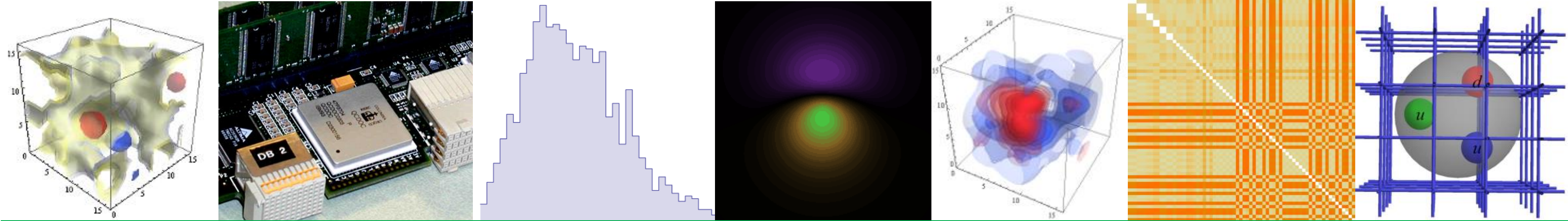


Challenges in Lattice Parton Distribution Functions

HUEY-WEN LIN

This work is supported by the NSF under grant PHY 1653405
"CAREER: Constraining Parton Distribution Functions for New-Physics Searches"

Outline

- § Parton Distribution Functions
- § LaMET and Some Example Results
- § Challenges Ahead

Thanks to MILC collaboration for sharing 2+1+1 HISQ lattices and RBC/UKQCD for sharing 2+1 DWF lattices

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

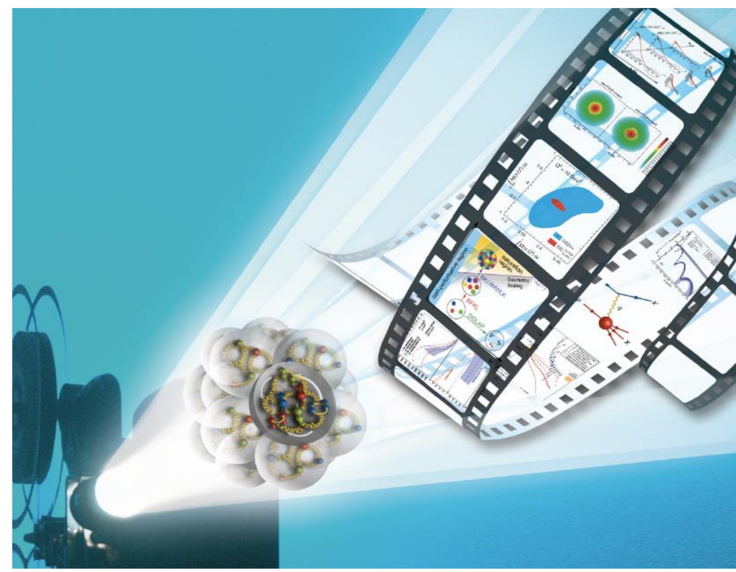
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …)

Electron Ion Collider:
The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701



Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$$

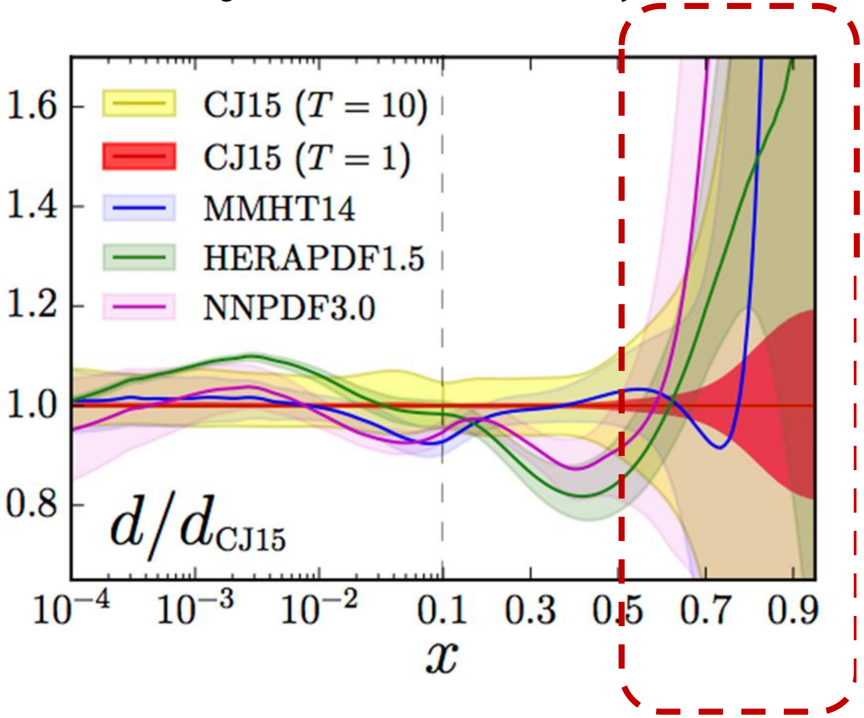
- Assumptions imposed

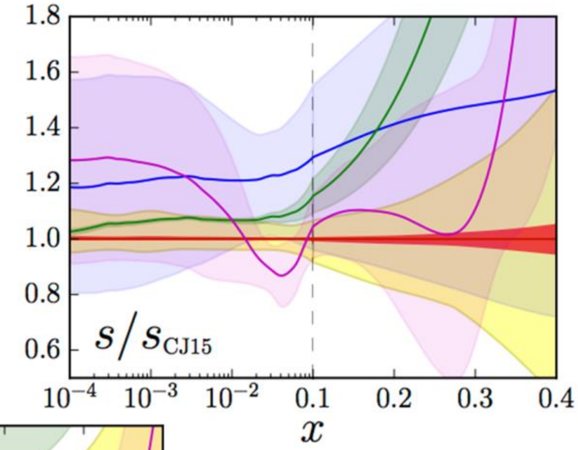
  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

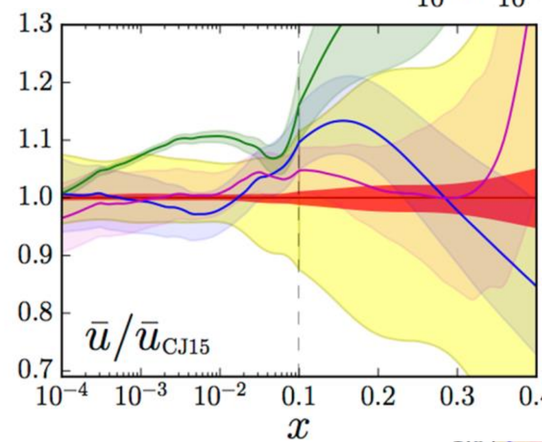
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

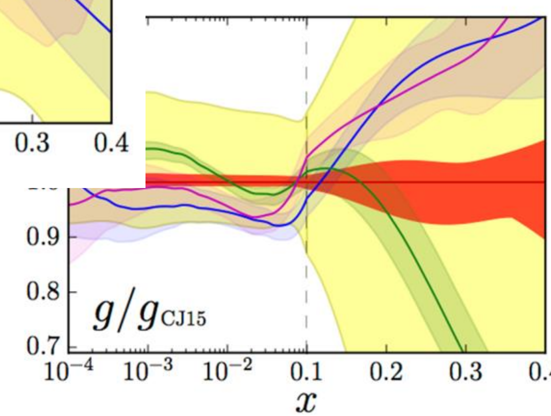
Global Analysis

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.









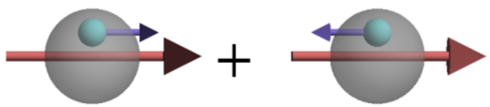
CTEQ-JLAB
https://www.jlab.org/theory/cj/

What can we do on the lattice?

PDFs on the Lattice

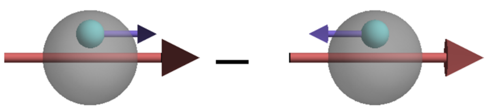
§ Lattice calculations rely on operator product expansion, only provide moments

most well known



Quark density/unpolarized

$$\langle x^n \rangle_q = \int_{-1}^{1} dx\, x^n q(x)$$



Helicity
longitudinally polarized

$$\langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^n \Delta q(x)$$

Transversity
transversely polarized

$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$

very poorly known

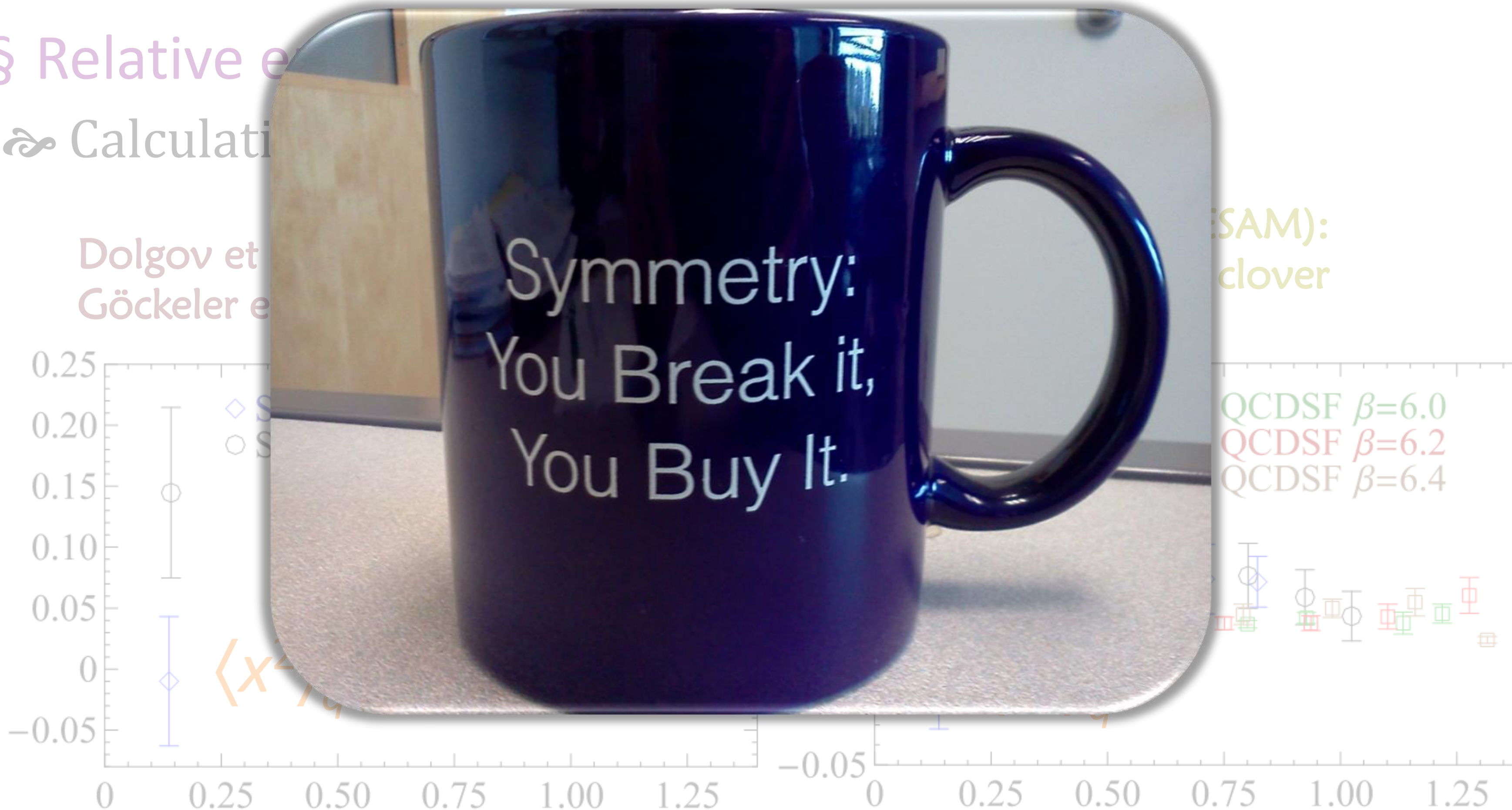
§ True distribution can only be recovered with all moments

