The muon anomalous magnetic moment

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(UR & BNL)

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The magnetic moment

- The magnetic moment $\vec{\mu}$ determines the shift of a particle's energy in the presence of a magnetic field $\vec{B}$

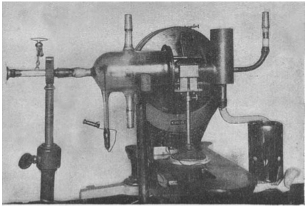
$$V = -\vec{\mu} \cdot \vec{B}$$

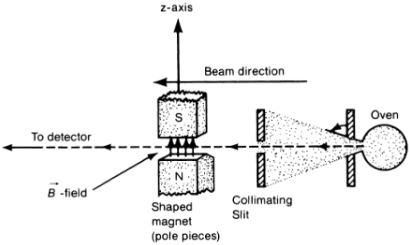
- The intrinsic spin $\vec{S}$ of a particle contributes

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{S}$$

with electric charge e, particle mass m, and Landé factor g.

Stern & Gerlach, 1922





$\vec{B} = 0$ $\vec{B} \neq 0$

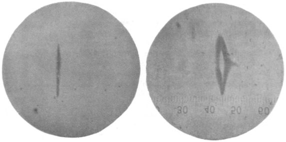


Fig. 2. Fig. 3.

- Send silver atoms through non-uniform magnetic field, $\vec{F} = -\vec{\nabla} V$
- Atoms electrically neutral $\Rightarrow$ spin effects can dominate
- Silver has single 5s electron and fully filled shells below $\Rightarrow$ observe μ of the electron
- $\vec{B} \neq 0$: two distinct lines $\Rightarrow$ quantized spin, distance of lines $\Rightarrow g_e$

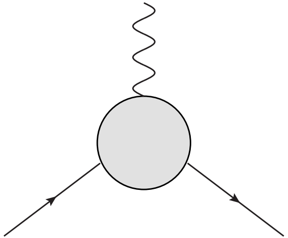
The anomalous magnetic moment

- 1924: Stern and Gerlach measured $g_e = 2.0(2)$
- 1928: Dirac shows that relativistic quantum mechanics yields $g_e = 2$
- 1947 (Phys. Rev. 72 1256, November 3): Kusch & Foley (Columbia) measure $g_e = 2.00229(8)$ in the Zeeman spectrum of gallium
- 1947 (Phys. Rev. 73 416, December 30): Schwinger calculates lowest-order radiative photon correction within quantum field theory (QFT): $g_e = 2 + \alpha/\pi = 2.00232\ldots$

  Define anomalous magnetic moment $a_e = (g_e - 2)/2$ exhibiting effects of QFT

The anomalous magnetic moment

- In QFT a can be expressed in terms of scattering of particle off a classical photon background



For external photon index μ with momentum q the scattering amplitude can be generally written as

$$(-ie)\left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2m}F_2(q^2)\right]$$

with $F_2(0) = a$.

Early measurements of a_μ

- Study of μ decays under varying magnetic field by Garwin, Lederman and Weinrich 1957 (Nevis Cyclotron, Columbia)

$$g_\mu = 2.0(2)$$

- Study of stopped muon precession by Garwin, Hutchinson, Penman, Shapiro 1960

$$a_\mu = 0.00113 + 0.00016 - 0.00012$$

- Crucial improvement (magic-momentum method) in CERN-3 experiment 1979

$$a_\mu = 0.001165924(9)\,.$$

Magic momentum method

- Send muon in storage ring with uniform magnetic field, observe decays as function of time
- Measure difference of cyclotron frequency ω_C and spin rotation frequency ω_S directly with

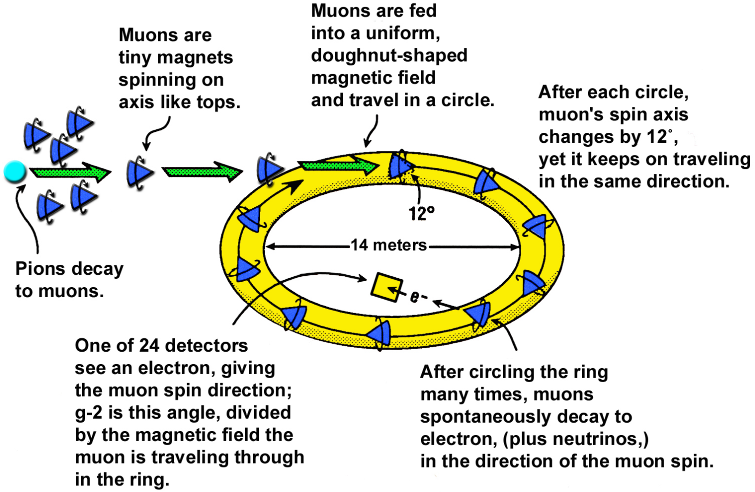
$$\vec{\omega_a} = \vec{\omega}_S - \vec{\omega}_C = -\frac{Qe}{m}\left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

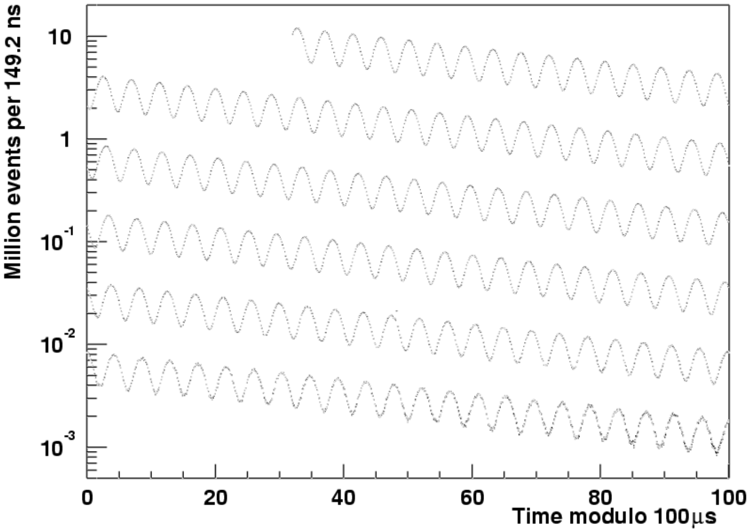
(Thomas 1927).

