Nucleon Matrix Element Results From the Extended Twisted Mass Collaboration

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The Cyprus Institute

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Contributors

ETM Collaboration

Nucleon Structure

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Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Jena, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool), US (Temple, PA)

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Outline

Preliminaries

- Simulation status/ensembles
- Analysis strategy and parameters
- Evaluation of disconnected contributions

Results

- Isovector charges
- Selected results with disconnected contributions
- Nucleon electromagnetic form factors
- Nucleon generalized form factors

Twisted Mass Lattice QCD

Light sector: Degenerate up & down quarks

$$S^{\ell}_{tm} = \sum_x \bar{\chi}_\ell(x) \left[D_W(U) + \frac{i}{4} c_{\rm sw} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + m + i\mu_\ell \tau^3 \gamma^5 \right] \chi_\ell(x)$$

Heavy sector: Strange and charm: $-\mu_\delta \tau_1 + i\mu_\sigma \tau^3 \gamma^5$

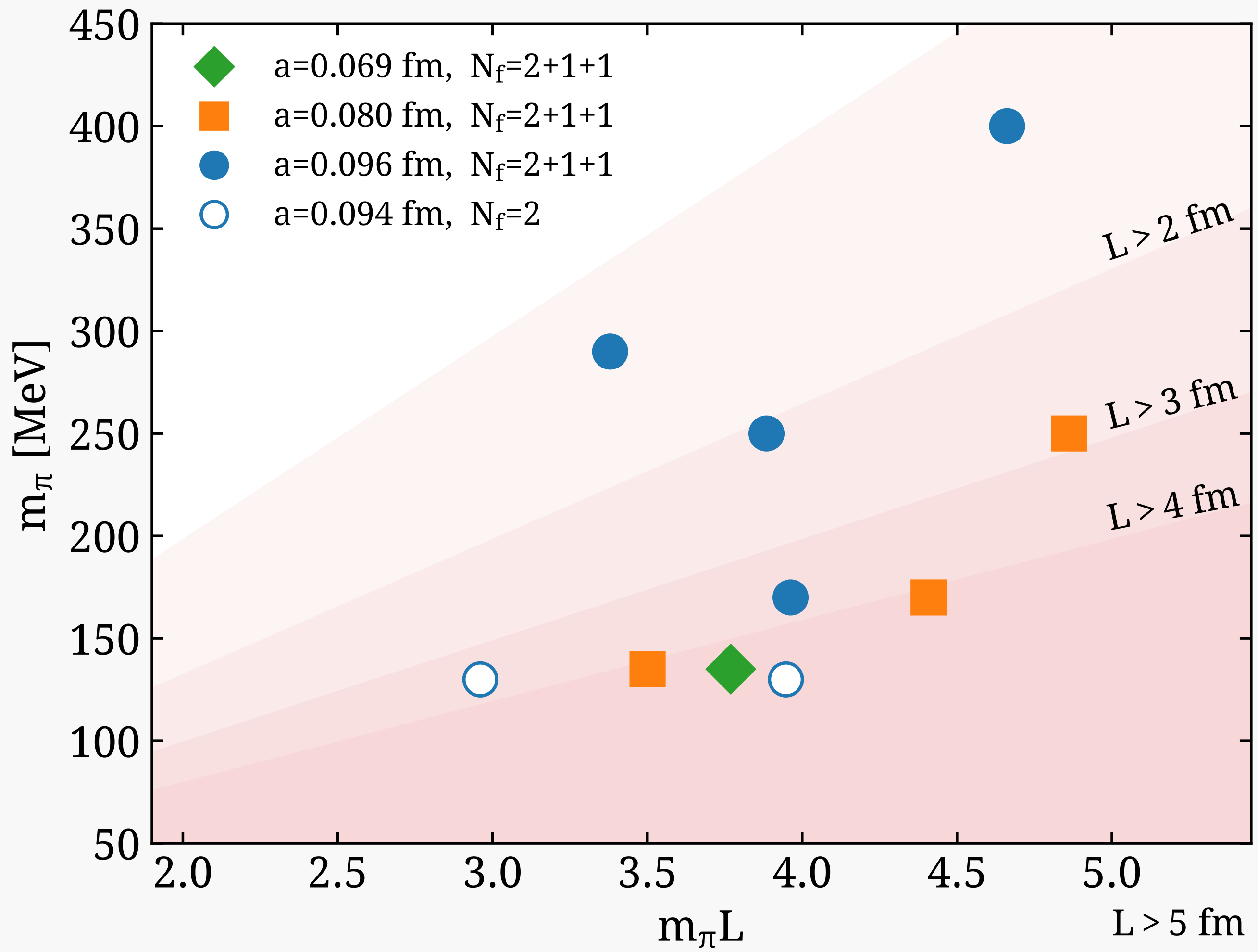
N_f=2+1+1

Formulation particularly attractive for nucleon structure

- Tuning procedure during simulation to reach "maximal twist" (tune m to $m_{\rm crit}$)
- *O*(*a*) improved operators without requiring further operator improvement

R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007

Twisted Mass/Clover Ensembles



Nucleon structure from three ensembles at physical point

- N_f=2, *a*=0.0938 fm, L=48 & 64 ➜ check for volume effects “`cA2.09.{48,64}`"
- N_f=2+1+1, a=0.0801 fm, L=64 “`cB211.072.64`"
- N_f=2+1+1, a≃0.069 fm being simulated (not analysed yet)

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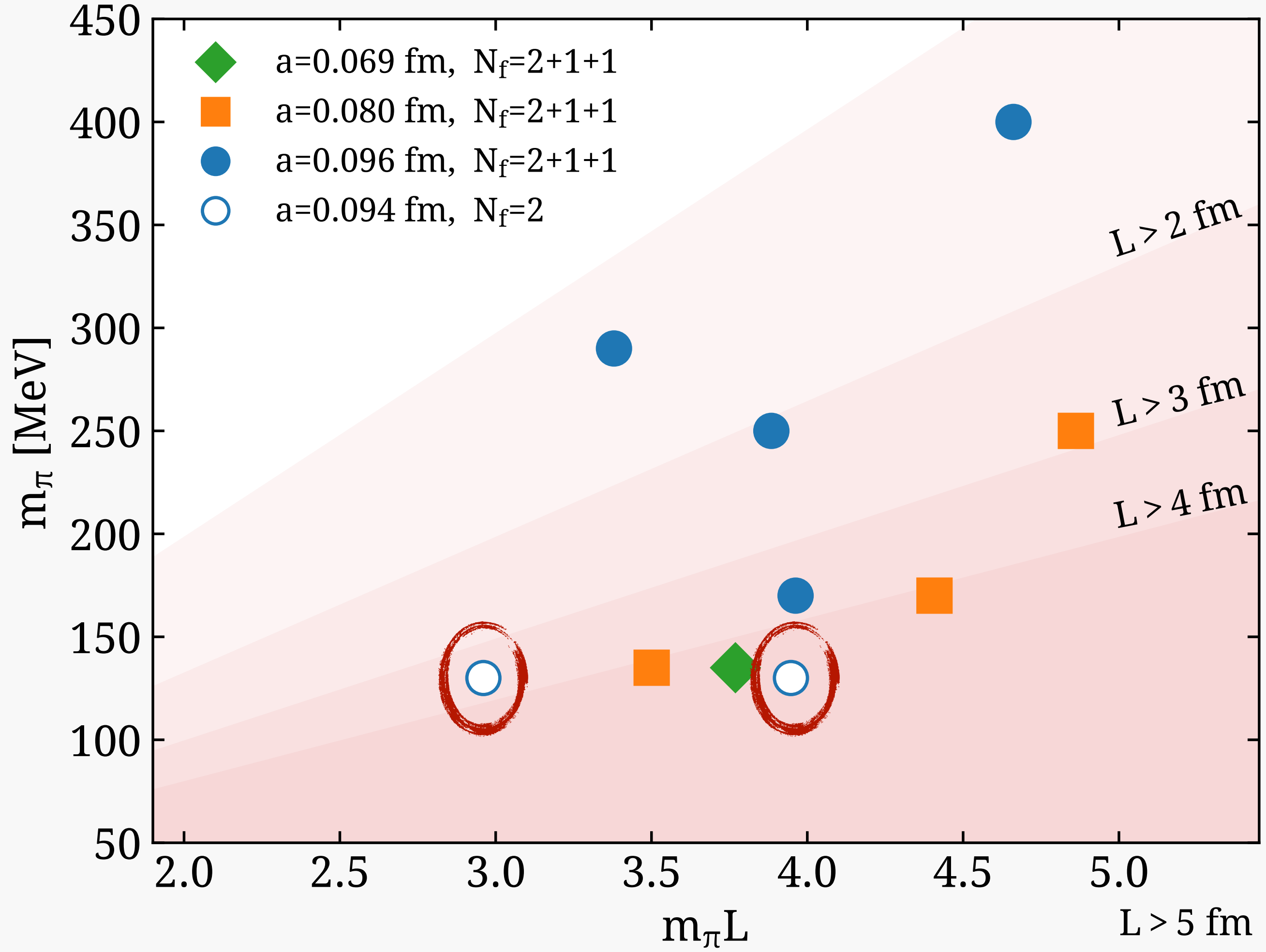

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Abdel-Rehim *et al.* Phys.Rev. D95 (2017) no.9, 094515, arXiv:1507.05068

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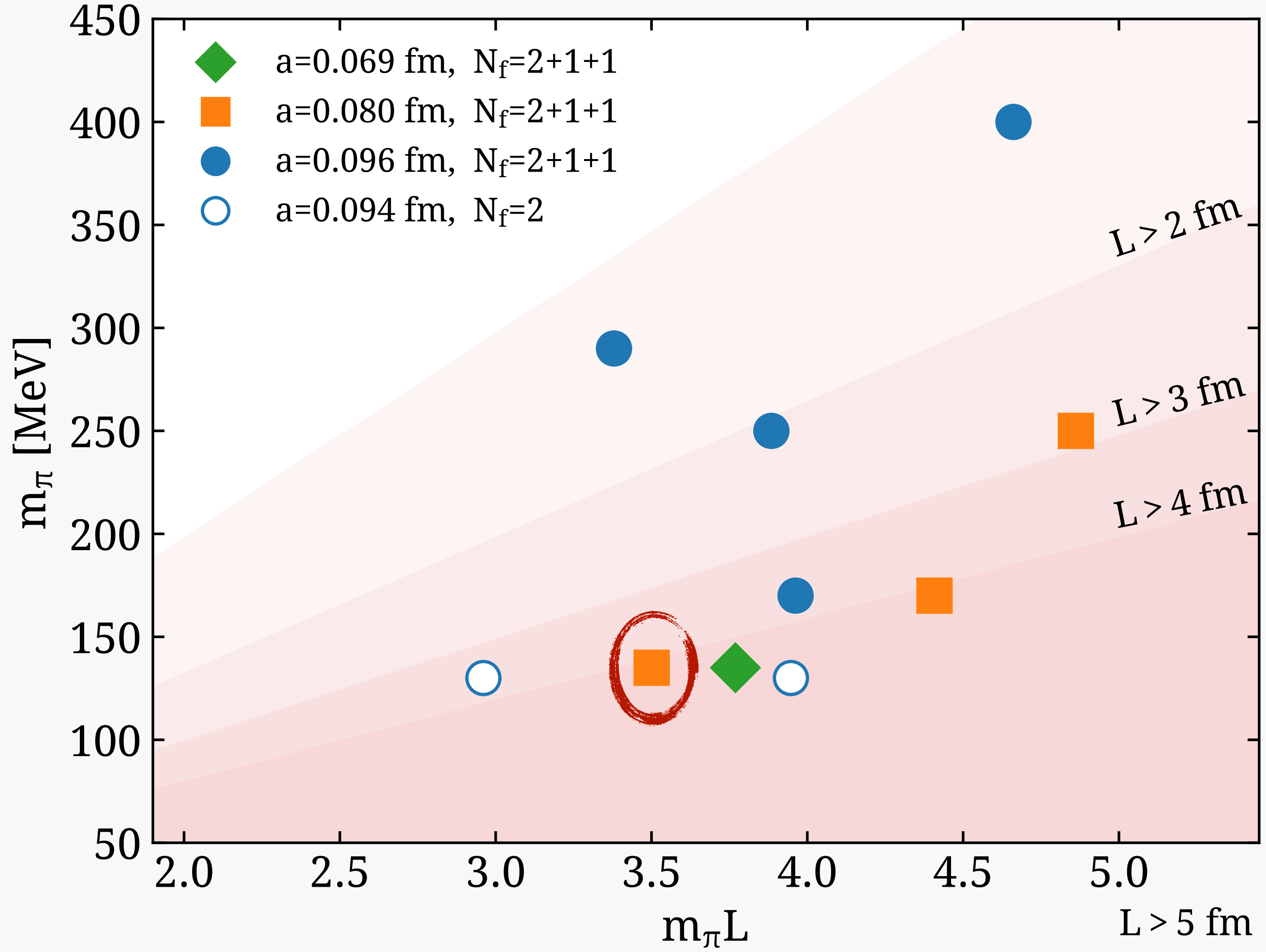

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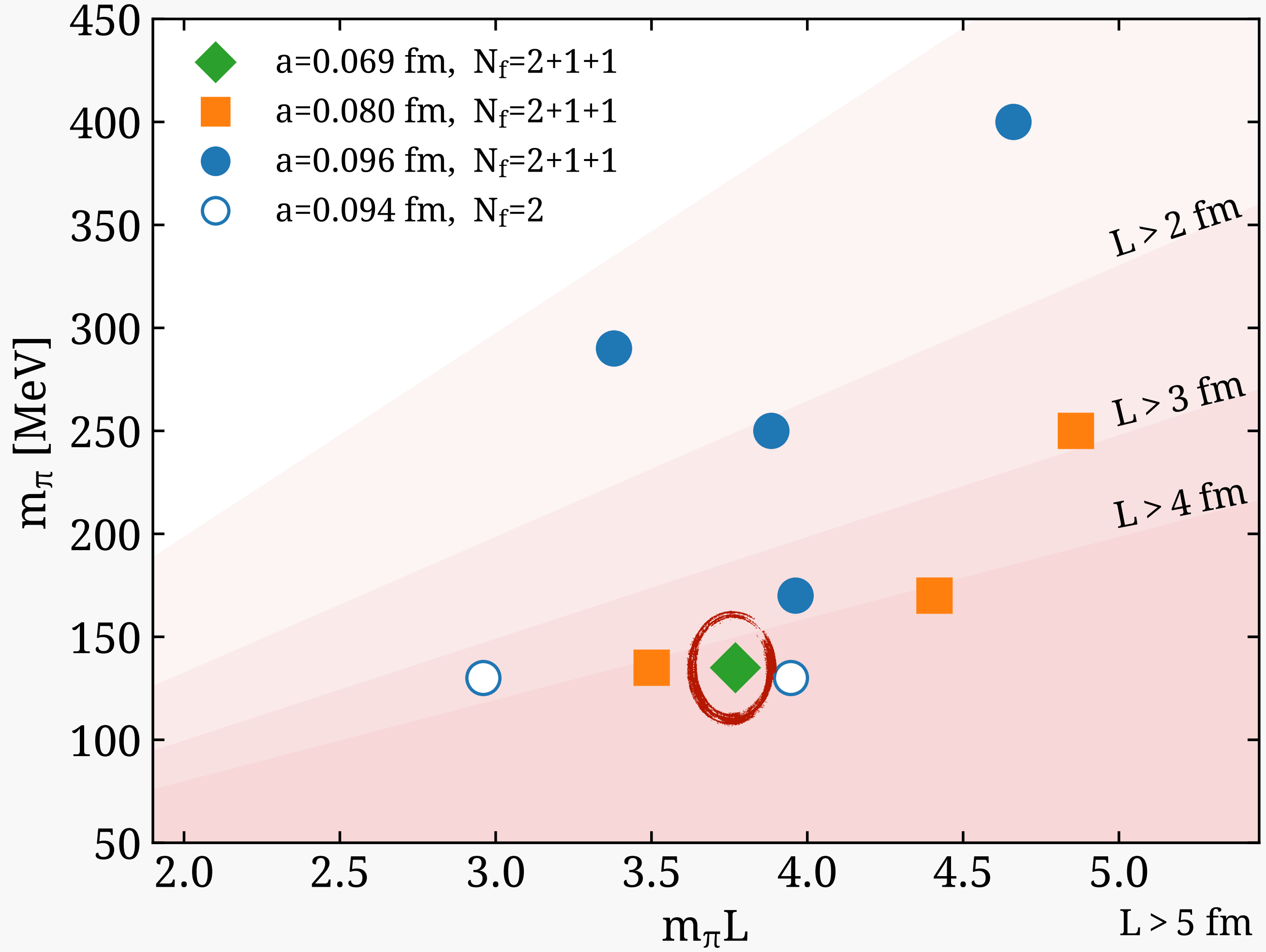
Alexandrou *et al.* Phys.Rev. D98 (2018) no.5, 054518, arXiv:1807.00495

