2$	$\vec{q}L/2\pi$ example	$F_A(q^2)$	$F_P(q^2)$
Q1	1	(1,0,0)	$R^0_{A_3}$	$R^{NZ}_{A_3}$, $R^0_{A_3}$
Q2	2	(1,1,0)	$R^0_{A_3}$	R_{A_j}
Q3	3	(1,1,1)	$R^{NZ}_{A_3}$, R_{A_j}	R_{A_j}
Q4	4	(2,0,0)	$R^0_{A_3}$	$R^{NZ}_{A_3}$, $R^0_{A_3}$



not significant contamination



significant contamination

Excited state contamination proportional to q_3

Contamination canceled in proper combination of $R^{NZ}_{A_3}$ and R_{A_j}

Excited state contamination in A_i matrix elements

Expected properties of excited state contamination

Excited state contamination proportional to q_3

Contamination canceled in proper combination of $R^{NZ}_{A_3}$ and R_{A_j}

Same properties are predicted in HBChPT.

LO HBChPT [Bär, PRD99:5:054506(2018)]

Leading πN contribution proportional to q_3

Cancellation of leading πN contributions can be shown using PPD assumption

Pion pole dominance (PPD): $F_P(q^2) = \dfrac{2M_N F_A(q^2)}{m_\pi^2 + q^2}$

Expected properties useful to develop new analysis method

Similar cancellation may happen in A_4 and A_i matrix elements.

Summary

Nucleon form factors by PACS Collaboration

large volume $>$ (8 fm)^3 (near) at physical pion mass

96^4 [PRD98:7:074510(2018)] and PACS10 [PRD99:1:014510(2019)] configurations

Isovector couplings and form factors

- g_A agrees with experiment, and g_T, g_S agree with FLAG19 average
- $G_M(q^2), F_A(q^2)$ agree well with experiment
- $F_P(q^2)$ has large $t_{\rm sep}$ dependence (new analysis method necessary)

Future works

- investigate discrepancy in $G_E(q^2)$
- continuum extrapolation
  PACS10 configuration 160^4 and 256^4
- disconnected diagram calculation

Back up

Isovector form factors

- Vector and induced tensor form factors

(elastic proton-electron scattering)

$$\langle N,p|V_\mu(q)|N,p'\rangle = \overline{u}_N(p)\left(F_1(q^2)\gamma_\mu + i\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2M_N}\right)u_N(p')$$

$$F_1(q^2), F_2(q^2) \to \begin{array}{l} G_E(q^2) = F_1(q^2) - \dfrac{q^2}{4M_N}F_2(q^2) \\ G_M(q^2) = F_1(q^2) + F_2(q^2) \end{array}$$

- Axialvector and induced pseudoscalar form factors

(β decay; muon capture on proton; neutrino-nucleon scattering; pion electroproduction)

$$\langle N,p|A_\mu(q)|N,p'\rangle = \overline{u}_N(p)\left(F_A(q^2)i\gamma_5\gamma_\mu + i\gamma_5 q_\mu F_P(q^2)\right)u_N(p')$$

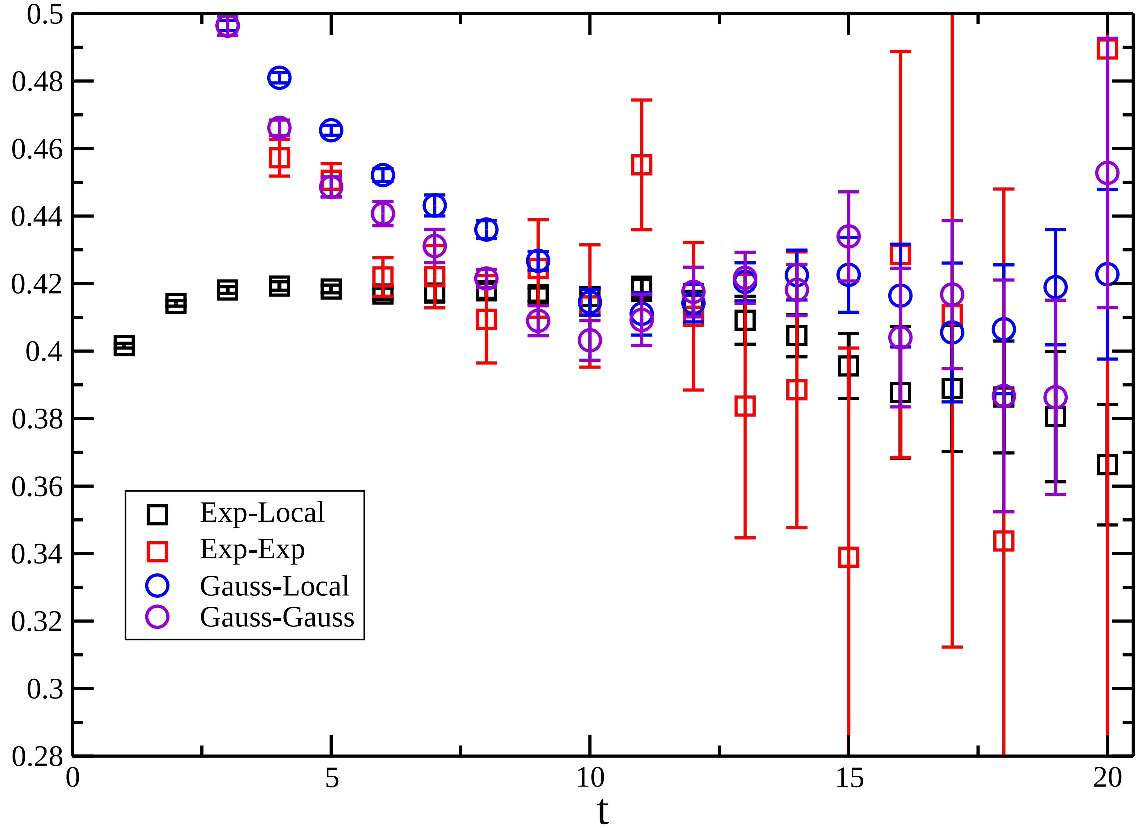
- Pseudoscalar form factor

$$\langle N,p|P(q)|N,p'\rangle = \overline{u}_N(p)\left(G_P(q^2)\gamma_5\right)u_N(p')$$

- Axial Ward-Takahashi identity

$$2M_N F_A(q^2) - q^2 F_P(q^2) = 2m_q G_P(q^2)$$

Effective mass on 128^4 lattice



Gauss source preliminary

Exponential and Gauss sources : plateau starts in $t \sim 10$

Exponential : easy to tune parameter

error of 3pt at the same t_{sep}: exponential $>$ Gauss

$\rightarrow$ error of 2pt : Exp-Exp $>$ Gauss-Gauss, *e.g.*, $t = 16$