Problem with Moments

- For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- Relative e
  - Calculati

Dolgov et
Göckeler e

SAM):
clover



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

LaMET

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

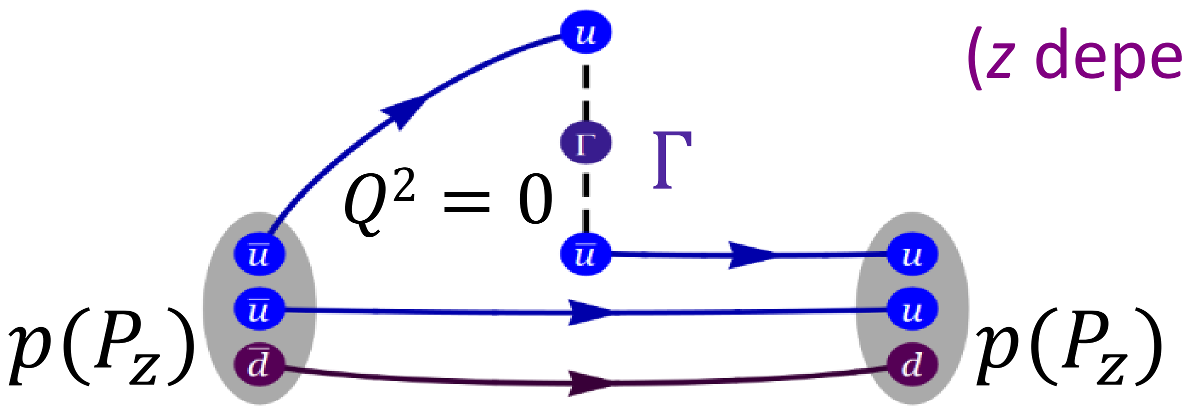
- Calculate the parton distributions through the infinite-momentum frame Feynman, Phys. Rev. Lett. 23, 1415 (1969)

LaMET Recipe

Large-Momentum Effective Theory (LaMET) X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice

(z dependence)



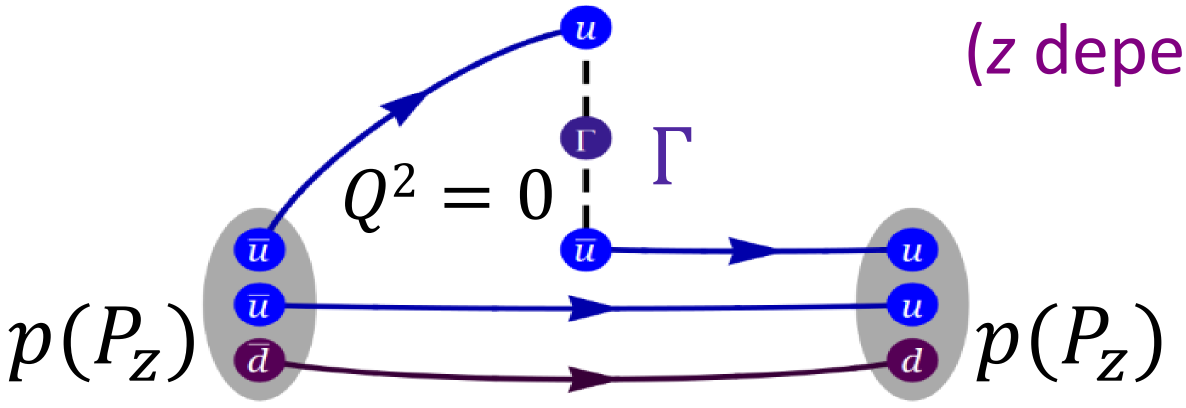
LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice

(z dependence)



2) Compute quasi-distribution via

$$\tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \bar{\psi}(z)\, \Gamma \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

$x = k_z/P_z$

Lattice z coordinate

hadron momentum $P_\mu = \{P_t, 0, 0, P_z\}$

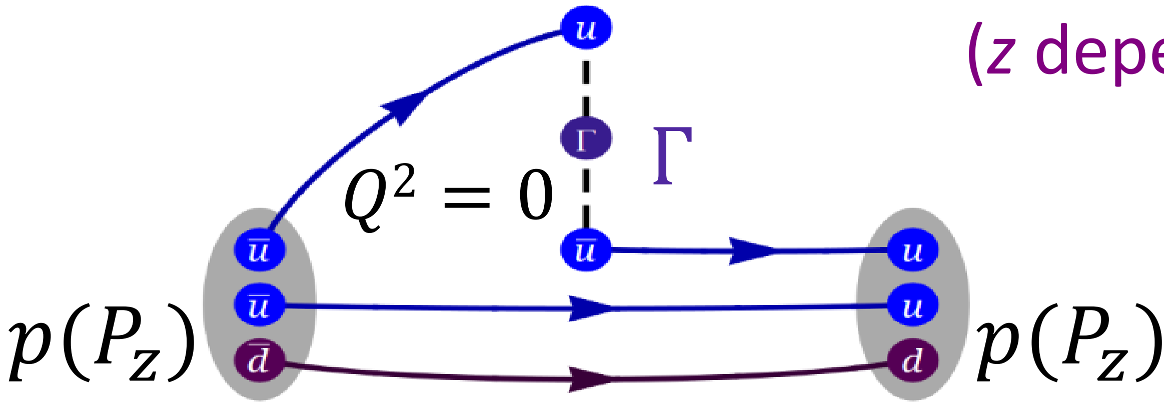
Product of lattice gauge links

LaMET Recipe

Large-Momentum Effective Theory (LaMET) X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice

(*z* dependence)



2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \Gamma \exp\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\rm QCD}^2/P_z^2\right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Progress in the theoretical development of LaMET

- **Renormalization:**

Ji and Zhang, 2015; Ishikawa et al., 2016, 2017; Chen, Ji and Zhang, 2016;

Xiong, Luu and Meiβner, 2017; Constantinou and Panagopoulos, 2017; Ji, Zhang, and Y.Z., 2017; J. Green et al., 2017; Ishikawa et al. (LP3), 2017; Wang, Zhao and Zhu, 2017; Spanoudes and Panagopoulos, 2018.

- **Factorization:**

Ma and Qiu, 2014, 2015, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **One-loop matching:**

Xiong, Ji, Zhang and Y.Z., 2014; Ji, Schaefer, Xiong and Zhang, 2015; Xiong and Zhang, 2015; Constantinou and Panagopoulos, 2017; I. Stewart and Y. Z., 2017; Wang, Zhao and Zhu, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **Power corrections:**

J.-W. Chen et al., 2016; A. Radyushkin, 2017.

- **Transvers momentum dependent parton distribution function:**

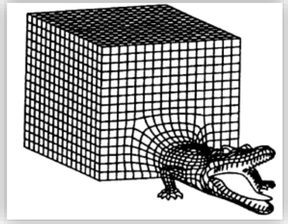
Ji, Xiong, Sun, Yuan, 2015; Ji, Jin, Yuan, Zhang and Y.Z., 2018; Ebert, Stewart and Y.Z., in progress.

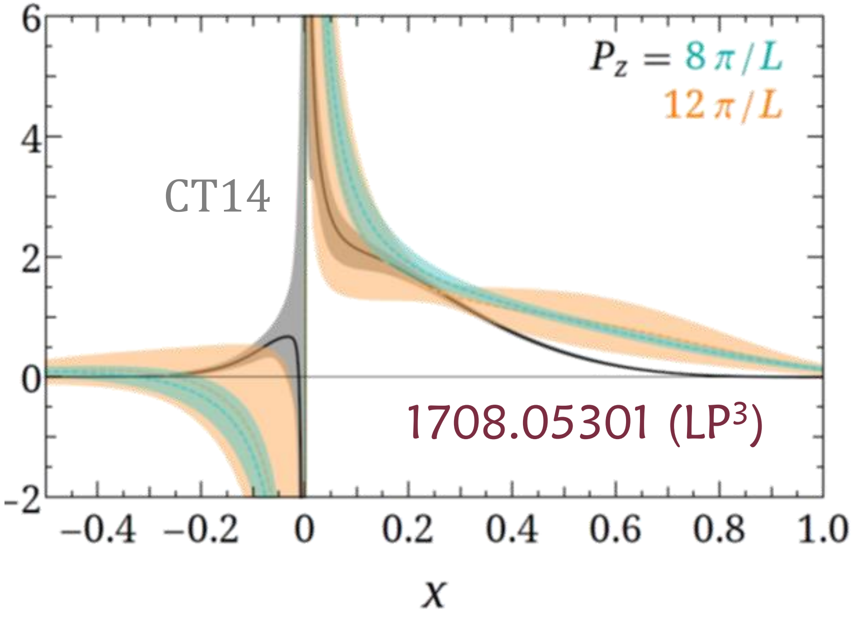
Slide credit: Yong Zhao, CIPANP 2018 Plenary talk; also se Y. Zhao's Lattice 2019 talk