Twisted Mass/Clover Ensembles



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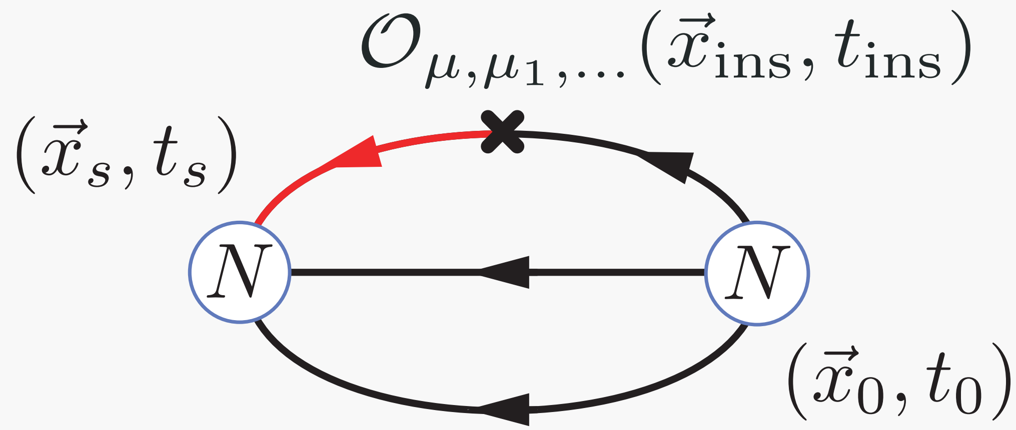
Setup



Connected:

- Sequential inversion through the sink
- Fix: Sink-source separation (t_s), nucleon polarisation (Γ), sink momentum *p'*

t_s/a	$N_{\text{src}} \times N_{\text{conf}}$		
	cB211.072.64	cA2.09.48	cA2.09.64
8	1×750	–	–
10	2×750	16×578	–
12	4×750	16×578	16×333
14	6×750	16×578	16×515
16	16×750	88×530*	32×515
18	48×750	88×725*	–
20	64×750	–	–
2-point	264×750	100×2153	32×515

* Only for Γ_0

- Four polarisations Γ_0, Γ_k
- Sink momentum fixed to $\vec{p}' = \vec{0}$
- More statistics for two-point function from disconnected calculation

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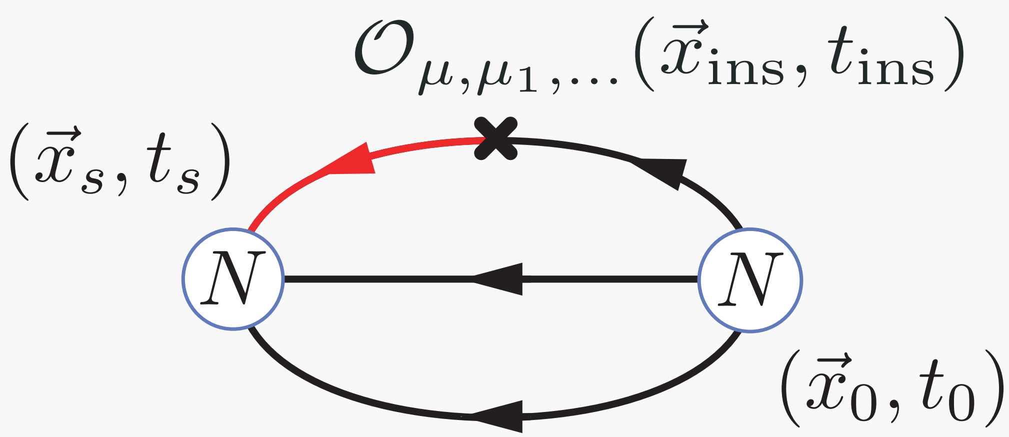
Setup



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* Only for Γ_0

- Four polarisations Γ_0, Γ_k
- Sink momentum fixed to $\vec{p}\,' = \vec{0}$
- More statistics for two-point function from disconnected calculation
- Increased statistics with increasing t_s

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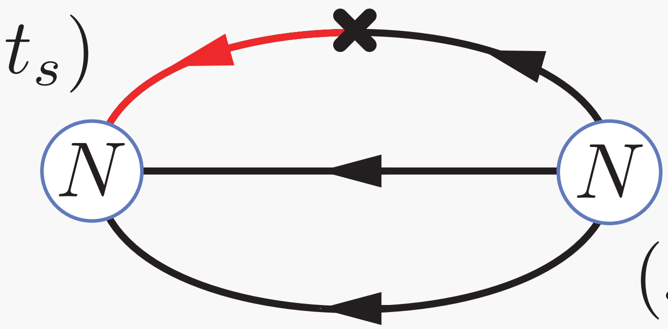
Treatment of excited states



- **Plateau**: $R(t_s, t_{\rm ins}, t_0) \xrightarrow[t_{\rm ins}-t_0\to\infty]{t_s-t_{\rm ins}\to\infty} \mathcal{M}[1+\mathcal{O}(e^{-\Delta(t_{\rm ins}-t_0)}, e^{-\Delta'(t_s-t_{\rm ins})})]$

  fit to constant w.r.t. $t_{\rm ins}$ for multiple values of t_s

- **Two- (or three-) state fit**: Fit two- and three-point function, including first (and when possible second) excited state

- **Summation:** Sum over $t_{\rm ins}$:

$$\sum_{t_{\rm ins}} R(t_s, t_{\rm ins}, t_0) \xrightarrow{t_s-t_0\to\infty} \mathrm{C} + \mathcal{M}(t_s - t_0) + \mathcal{O}(t_s e^{-\Delta t_s})$$

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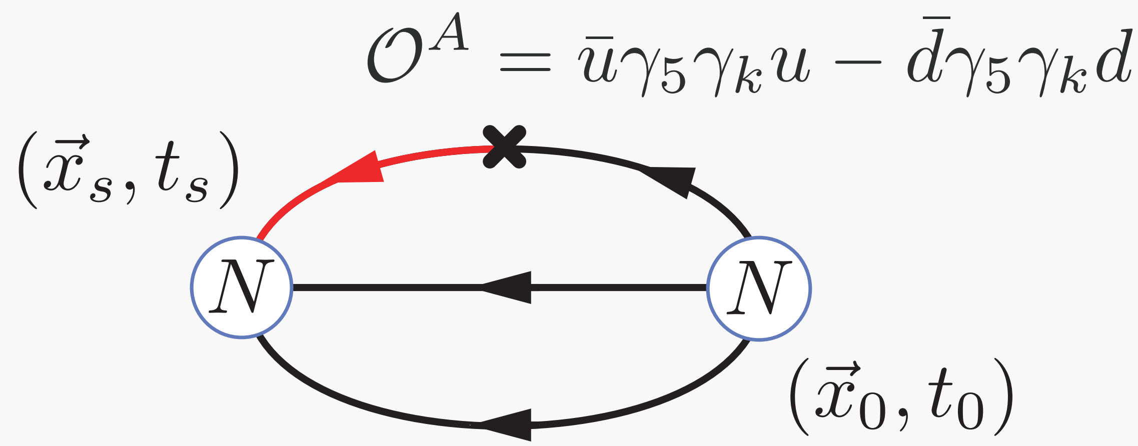
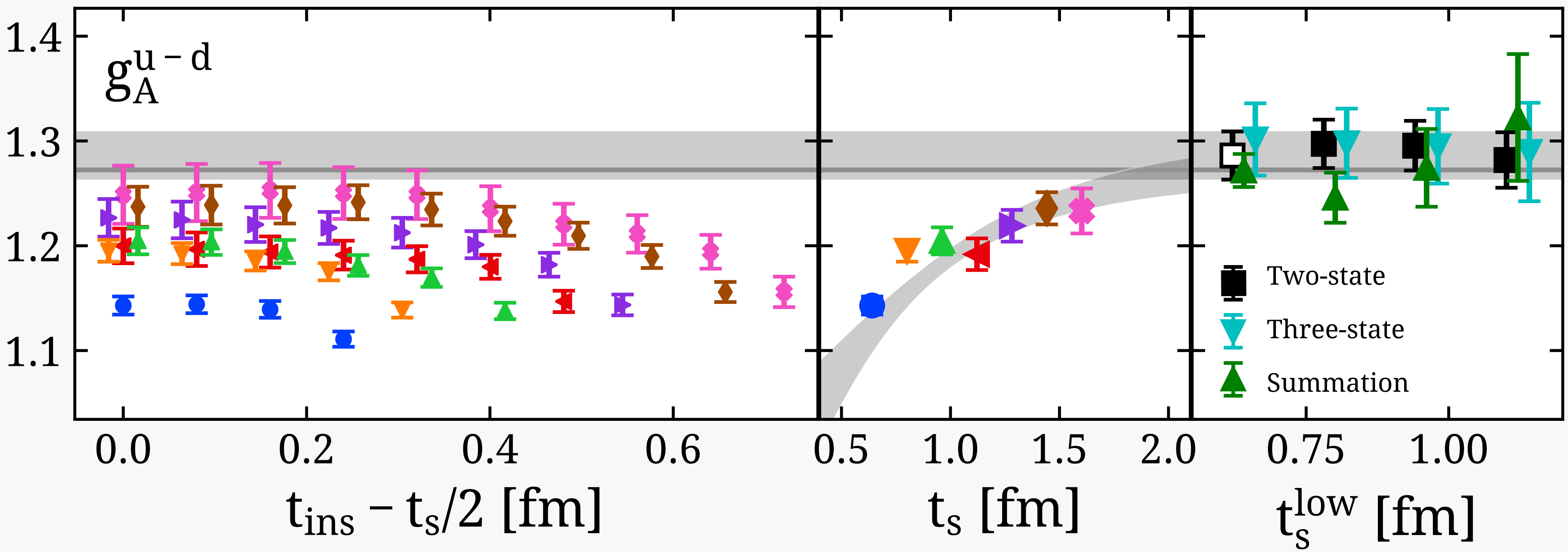
Axial charge

Isovector axial charge

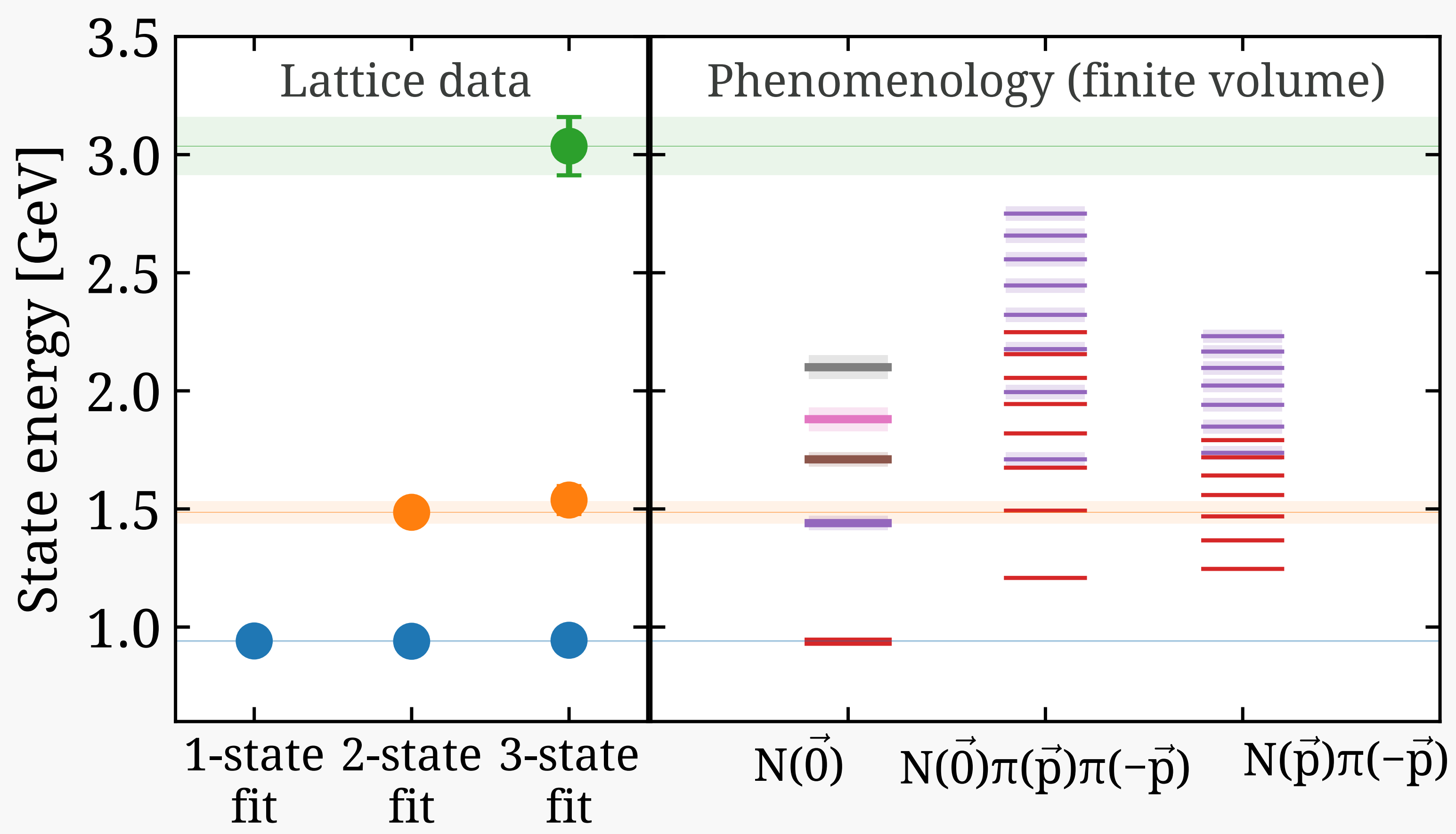
- Two- and three-state fits possible
- Two-state fit analysis reveals slow suppression of excited states up to $t_s \approx 2$ fm





Treatment of excited states

- Analysis of nucleon two-point correlation function



Excited state consistent between 2- and 3-state fit

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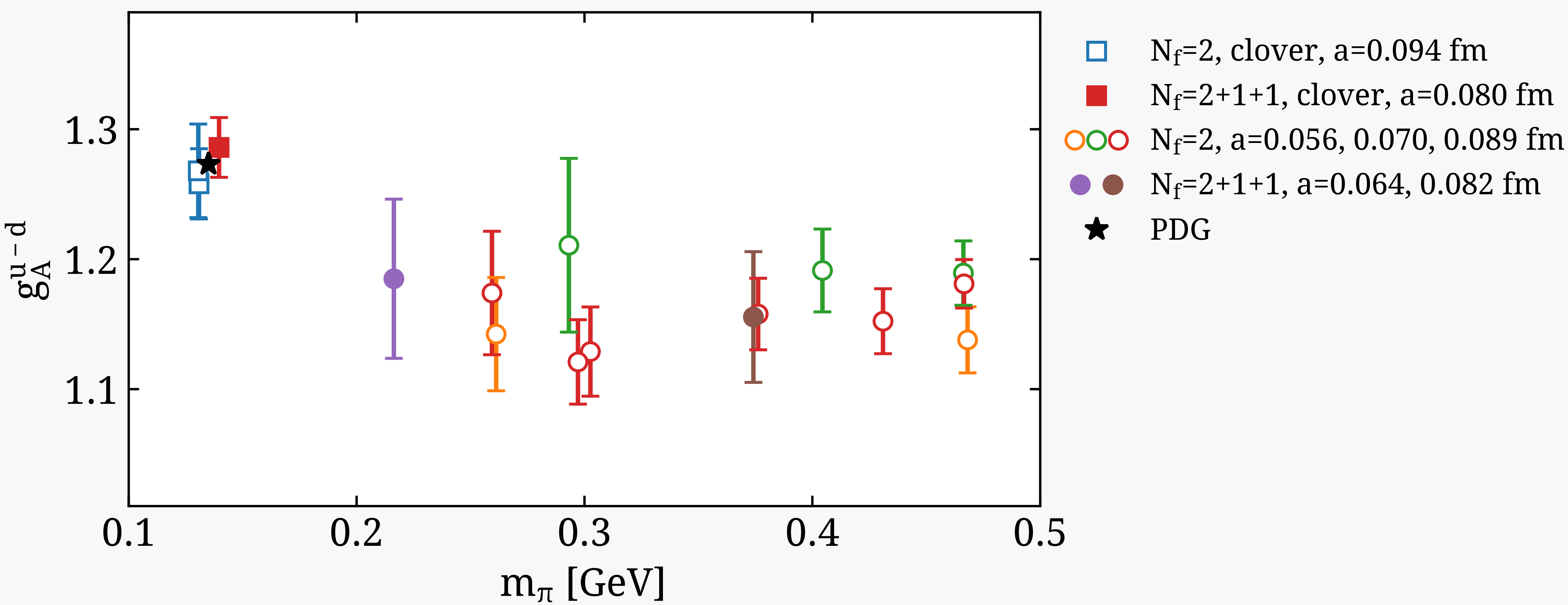
Axial charge

Isovector axial charge



- ETMC data including non-clover N_f=2 and N_f=2+1+1
- Agreement with PDG value at physical point
- Mild pion mass dependence for $m_\pi \gtrsim 200$ MeV