- Minimize uncertainty by tuning $\gamma^2 - 1 \approx 1/a_\mu$ or $p_\mu \approx 3.09$ GeV to suppress effect of electric field; treat $\vec{\beta}\cdot\vec{B}$ term as perturbation
- All experiments discussed in the following use this method

The BNL E821 experiment (2006)

http://www.g-2.bnl.gov/physics/index.html



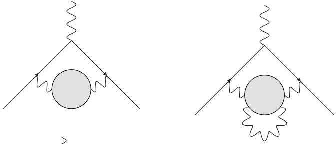


$$a_\mu^{\mathrm{E821}} = 0.00116592089(54)_{\mathrm{stat}}(33)_{\mathrm{sys}}$$

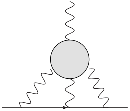
There is a tension of 3.7σ for the muon

$$a_\mu^{\mathrm{E821}} - a_\mu^{\mathrm{SM}} = 27.4\,\underbrace{(2.7)}_{\mathrm{HVP}}\,\underbrace{(2.6)}_{\mathrm{HLbL}}\,\underbrace{(0.1)}_{\mathrm{other}}\,\underbrace{(6.3)}_{\mathrm{E821}}\times 10^{-10}$$

Hadronic Vacuum Polarization (HVP)



Hadronic Light-by-Light (HLbL)



New experiment: Fermilab E989

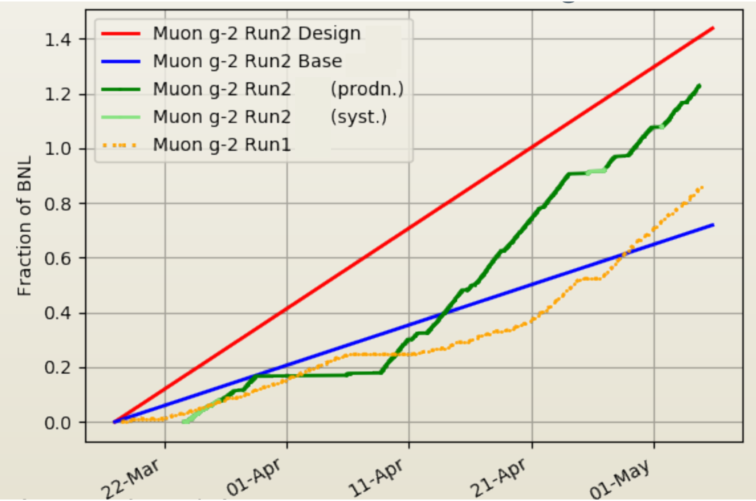
$$a_\mu^{\mathrm{E821}} - a_\mu^{\mathrm{SM}} = 27.4\,\underbrace{(2.7)}_{\mathrm{HVP}}\,\underbrace{(2.6)}_{\mathrm{HLbL}}\,\underbrace{(0.1)}_{\mathrm{other}}\,\underbrace{(6.3)}_{\mathrm{E821}} \times 10^{-10}$$

$$\delta a_\mu^{\mathrm{E989},\ \mathbf{2019}} = 4.5 \times 10^{-10}\,, \qquad \delta a_\mu^{\mathrm{E989},\ 2021} = 1.6 \times 10^{-10}$$

Need to improve uncertainties on HVP and HLbL contributions

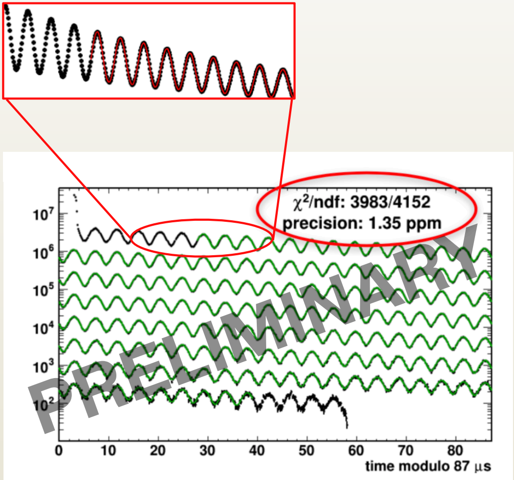
Experiment

Statistics Run 1 in 2018 and Run 2 in 2019 (talk by N. Tran at FPCP 2019):



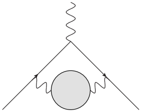
Run 1 fit (talk by N. Tran at FPCP 2019):

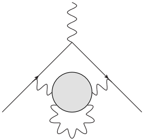
$$N(t) = N_0 e^{-t/\tau}\left[1 - A\cos\left(\omega_a t + \phi\right)\right]$$



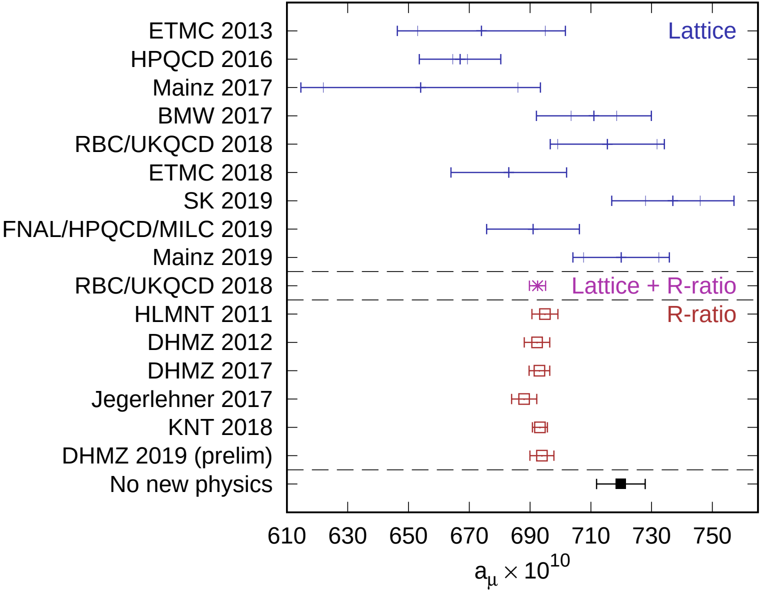
Relative unblinding of 6 analyzing groups successful!

HVP contribution

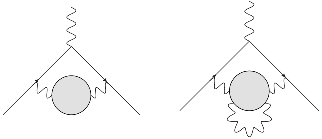


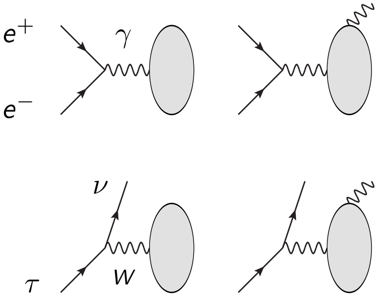


Status of HVP determinations



The HVP from dispersion relations





$$e^+e^- \to \mathrm{hadrons}(\gamma)$$

$$J_\mu = V_\mu^{I=1,I_3=0} + V_\mu^{I=0,I_3=0}$$

$$\tau \to \nu\mathrm{hadrons}(\gamma)$$

$$J_\mu = V_\mu^{I=1,I_3=\pm1} - A_\mu^{I=1,I_3=\pm1}$$

Knowledge of isospin-breaking corrections and separation of vector and axial-vector components needed to use τ decay data.