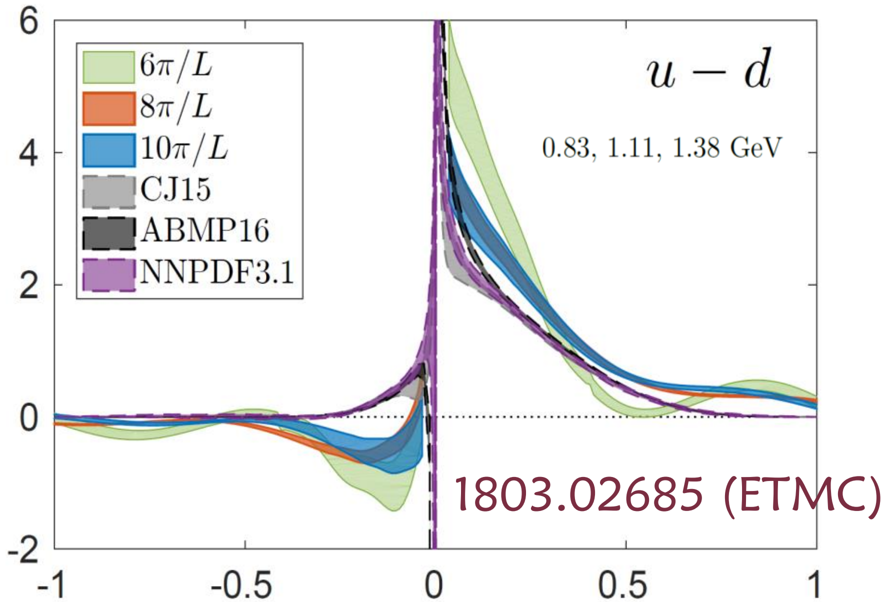
Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed





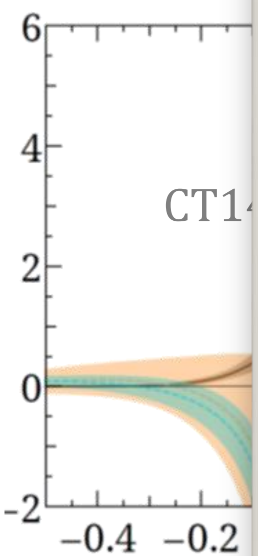


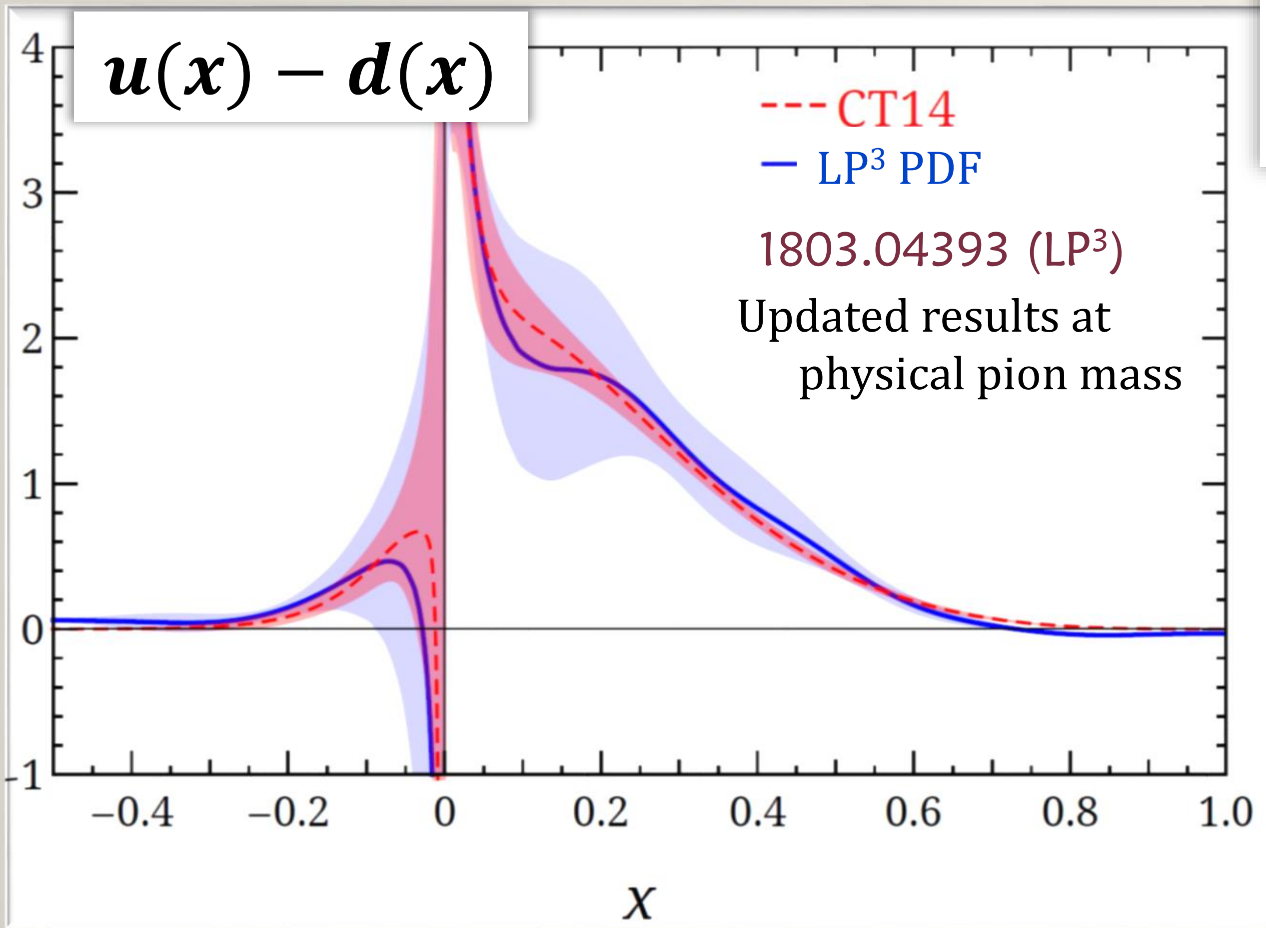
Not using parametrization (e.g. $xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$)
Less pretty results;
less likely to exactly coincide with global fits.

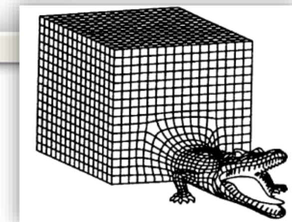
Physical Pion Mass Results

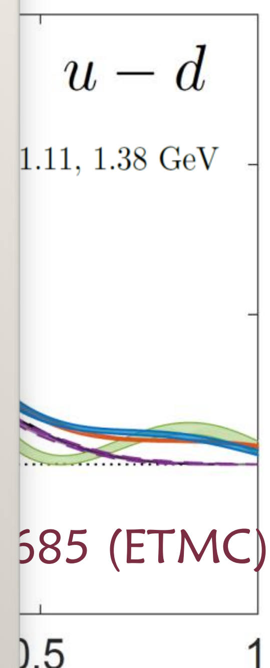
§ Exciting! Two collaborations' results at physical pion mass

