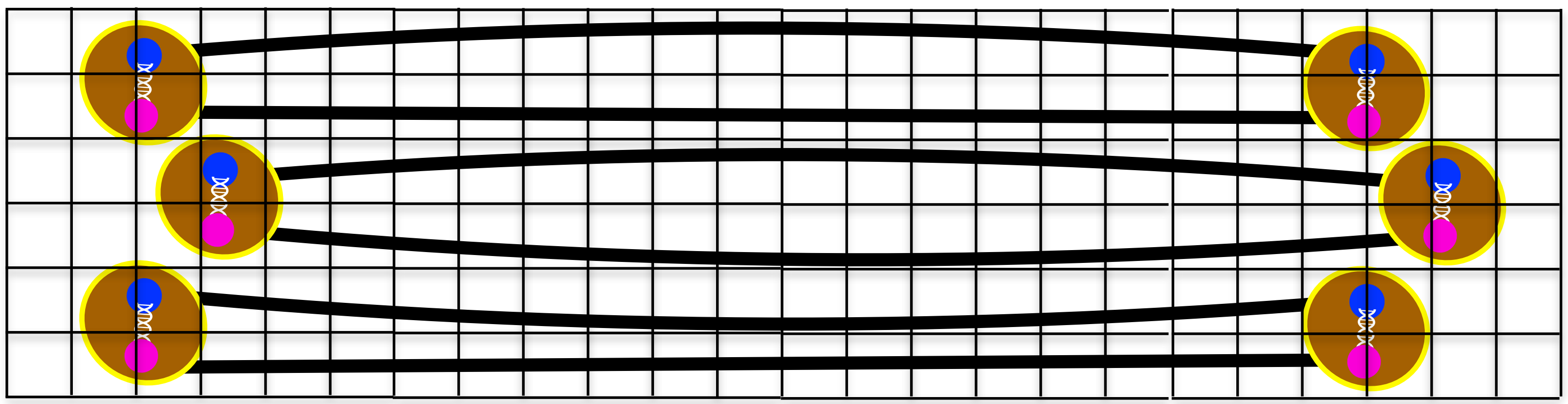
Charged hadron interactions in QCD+QED

Michael Wagman



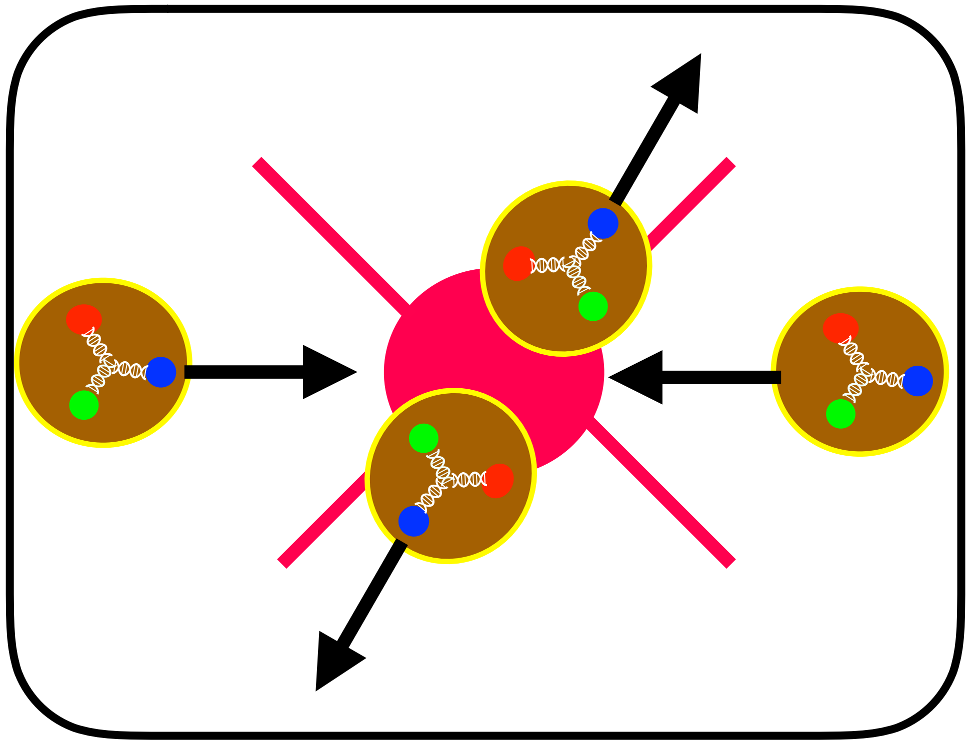
Work in progress from NPLQCD/CSSM/QCDSF/UKQCD

Aug 27, 2019

Sante Fe

QCD+QED Phenomenology

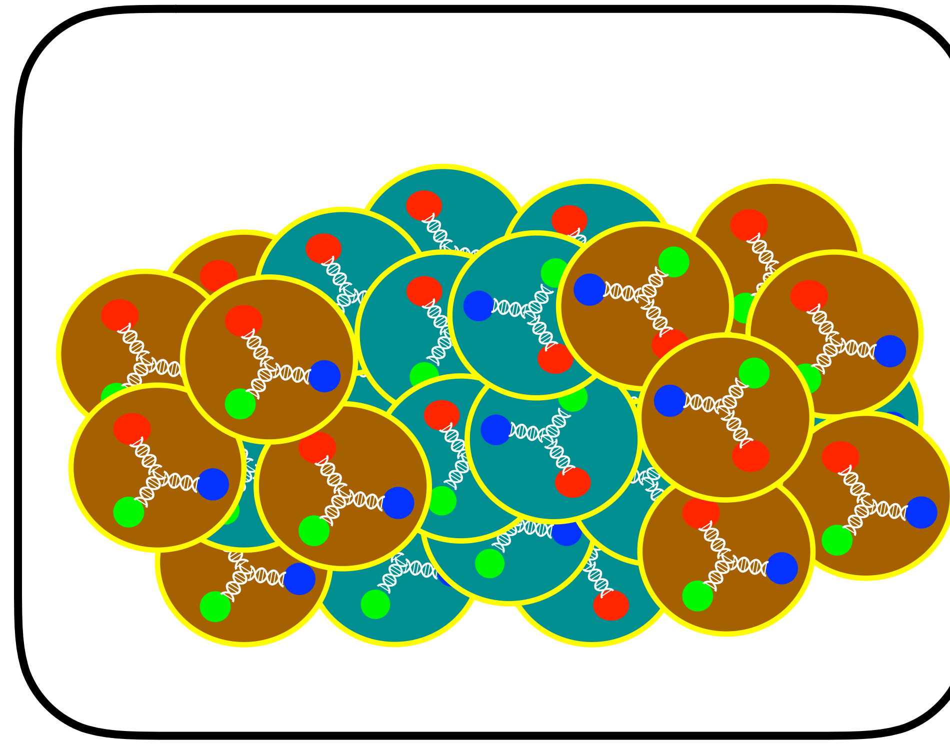
Charged hadron scattering



LQCD+QED can separate strong and electromagnetic isospin violation in nucleon-nucleon scattering lengths

Precise predictions of isospin-violating differences between *pp*, *pn*, and *nn* scattering lengths would improve nuclear EFTs/models

High charge-density systems



Boundaries of periodic table determined by competition between QCD and QED

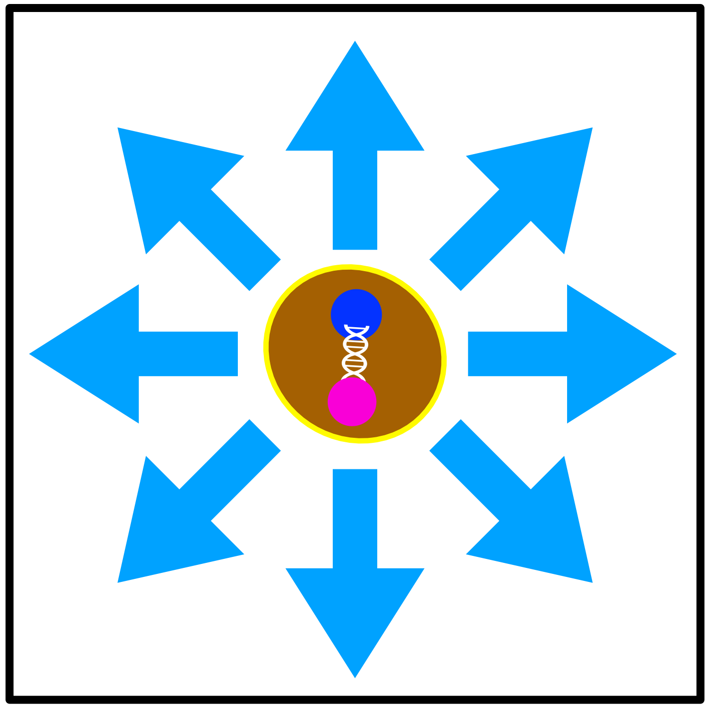
Nonperturbative relativstic QED effects important

LQCD+QED calculations can disentangle nonperturbative QED and QCD effects

QCD+QED Theory

LQCD+QED for $\alpha Z > 1$ nuclei faces several obstacles:

Charged systems in a box with periodic boundary conditions violate Gauss's law



— *Either boundary conditions or photon field must be modified*

Review: Patella, PoS LATTICE 2016 (2017)

— *QED_L removes spatial zero-mode of photon, equivalent to adding uniform density of opposite charge*

Duncan et al, PRL 76 (1996) Blum et al, PRD 76 (2007) Hayakawa and Uno, Prog. Theor. Phys. 120 (2008)

QCD+QED_L preserves symmetries of QCD+QED but violates locality

— *Subtleties of nonlocal field theory must be understood in order to match LQCD+QED_L results to hadronic EFTs*

Exponential signal-to-noise degradation with increasing baryon number

— *Not this talk. Start with LQCD+QED_L for charged mesons with unphysically large α to explore high charge density systems at lower cost*