Dispersive method - e^+e^- status

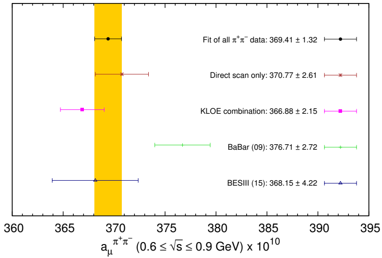
Recent results ($\times 10^{10}$) by Keshavarzi et al. 2018, Davier et al. 2017:

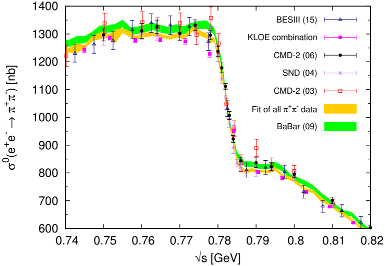
Channel	This work (KNT18)	DHMZ17 [78]	Difference
Data based channels ($\sqrt{s} \leq 1.8$ GeV)			
$\pi^0\gamma$ (data + ChPT)	4.58 ± 0.10	4.29 ± 0.10	0.29
$\pi^+\pi^-$ (data + ChPT)	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$ (data + ChPT)	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
...			
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

Good agreement for total, individual channels disagree to some degree. Surprising since they use the same experimental input.

Dispersive method - e^+e^- status

Tension in 2π experimental input. BaBar and KLOE central values differ by $\delta a_\mu = 9.8(3.5) \times 10^{-10}$, compare to quoted total uncertainties of dispersive results of order $\delta a_\mu = 3 \times 10^{-10}$.





Conflicting input limits the precision and reliability of the dispersive results.
First-principles calculation to remove dependence on conflicting input data desirable.
(RBC/UKQCD 2018)

Looking for more data and insight: energy-scans update from CMD-3 in Novosibirsk and ISR updates from KLOE2, BaBar, Belle, BESIII and BelleII.

Talk by Zhang at EPS 2019 (DHMZ 2019 prelim):

Combined Results Fit [<0.6 GeV] + Data [0.6-1.8 GeV]

$\sqrt{s}$ range [GeV]	a_μ^{had} $[10^{-10}]$ All data	a_μ^{had} $[10^{-10}]$ All but BABAR	a_μ^{had} $[10^{-10}]$ All but KLOE
threshold - 1.8	$506.9 \pm 1.9_{total}$	$505.0 \pm 2.1_{total}$	$510.6 \pm 2.2_{total}$

⇒ The difference "All but BABAR" and "All but KLOE" = 5.6
to be compared with 1.9 uncertainty with "All data"

- The local error inflation is not sufficient to amplify the uncertainty
- Global tension (normalisation/shape) not previously accounted for
- Potential underestimated uncertainty in at least one of the measurements?
- Other measurements not precise enough and are in agreement with BABAR or KLOE

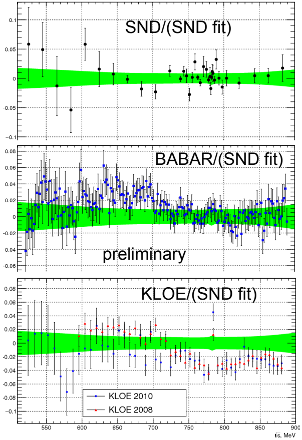
⇒ Given the fact we do not know which dataset is problematic, we decide to

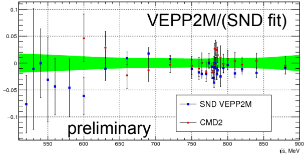
- Add half of the discrepancy (2.8) as an additional uncertainty (correcting the local PDG inflation to avoid double counting)
- Take the mean value "All but BABAR" and "All but KLOE" as our central value

EPS 2019, Ghent, July 10-17, 2019 Zhiqing Zhang (LAL, Orsay) 10/14+3

Talk by Druzhinin at EPS 2019 (SND experiment preliminary):

$$e^+e^- \to \pi^+\pi^-$$





$$0.53 < \sqrt{s} < 0.88 \text{ GeV}$$

	$a_\mu(\pi^+\pi^-) \times 10^{10}$
SND & VEPP-2000	$411.8 \pm 1.0 \pm 3.7$
SND & VEPP-2M	$408.9 \pm 1.3 \pm 5.3$
BABAR	$414.9 \pm 0.3 \pm 2.1$

Dispersive method - τ status

Experiment	$a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10}) $2m_{\pi^\pm} - 0.36$ GeV	$0.36 - 1.8$ GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

Davier et al. 2013: $a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau] = 516.2(3.5) \times 10^{-10}$ $(2m_\pi^\pm - 1.8$ GeV$)$

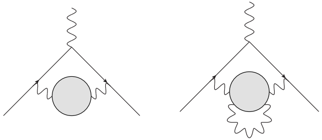
Compare to e^+e^-:

- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 507.1(2.6) \times 10^{-10}$ (DHMZ17, $2m_\pi^\pm - 1.8$ GeV)
- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 503.7(2.0) \times 10^{-10}$ (KNT18, $2m_\pi^\pm - 1.937$ GeV)

Here treatment of isospin-breaking to relate matrix elements of $V_\mu^{I=1,I_3=1}$ to $V_\mu^{I=1,I_3=0}$ crucial.

Can calculate from first-principles in lattice QCD+QED (Bruno, Izubuchi, CL, Meyer 2018)

Euclidean Space Representation



Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3}\sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x},t) J_j(0)\rangle$$

and w_t capturing the photon and muon part of the HVP diagrams.

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, charm, and bottom quark contributions.

Window method (RBC/UKQCD 2018)

We therefore also consider a window method

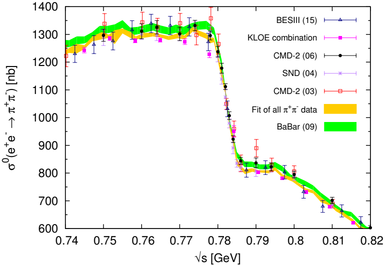
$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

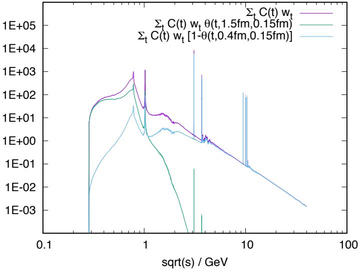
with

$$\begin{aligned}
a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\
a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\
a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\
\Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]\,/2\,.
\end{aligned}$$

In this version of the calculation, we use $C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})$ to compute a_μ^{SD} and a_μ^{LD} and Lattice QCD+QED for a_μ^{W}.