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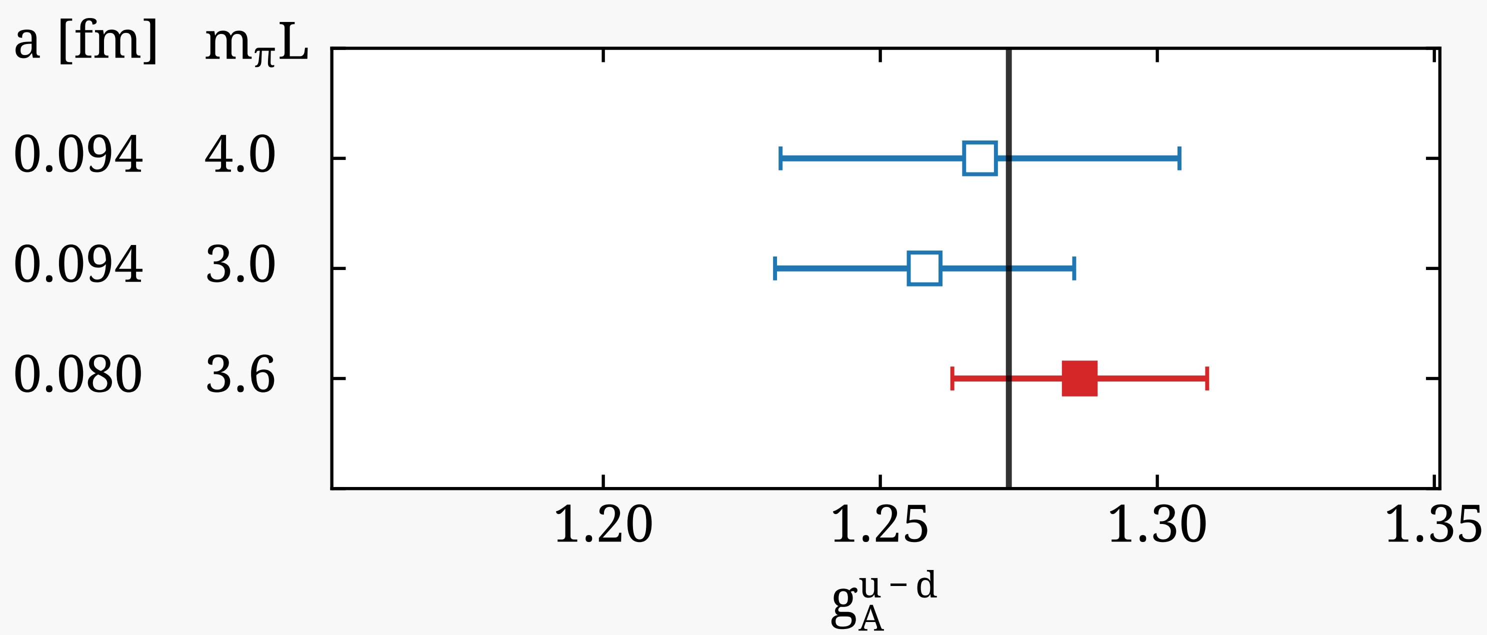
Axial charge

Isovector axial charge



- Three ensembles at physical point, agreement with PDG value
- Volumes: L=4.5 to 6 fm, no detectable volume effects
- Two lattice spacings: *a*=0.0938 and 0.0801 fm, no detectable cut-off effects

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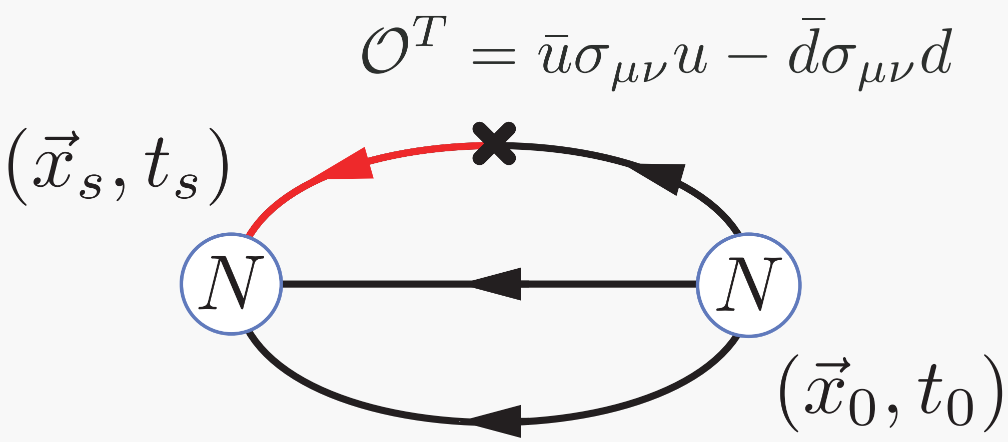
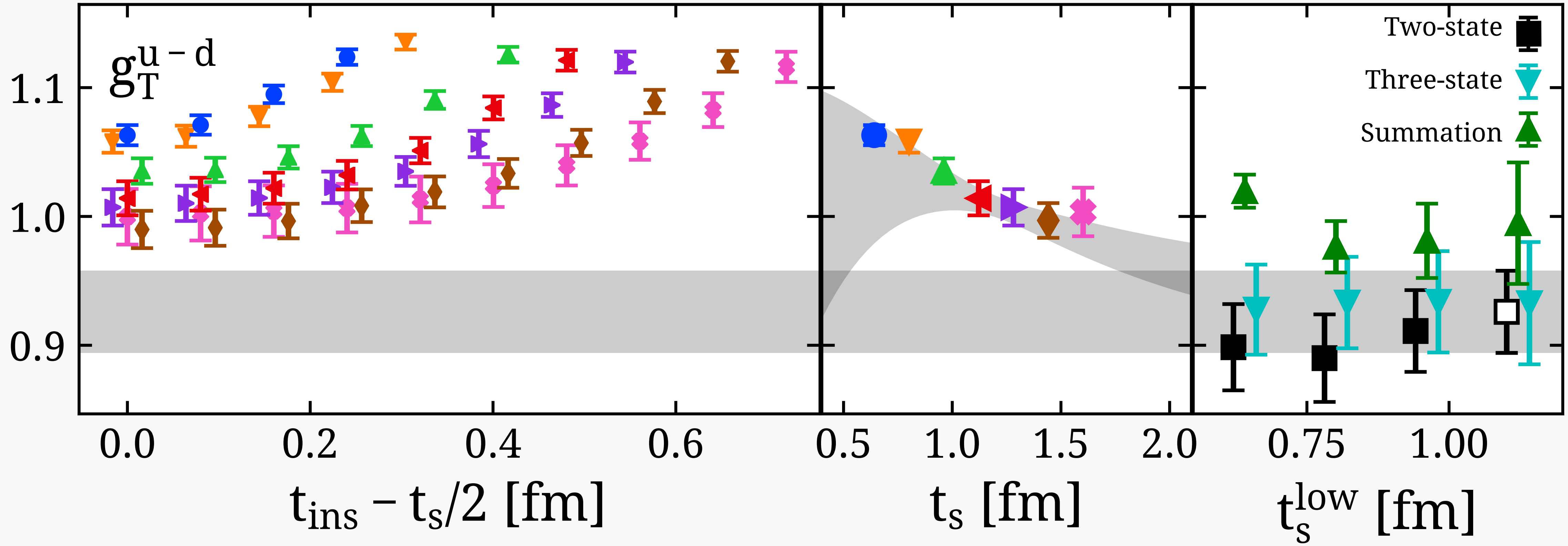
Tensor charge

Isovector tensor charge

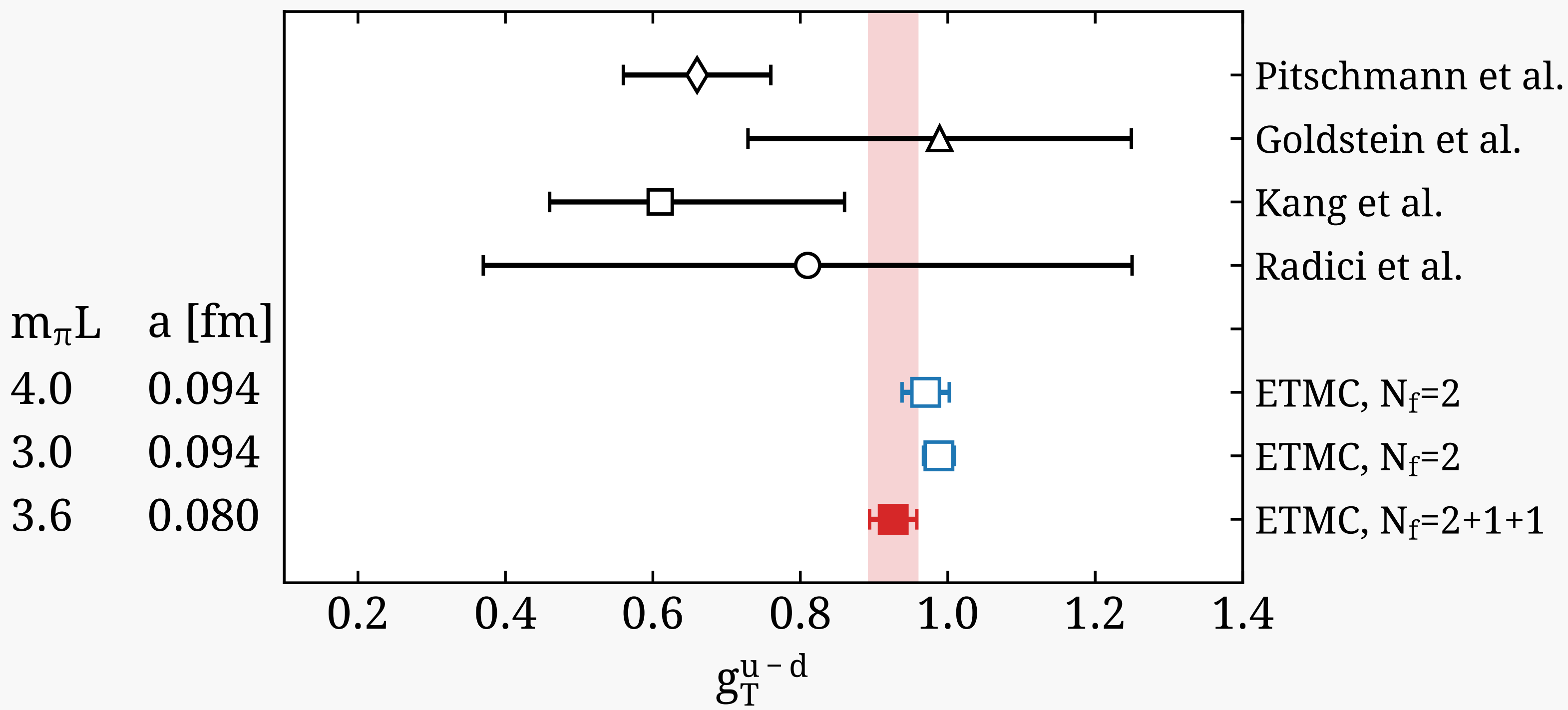
- Slower convergence to ground state compared to g_A





Tensor charge

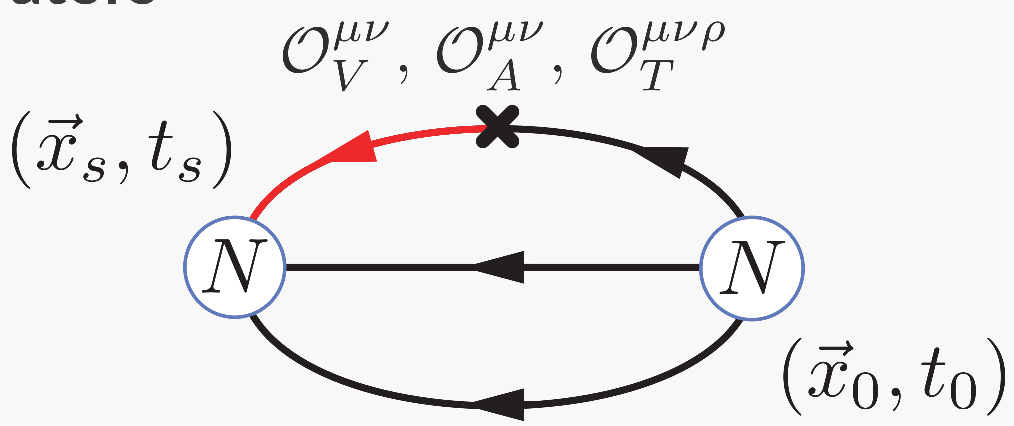
Isovector tensor charge



- Tensor yields good signal for all three ensembles
- No detectable volume effects between L=4.5 to 6 fm
- Central value closer to 1 compared to experimental extractions

Moments of PDFs

Isovector matrix element of 1st-derivative operators



- Unpolarized (momentum fraction)

$$\mathcal{O}_V^{\mu\nu} = \bar{\psi}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi,\ \Pi_V^{44}(\Gamma_0) = -\frac{3m_N}{4}\langle x\rangle_{u-d}$$

- Helicity

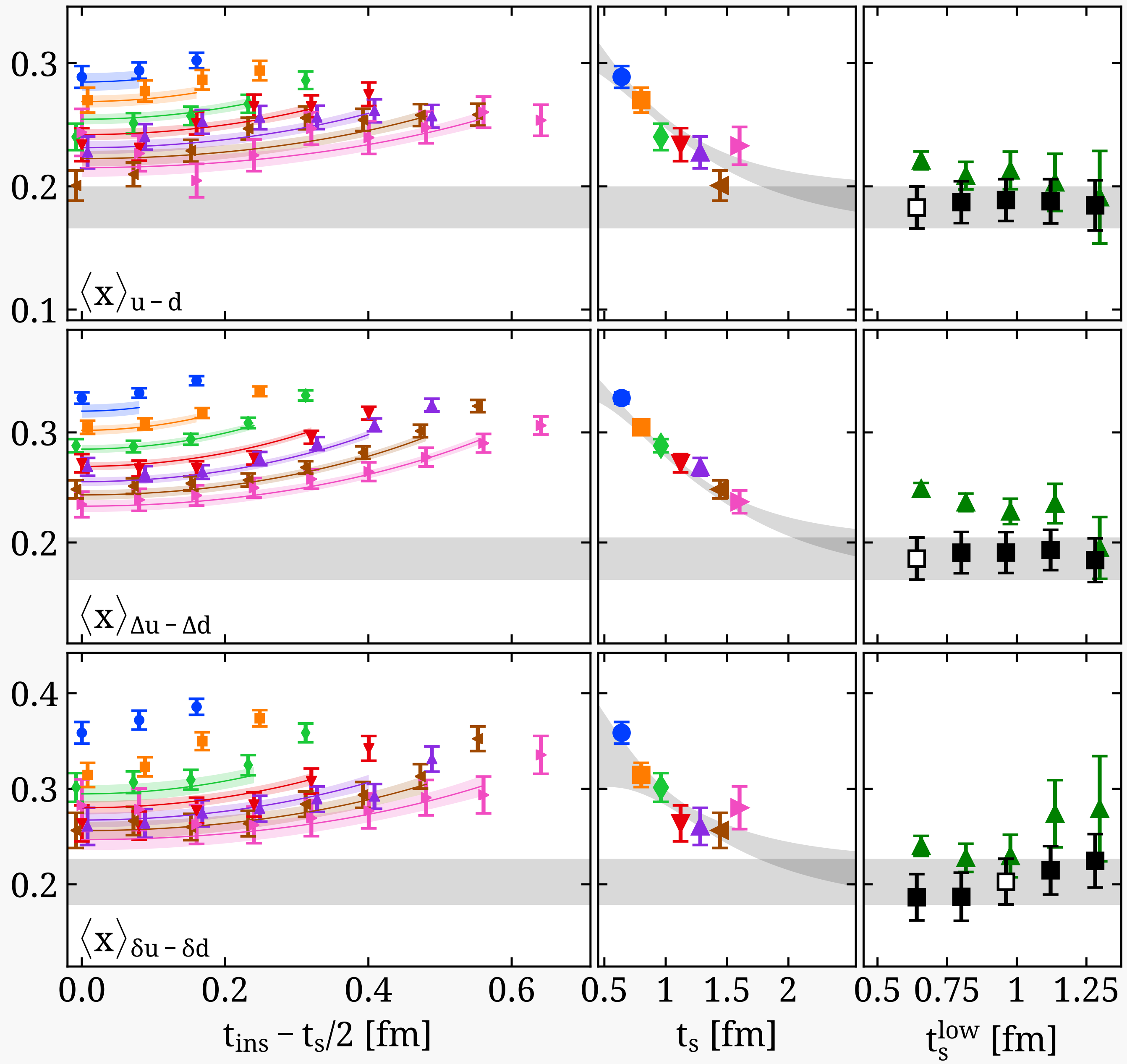
$$\mathcal{O}_A^{\mu\nu} = \bar{\psi}\gamma_5\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi,\ \Pi_A^{j4}(\Gamma_k) = -\frac{im_N}{2}\delta_{jk}\langle x\rangle_{\Delta u-\Delta d}$$

- Transversity

$$\mathcal{O}_T^{\mu\nu\rho} = \bar{\psi}\sigma^{[\mu\{\nu]}\overleftrightarrow{D}^{\rho\}}\psi,\ \Pi_T^{\mu\nu\rho}(\Gamma_k) = i\epsilon_{\mu\nu\rho k}\frac{m_N}{8}(2\delta_{4\rho} - \delta_{4\mu} - \delta_{4\nu})\langle x\rangle_{\delta u-\delta d}$$