NRQED

Relativistic formalism for FV effects on generic particle-number systems unknown

Non-Relativistic QED (NRQED) describes low-energy QED for electromagnetic bound states, charged particle scattering, hadronic reactions and decays, …

Caswell and Lepage, Phys. Lett. 167B (1985)

Kong and Ravndal, Nucl. Phys. A 665 (2000)

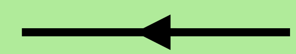
Carrasco et al, PRD 91 (2015)

…

Dual expansion:

$\alpha \qquad p/M$

Power counting (on-shell matter particles):

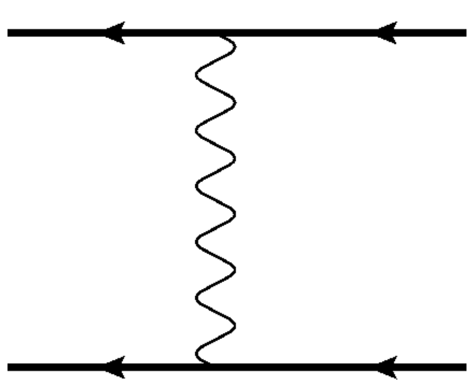


$$\sim \frac{1}{E-(p+k)^2/M} \sim \frac{M}{p^2}$$

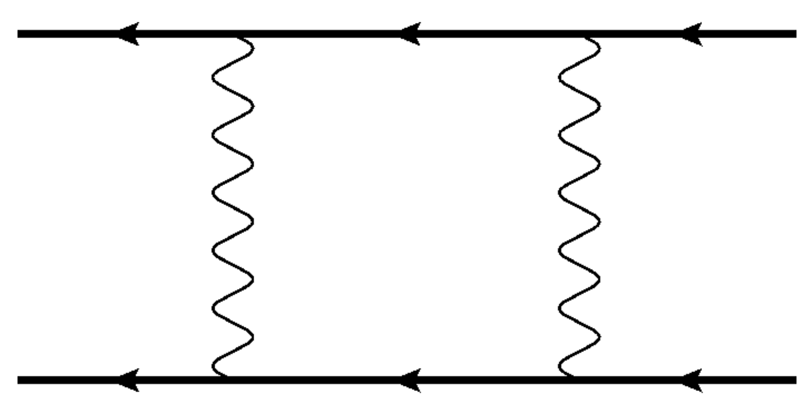
$$\sim \frac{1}{p^2}$$

$$\int dE \int d^3k \sim \frac{p^5}{M}$$

With multiple charged particles, loop expansion does not match EFT expansion, Coulomb ladder diagrams must be resumed for $\eta = \frac{\alpha M}{2p} > 1$



$$\sim \frac{\alpha}{p^2}$$



$$\sim \frac{\alpha}{p^2}\left(\frac{\alpha M}{p}\right)$$

Two Charged Particles in a Box

In finite-volume with PBCs, momenta are quantized $p = \frac{2\pi}{L} n, \; n \in \mathbb{Z}^3$

Coulomb ladder diagrams must be nonperturbatively resumed for large enough volumes that $\alpha M/p \sim \alpha M L \gg 1$

— technically challenging unsolved problem

For intermediate volumes $\frac{1}{m_\pi} \ll L \ll \frac{1}{\alpha M}$ Coulomb effects are perturbative

Beane and Savage, PRD 90 (2014)

Beane and Savage derived generalized Lüscher quantization condition relating FV energy shifts and scattering phase shifts accurate to $\mathcal{O}(\alpha), \; \mathcal{O}(\alpha M L)$

$$C^2_{\eta(p)} p \cot\delta + \alpha M h(\eta) = -\frac{1}{a_C} + \frac{1}{2} r_0 p^2 + \ldots = \frac{1}{\pi L} \mathcal{S}^C \left(\frac{pL}{2\pi} \right) + \alpha M \left[\ln \left(\frac{4\pi}{\alpha M L} \right) - \gamma_E \right]$$

Range of validity of Beane-Savage regime remains to be explored in LQCD+QED$_L$

Lattice QCD+QED$_L$

Dynamical QCD + QED$_L$ ensembles generated by CSSM/QCDSF/UKQCD collaboration

Horsley et al, J. Phys. G 43 (2016)   Horsley et al, JHEP 1604 (2016)   Horsley et al, arXiv:1904.02304

$\alpha = 0.1$ $a = 0.068(2)$ fm $m_K = 404(1)(11)$ MeV $aL = 24,\ 32,\ 48$

$N_f = 1 + 2$ quark flavors with masses tuned to symmetric point where all neutral pseudoscalar mesons are degenerate, $m_{\overline{u}u} \approx m_{\overline{d}d} = m_{\overline{s}s}$

Gauge-fixing required for non-zero charged particle correlation functions

Landau gauge-fixing used with residual gauge degrees of freedom removed by adding multiples of $2\pi/(e_q L_\mu)$ to A_μ to satisfy

$$-\frac{\pi}{|e_q|L_\mu} < \frac{1}{V}\sum_x A_\mu(x) \leq \frac{\pi}{|e_q|L_\mu}$$

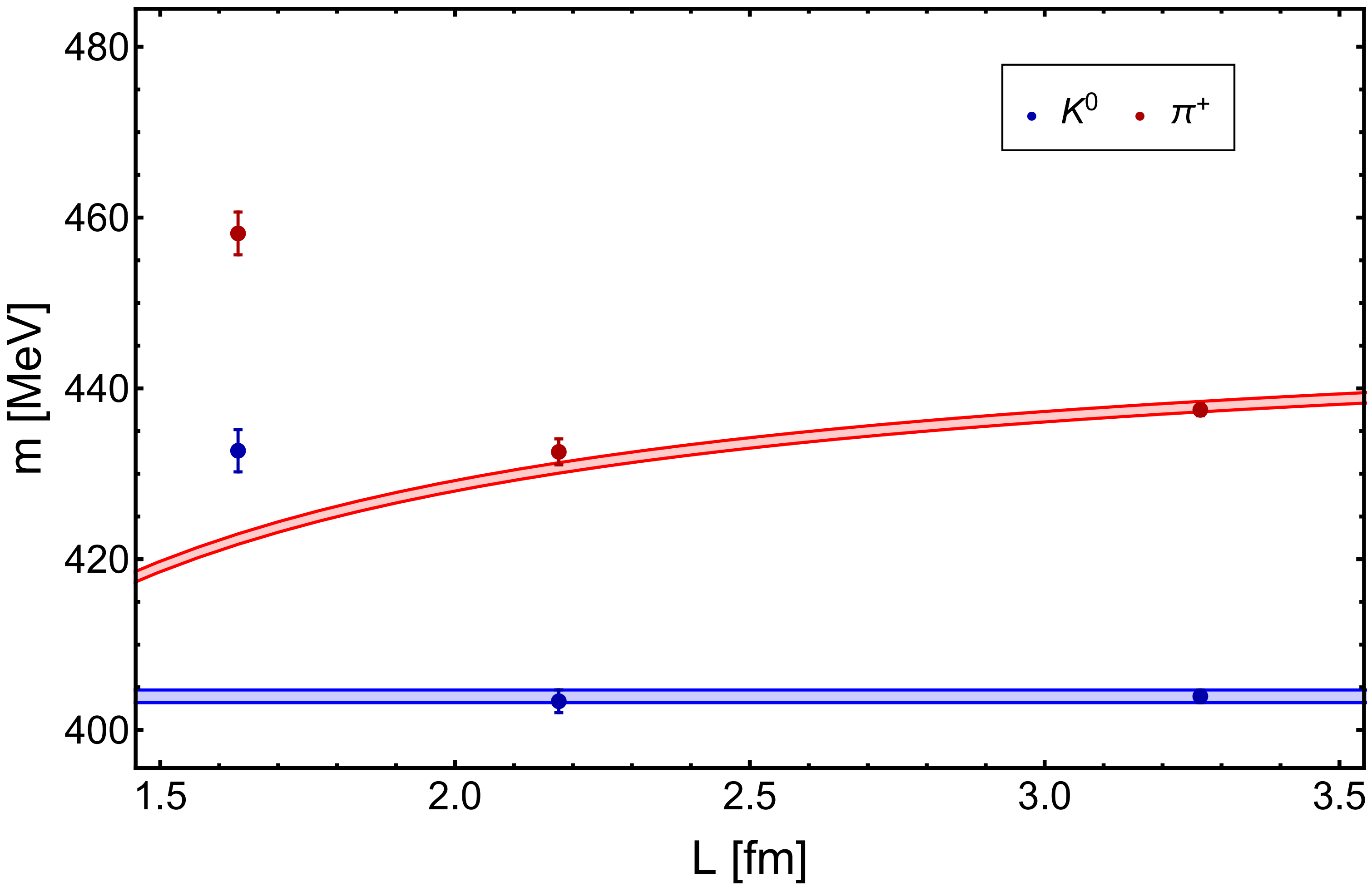
Quark determinant unaffected by this gauge-fixing procedure

Photon zero-modes removed from each timeslice for valence quark propagators, FV effects of zero-mode on quark determinant add small systematic uncertainty

Charged and Neutral Mesons

QED effects explored by comparing systems of 1-12 π^+ mesons with systems of 1-12 K^0 mesons

Quark mass tuning removes strong isospin breaking between π^+ and K^0 masses, remaining mass difference pure QED