How does this translate to the time-like region?





Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.

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Editors' Suggestion

Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

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We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is $a_\mu^{\text{HVP LO}} = 715.4(18.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with R-ratio data, we significantly improve the precision to $a_\mu^{\text{HVP LO}} = 692.5(2.7) \times 10^{-10}$. This is the currently most precise determination of $a_\mu^{\text{HVP LO}}$.

This method allows us to reduce HVP uncertainty over next years to $\delta a_\mu^{\text{LO HVP}} \sim 1 \times 10^{-10}$, below Fermilab E989 uncertainty

Overview of individual contributions

Diagrams – Isospin limit

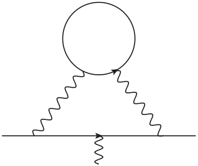
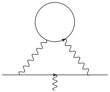
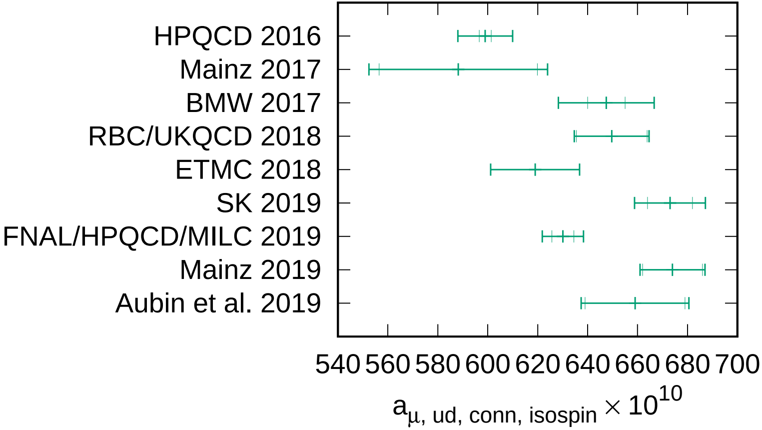
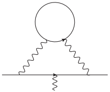
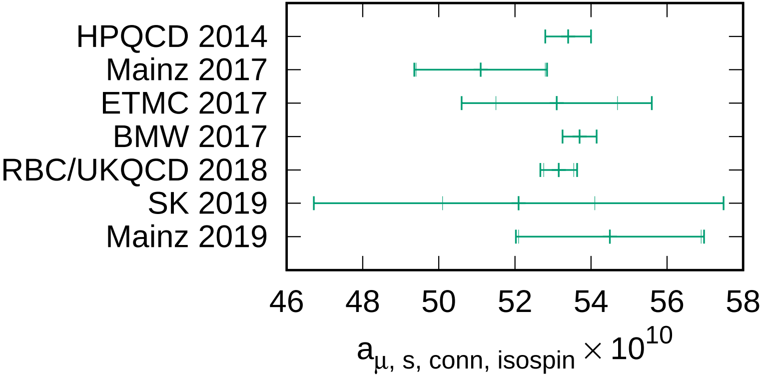


FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\mathrm{HVP\ LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.




HPQCD 2016
Mainz 2017
BMW 2017
RBC/UKQCD 2018
ETMC 2018
SK 2019
FNAL/HPQCD/MILC 2019
Mainz 2019
Aubin et al. 2019
540 560 580 600 620 640 660 680 700
$a_{\mu,\text{ ud, conn, isospin}} \times 10^{10}$




HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
SK 2019
Mainz 2019
46
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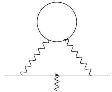
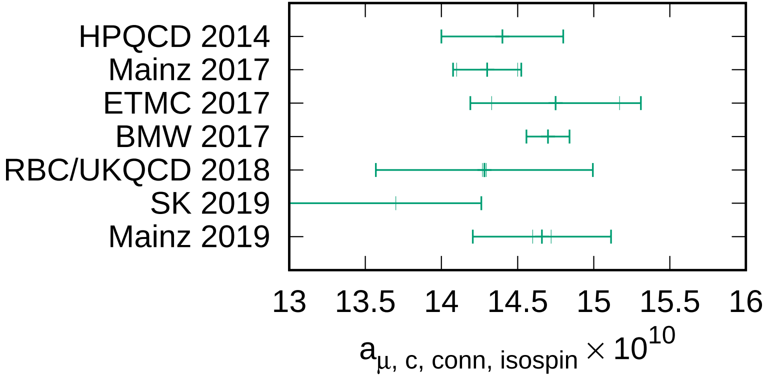
58
$a_{\mu,\text{ s, conn, isospin}} \times 10^{10}$




HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
SK 2019
Mainz 2019
13
13.5
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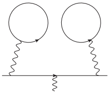
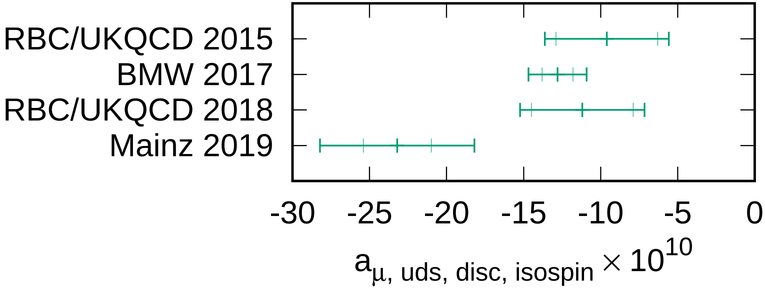
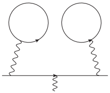
16
$a_{\mu,\,c,\,\mathrm{conn,\,isospin}} \times 10^{10}$





RBC/UKQCD 2015
BMW 2017
RBC/UKQCD 2018
Mainz 2019
-30
-25
-20
-15
-10
-5
0
$a_{\mu,\ \mathrm{uds,\ disc,\ isospin}} \times 10^{10}$



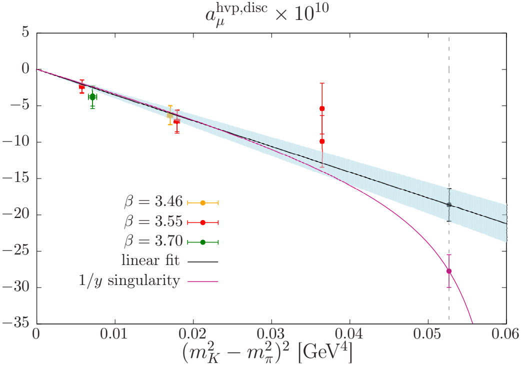
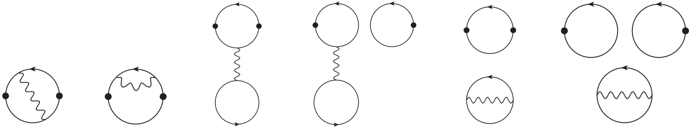


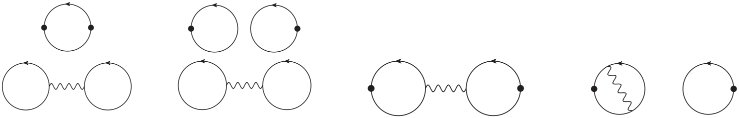
FIG. 9: Extrapolation of the disconnected contribution to a_μ^{hvp} in the SU(3)-breaking variable $\Delta_2 \equiv m_K^2 - m_\pi^2$. The data points for the local-local and the local-conserved discretizations are shown. A linear fit (straight black line), as well as a fit based on ansatz (30) are shown.