Moments of PDFs

Isovector matrix element of 1st-derivative operators



- Similar dependence on excited states as charges
- Helicity especially slow to converge

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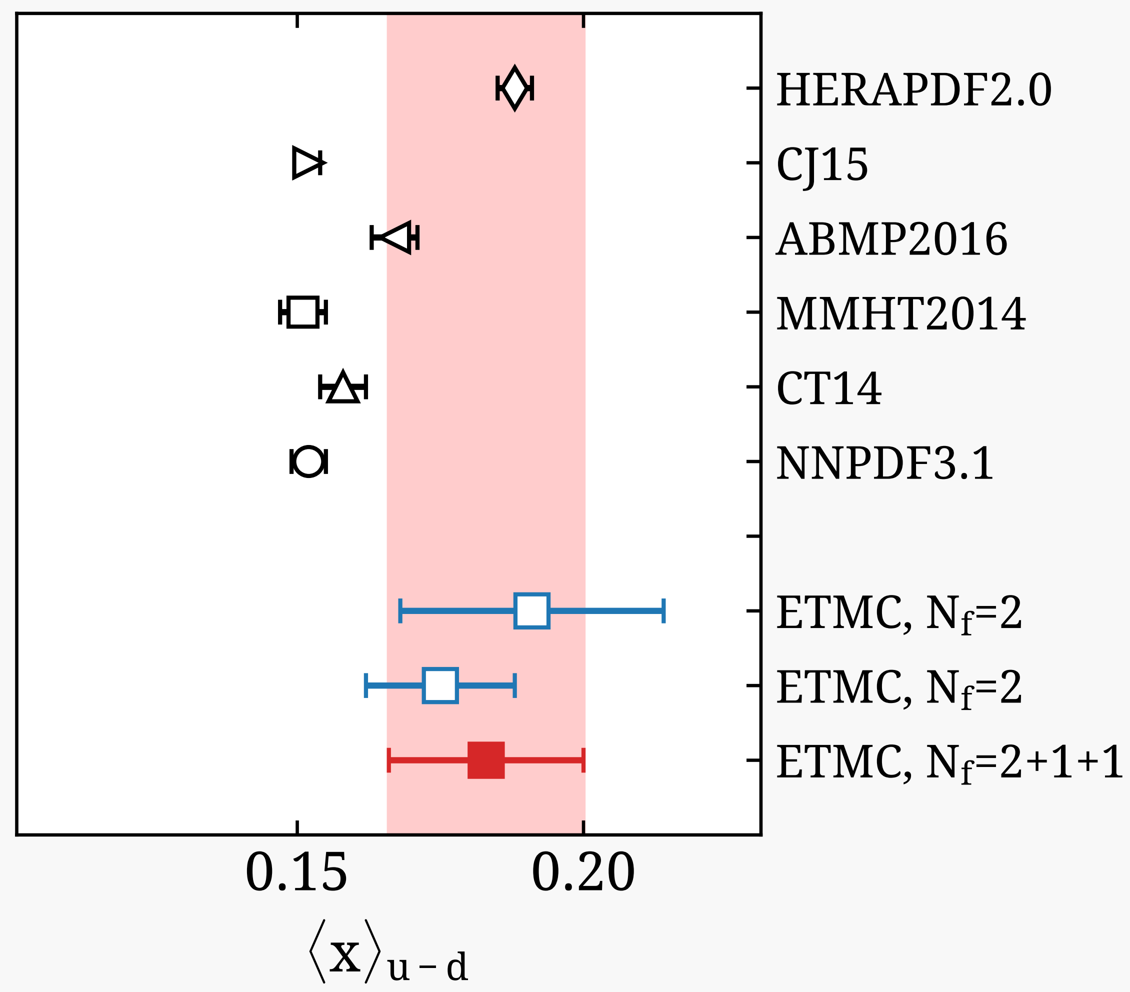
Moments of PDFs

Isovector matrix element of 1st-derivative operators

$m_\pi L$	a [fm]
4.0	0.094
3.0	0.094
3.6	0.080



- Check of volume effects
- Slightly higher value compared to most phenomenological extractions

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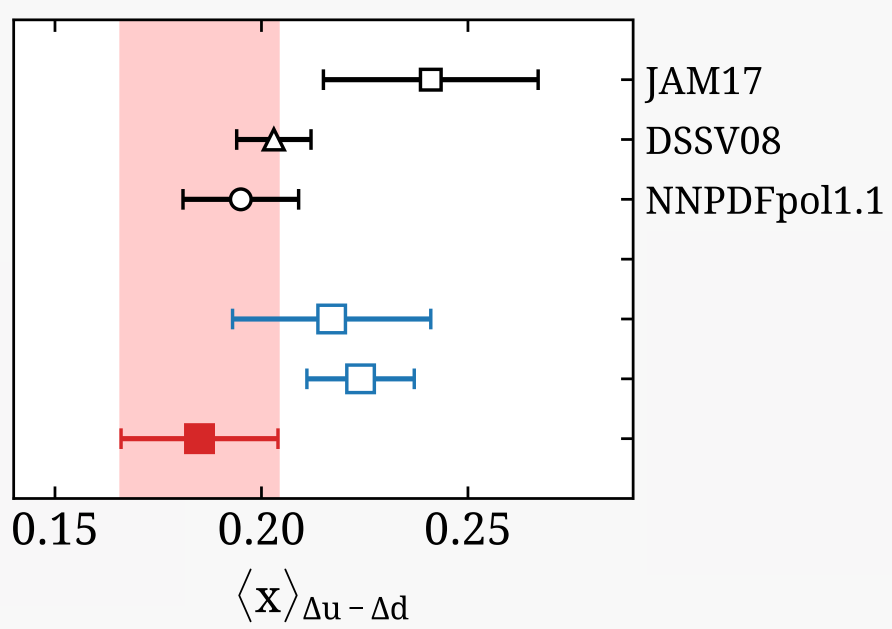
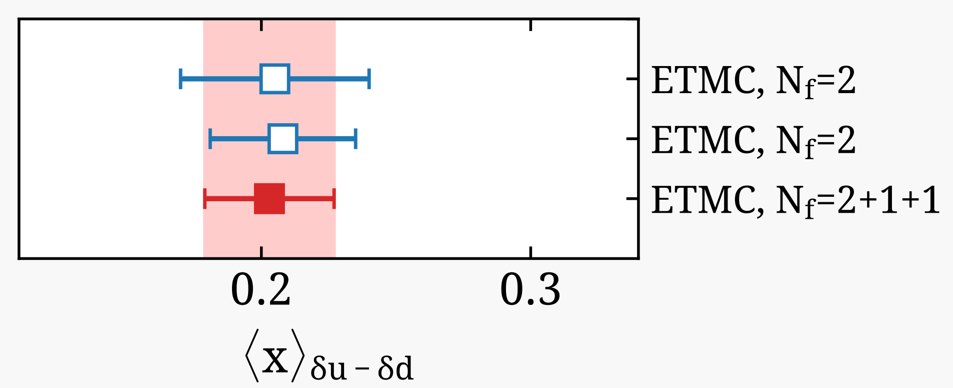
Moments of PDFs

Isovector matrix element of 1st-derivative operators

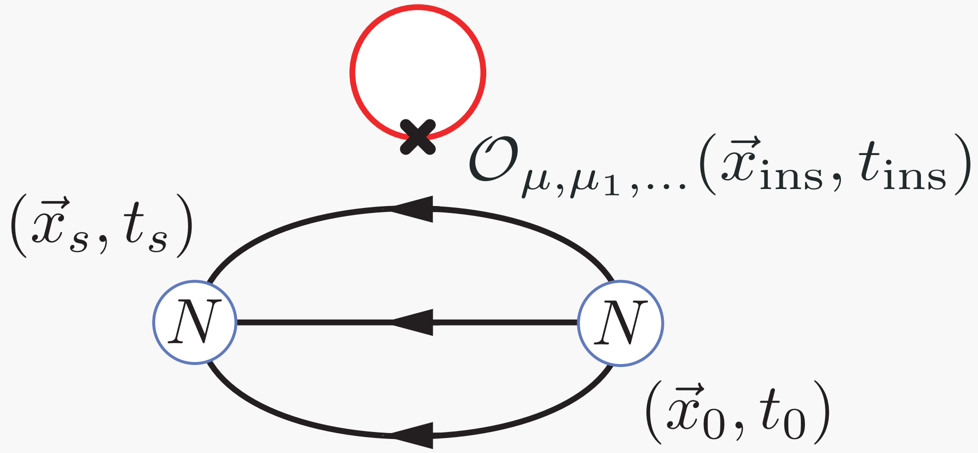




- Not shown: One other lattice study at 150 MeV: RQCD, arXiv:1812.08256, PRD
  - Extracted from single separation ~1.1 fm
  - Consistent with our plateau at similar separation

Disconnected contributions

t_s/a	$N_{\rm src} \times N_{\rm conf}$	
	cB211.072.64	cA2.09.48
2-point	264×750	100×2153



Loops calculated on one N_f=2 and one N_f=2+1+1 ensemble

Disconnected, combination of:

- Hierarchical probing (distance **8** for light, distance **4** for strange and charm)
- Spin & color dilution
- Exact eigenvalue deflation of 200 low-modes for light

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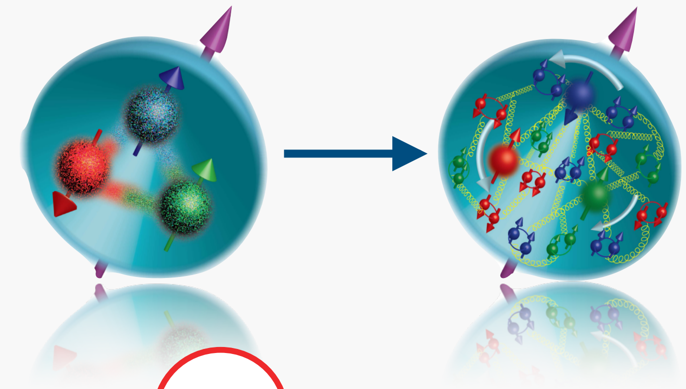
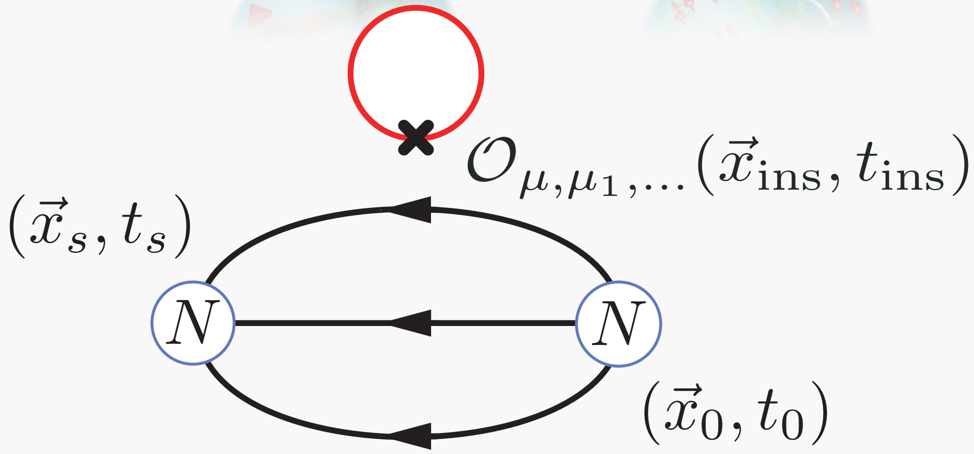
Nucleon spin



Quark intrinsic spin contributions to nucleon spin

$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}\sum_{q=u,d,s,\ldots} g_A^q$$



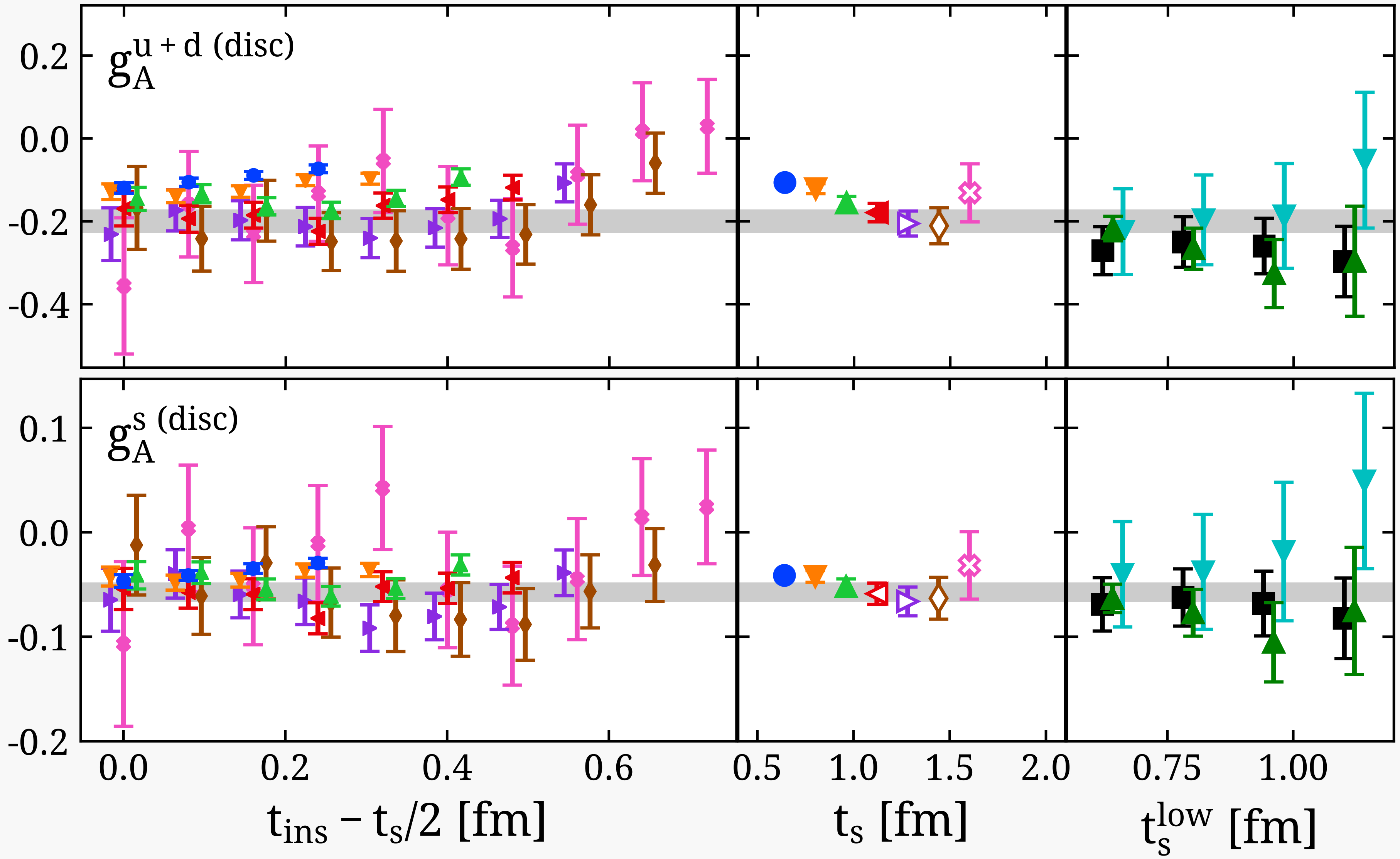
Quark intrinsic spin contributions to nucleon spin

- Need linear combination of isovector and isoscalar contributions for individual up- and down-quarks
- Strange quark contribution is sea-quark contribution only (disconnected diagrams)
- Need O(10) - O(100) times more statistics

Isoscalar and strange g_A

Disconnected axial current:

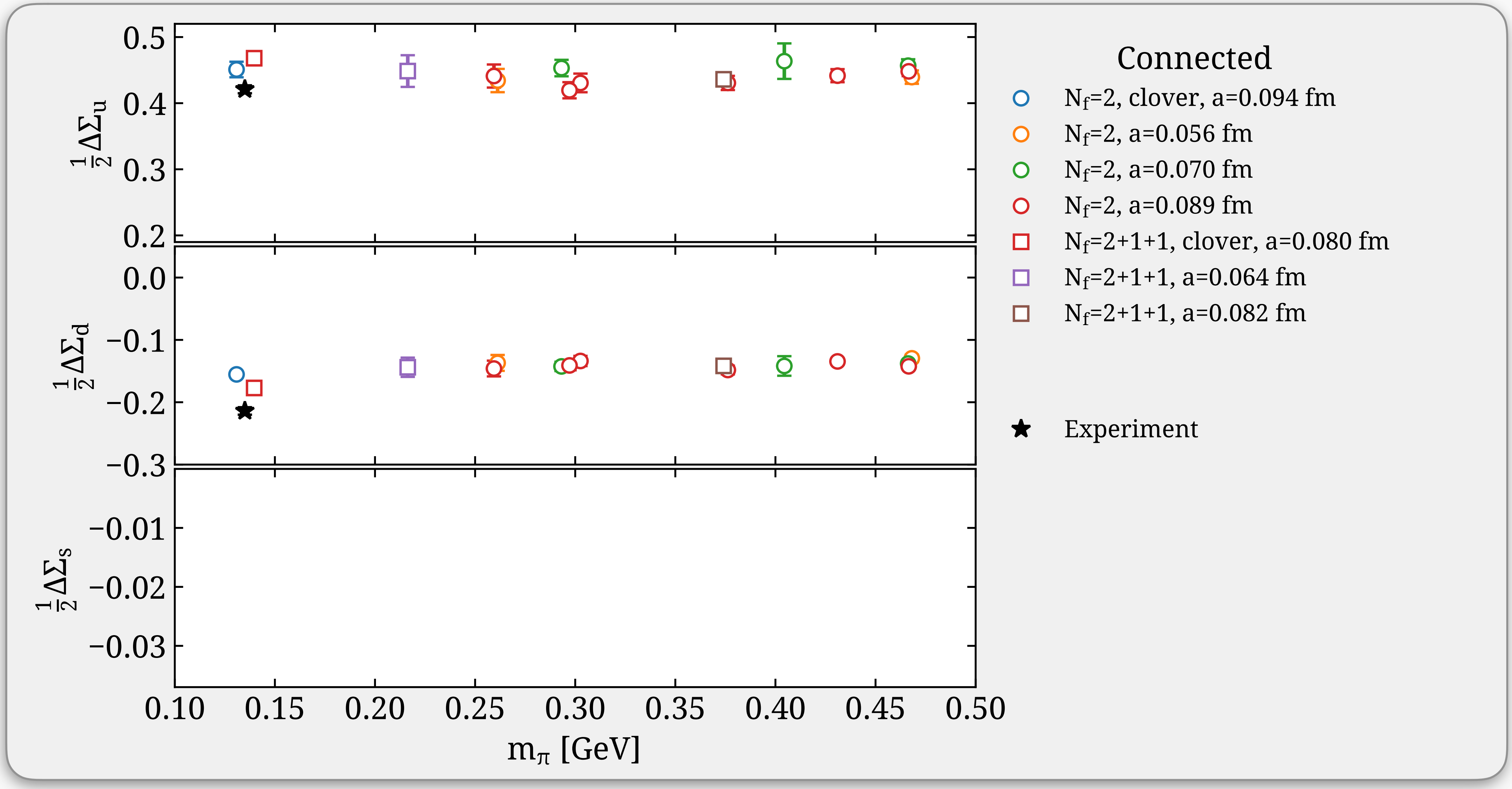
- Non-zero signal, ~10% of connected
- Take plateau when converged



Quark intrinsic spin contributions

Quark intrinsic spin contributions to nucleon spin

- Mild cut-off effects
- Strange and down-quark contributions negative



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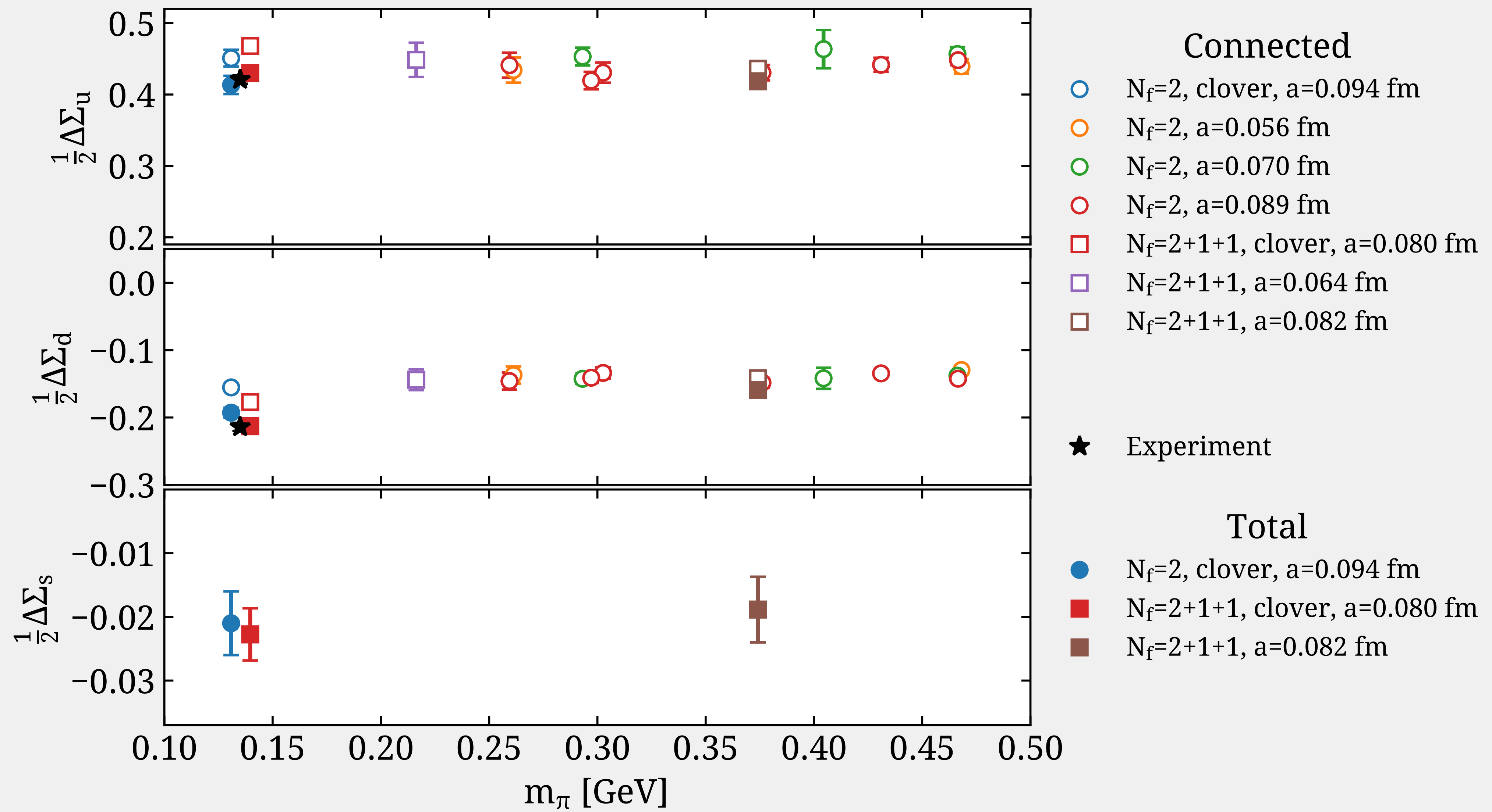

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Quark intrinsic spin contributions

Quark intrinsic spin contributions to nucleon spin

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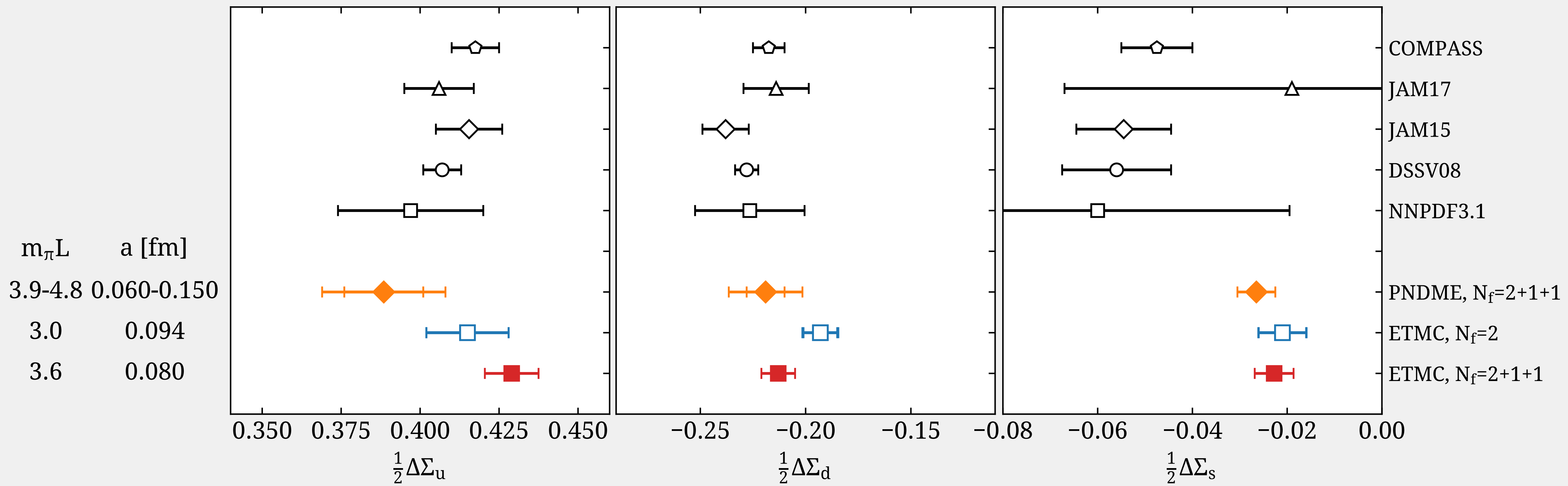

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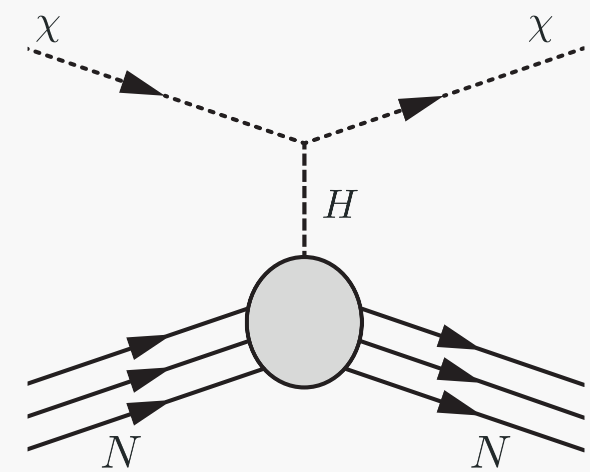
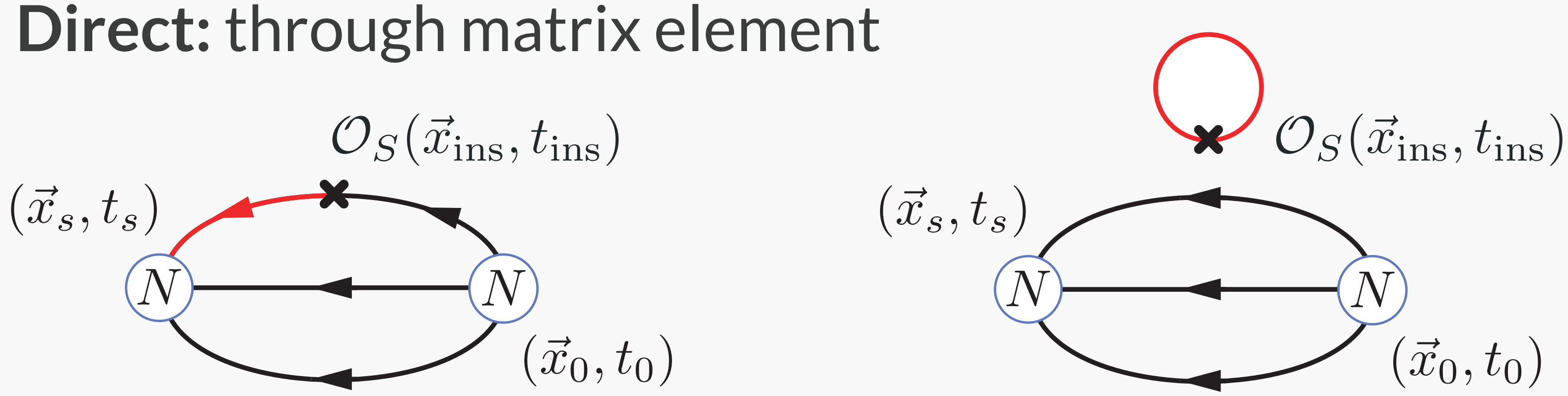
Nucleon σ-terms

- Pion nucleon σ-term: $\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$
- Strange σ-term: $\sigma_s = m_s\langle N|\bar{s}s|N\rangle$
- Scattering cross sections of scalars with nucleon (e.g. neutralino through Higgs)

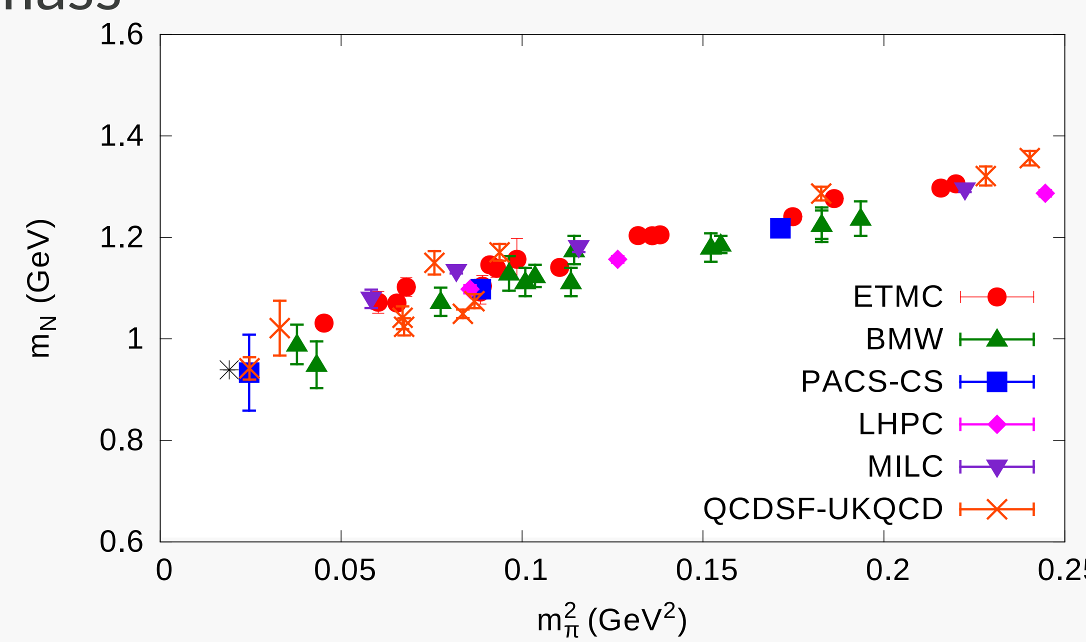


Direct: through matrix element



Feynman - Hellmann: dependence on quark mass

- Requires modelling of m_q dependence
- Weak m_s dependence



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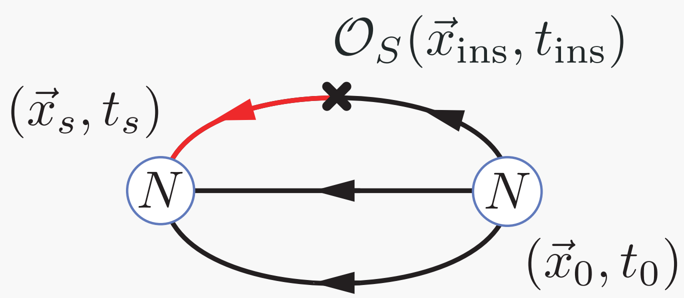
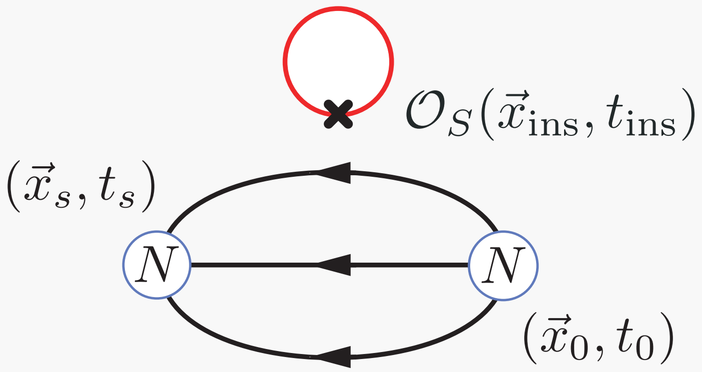
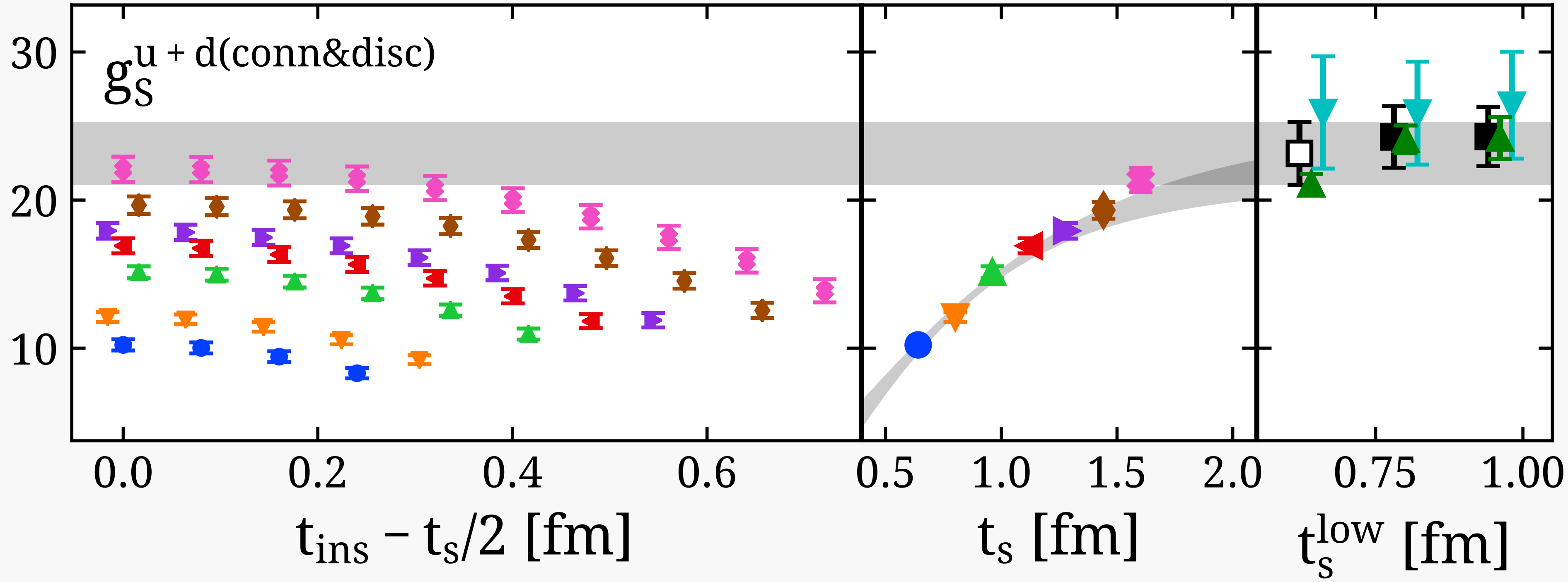
Nucleon σ-terms