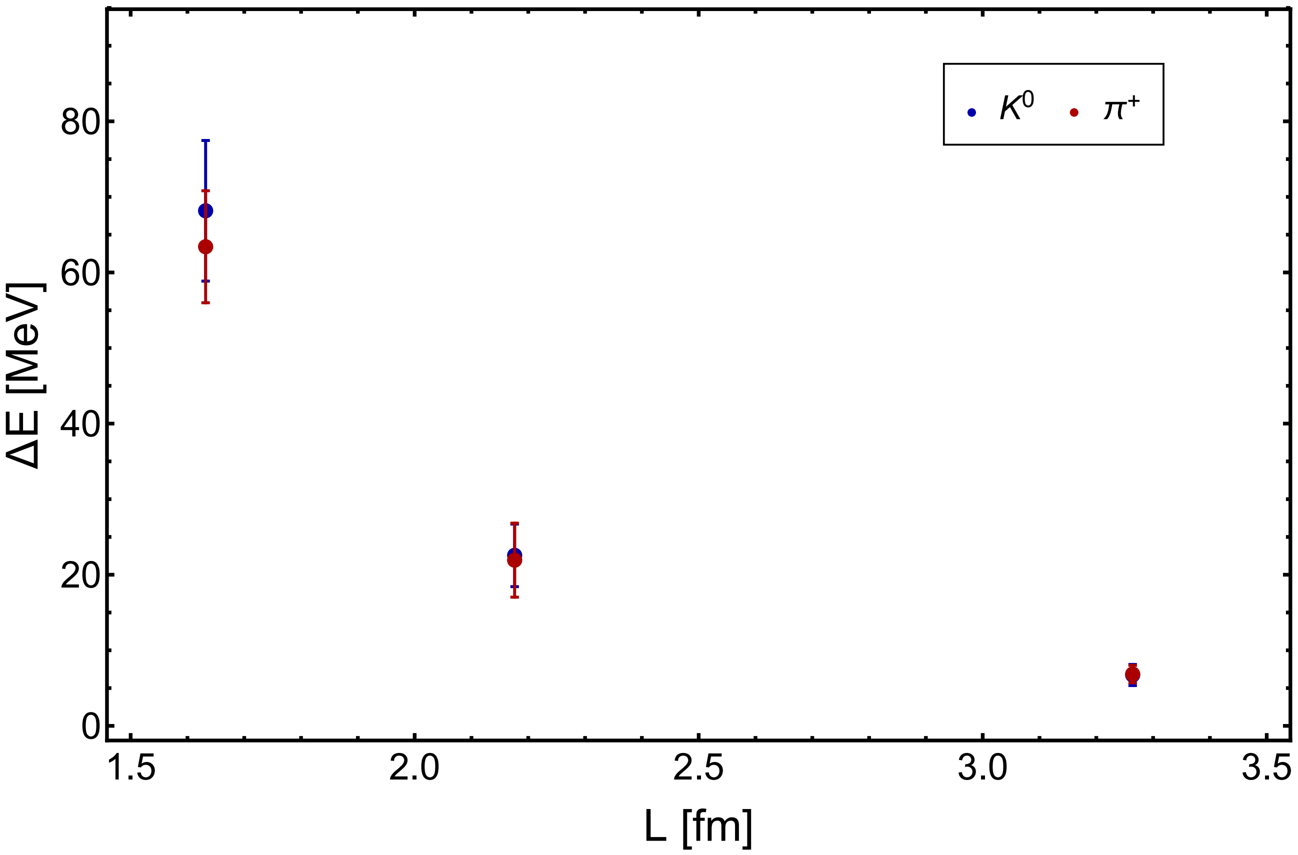
FV effects on π^+ mass described by NRQED$_L$ at NLO (1 fit parameter) for L = 32, 48

e^{-mL} FV effects clearly present at L=24 for π^+ and K^0, volume excluded from analysis

Same conclusion as

Horsley et al, J. Phys. G 43 (2016)

Charged Particle Interactions



Coulomb effects on $\pi^+\pi^+$ FV energy shift might be expected to be large since

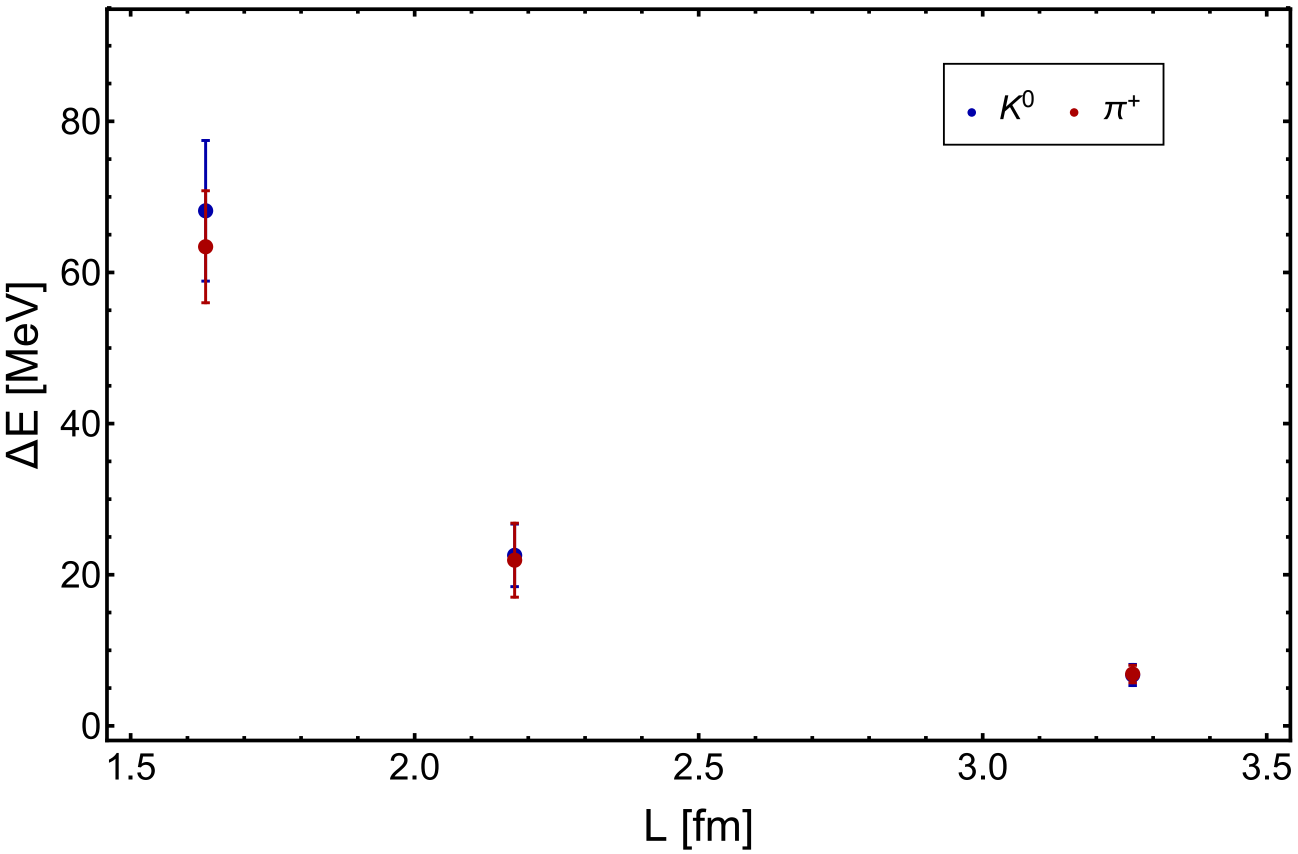
$$\alpha m_{\pi^+} L \sim 0.38,\ 0.48,\ 0.72$$

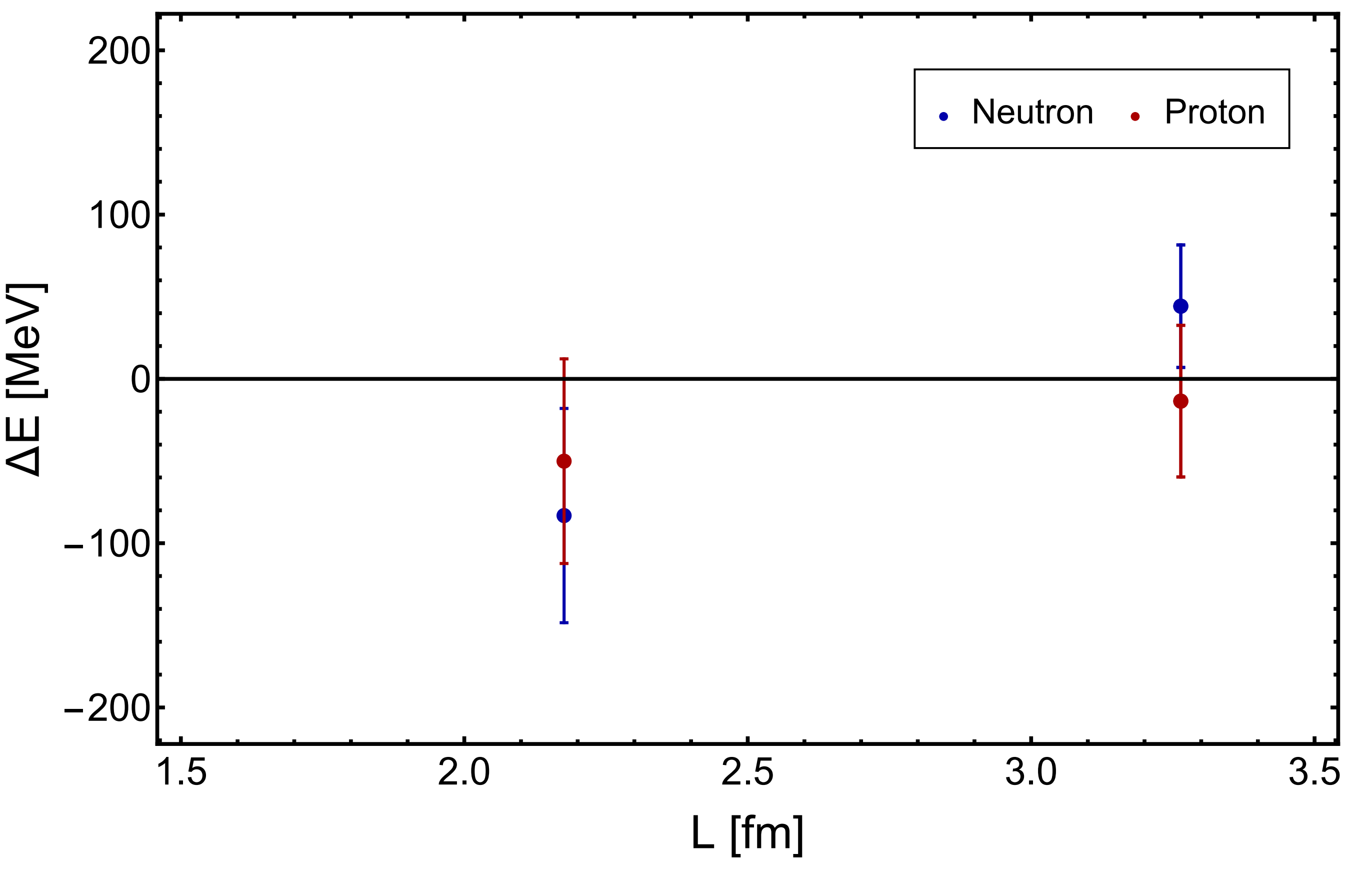
QED effects < 10% of total shift, even at largest volume