Mainz 2019: arXiv:1904.03120; better control of chiral extrapolation could be helpful

Diagrams – QED corrections

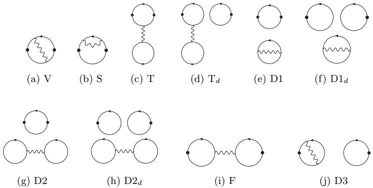


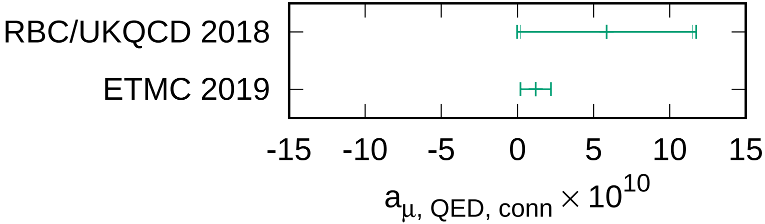
(a) V (b) S (c) T (d) T_d (e) D1 (f) $\mathrm{D1}_d$

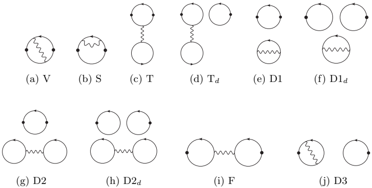


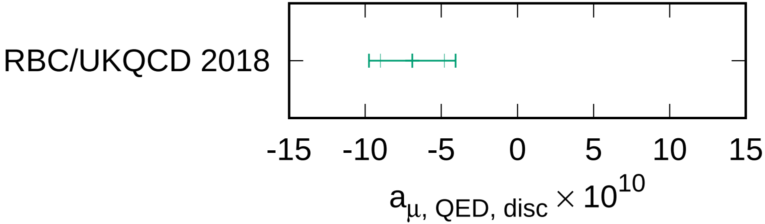
(g) D2 (h) $\mathrm{D2}_d$ (i) F (j) D3

For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.

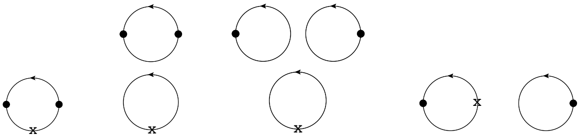

(a) V
(b) S
(c) T
(d) T_d
(e) D1
(f) $D1_d$
(g) D2
(h) $D2_d$
(i) F
(j) D3


RBC/UKQCD 2018
ETMC 2019
-15
-10
-5
0
5
10
15
$a_{\mu,\ \mathrm{QED,\ conn}} \times 10^{10}$


(a) V
(b) S
(c) T
(d) T_d
(e) D1
(f) $D1_d$
(g) D2
(h) $D2_d$
(i) F
(j) D3

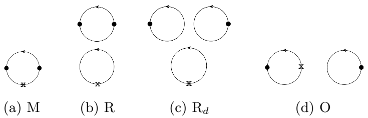

RBC/UKQCD 2018
-15
-10
-5
0
5
10
15
$a_{\mu,\,\mathrm{QED,\,disc}} \times 10^{10}$

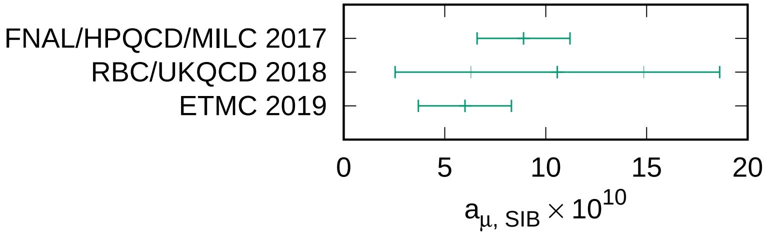
Diagrams – Strong isospin breaking



(a) M (b) R (c) R_d (d) O

For the HVP R is negligible since $\Delta m_u \approx -\Delta m_d$ and O is SU(3) and $1/N_c$ suppressed.


(a) M
(b) R
(c) R$_d$
(d) O


FNAL/HPQCD/MILC 2017
RBC/UKQCD 2018
ETMC 2019
0
5
10
15
20
$a_{\mu,\,\mathrm{SIB}} \times 10^{10}$

Status of RBC/UKQCD HVP effort

The pure lattice calculation of RBC/UKQCD 2018:

$$
\begin{aligned}
10^{10} \times a_\mu^{\mathrm{HVP\ LO}} &= 715.4(18.7) \\
&= 715.4(16.3)_{\mathrm{S}}(7.8)_{\mathrm{V}}(3.0)_{\mathrm{C}}(1.9)_{\mathrm{A}}(3.2)_{\mathrm{other}}
\end{aligned}
$$

(S) statistics, (V) finite-volume errors, (C) the continuum limit extrapolation, (A) scale setting uncertainty;

other $\supset$ neglected diagrams for QED and SIB, estimate of bottom quark contribution

Statistical noise mostly from isospin symmetric light quark connected (14.2) and disconnected (3.3), QED (5.7), SIB (4.3)

RBC/UKQCD 2019 update (in preparation):

- Improved methodology
- A lot of new data

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

CERN

Mattia Bruno

Columbia University

Ryan Abbot
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University

Peter Boyle
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Regensburg

Christoph Lehner (BNL)

University of Southampton

Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Aaron & Mattia joined since 2018 paper

Improved statistics and systematics – Bounding Method

BMW/RBC/UKQCD 2016

The correlator in finite volume

$$C(t) = \sum_n |\langle 0|V|n\rangle|^2 e^{-E_n t} \,.$$

We can bound this correlator at each t from above and below by the correlators

$$\tilde{C}(t; T, \tilde{E}) = \begin{cases} C(t) & t < T \,, \\ C(T) e^{-(t-T)\tilde{E}} & t \geq T \end{cases}$$

for proper choice of $\tilde{E}$. We can chose $\tilde{E} = E_0$ (assuming $E_0 < E_1 < \ldots$) to create a strict upper bound and any $\tilde{E}$ larger than the local effective mass to define a strict lower bound.