- Boost $P \sim$ 1.4 GeV
- Study





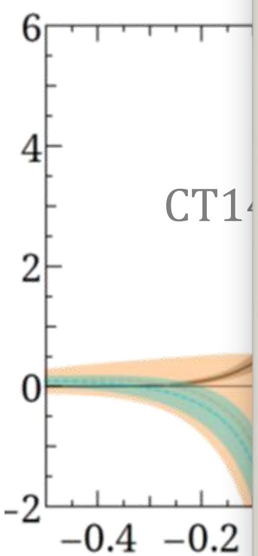


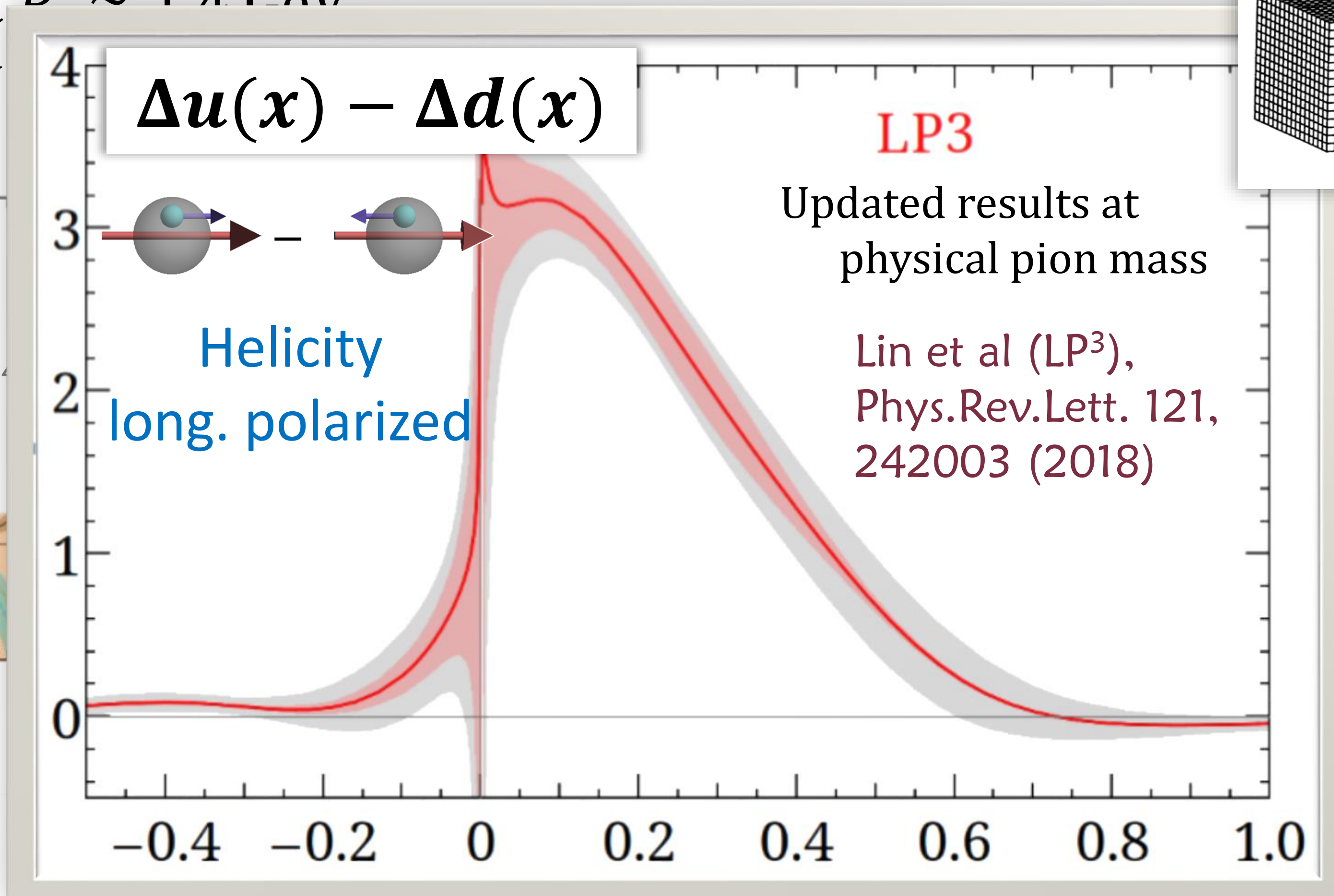


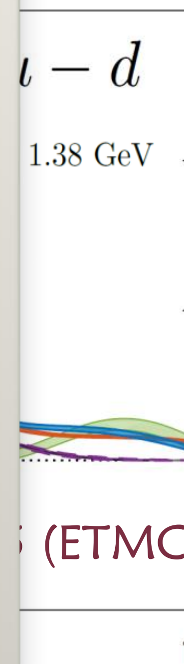
Physical Pion Mass Results

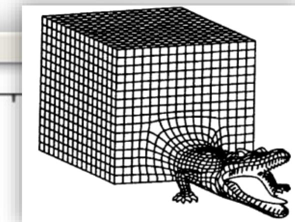
§ Exciting! Two collaborations' results at physical pion mass

- Boost $P_z \sim 1.4$ GeV
- Study





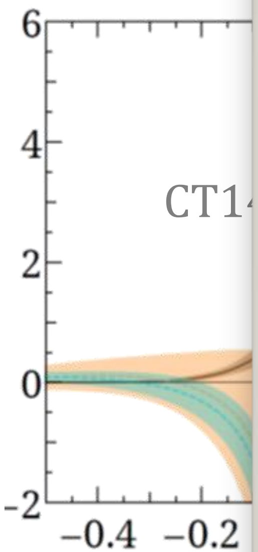


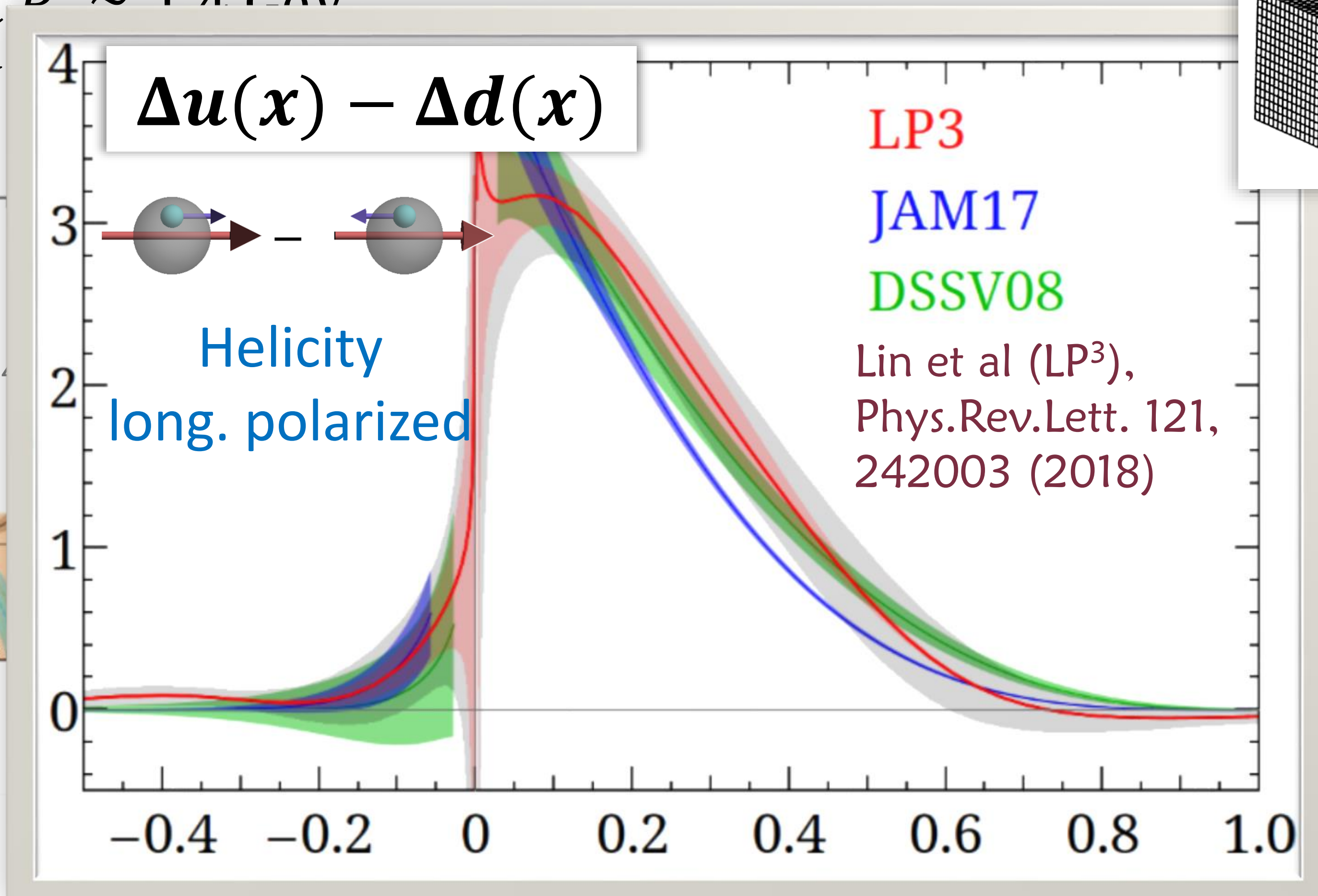


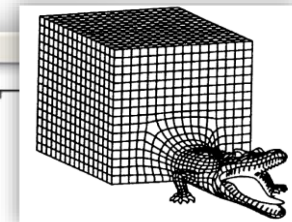
Physical Pion Mass Results

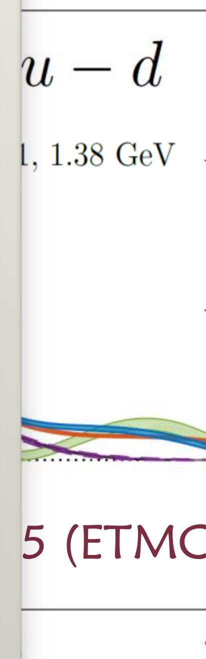
§ Exciting! Two collaborations' results at physical pion mass