$$\Delta E_{\pi^+\pi^+} = 6.9(1.1)\ \mathrm{MeV}$$

$$\Delta E_{\pi^+\pi^+} - \Delta E_{K^0 K^0} = 0.15(50)\ \mathrm{MeV}$$

Charged Particle Interactions





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Nucleon-nucleon systems have stronger Coulomb effects, still not resolved despite

$$\alpha M_p L \sim 1.3,\ \ 1.9$$

Is this consistent with NRQED_L?

Nonlocality

Details of NRQED_L must be understood to interpret LQCD+QED_L results in EFT

Significant recent progress in understanding NRQED_L subtleties for single hadrons

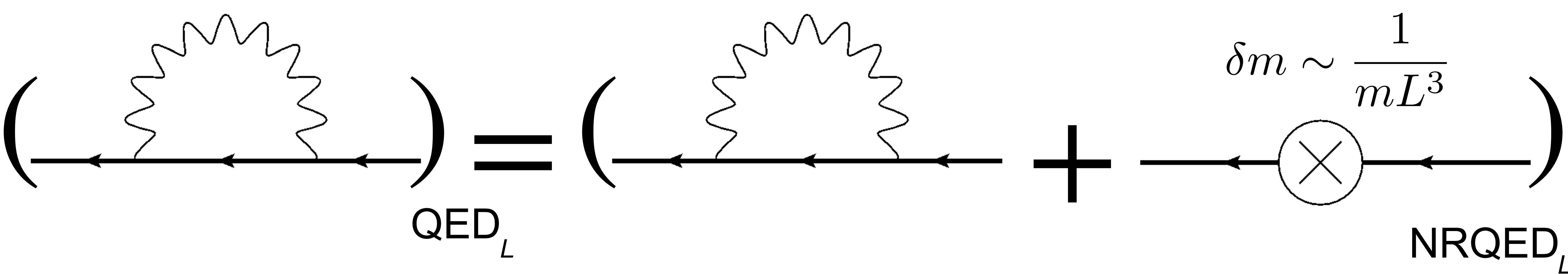
Davoudi and Savage, PRD 90 (2014) Borsanyi et al, Science, vol. 347 (2015) Fodor et al, Phys. Lett. B 755 (2016)

Lee and Tiburzi, PRD 93 (2016) Matzelle and Tiburzi, PRD 95 (2017)

Davoudi, Harrison, Jüttner, Portelli, and Savage, PRD 99 (2019)

— locality violation from zero-mode subtraction allows IR scales such as L to appear in coefficients of UV divergences and renormalized couplings

— careful matching between QED_L and NRQED_L allows nonlocal counterterms to be determined



Remaining questions for multi-particle systems:

How do non-local counterterms modify 2-body interactions?

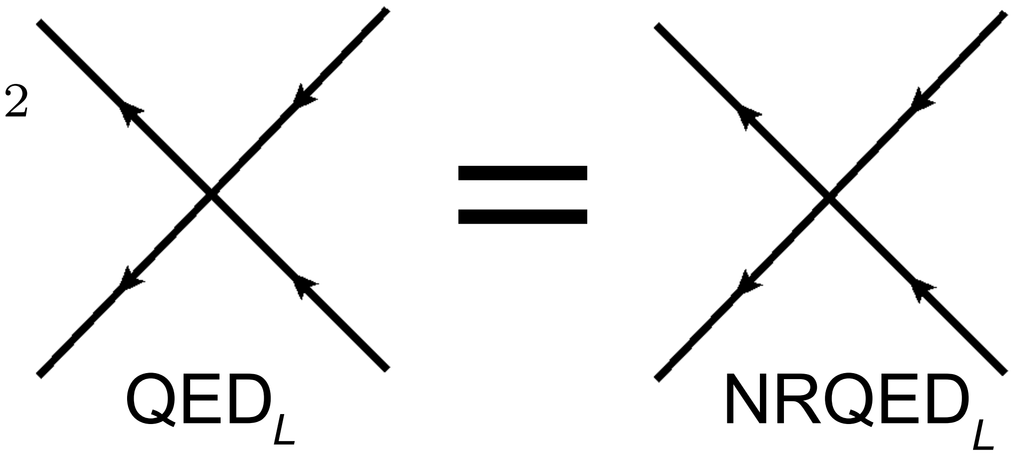
Are non-local 2-body effects enhanced by Coulomb ladder diagrams?

Matching NRQED$_L$ and QED$_L$

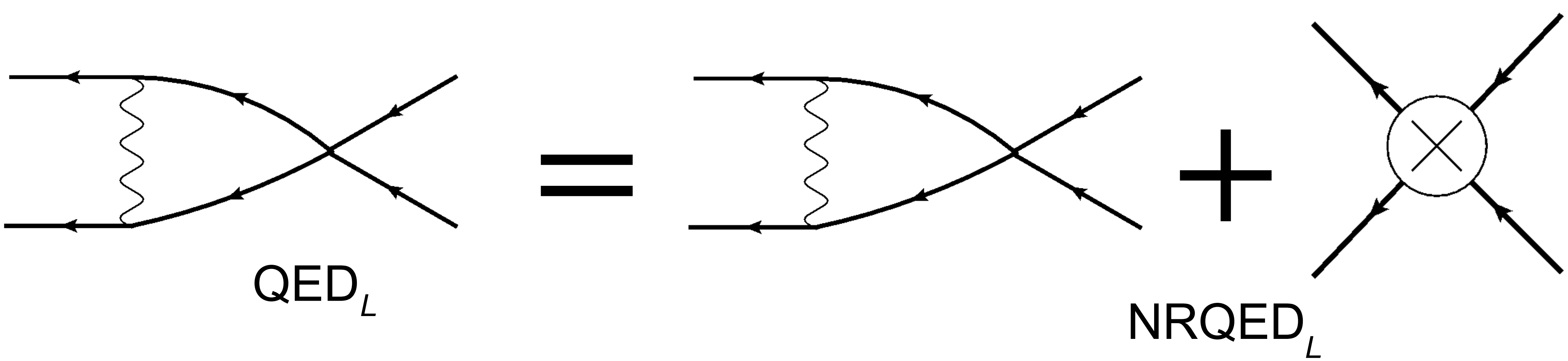
Tree-level QED$_L$ 2 -> 2 amplitude reproduced by NRQED$_L$ with $\psi = \sqrt{2M}e^{iMt}\varphi$

$$\mathcal{L}^{\varphi}_{QED_L} = -D_\mu \varphi^\dagger D^\mu \varphi - M^2 \varphi^2 \varphi - 16\pi a M (\varphi^\dagger \varphi)^2$$

$$\mathcal{L}^{\psi}_{NRQED_L} = \psi^\dagger \left(iD_0 - \frac{D_i D^i}{2M} \right) \psi - \frac{4\pi a}{M} (\psi^\dagger \psi)^2$$



One-level QED$_L$ 2 -> 2 amplitude includes antiparticle pole contributions not present in NRQED$_L$



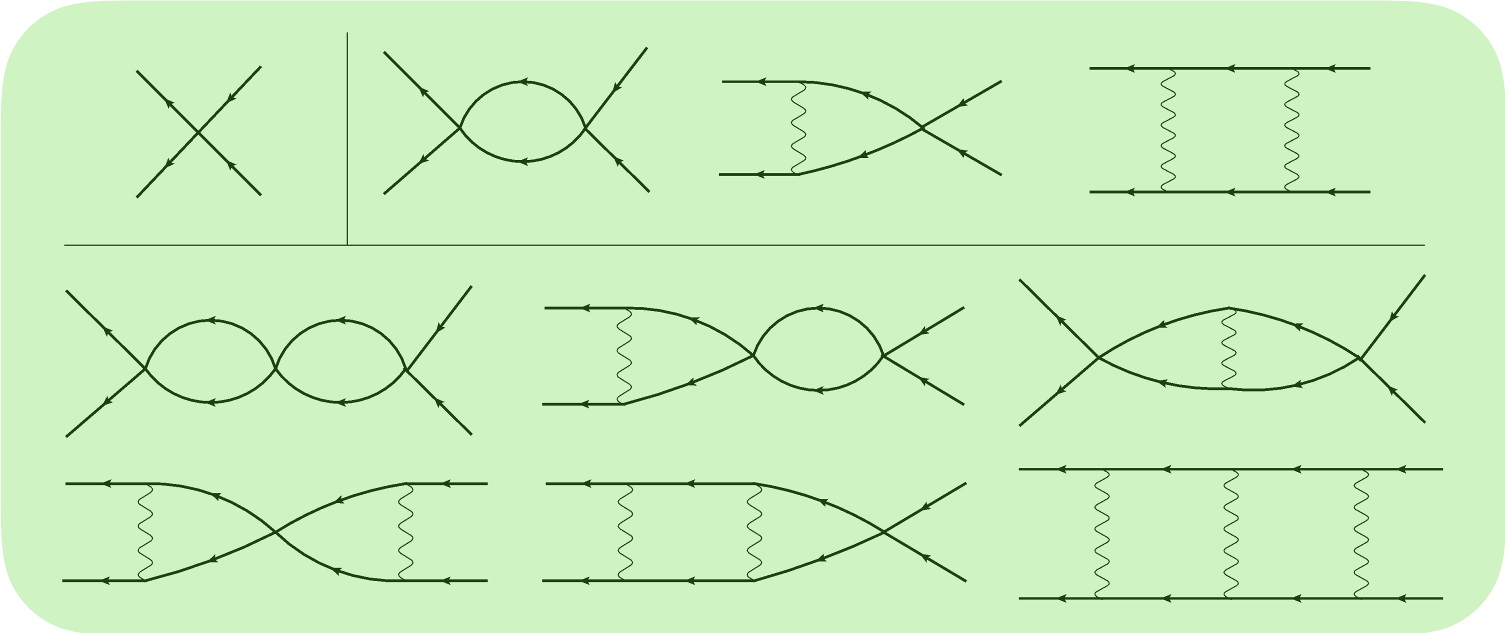
Explicit calculation shows antiparticle pole contribution suppressed by $\frac{\alpha}{(ML)^5}$