Improved Bounding Method

RBC/UKQCD 2018

Therefore if we had precise knowledge of the lowest $n = 0, \ldots, N$ values of $|\langle 0|V|n\rangle|$ and E_n, we could define a new correlator

$$C^N(t) = C(t) - \sum_{n=0}^{N} |\langle 0|V|n\rangle|^2 e^{-E_n t}$$

which we could bound much more strongly through the larger lowest energy $E_{N+1} \gg E_0$.

New method: do a GEVP study of FV spectrum to perform this subtraction

- 10 operator basis including two 4π operators
- Automatic group theory by A. Meyer
- Automatic contractions/evaluations using distillation: `https://github.com/lehner/Wick`

Reduces statistical error of light quark contribution by more than a factor of 3.

Other improvements:

- FV corrections both directly calculated at physical pion mass ($a_\mu(L = 6.22\text{ fm}) - a_\mu(L = 4.66\text{ fm})$), GSL^2 method, update of Hansen and Patella.
- HVP QED from re-analysis of HLbL point-source data (see also RBC/UKQCD τ project, Bruno et al. 1811.00508) reduces statistical noise by $\approx 10\times$ for V and S
- Infinte-volume and continuum limit also for diagram V, S, and F
- First results for T, D1, and R; other sub-leading in preparation
- Global fit combined with calculation of mass derivatives gives much reduced uncertainty for diagrams M and O (connected and disconnected SIB)

Ensembles at physical pion mass:

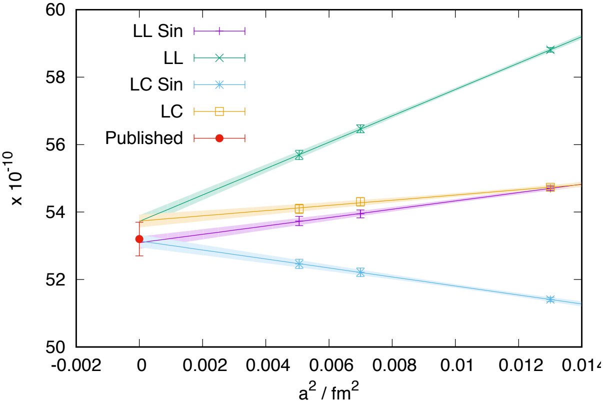
48I (1.73 GeV, 5.5fm), 64I (2.359 GeV, 5.4fm), 24ID (1 GeV, 4.7fm), 32ID (1 GeV, 6.2fm), 48ID (1 GeV, 9.3fm), 32IDf (1.37 GeV, 4.6fm)

RBC/UKQCD 2019 (data for light quarks, changes from 2018):

- A2A data for connected isospin symmetric: 48I (127 conf $\rightarrow$ 400 conf), 64I (160 conf $\rightarrow$ 250 conf), 24ID (new 130 conf, multi mass), 32ID (new 88 conf, multi mass)
- A2A data (tadpole fields) for disconnected: 48I (33 conf), 24ID (new 260 conf, multi mass), 32IDf (new 103 conf)
- QED and SIB corrections to meson and Ω masses, Z_V: 48I (30 conf) and 64I (new 30 conf)
- QED and SIB from HLbL point sources on 48I, 24ID, 32ID, 32IDf (on order of 20 conf each, 2000 points per config)
- Distillation data on 48I (33 conf), 64I (in progr.), 24ID (33 conf), 32ID (11 conf, multi-mass)
- New Ω mass operators (excited states control): 48I (130 conf)

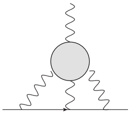
Add $a^{-1} = 2.77$ GeV lattice spacing

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):

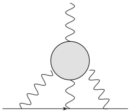


- For light quark need new ensemble at physical pion mass. Started run on Summit Machine at Oak Ridge this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).

HLbL contribution



Current HLbL value is model estimate



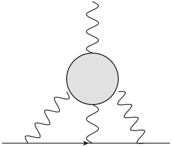
Contributions to $a_\mu^{\mathrm{HLbL}} \times 10^{10}$

	PdRV09	JN09	FJ17
π^0, η, η'	11.4(1.3)	9.9(1.6)	9.5(1.2)
π, K loops	-1.9(1.9)	-1.9(1.3)	-2.0(5)
axial-vector	1.5(1.0)	2.2(5)	0.8(3)
scalar	-0.7(7)	-0.7(2)	-0.6(1)
quark loops	0.2 (charm)	2.1(3)	2.2(4)
tensor			0.1(0)
NLO			0.3(2)
Total	10.5(4.9)	11.6(3.9)	10.3(2.9)
	10.5(2.6) (quadrature)		

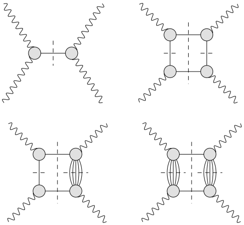
Potential double-counting and ad-hoc uncertainties

Two new avenues for a model-independent value for the HLbL

Dispersive analysis +
Experimental/lattice input

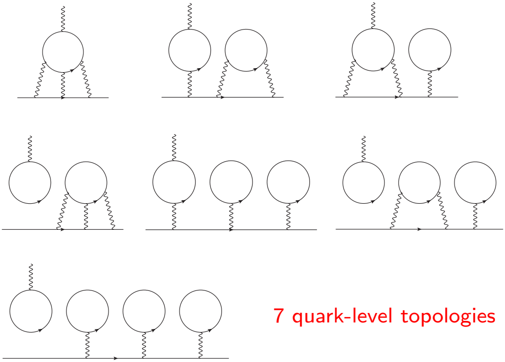


Direct lattice calculation



...

How to estimate uncertainty
of truncation of cuts/states?