- Boost $P \sim 1.4$ GeV
- Study





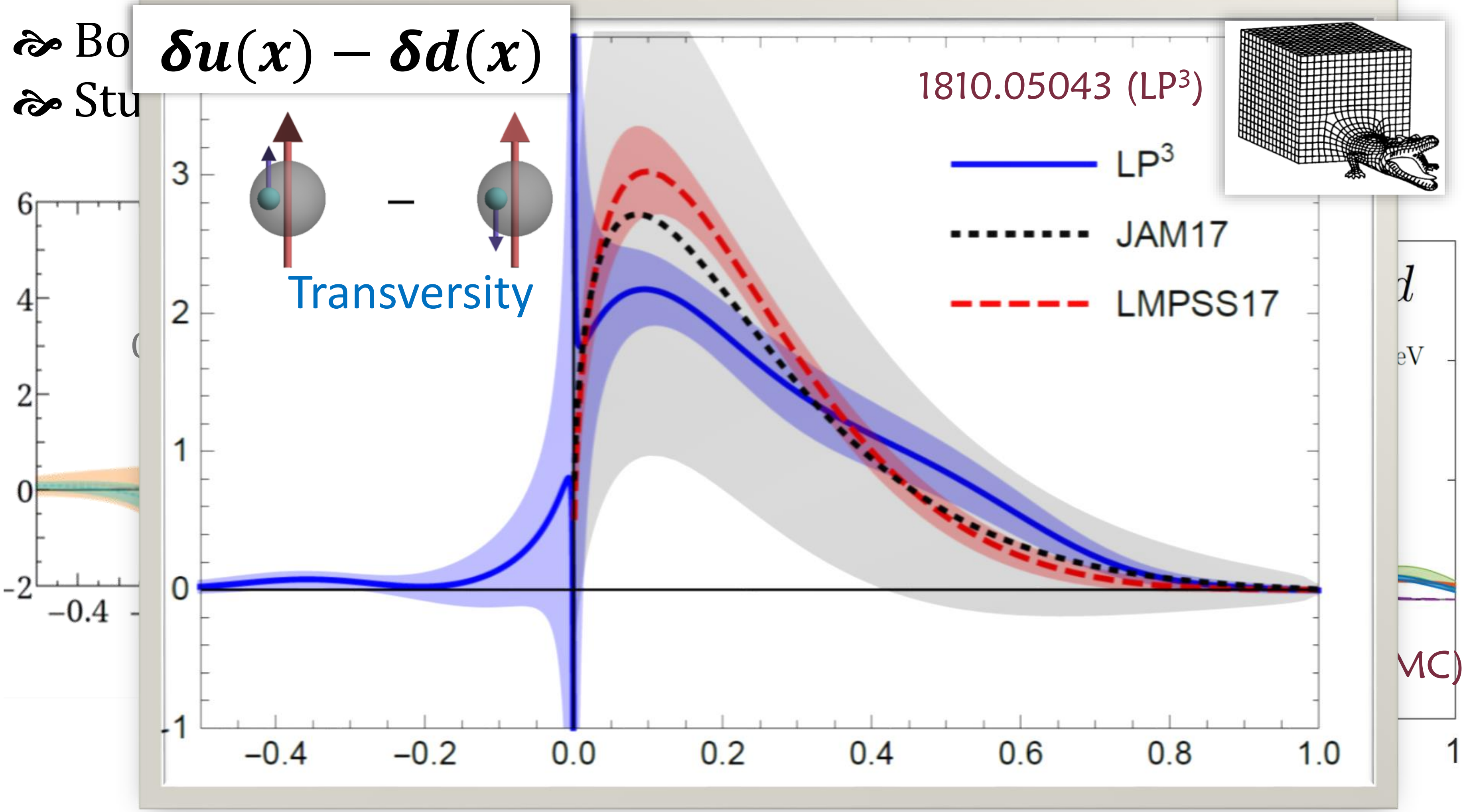




Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Bo
- Stu

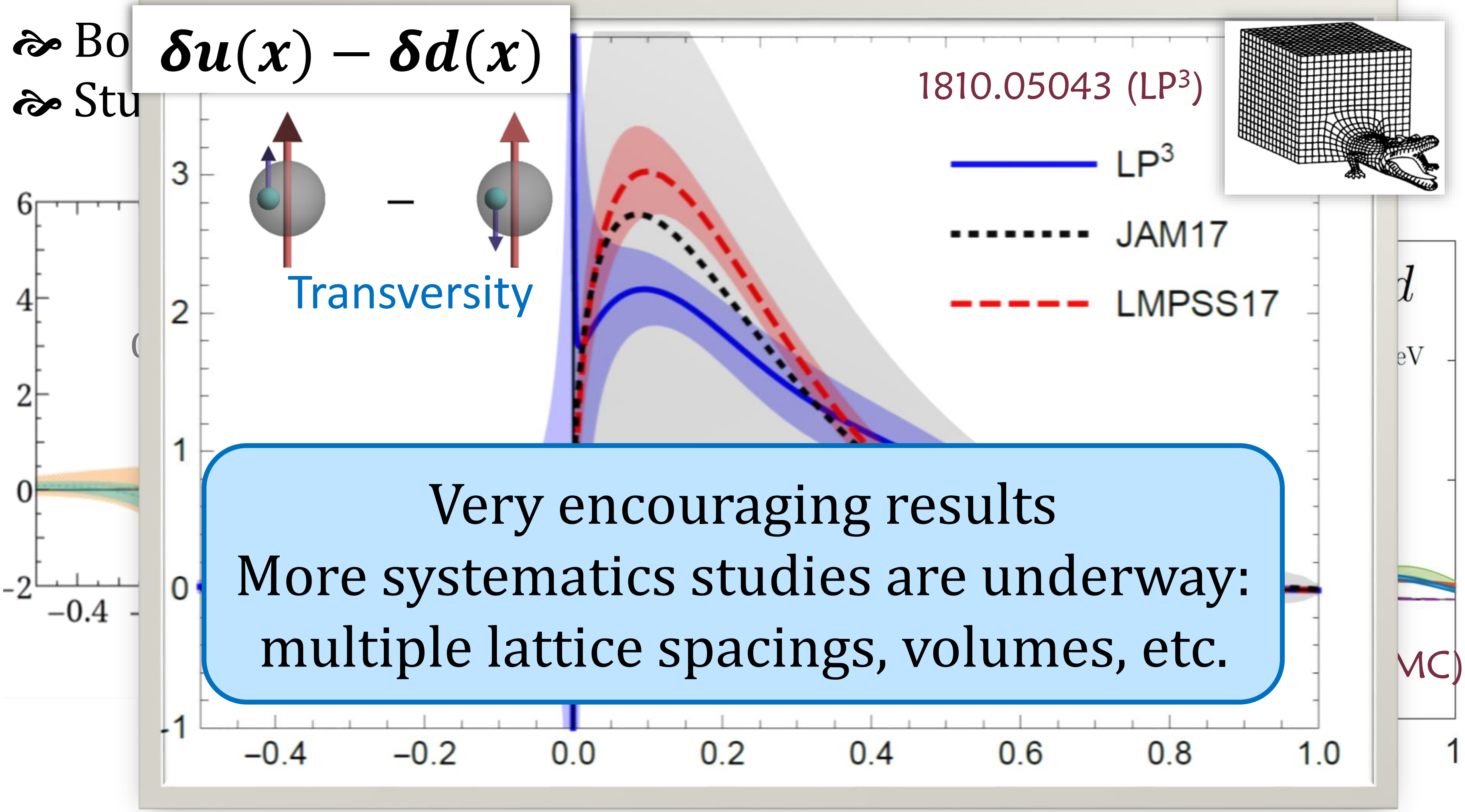


Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Bo
- Stu

$\boldsymbol{\delta u(x) - \delta d(x)}$

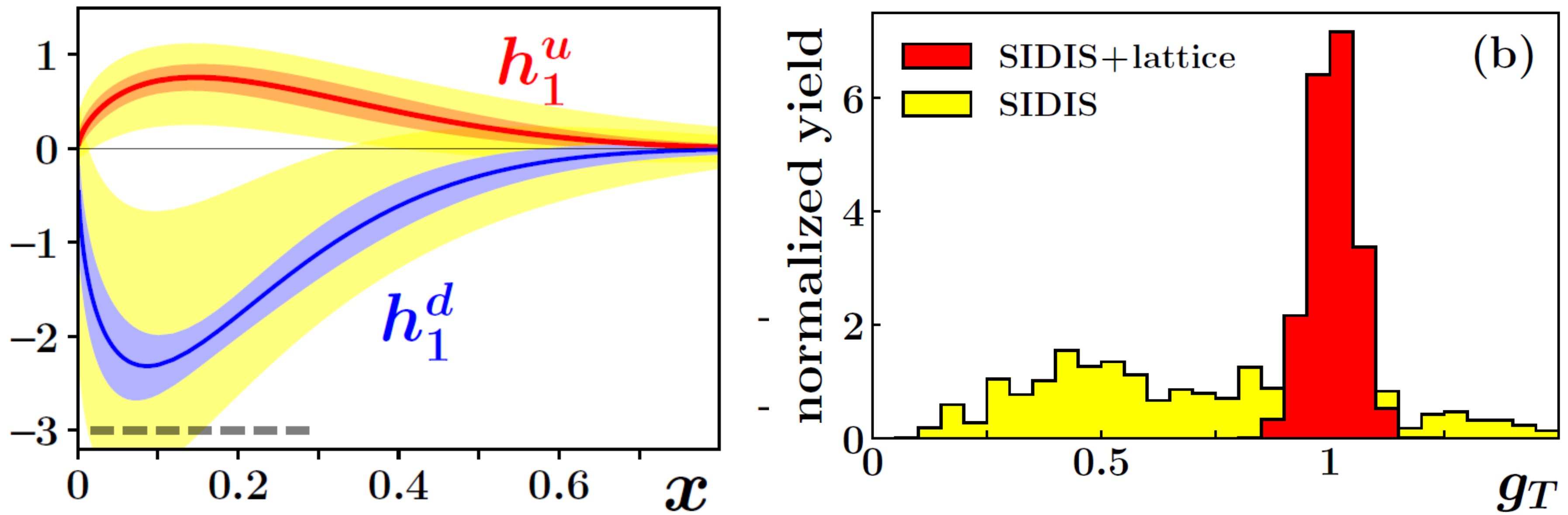


Very encouraging results
More systematics studies are underway:
multiple lattice spacings, volumes, etc.

Lattice Constraints to PDFs

§ Improved **transversity** distribution with LQCD g_T

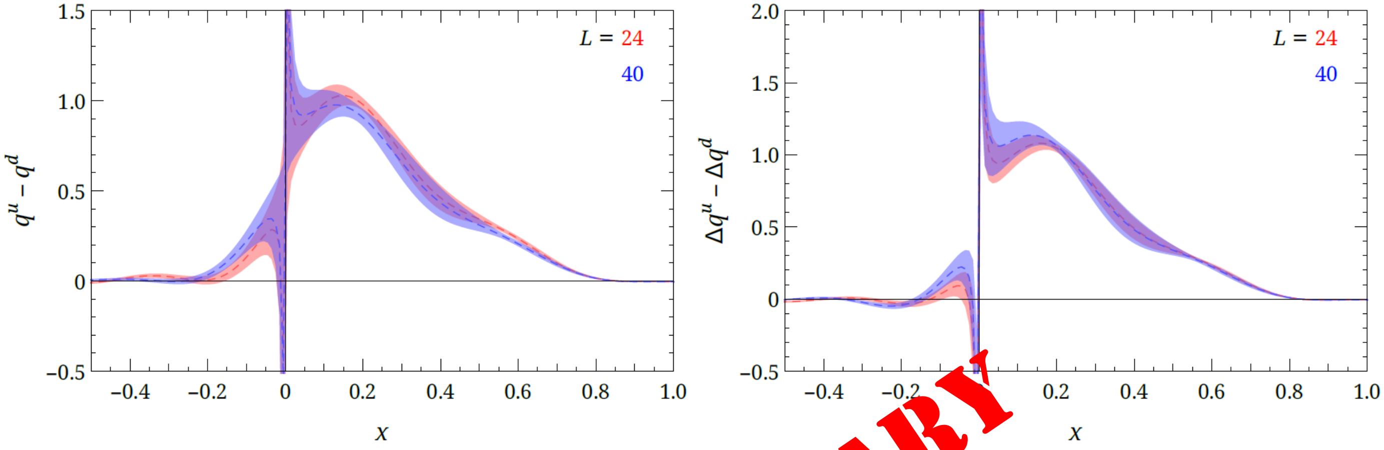
- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
  - Including only multiple lattice spacing, volumes works
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets

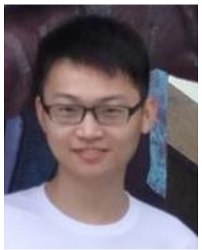


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Systematics Study Underway

§ Finite-volume study

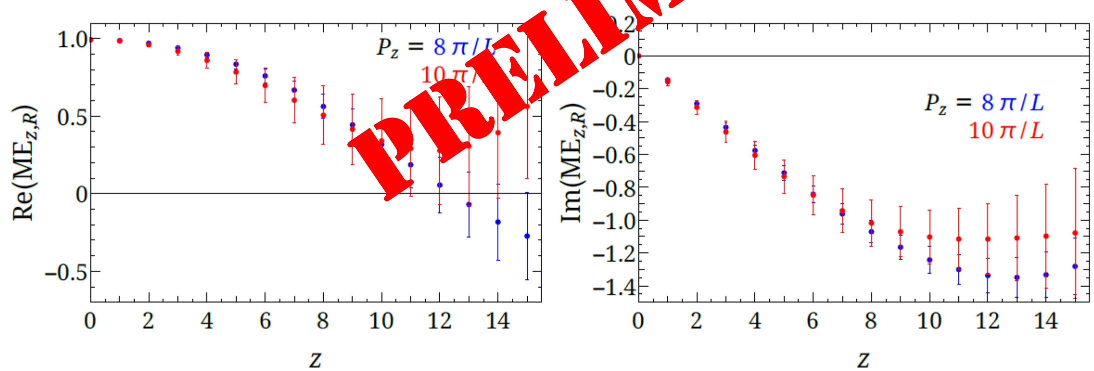




R. Zhang

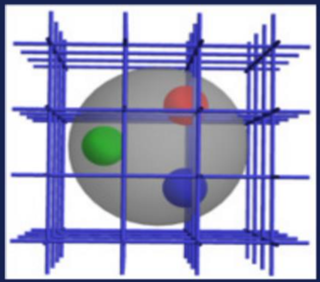
§ Looking into fine lattice spacing a = 0.042 fm

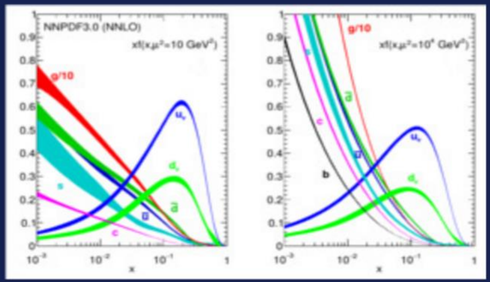
PRELIMINARY



Zhouyou Fan

+BNL group





Parton Distributions and Lattice Calculations in the LHC era
(PDFLattice 2017)

22-24 March 2017, Oxford, UK

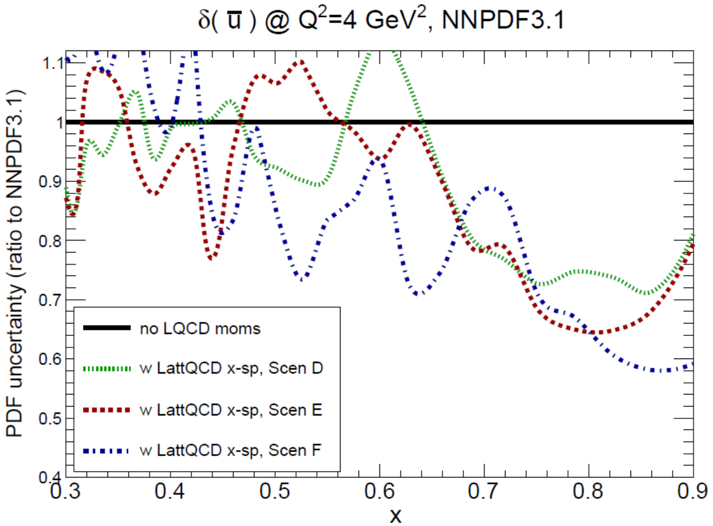
§ Implementing the pseudo-data from LQCD with $x=0.7$–0.9

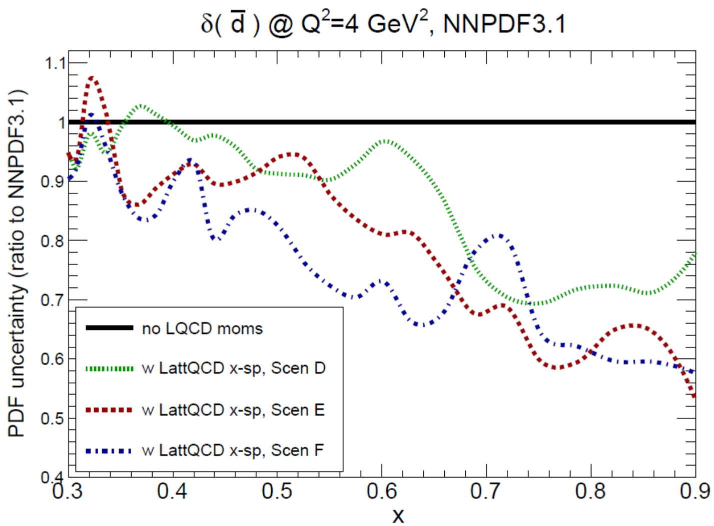
$$u(x_i,Q^2)-d(x_i,Q^2)\text{ and }\bar{u}(x_i,Q^2)-\bar{d}(x_i,Q^2)$$

D: 12%

E: 6%

F: 3%





Lin et al, Progress in Particle and Nuclear Physics 100, 106 (2018)

Challenge:

The Necessity of Large Boosted Momentum

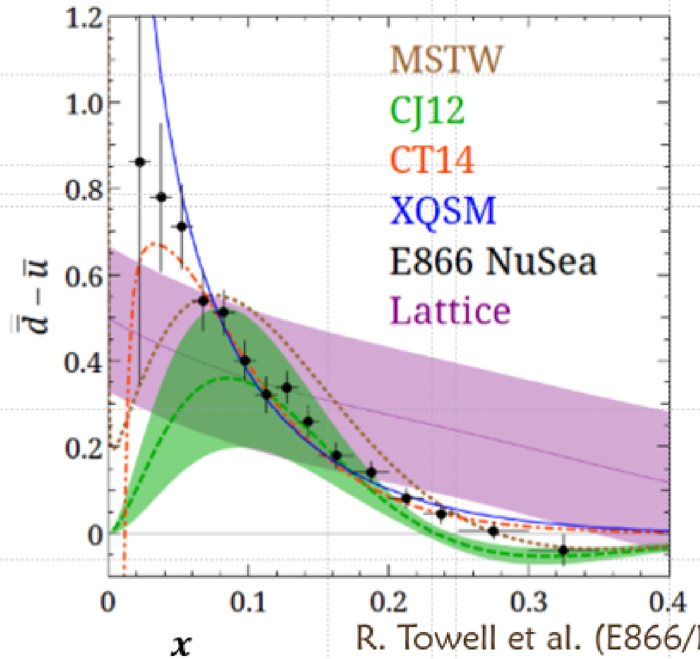
Backstory

- Many of you are old enough to remember this:

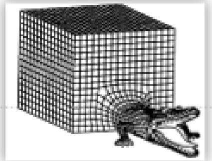
Sea Flavor Asymmetry

- First time in LQCD history to study antiquark distribution!
  - $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm**

HWL et al. 1402.1462



$\bar{q}(x) = -q(-x)$



Lost resolution in small-x region

Future improvement: larger lattice volume

$$\int dx\,(\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

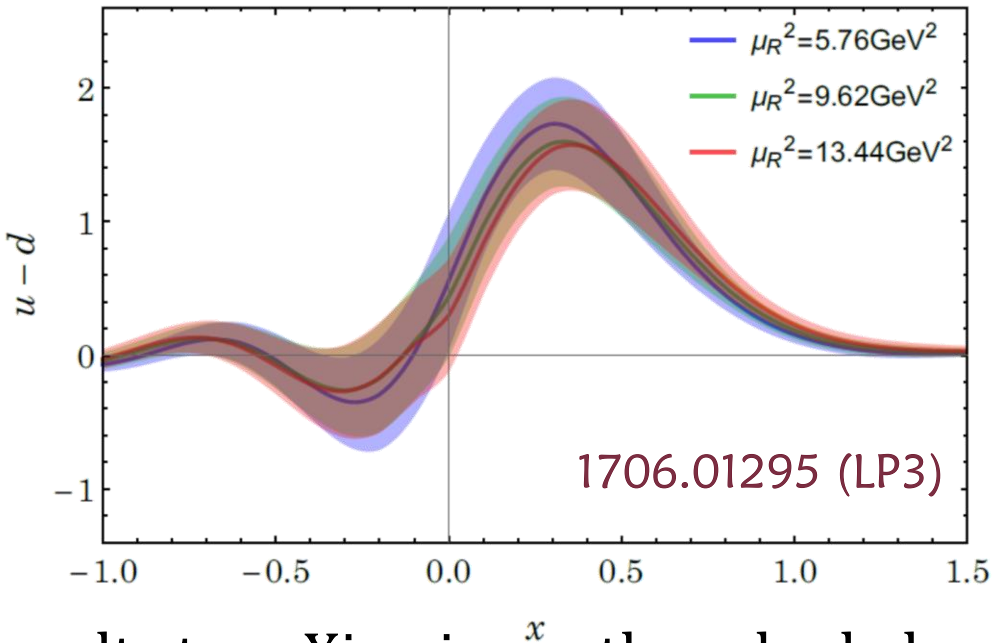
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Caveat: These matrix elements are not properly renormalized

Backstory

§ Efforts by multiple collaborations have been devoted into working on lattice renormalization

- We finally obtained the renormalized ME, and the renormalized PDF results puzzled us for months!