Antiparticle poles in Coulomb ladder diagrams further suppressed, nonlocal quartic interactions in NRQED$_L$ only arise as high-order relativistic corrections

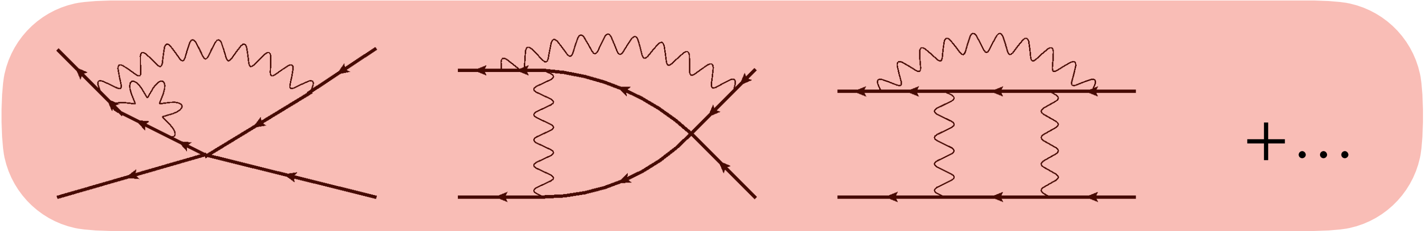
Matching pNRQED$_L$ and NRQED$_L$

Dominant contributions arise from diagrams with Coulomb photon exchange between space-like separated particles

All nonlocal effects suppressed by powers of $1/(ML)$



Contributions from radiation photon diagrams suppressed by $1/(ML)$



Coulomb photon contributions reproduced in Rayleigh-Schrödinger perturbation theory with potential

$$\langle p_1', p_2' | V | p_1, p_2 \rangle = \left[\left(\frac{4\pi a}{ML^3} \right) + \frac{\alpha}{\pi L} \sum_{i \in \mathbb{Z}^3}{}' \frac{1}{|i|^2} \right] \delta_{p_1 + p_2 - p_1' - p_2'}$$

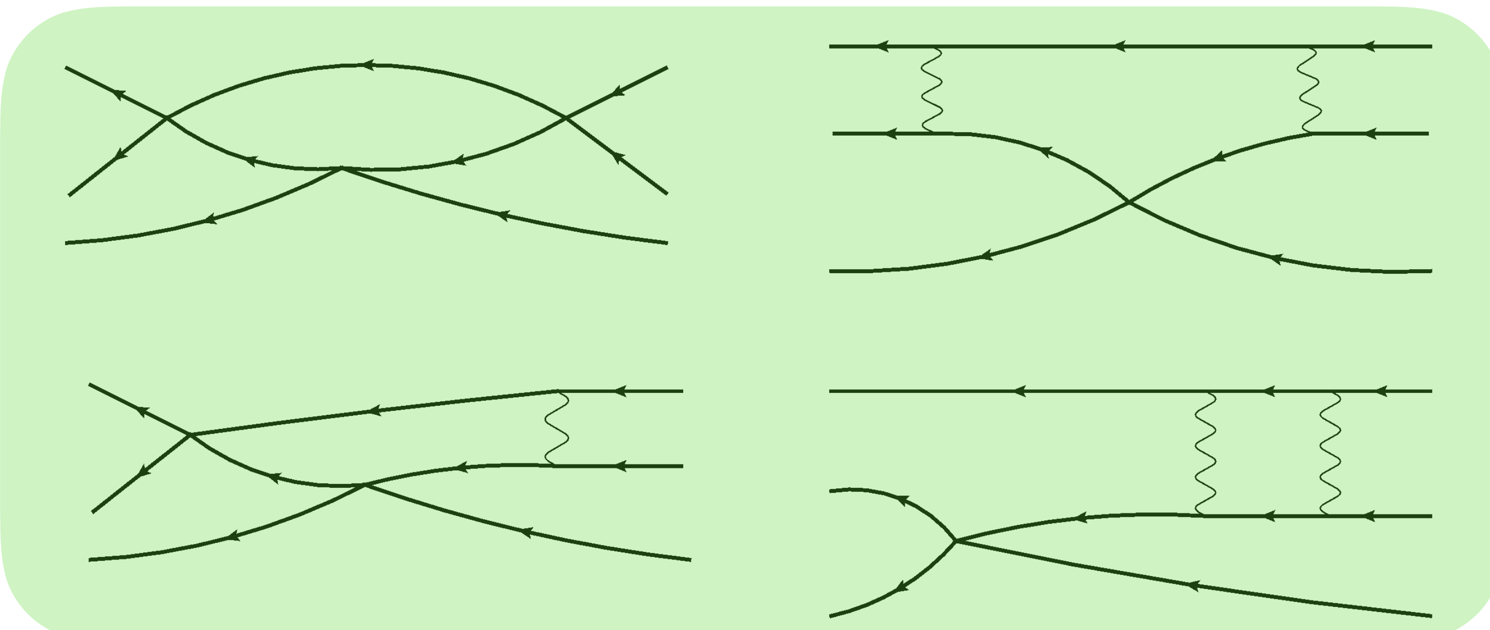
Defines leading-order potential NRQED$_L$ (pNRQED$_L$) accurate up to $1/(ML)$ effects, higher-order potential computable as in infinite-volume pNRQED/pNRQCD

Pineda and Soto, Nucl. Phys. Proc. Suppl. 64 (1998) Pineda and Soto, PRD 59 (1999)

Many Charged Particles in a Box

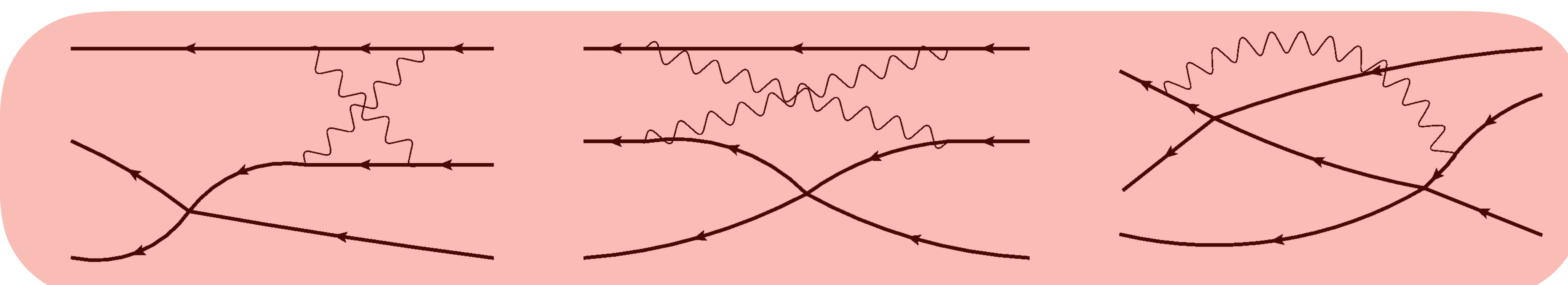
3+ particle systems achieve higher charge density in fixed volume, eventually probe nonperturbative relativistic QED effects

— comparison of LQCD+QED_L and $NRQED_L$ for multi-pion systems provides strong tests of validity of $NRQED_L$



Coulomb photon contributions accessible to $pNRQED_L$ dominant

Radiation photon diagrams again suppressed by $1/(ML)$



$pNRQED_L$ suitable for calculations of many-pion energy shifts up to $1/(ML)$ effects

FV Energy Shifts in pNRQED$_L$

n-boson energy shifts straightforwardly computable in pNRQED$_L$

$$\Delta_0 = \frac{4\pi a_C}{ML^3}\binom{n}{2}\left\{1 - \left(\frac{a_C}{\pi L}\right)\mathcal{I} + \left(\frac{a_C}{\pi L}\right)^2\left[\mathcal{I}^2 + (2n-5)\mathcal{J}\right]\right\}$$