7 quark-level topologies

Dispersive analysis - recent results

- PRD94(2016)074507 (Mainz): Pion-pole contribution
  $a_\mu^{\pi-pole} = 6.50(83) \times 10^{-10}$ using a model parametrization of the $\pi \to \gamma^*\gamma^*$ form factor constrained by lattice data

$$\mathcal{F}^{\mathrm{LMD+V}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{\widetilde{h}_0\, q_1^2 q_2^2 (q_1^2+q_2^2) + \widetilde{h}_1 (q_1^2+q_2^2)^2 + \widetilde{h}_2\, q_1^2 q_2^2 + \widetilde{h}_5\, M_{V_1}^2 M_{V_2}^2\, (q_1^2+q_2^2) + \alpha\, M_{V_1}^4 M_{V_2}^4}{(M_{V_1}^2 - q_1^2)(M_{V_2}^2 - q_1^2)(M_{V_1}^2 - q_2^2)(M_{V_2}^2 - q_2^2)}$$

- JHEP1704(2017)161 (Colangelo et al.): Pion-box plus S-wave rescattering
  $a_\mu^{\pi-box} + a_\mu^{\pi\pi,\pi-pole\ LHC,J=0} = -2.4(1) \times 10^{-10}$

- PRL121(2018)112002 (Hoferichter et al.); 1808.04823: Pion-pole contribution
  $a_\mu^{\pi-pole} = 6.26(30) \times 10^{-10}$ reconstructing $\pi \to \gamma^*\gamma^*$ form factor from $e^+e^- \to 3\pi$, $e^+e^-\pi^0$ and $\pi^0 \to \gamma\gamma$ width

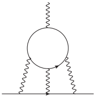
Combining these results one finds: $a_\mu^{\pi-pole} + a_\mu^{\pi-box} + a_\mu^{\pi\pi} = 3.9(3) \times 10^{-10}$

Further estimates: $a_\mu^{\eta,\eta'} \approx 3 \times 10^{-10}$, $a_\mu^{\text{axial vector}} \approx 1 \times 10^{-10}$, $a_\mu^{\text{short distance}} \approx 1 \times 10^{-10}$

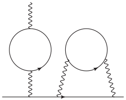
Control of truncation error very important.

7 quark-level topologies of direct lattice calculation

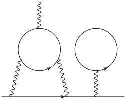
Hierarchy imposed by QED charges of dominant up- and down-quark contribution

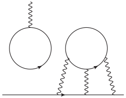


$Q_u^4 + Q_d^4 = 17/81$

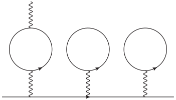


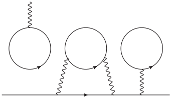
$(Q_u^2 + Q_d^2)^2 = 25/81$



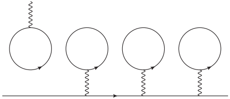


$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$





$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$

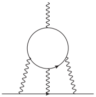


$(Q_u + Q_d)^4 = 1/81$

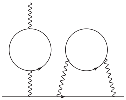
Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

7 quark-level topologies of direct lattice calculation

Hierarchy imposed by QED charges of dominant up- and down-quark contribution

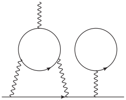


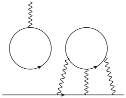
$Q_u^4 + Q_d^4 = 17/81$



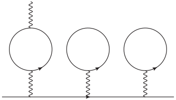
$(Q_u^2 + Q_d^2)^2 = 25/81$

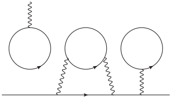
Dominant diagrams in top row: connected and leading disconnected diagram



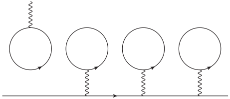


$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$





$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$

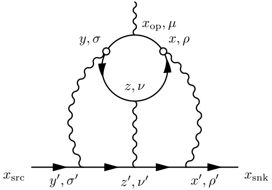


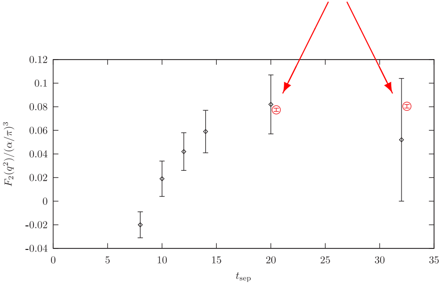
$(Q_u + Q_d)^4 = 1/81$

Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

PRD93(2015)014503 (Blum, Christ, Hayakawa, Izubuchi, Jin, and CL):

New sampling strategy with 10x reduced noise for same cost (red versus black):



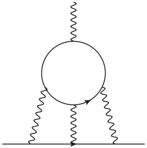


Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

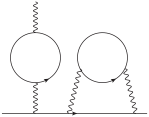
PRL118(2016)022005 (Blum, Christ, Hayakawa, Izubuchi, Jin, Jung, and CL):

- Calculation at physical pion mass with finite-volume QED prescription ($\mathrm{QED_L}$) at single lattice cutoff of $a^{-1} = 1.73$ GeV and lattice size $L = 5.5$ fm.
- Connected diagram:



$$a_\mu^{\mathrm{cHLbL}} = 11.6(0.96) \times 10^{-10}$$

- Leading disconnected diagram:

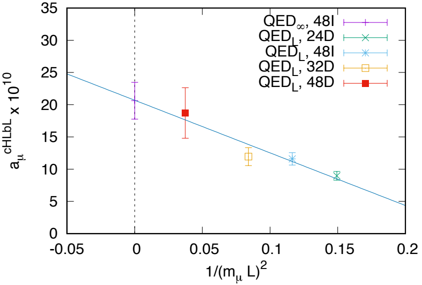


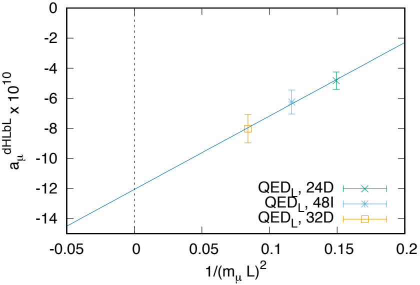
$$a_\mu^{\mathrm{dHLbL}} = -6.25(0.80) \times 10^{-10}$$

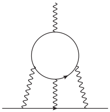
- Large cancellation expected from pion-pole-dominance considerations is realized: $a_\mu^{\mathrm{HLbL}} = a_\mu^{\mathrm{cHLbL}} + a_\mu^{\mathrm{dHLbL}} = 5.35(1.35) \times 10^{-10}$

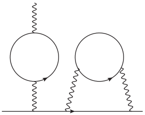
Potentially large systematics due to finite-volume QED!