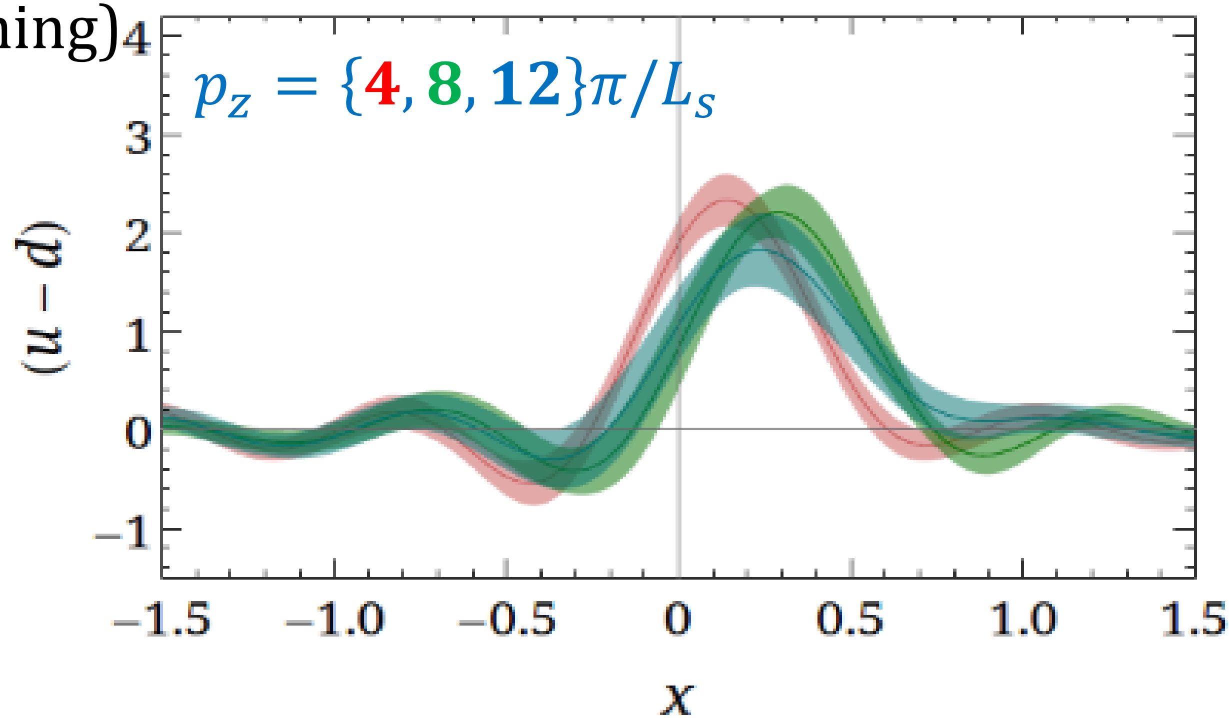


- We finally posted the results to arXiv, since others had already posted their renormalized result

Backstory

§ Immediately, we checked a different lattice and observed the same thing and worse, since we had more "z" data!

- Results from 2017 Summer at physical pion mass (before matching)

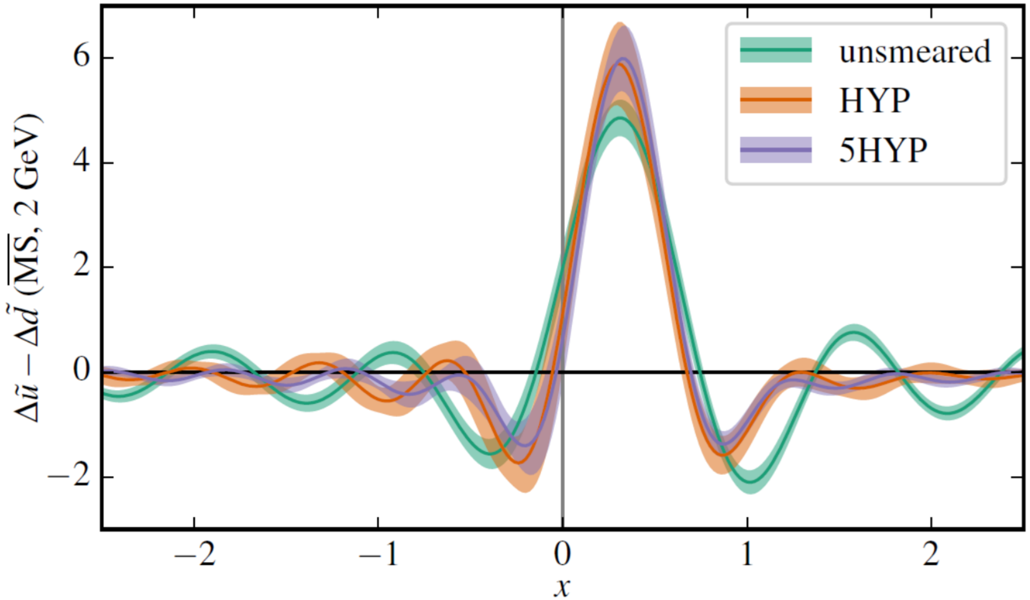


Backstory

§ Something is obviously wrong!

- This effect was missed by earlier ETMC work due to the small *z* and momentum combination

J. Green et al 1707.07152

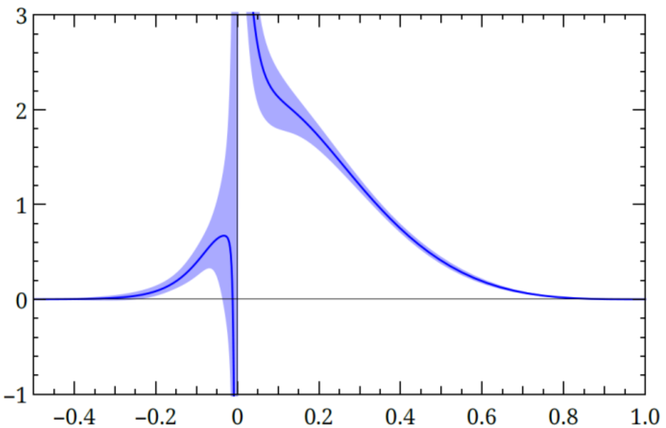


- We called it "oscillation" and F.T. truncation artifacts
  - "Inversion problem" name stick
- For a while, people had no idea what we are talking about

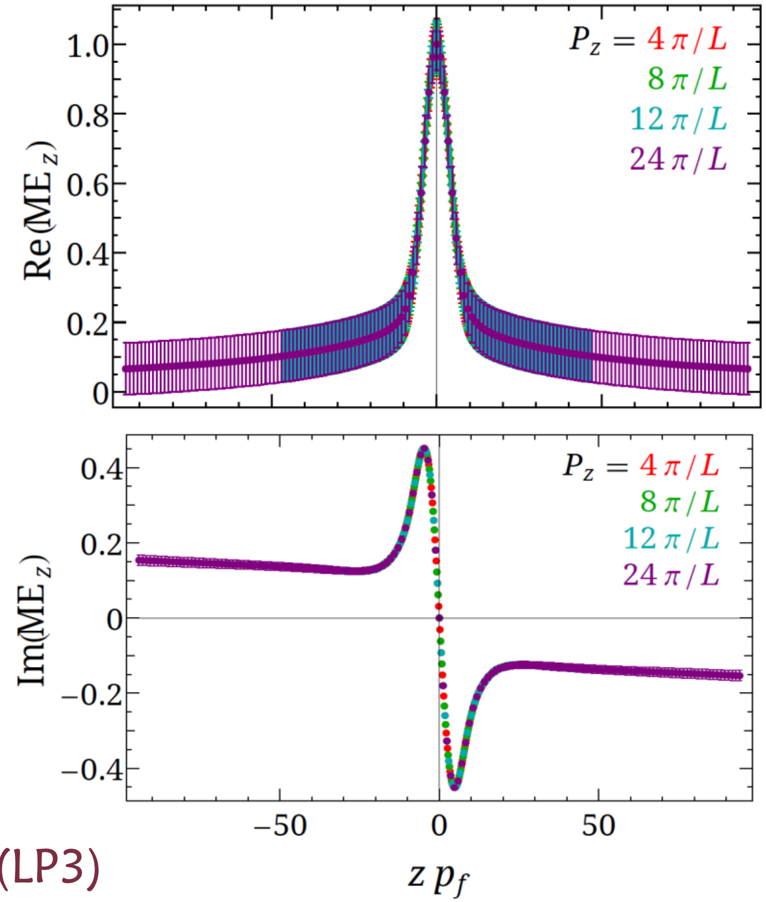
Focus on Continuum

- § Not a lattice problem but a Fourier-transform issue
- § Simple exercise with CT14 PDF 1506.07443

Pick your favorite PDF (CT14 here)



Fourier to coordinate space at some momentum



1708.05301 (LP3)