Huang and Yang, Phys. Rev. 105 (1957)

$$+ \frac{\alpha}{2\pi L}\binom{n}{2}\left\{-\left(\frac{a_C}{\pi L}\right)4\mathcal{J} + \left(\frac{a_C}{\pi L}\right)^2\left[4\mathcal{I}\mathcal{J} + (8n-20)\mathcal{K} + 2\mathcal{R}_{22} - 8\pi^4\left(\ln\left(\frac{4\pi}{\alpha ML}\right) - \gamma_E\right)\right]\right.$$

$$\left. - \left(\frac{\alpha ML}{4\pi^3}\right)2\mathcal{K} + \left(\frac{\alpha ML}{4\pi^3}\right)\left(\frac{a_C}{\pi L}\right)\left[2\mathcal{J}^2 + 4\mathcal{R}_{24} + (4n-10)\mathcal{L}\right] + \left(\frac{\alpha ML}{4\pi^3}\right)^2 2\mathcal{R}_{44}\right\}.$$

$n = 2,\ O(\alpha)$
Beane and Savage, PRD 90 (2014)

Important features:

Log involves FV analog of Coulomb expansion parameter $\eta = \frac{\alpha M}{2p} = \frac{\alpha ML}{4\pi}$

Coulomb photon effects enter as power series in η with O(1) coefficients

→ FV Coulomb perturbative when $\frac{\alpha ML}{4\pi} \ll 1$, including $\alpha ML \sim 1$

Physical quark mass protons with $\alpha = 0.1$

$\alpha ML \ll 1$ → $L \ll 2.1\ \text{fm}$

$\frac{\alpha ML}{4\pi} \ll 1$ → $L \ll 26\ \text{fm}$

Power Counting

With $m_\pi = 437(1)(13)\ \text{MeV}$ and volumes $L = 2.2\ \text{fm},\ 3.2\ \text{fm}$

$$\frac{\alpha m_\pi L}{4\pi} = 0.04,\ 0.06 \qquad\qquad \frac{a_{\pi\pi}}{L} = 0.11,\ 0.07$$

Hierarchy between QCD and QED effects volume dependent, two reasonable power counting for this range of volumes

PC1 $$\frac{\alpha m_\pi L}{4\pi} \sim \frac{a_{\pi\pi}}{L}$$

PC2 $$\frac{\alpha m_\pi L}{4\pi} \sim \left(\frac{a_{\pi\pi}}{L}\right)^2$$

Previous result ordered by loop expansion appropriate for PC1 at NNLO

Known higher-order results for pure QCD allow PC2 to be extend to N^3LO

Beane, Detmold, and Savage, PRD 76 (2007)

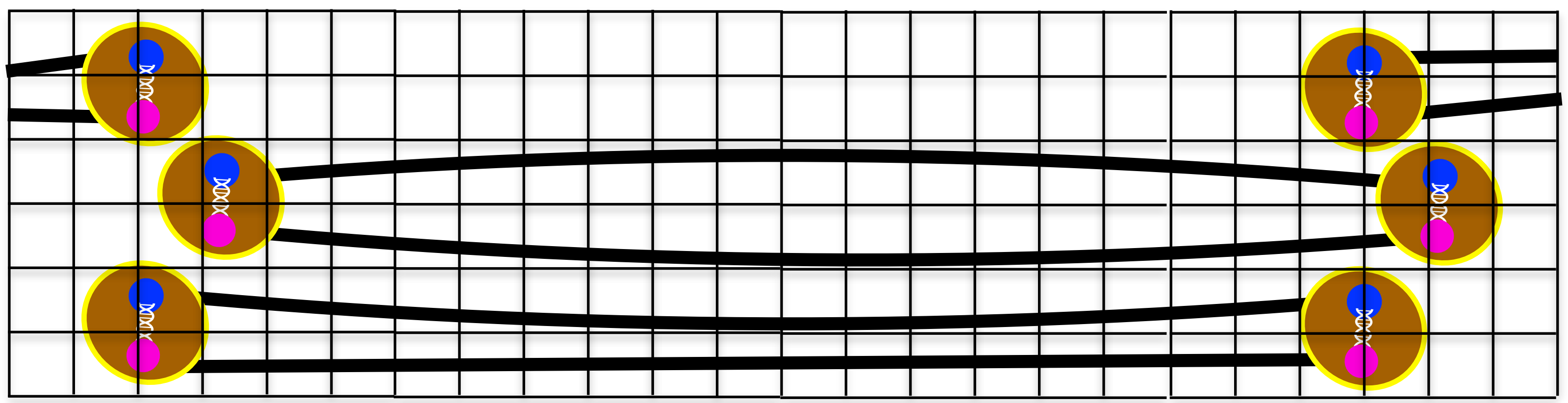
Many Charged Mesons

Correlation functions for up to N=12 charged pions can be computed from one pair of point-to-all quark propagators, as in previous NPLQCD studies without QED

Beane et al, PRL 100 (2008) Detmold et al, PRD 78 (2008) Detmold and Savage, PRD 82 (2010)

Allows study of large charge densities: $Z\alpha \leq 1.2$ $\quad \frac{Z\alpha}{L^3} \lesssim 0.12 \text{ fm}^{-3}$

Thermal effects become increasingly important for larger particle number



Fit results for k < N particles used to determine thermal effects for N particles

$$G(N,t) = \sum_n Z_n \left(e^{-E_n(N)t} + e^{-E_n(N)(\beta-t)} \right)$$

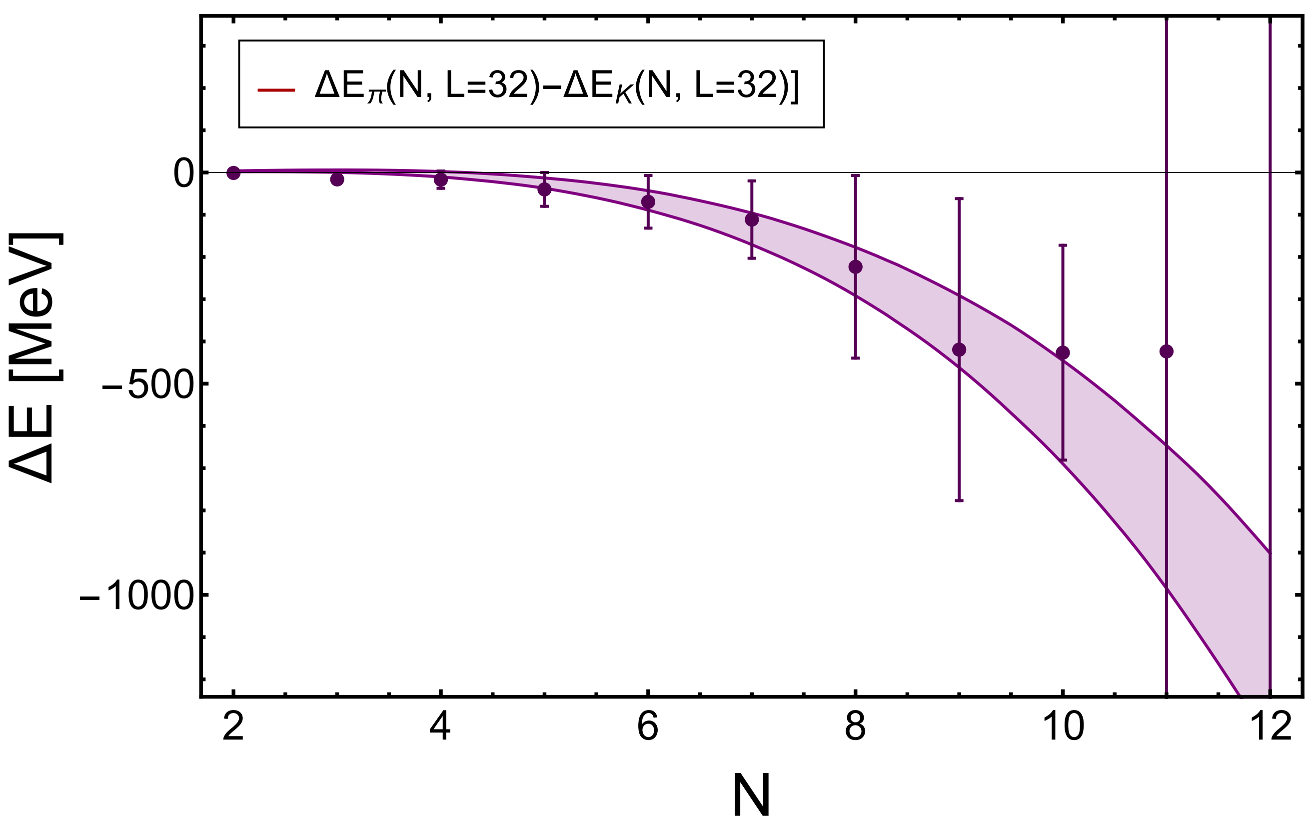
$$+ \sum_{k=1}^{[n/2]} \widetilde{Z}_0^k \left(e^{-E_0(k)t} e^{-E_0(N-k)(\beta-t)} + e^{-E_0(k)(\beta-t)} e^{-E_0(N-k)t} + \ldots \right)$$