Preliminary results for infinite-volume extrapolation



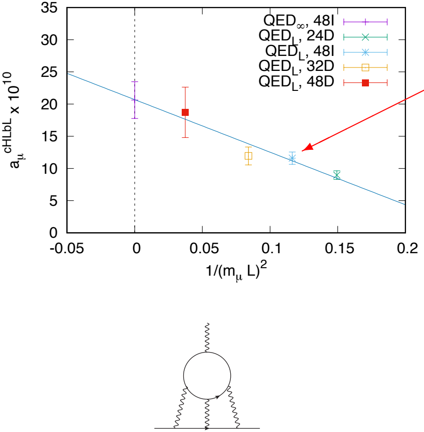


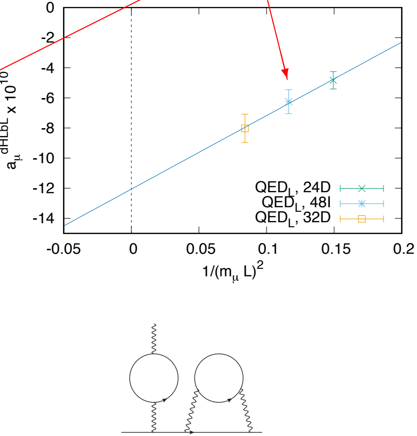




Preliminary results for infinite-volume extrapolation

Data used for finite-volume result in PRL118(2016)022005





Preliminary QED_L result in proceedings: 1907.00864, paper in preparation

Hadronic light-by-light contribution to the muon anomalous magnetic moment from lattice QCD

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We report preliminary results for the hadronic light-by-light scattering contribution to the muon anomalous magnetic moment. Several ensembles using 2+1 flavors of Möbius domain-wall fermions, generated by the RBC/UKQCD collaborations, are employed to take the continuum and infinite volume limits of finite volume lattice QED+QCD. We find $a_\mu^{\text{HLbL}} = (7.41 \pm 6.33) \times 10^{-10}$.

Next steps in first-principles calculation of HLbL

- Further reduce statistical and finite-volume errors

- Take infinite-volume limit also with finite-volume QCD+infinite-volume QED mixed approach
PRD96(2017)034515 (Blum, Christ, Hayakawa, Izubuchi, Jin, Jung, and CL)

Continued effort using these methods to reduce HLbL uncertainty over next years to $\delta a_\mu^{\mathrm{HLbL}} \sim 1 \times 10^{-10}$, below Fermilab E989 uncertainty

g-2 theory initiative

Muon g-2 Theory Initiative – Goals

Theory Support for the Fermilab E989 experiment to maximize its impact:

- Work towards reduction and scrutiny of uncertainties of hadronic contributions
- Provide summary of theory calculations of the hadronic contributions

  $\Rightarrow$ Write report (whitepaper) before Fermilab experiment has first results (target December 2019)

- Steering Committee: Colangelo, Davier, Eidelman, El-Khadra, Lehner, Mibe, Nyffeler, Roberts, Teubner

Muon g-2 Theory Initiative – Workshops and Whitepaper

- Plenary and working-group workshops:
  - 3-6 June 2017, near Fermilab, first plenary workshop
  - 12-14 February 2018, KEK, HVP working group workshop
  - 12-14 March 2018, University of Connecticut, HLbL WG workshop
  - 18-22 June 2018, Mainz, 2018 plenary meeting
  - 9-13 September 2019, Seattle, 2019 workshop with focus on whitepapers

As whitepapers are being finalized, there are still opportunities to participate in the effort!

A tale of two anomalies

Assuming further improvements solidify the tensions

$$a_e^{\mathrm{EXP}} - a_e^{\mathrm{SM}} = -88\,\underbrace{(23)}_{\alpha}\,\underbrace{(02)}_{\mathrm{SM}}\,\underbrace{(28)}_{\mathrm{EXP}} \times 10^{-14}$$

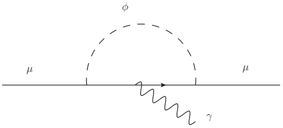
and

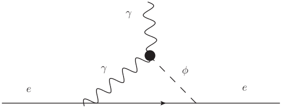
$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = 27.4\,\underbrace{(2.7)}_{\mathrm{HVP}}\,\underbrace{(2.6)}_{\mathrm{HLbL}}\,\underbrace{(0.1)}_{\mathrm{other}}\,\underbrace{(6.3)}_{\mathrm{EXP}} \times 10^{-10}\,,$$

is there a plausible BSM scenario?

Davoudiasl & Marciano 2018: Light new physics

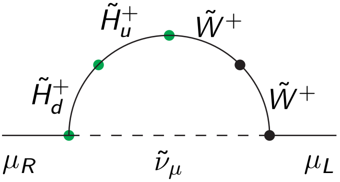
$$\mathcal{L}_\phi = -\frac{1}{2}m_\phi^2\phi^2 - \sum_f \lambda_f \phi\, \bar{f} f - \frac{\kappa_\gamma}{4}\phi\, F_{\mu\nu}F^{\mu\nu}$$





- 1-loop Δa_μ, 2-loop (Barr-Zee) for Δa_e gives opposite signs!
- Real scalar ϕ; $\phi\gamma\gamma$ coupling from integrating out heavy fermion
- For $m_\phi = 250$ MeV, $\lambda_\mu = 10^{-3}$, $\lambda_e = 4 \times 10^{-6}$, $\lambda_\tau = 0.06$, can obtain both anomalies. This parameter space is not yet ruled out by other experiments.
- This model can be tested in $e^+e^- \to \tau^+\tau^-\phi \to \tau^+\tau^-\ell^+\ell^-$ decays at Belle II (Batell et al. 2016)

Stöckinger et al. 2015: MSSM ($\tan\beta \to \infty$) with radiative muon mass



- $m_\mu^{\mathrm{Pole}} \sim y_\mu v_d + y_\mu v_u \times \mathrm{loop}$ and $a_\mu^{\mathrm{SUSY}} \sim y_\mu v_u \times \mathrm{loop}$
- Idea: $v_d = 0$ then mass and a_μ diagrams scale identically
- $M_{\mathrm{SUSY}} = \ldots = m_{\tilde{e}_R} = 500\ \mathrm{GeV}$ and $m_{\tilde{\mu}_R} \approx 10 \times M_{\mathrm{SUSY}}$, then

$$\Delta a_e = -7 \times 10^{-13}\,, \qquad \Delta a_\mu = 30 \times 10^{-10}\,.$$

Conclusions and Outlook

- Expect experimental results from Fermilab E989 before end of year
- Concerted effort of theory community both lattice and non-lattice methods (g-2 theory initiative whitepaper to appear before experimental result)
- Interplay of lattice and non-lattice methods for both HVP and HLbL useful to address leading systematics in dispersive approaches
- Pure lattice QCD calculations for HVP have made significant progress and may soon rival precision of dispersive approach
- RBC/UKQCD:
  - HVP: New methods to reduce statistical and systematic errors and a lot of additional data, by end of year first-principles lattice result could have uncertainty of $O(5 \times 10^{-10})$
  - HLbL: First ab-initio calculation with complete error budget in preparation with uncertainty of $O(5 \times 10^{-10})$, publish before end of year