Direct: through matrix element





Unrenormalized g_S



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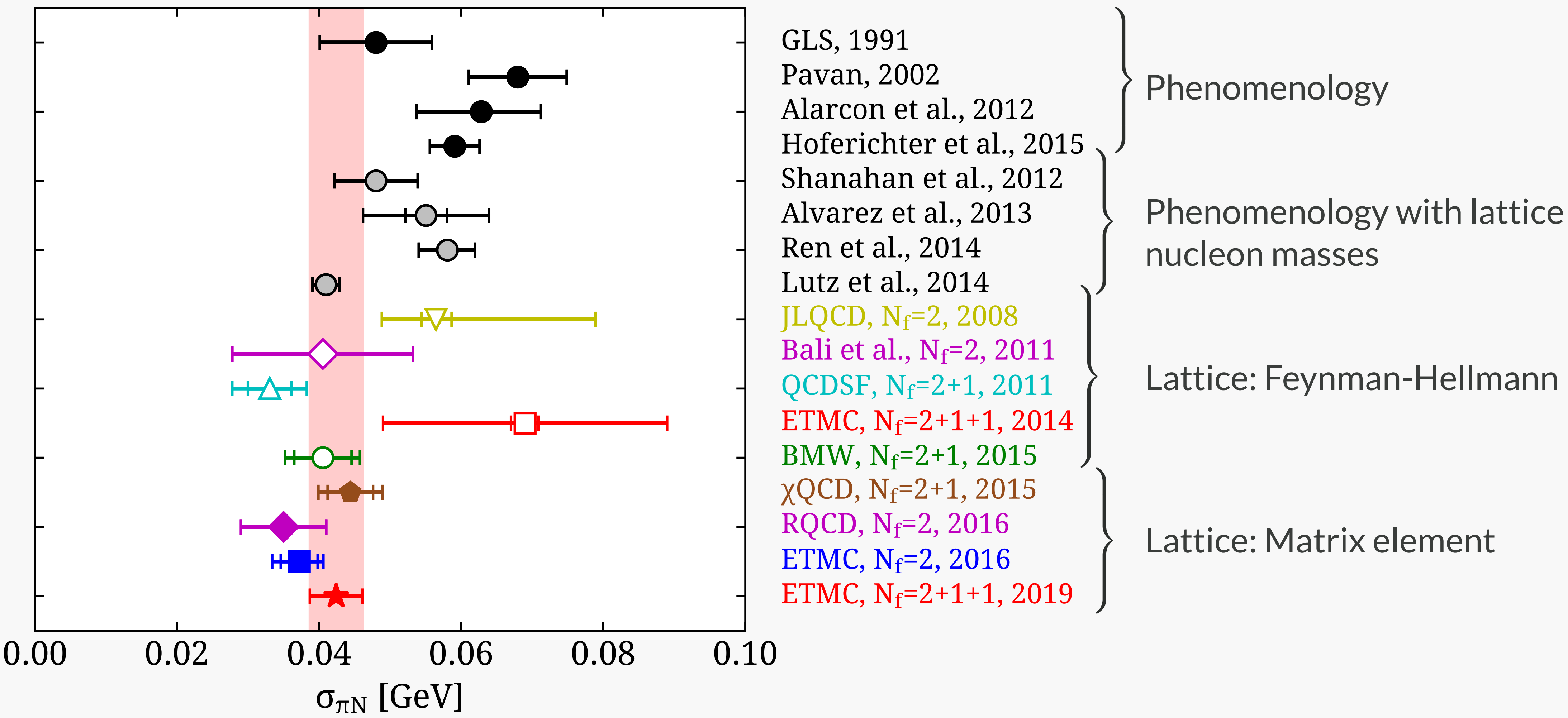
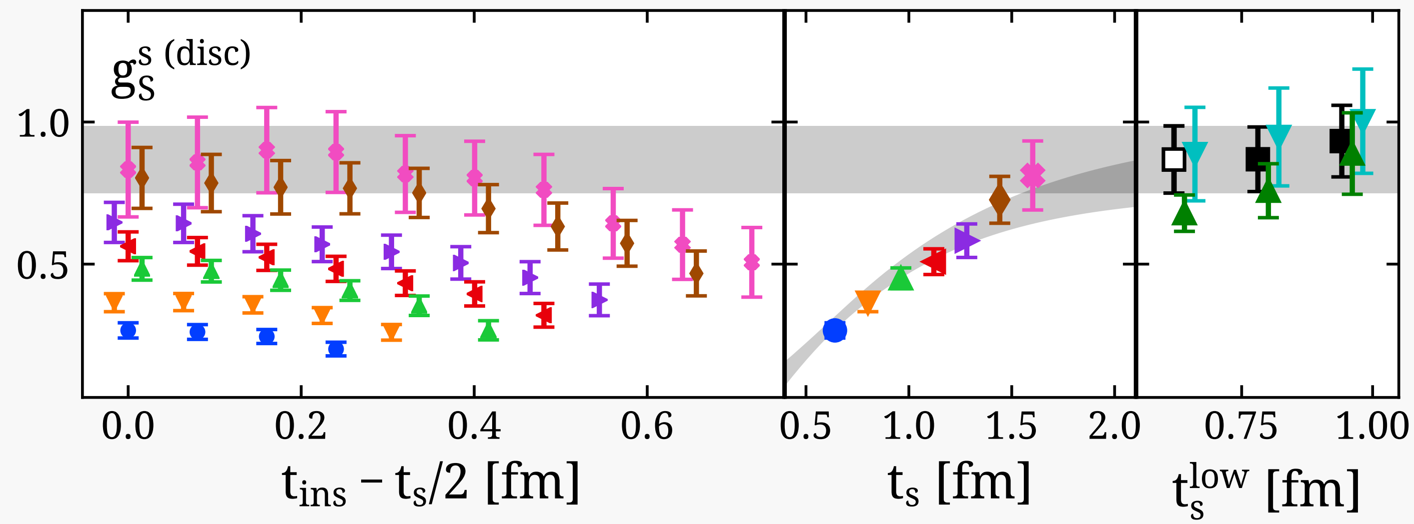
Nucleon σ-terms

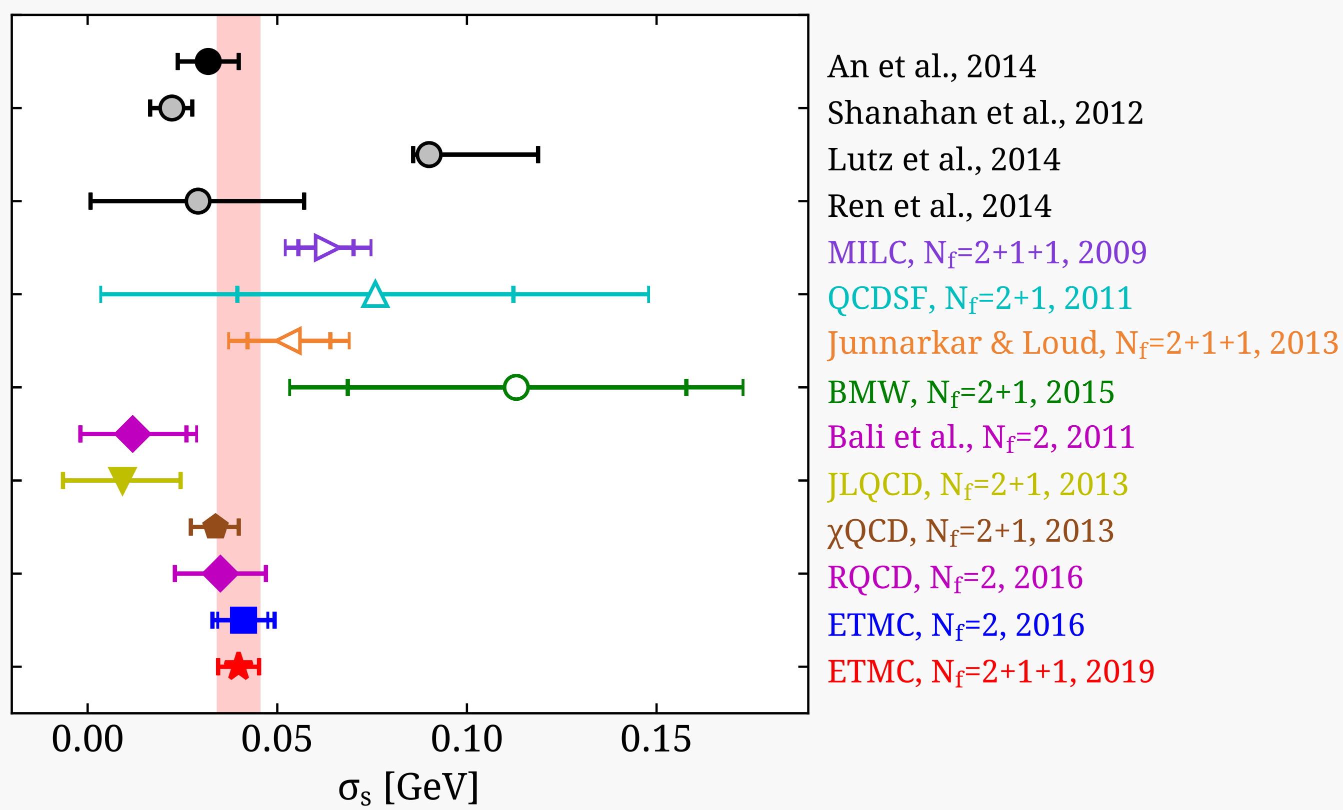
Light σ-term $\sigma_{\pi N}$



Nucleon σ-terms

Good signal also for strange and charm σ-terms





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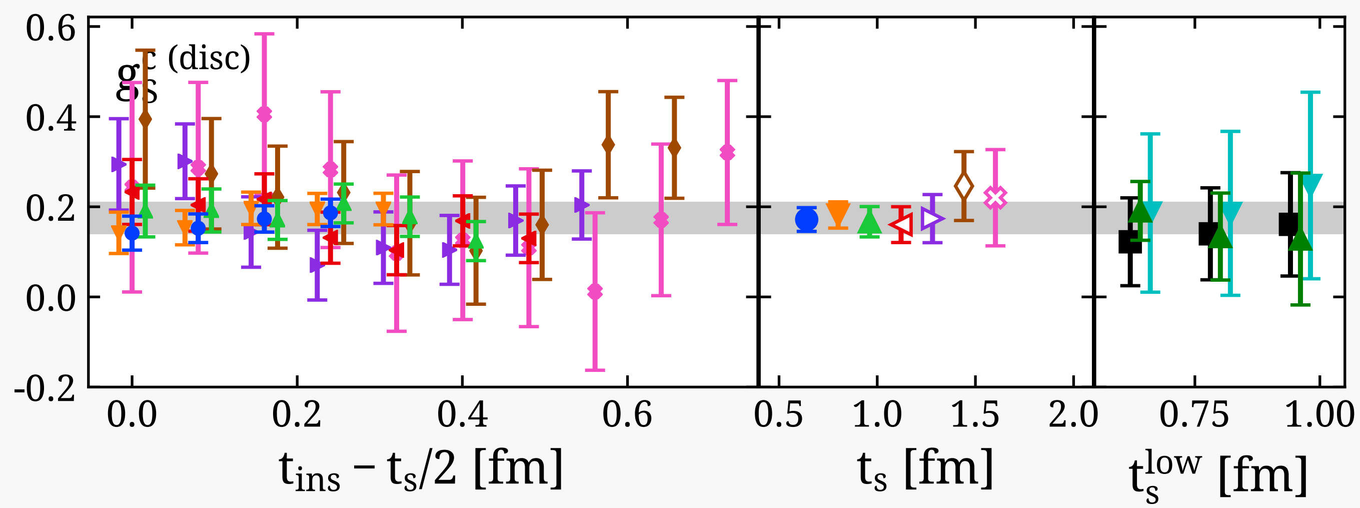

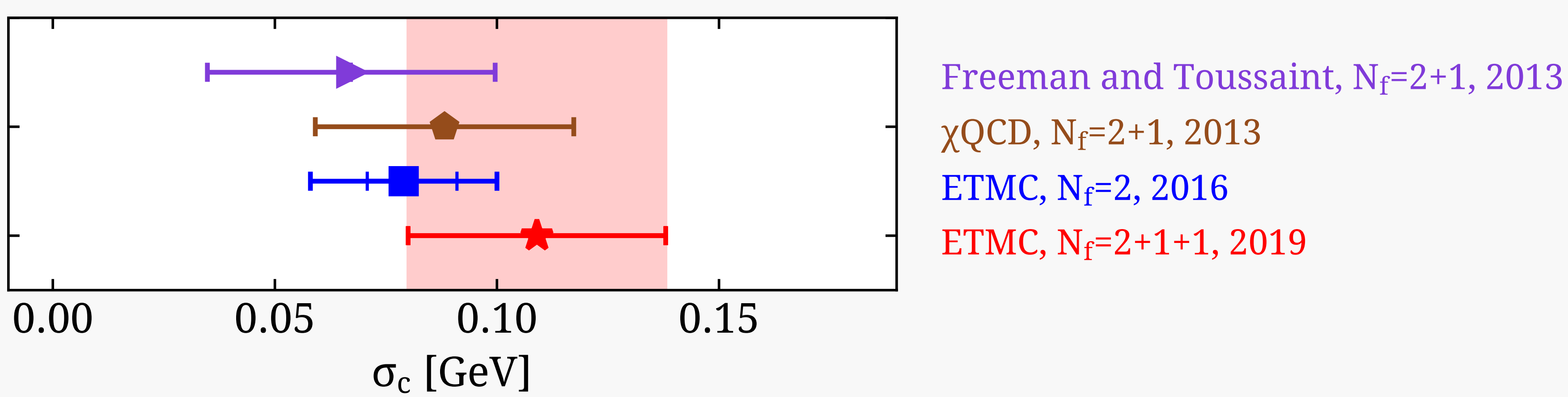

Nucleon Electromagnetic Form-Factors

Matrix element:

$$\langle N(p',s')|j^\mu|N(p,s)\rangle = \sqrt{\frac{M_N^2}{E_N(\mathbf{p}')E_N(\mathbf{p})}}\bar{u}(p',s')\mathcal{O}^\mu u(p,s)$$

$$\mathcal{O}^\mu = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M_N}F_2(q^2), \quad q = p' - p$$

Sachs form-factors:

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2M_N)^2}F_2(q^2)$$

Isovector & Isoscalar combinations:

$$j_\mu^v = \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d,\ j_\mu^s = \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$$

$$F^p - F^n = F^u - F^d$$

$$F^p + F^n = \frac{1}{3}(F^u + F^d)$$

Assuming flavour isospin symmetry

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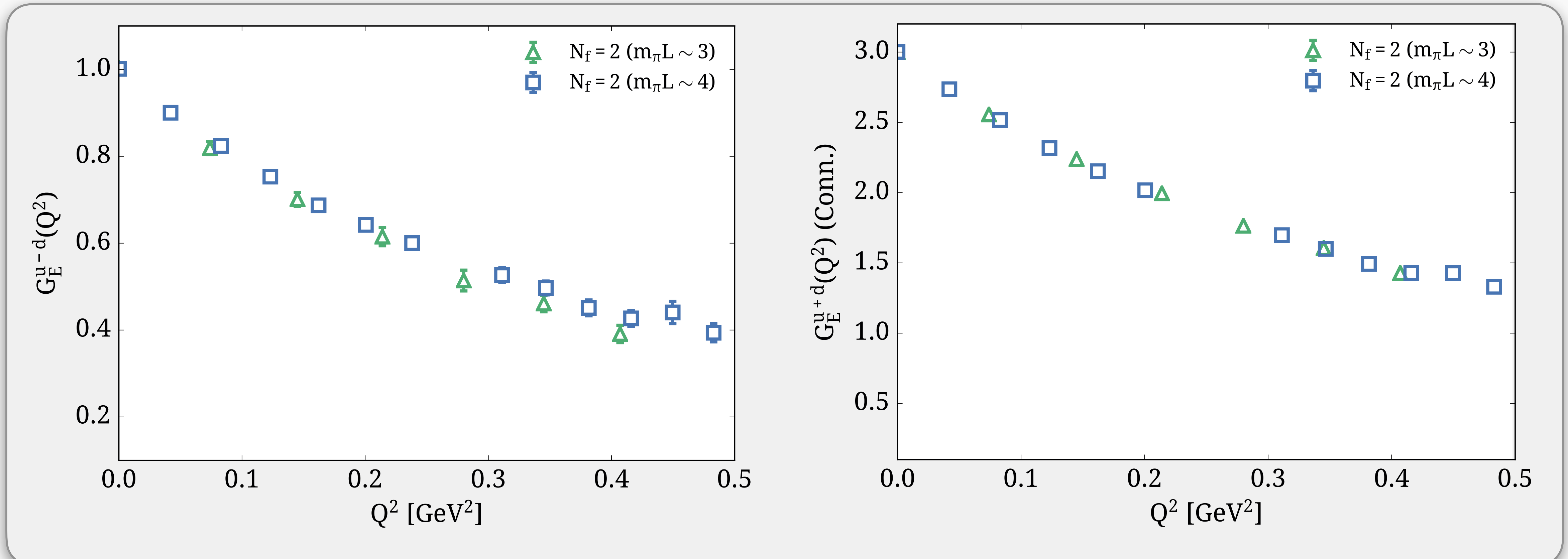
Nucleon Electromagnetic Form-Factors



- Comparison for two volumes with N_f=2
- No detectable volume effects
- Same for G_M

C. Alexandrou et al. arXiv:1812.10311 PRD

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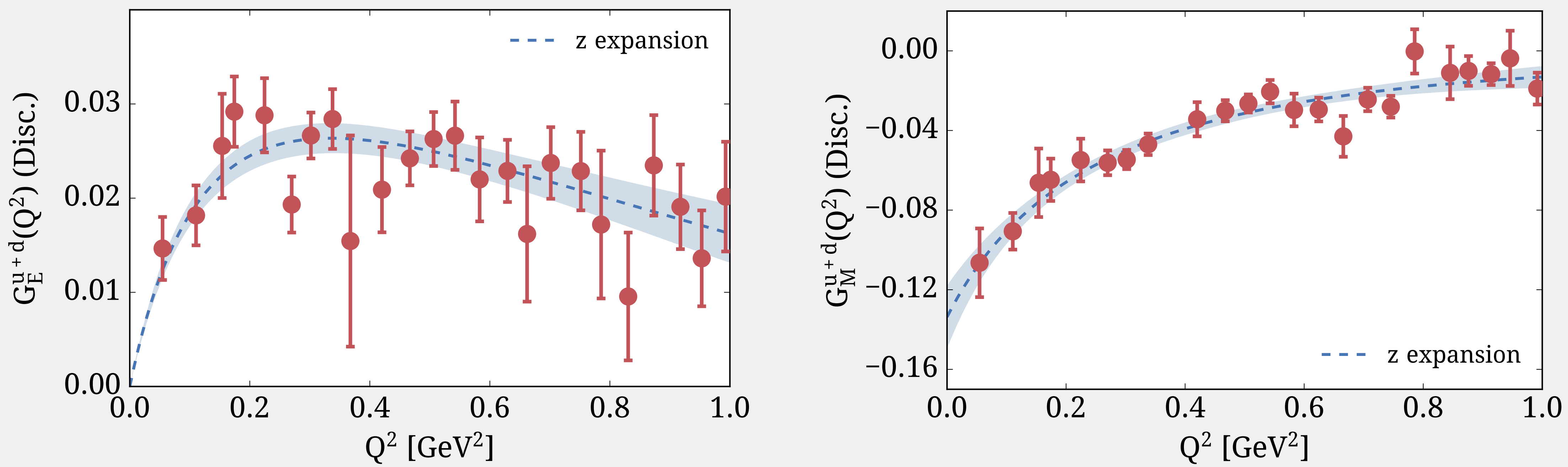

Disconnected contributions

- Good signal thanks to hierarchical probing and use of boosted frames
- Small in magnitude (few percent level)

C. Alexandrou et al. arXiv:1812.10311 PRD

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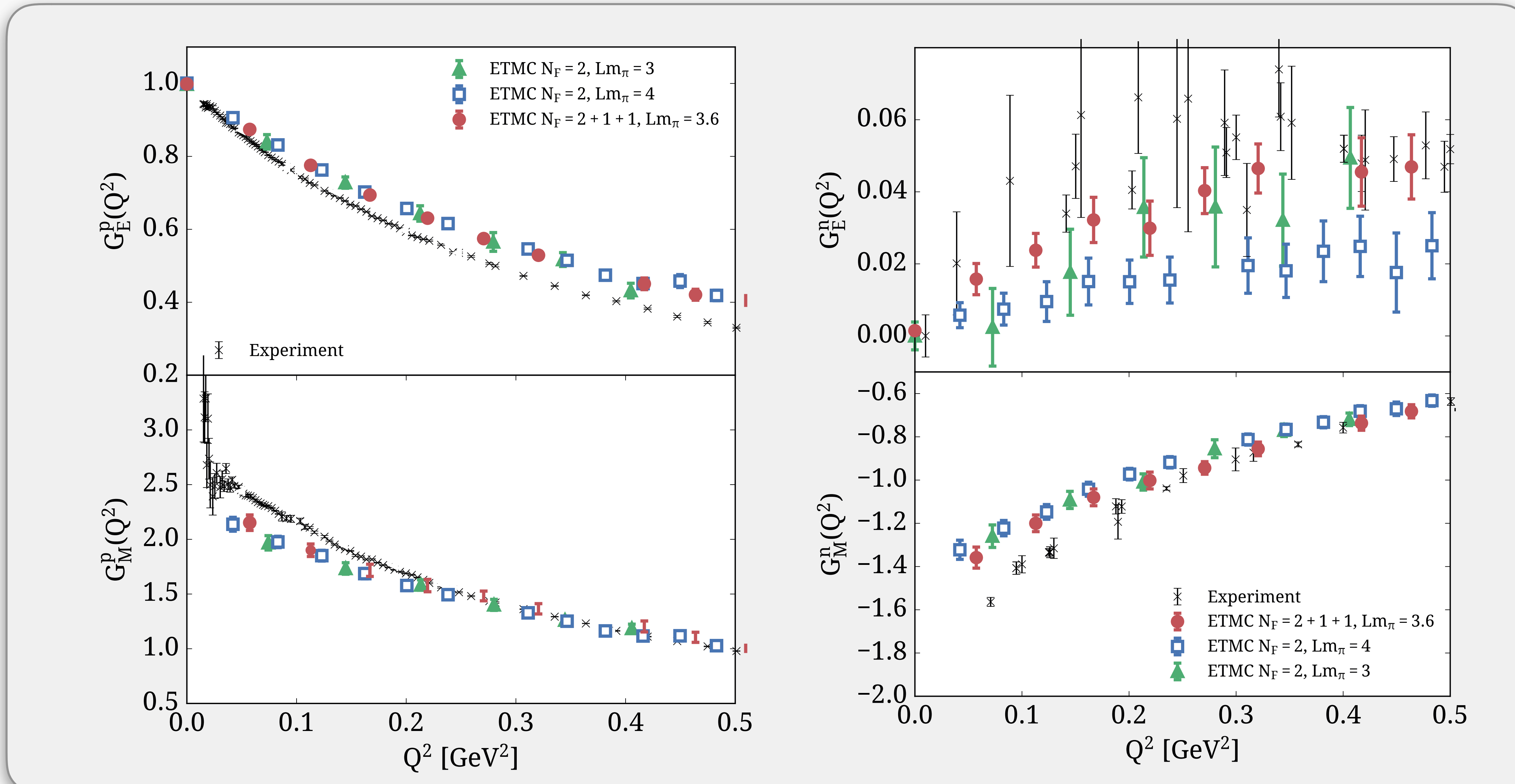

Proton/neutron separated EM form-factors

- Including disconnected; very small effect
- G_E: slight tension at high Q^2
- G_M: slight tension at small Q^2

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Nucleon Generalized Form-Factors

Matrix element:

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{V,A}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\Big[...\Big]u_N(p,s)$$

Three vector and two axial GFFs:

$$\text{Vector}: A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}$$

$$\text{Axial}: \tilde{A}_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde{B}_{20}(q^2)\,\frac{q^{\{\mu}P^{\nu\}}}{2m_N}\gamma^5$$

Ji spin sum: $J^q = \frac{1}{2}[A^q_{20}(0) + B^q_{20}(0)]$

$A^{u-d}_{20}(0) = \langle x\rangle_{u-d}$: directly calculated at Q^2=0

$B^{u-d}_{20}(0)$: need to model $B^{u-d}_{20}(Q^2)$ and take $Q^2 \longrightarrow 0$

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Nucleon Generalized Form-Factors

Solve via SVD of matrix of kinematics:

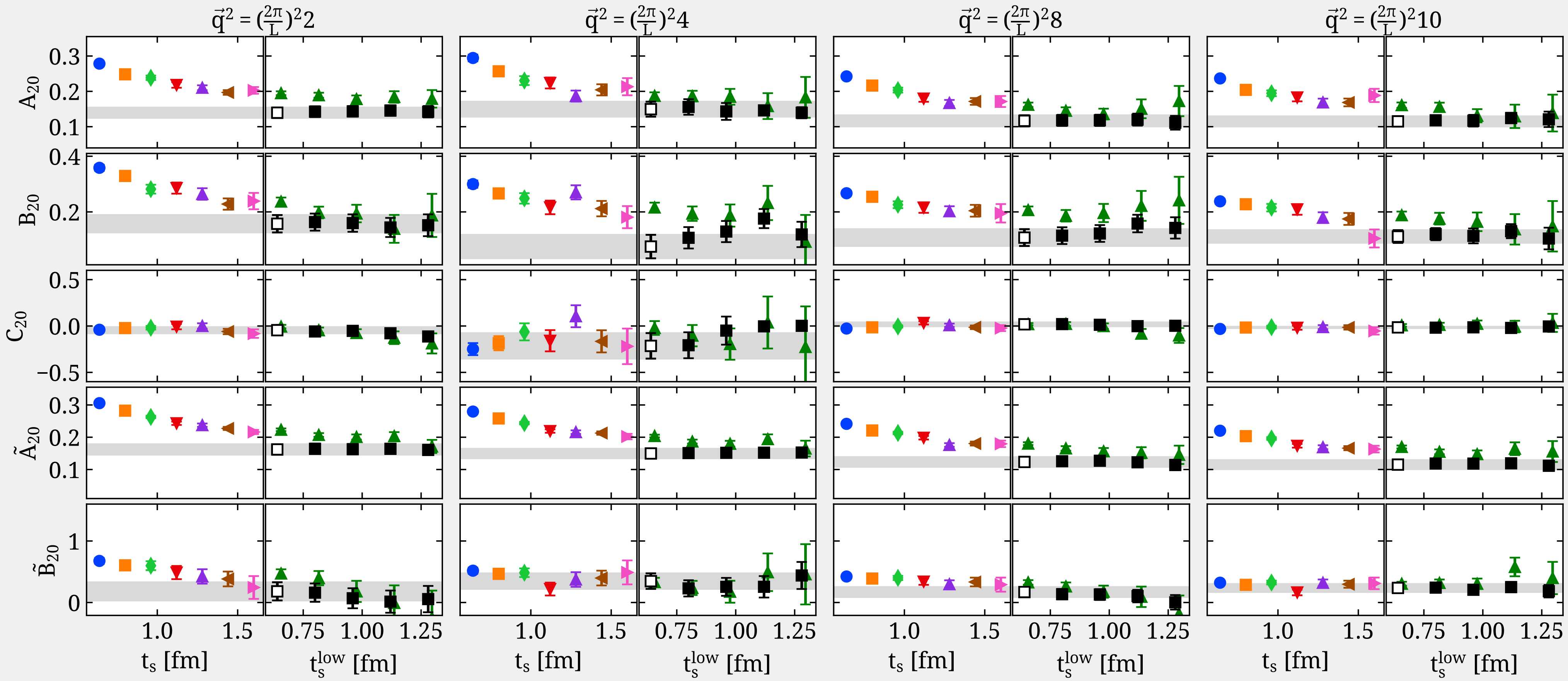
$$\Pi^{\mu\nu}(\Gamma;\vec{q}) = \mathcal{G}^{\mu\nu}(\Gamma;\vec{q})F(Q^2)$$

$\boldsymbol{\Pi}$ lattice measurements, $\mathcal{G}$ kinematics, $\boldsymbol{F}$ vector of GFFs

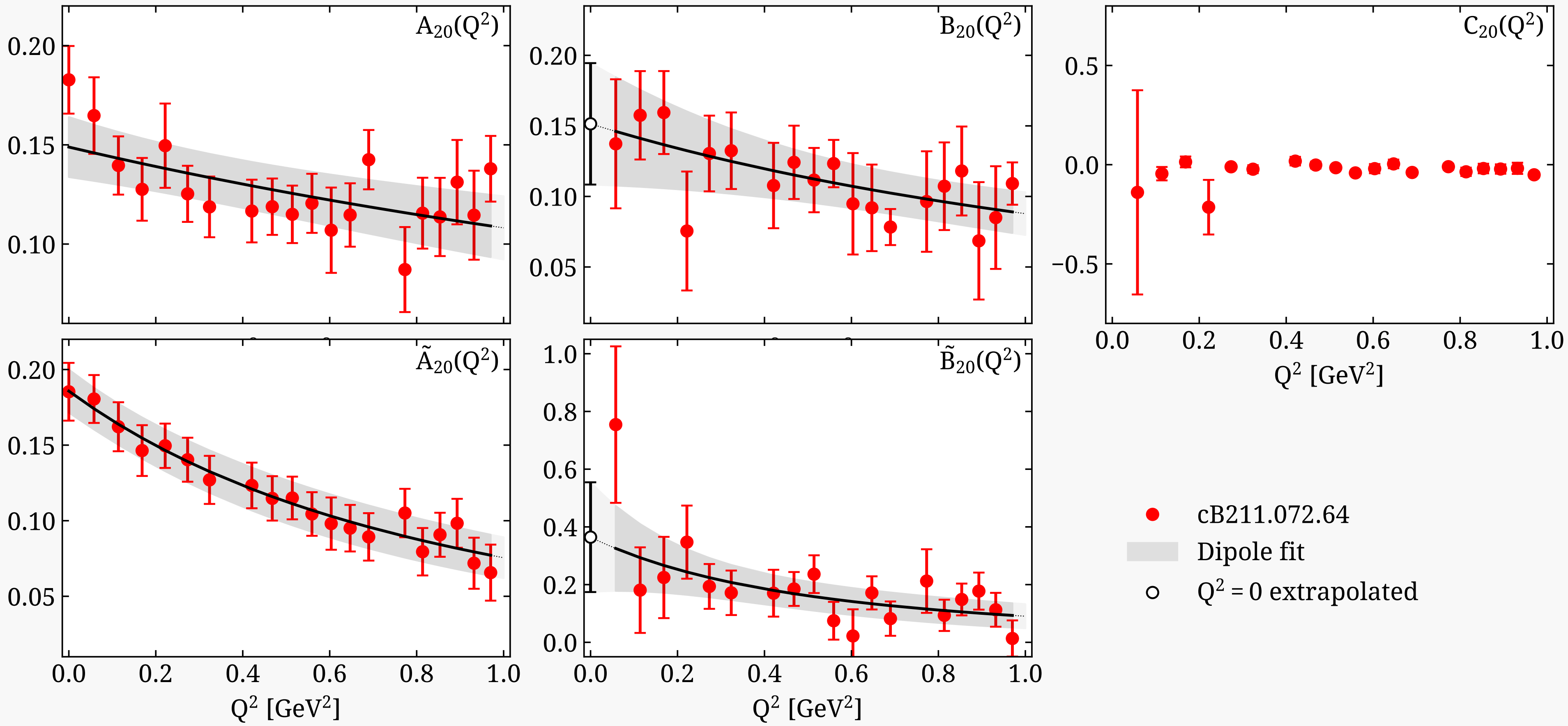
"Single-step" approach for combining fit with SVD

Bali *et al.*, arXiv:1812.08256, PRD

Same excited stat analysis as charges but for each Q^2 separately



Nucleon Generalized Form-Factors



Preliminary

- Dipole fits model well B_{20} and $\tilde{A}_{20}$
- Determination of $J^{u\text{-}d}$ and systematic still ongoing