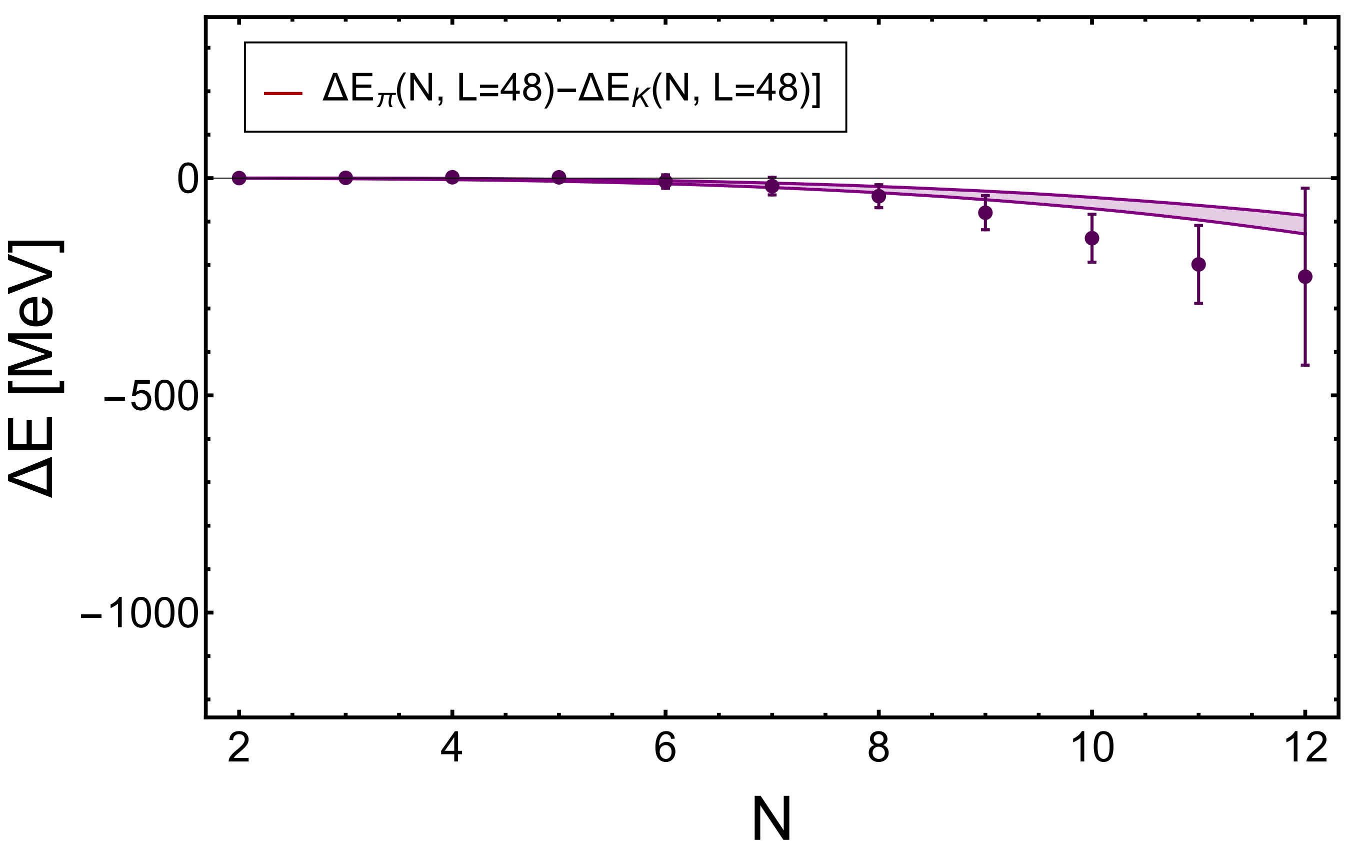
EFT Fits

EFT parameters $a^{K^0K^0}$, $a_C^{\pi^+\pi+}$ and three-body couplings $\eta^{K^0K^0K^0}$, $\eta^{\pi^+\pi^+\pi^+}$ determined from global fit to all FV shifts with 2-12 mesons and correlated differences $\Delta E_{N\pi^+} - \Delta E_{NK^0}$

NNLO fits with both power counting give $\chi^2/\mathrm{dof} \sim 2.0$

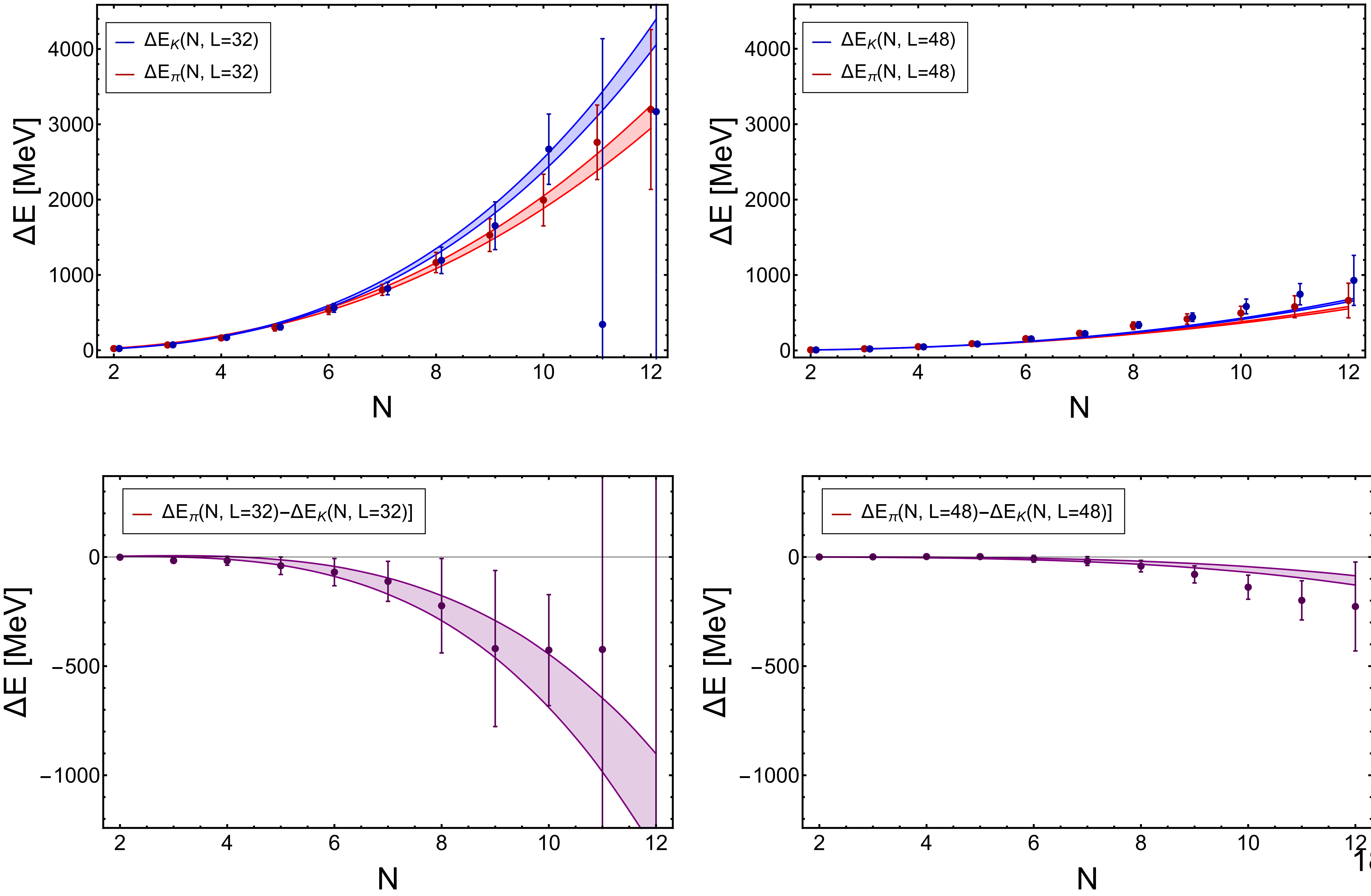
N^3LO fits (pc 2) add three-body forces, better agreement $\chi^2/\mathrm{dof} \sim 1.2$





Negative sign arises from zero-mode subtraction, predicted by NRQED_L

EFT Fits



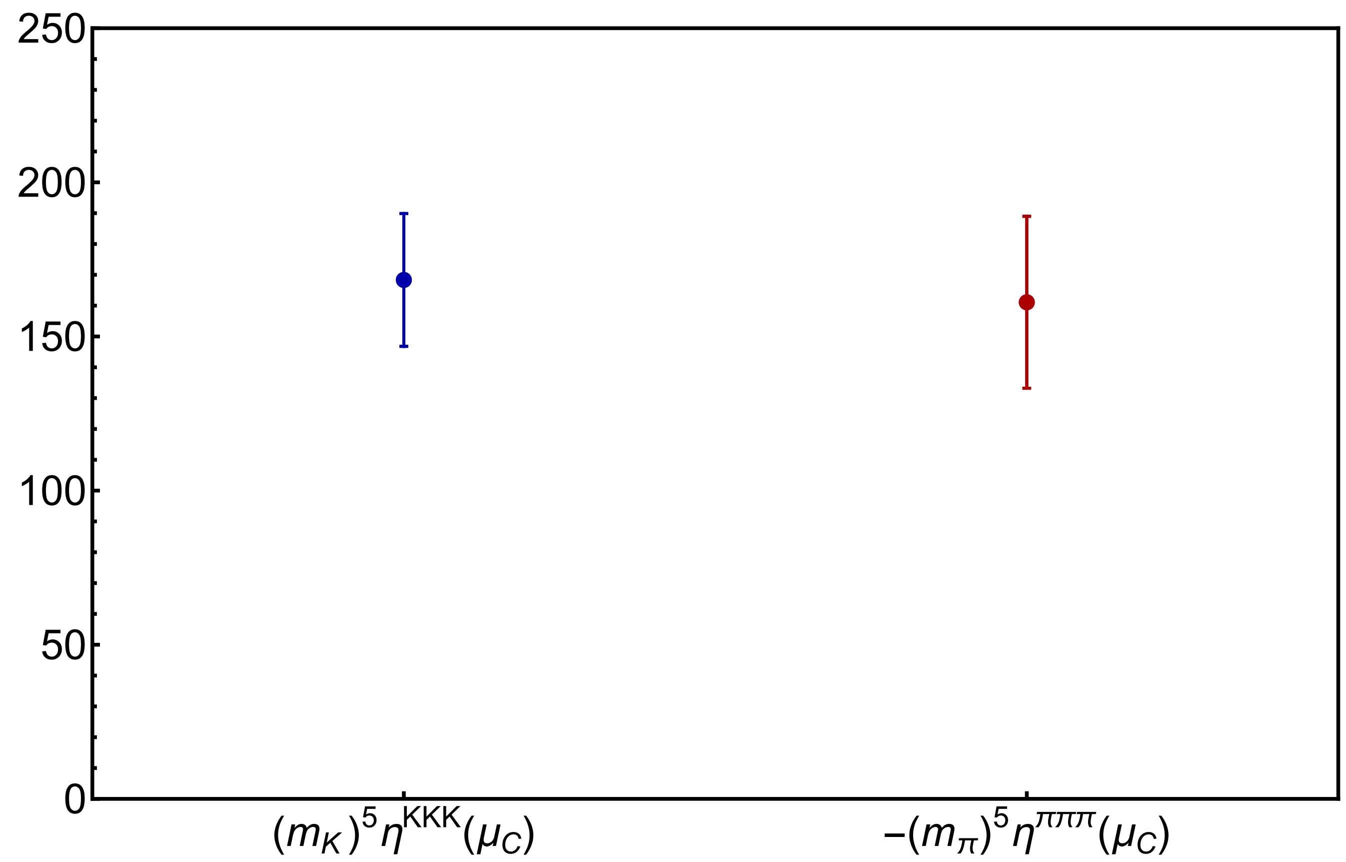
ΔE_K(N, L=32)
ΔE_π(N, L=32)
ΔE [MeV]
N
ΔE_K(N, L=48)
ΔE_π(N, L=48)
ΔE [MeV]
N
ΔE_π(N, L=32)−ΔE_K(N, L=32)]
ΔE [MeV]
N
ΔE_π(N, L=48)−ΔE_K(N, L=48)]
ΔE [MeV]
N

EFT Results

Coulomb scattering length for $\pi^+\pi^+$ differs from K^0K^0 scattering length

$$m_{K^0} a^{K^0 K^0} = 0.3376(96) \qquad m_{\pi^+} a_C^{\pi^+\pi^+} = 0.5241(93)$$

Three-body forces clearly resolved, no QED effects visible on strength of three-body forces



(all results valid for unphysical quark masses used in calculation)

Summary

Nonlocal counterterms in NRQED$_L$ are not enhanced by Coulomb ladders (at least at 2-loop order), appear only as relativistic corrections

FV effects on interacting charged hadron systems in LQCD+QED$_L$ consistent with NRQED$_L$ up to $Z\alpha \leq 1.2$, $Z\alpha/L^3 \lesssim 0.12 \text{ fm}^{-3}$ with $m_\pi \sim 437 \text{ MeV}$, no leptons

Size of QED effects consistent with Coulomb expansion parameter

$$\eta = \frac{\alpha M}{2p} = \frac{\alpha M L}{4\pi} \ll \alpha M L$$

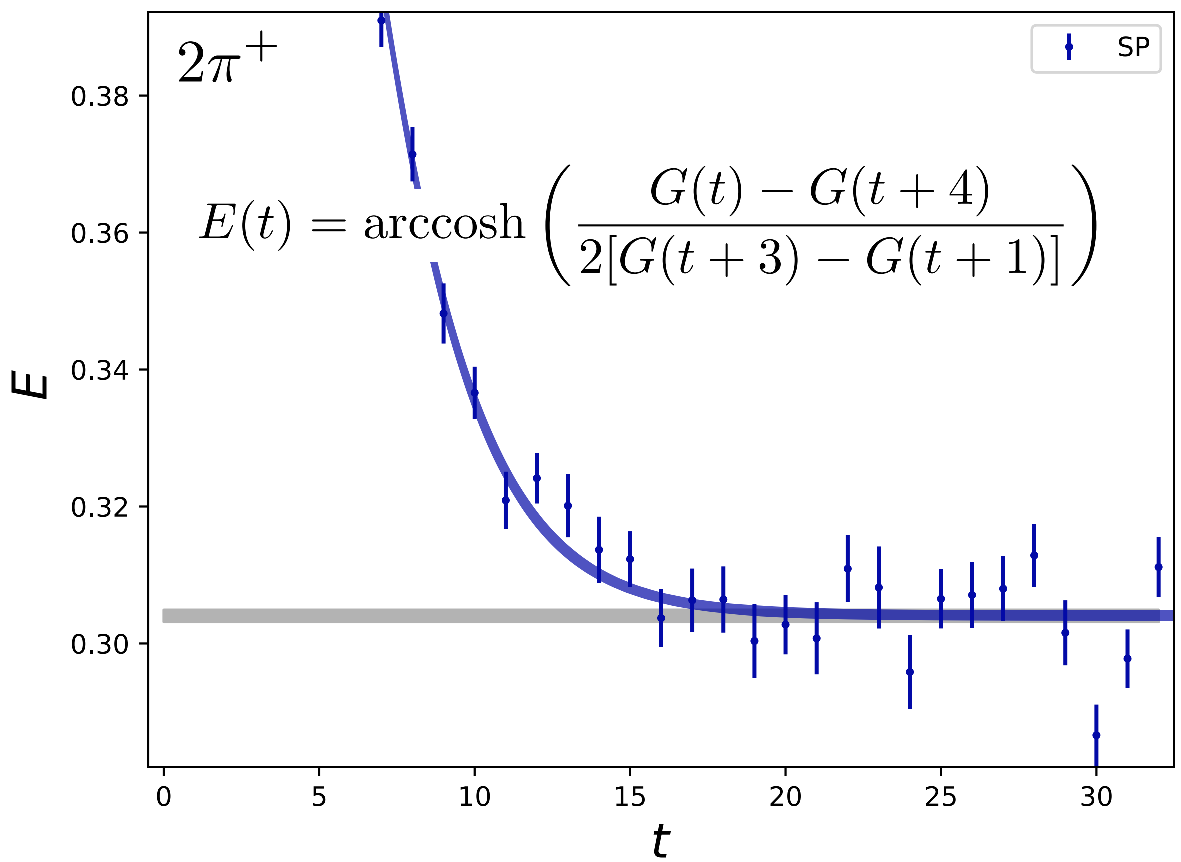
NRQED$_L$ with perturbative Coulomb should converge for proton-proton systems at the physical point with $L \ll 26 \text{ fm}$

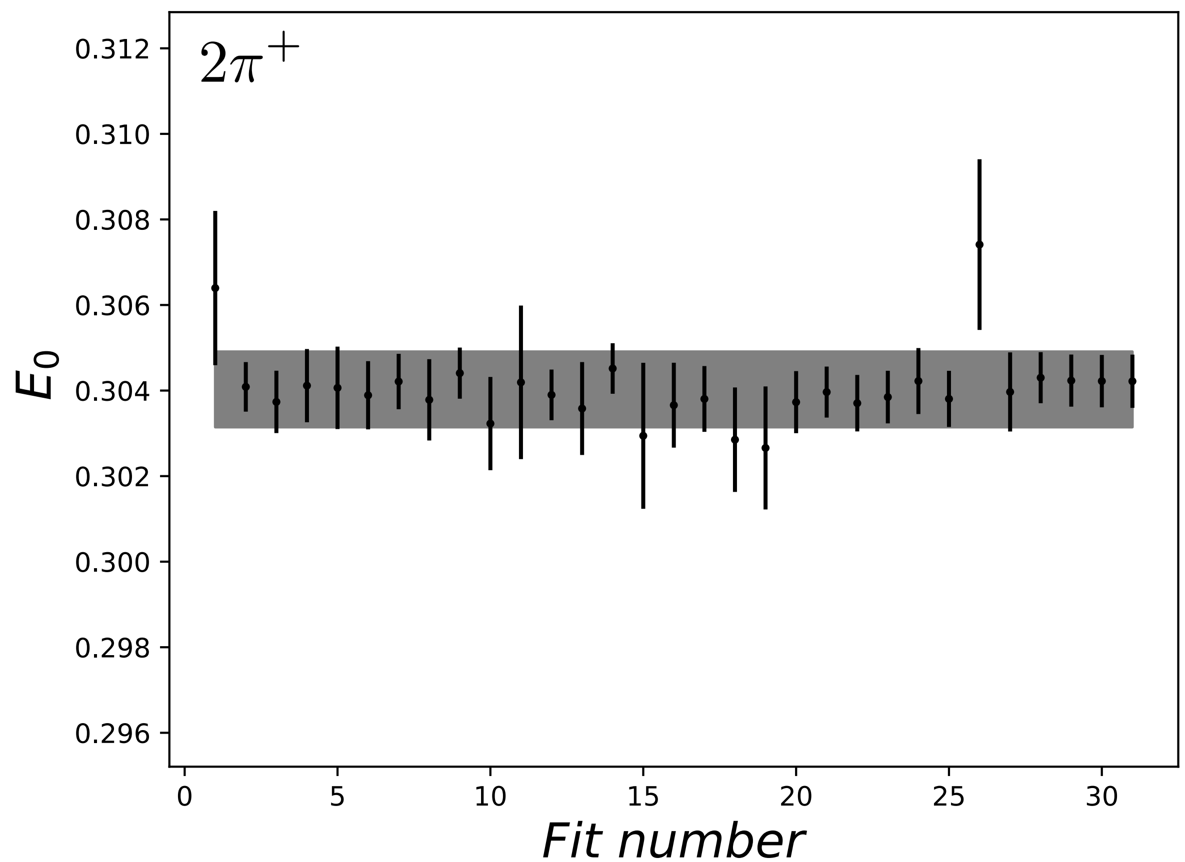
Backup

Correlator Fits

$N_{src} = 35\,010,\ 1711$

- All possible fit ranges sampled with $t_{min} \geq 2$ and $t_{max} = \frac{3\beta}{8}$ to avoid thermal excited states near midpoint (or noise threshold)
- Number of states included chosen by information criterion $\Delta\text{AIC} \leq -2$
- Optimal shrinkage used to estimate covariance matrix — Ledoit, Wolf, Journal of Multivariate Analysis 88 (2004)
- Fits rejected if two numerical optimizers, correlated vs uncorrelated fits, or central value too far from bootstrap median
- Weighted average of all acceptable fits determines final result and uncertainties





Result of fit completely determined after specifying weights and tolerances