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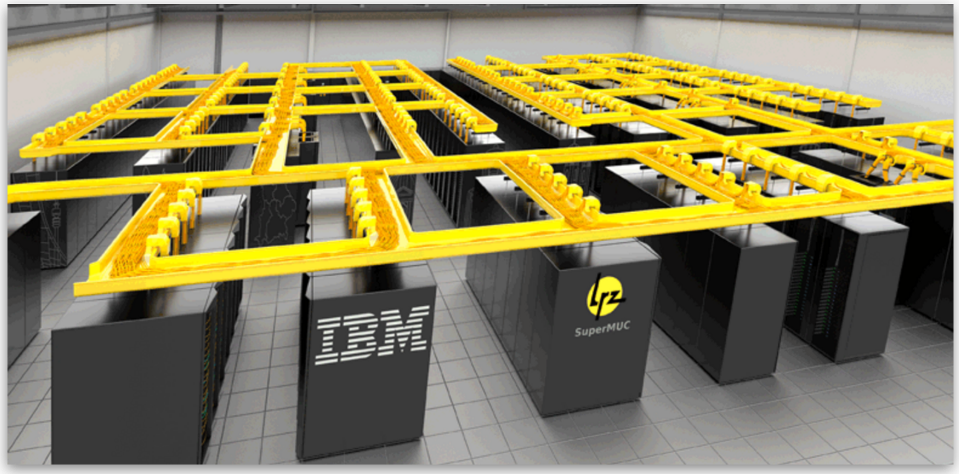
Conclusions and outlook

- Analysis of new N_f=2+1+1 ensemble at physical point *a*=0.08 fm
- High statistics and seven sink-source separations
- Thorough study of excited states. Use of two-state fit necessary to obtain asymptotic value
- Results:
  - Isovector: g_A, g_T, moments of PDFs
  - No detectible volume or cut-off effects; analysis of finer lattice under way
- Good signal for Disconnected contributions:
  - Intrinsic quark spin contributions and σ-terms from matrix element
  - Proton and neutron separated EM form-factors
- Ongoing:
  - Further analysis of disconnected contributions
  - Finer lattice and larger volumes

Acknowledgements

Gauge Field generation:
SuperMUC @ LRZ



Propagators & Contractions:
Piz Daint @ CSCS

- tmLQCD: `github.com/etmc/tmLQCD`
- DDalphaAMG adapted to twisted mass: `github.com/sbacchio/DDalphaAMG`

- QUDA Multi-Grid
- Custom contraction code: `github.com/ETMC-QUDA/quda-QKXTM-Multigrid-PlugIn`

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BACKUP

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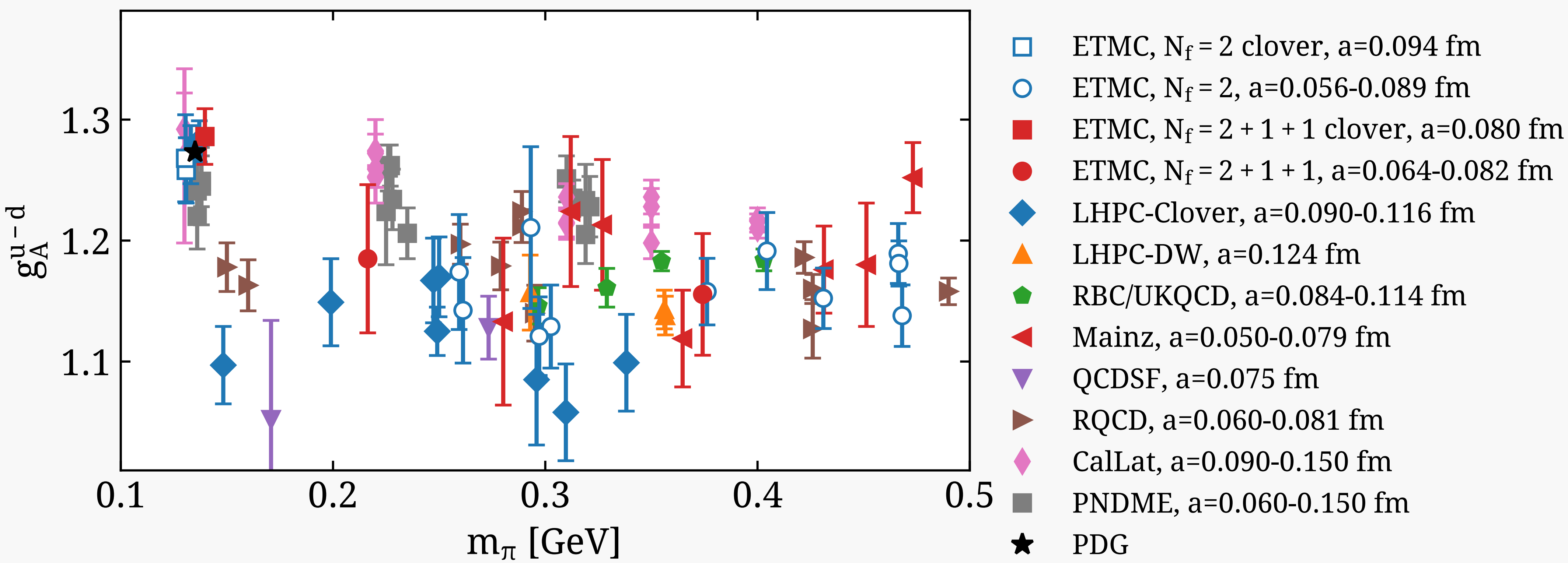

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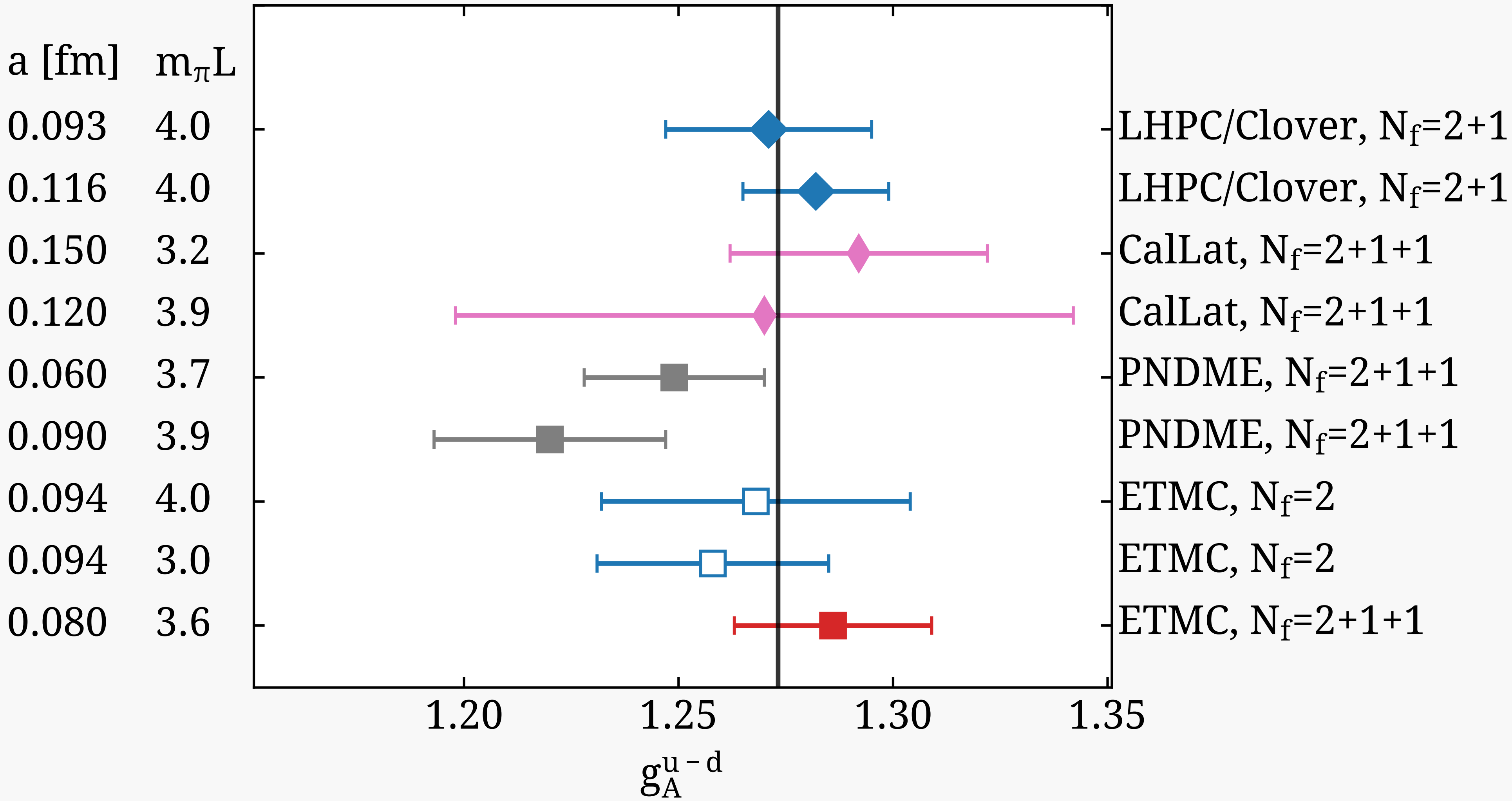

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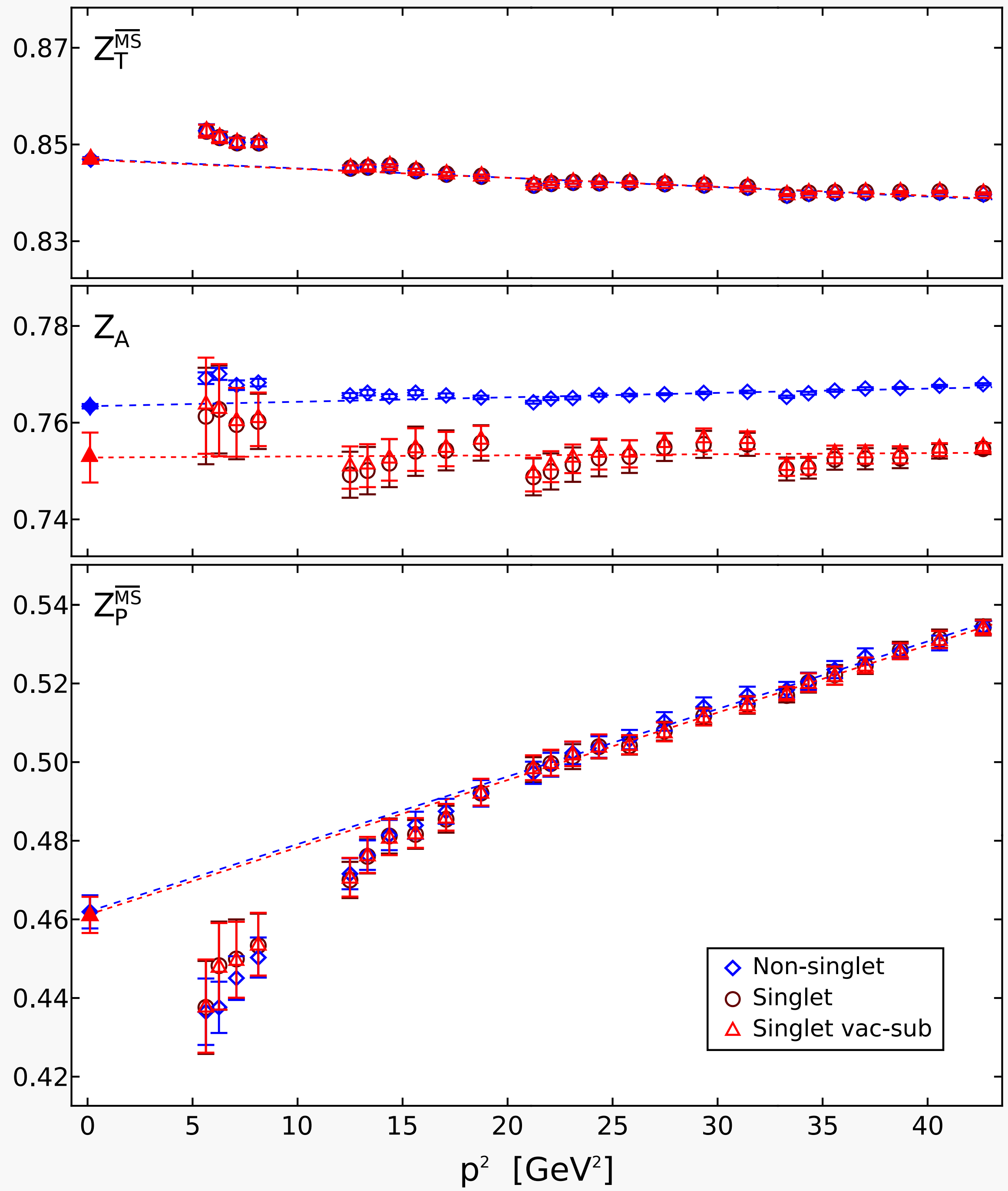

- RI'MOM scheme
- Dedicated N_f=2 and N_f=4 ensembles for taking chiral limit
- Singlet – non-singlet difference calculated non-perturbatively, computing the relevant disconnected contributions
- $\overline{\text{MS}}$ scheme at μ =2 GeV. Conversion factor computed to two loops
- Subtraction of lattice artefacts to $\mathcal{O}(g^2 a^\infty)$ perturbatively

$$\left.\begin{array}{l} Z^{ns} g^{u-d} \\ Z^{ns} g^{u+d-2s} \\ Z^{ns} g^{u+d+s-3c} \\ Z^{s} g^{u+d+s+c} \end{array}\right\} \text{Solve for } u, d, s, c$$

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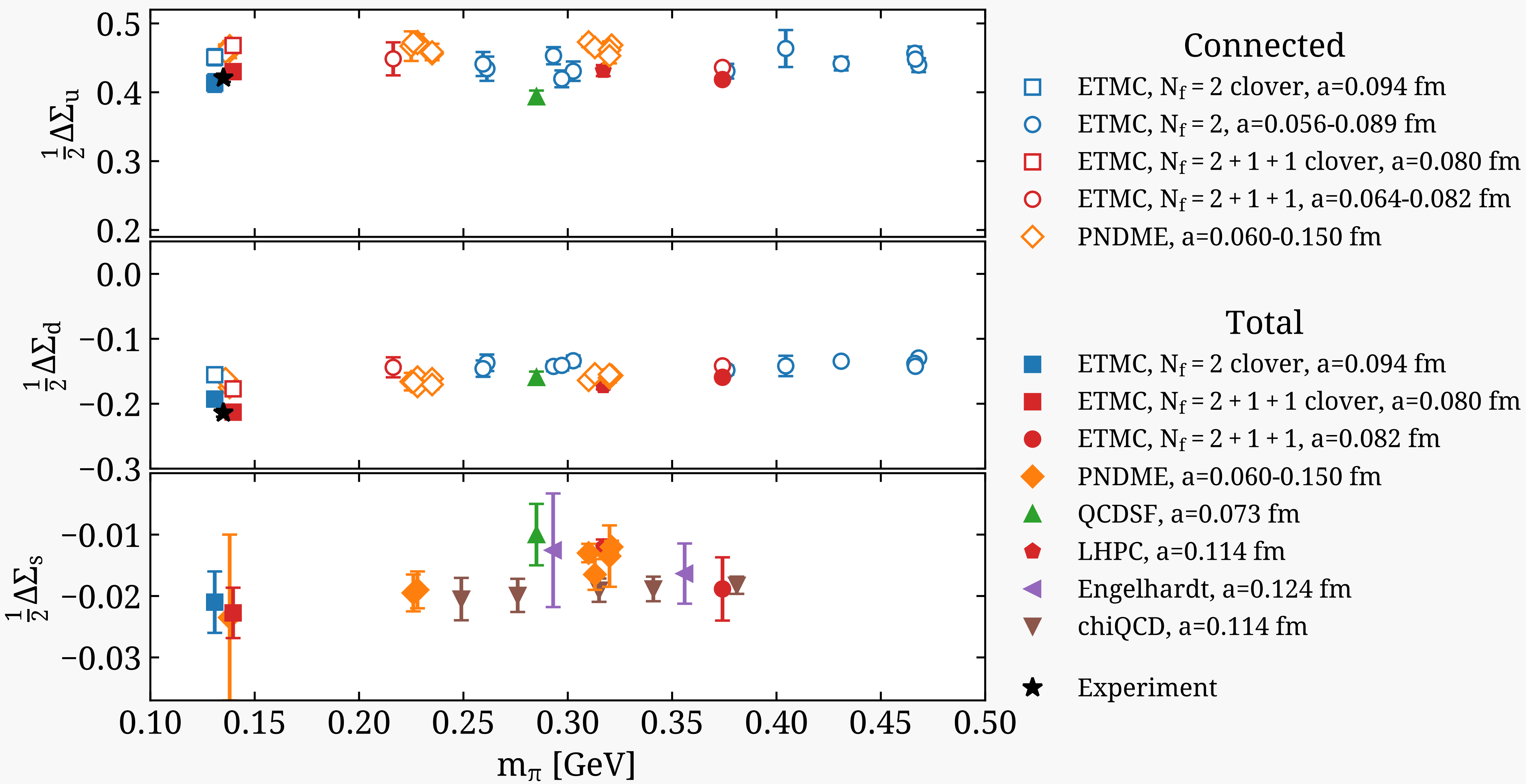

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