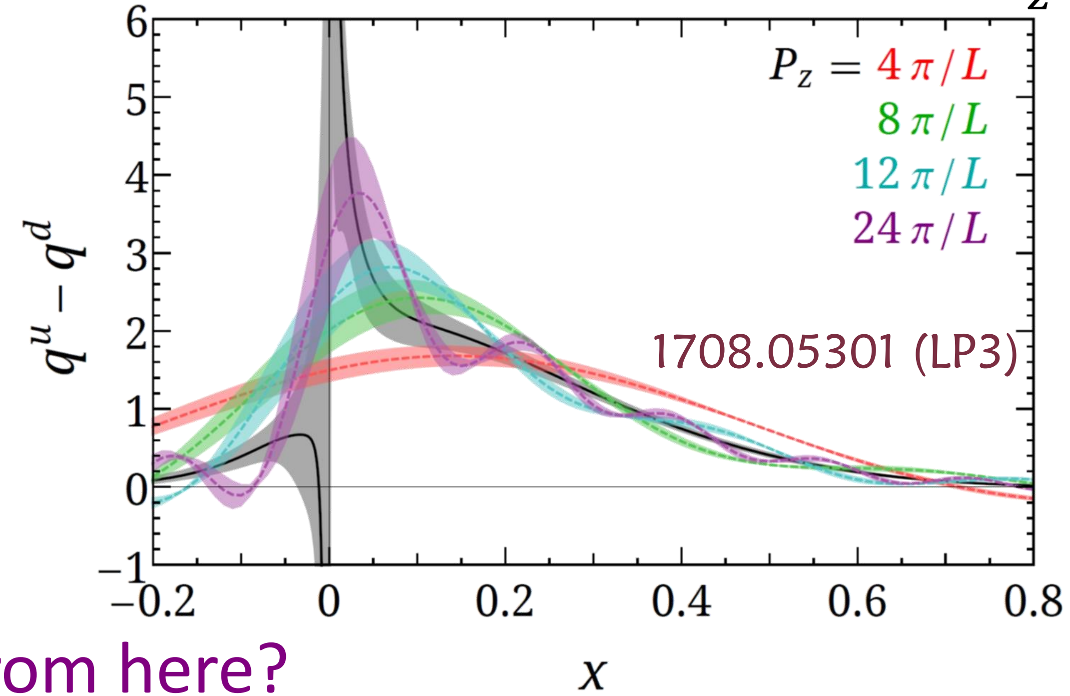
Focus on Continuum

§ Not a lattice problem but a Fourier-transform issue

§ Simple exercise with CT14 PDF

Fixed $L_z = 32$

Inverse Fourier transform back to momentum space



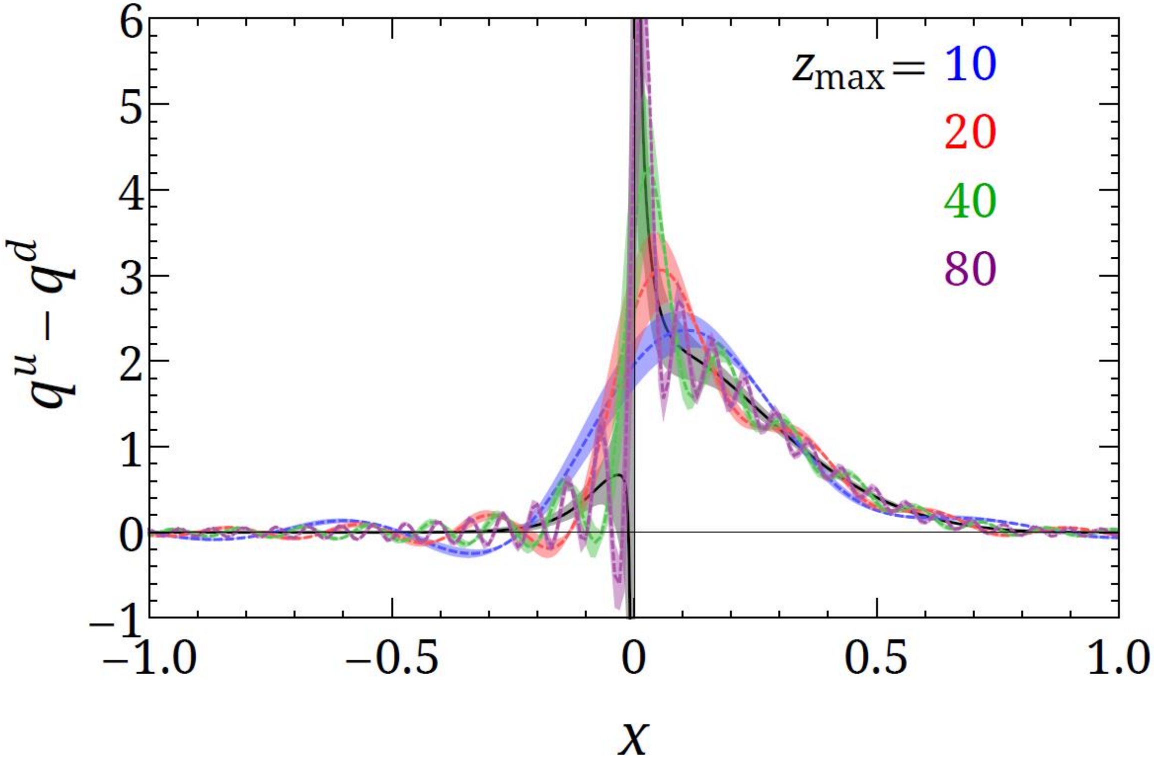
§ What do you do from here?

- Throw up your hands and add 100% errorbars across all x
- Find some way to salvage as much information as possible

Continuum

§ The distribution is z_{max} dependent

Fixed $P_z = 24\pi/L$



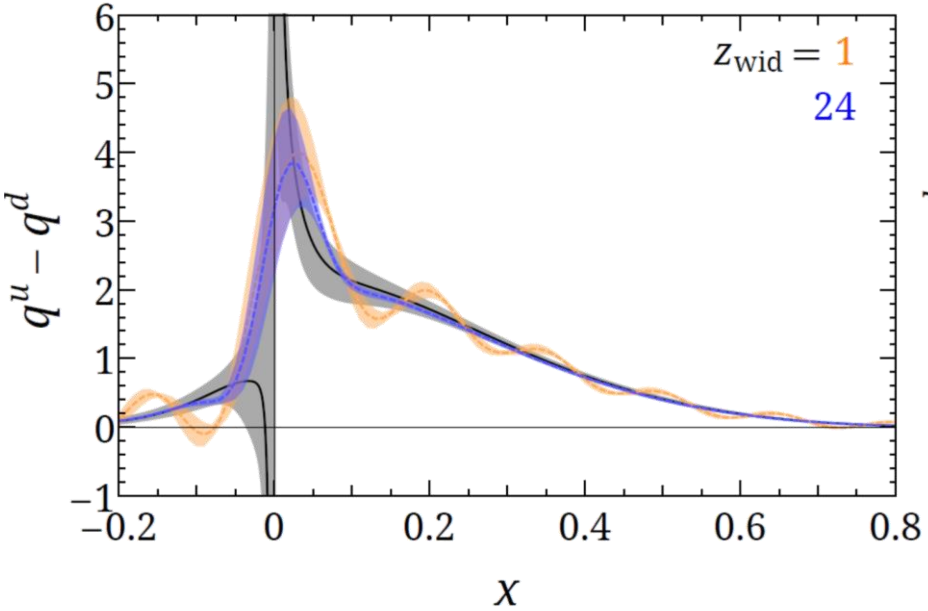
1708.05301 (LP[3])

Continuum

- Luckily, we know the answer
- Two possible solutions proposed (likely more) 1708.05301 (LP[3])

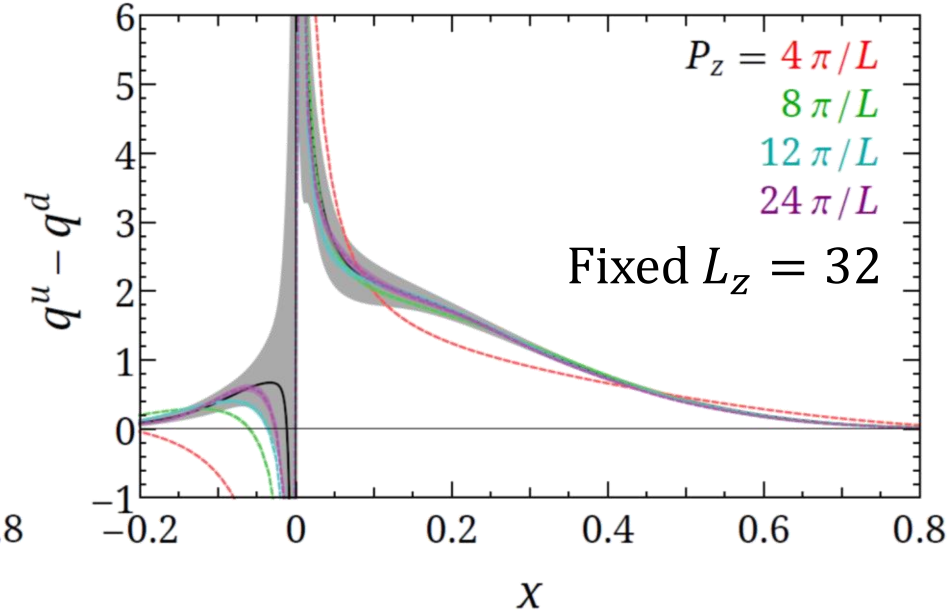
Filter approach

$$F(z_{\rm lim}, z_{\rm wid}) = \frac{1+\mathrm{erf}\left(\frac{z+z_{\rm lim}}{z_{\rm wid}}\right)}{2}\frac{1-\mathrm{erf}\left(\frac{z-z_{\rm lim}}{z_{\rm wid}}\right)}{2}$$

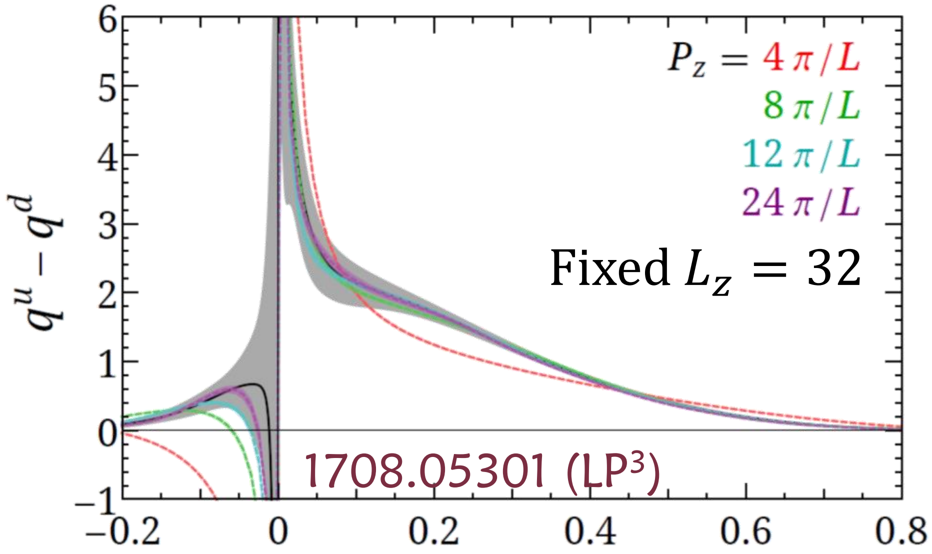


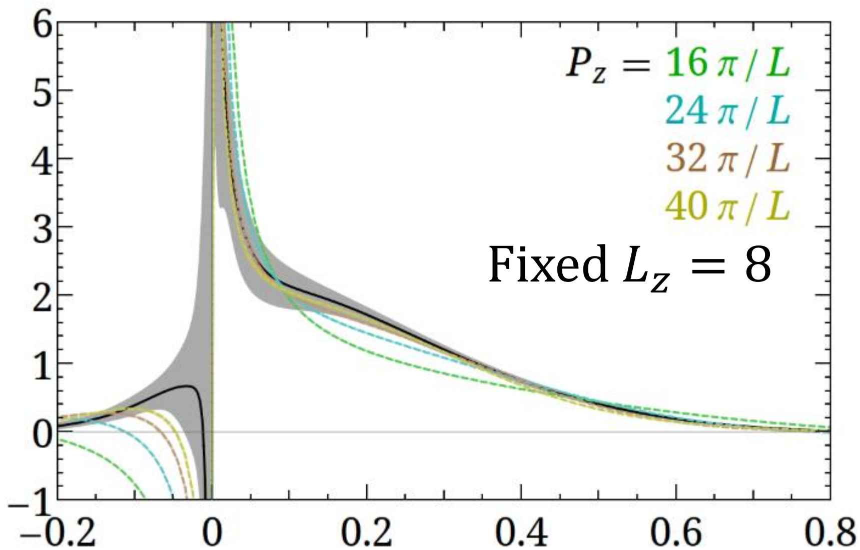
Derivative approach

$$q(x) = \int_{-z_{\rm max}}^{+z_{\rm max}} dz \frac{-P_z}{2\pi}\frac{e^{ixP_z z}}{iP_z x} h'(z)$$



Continuum



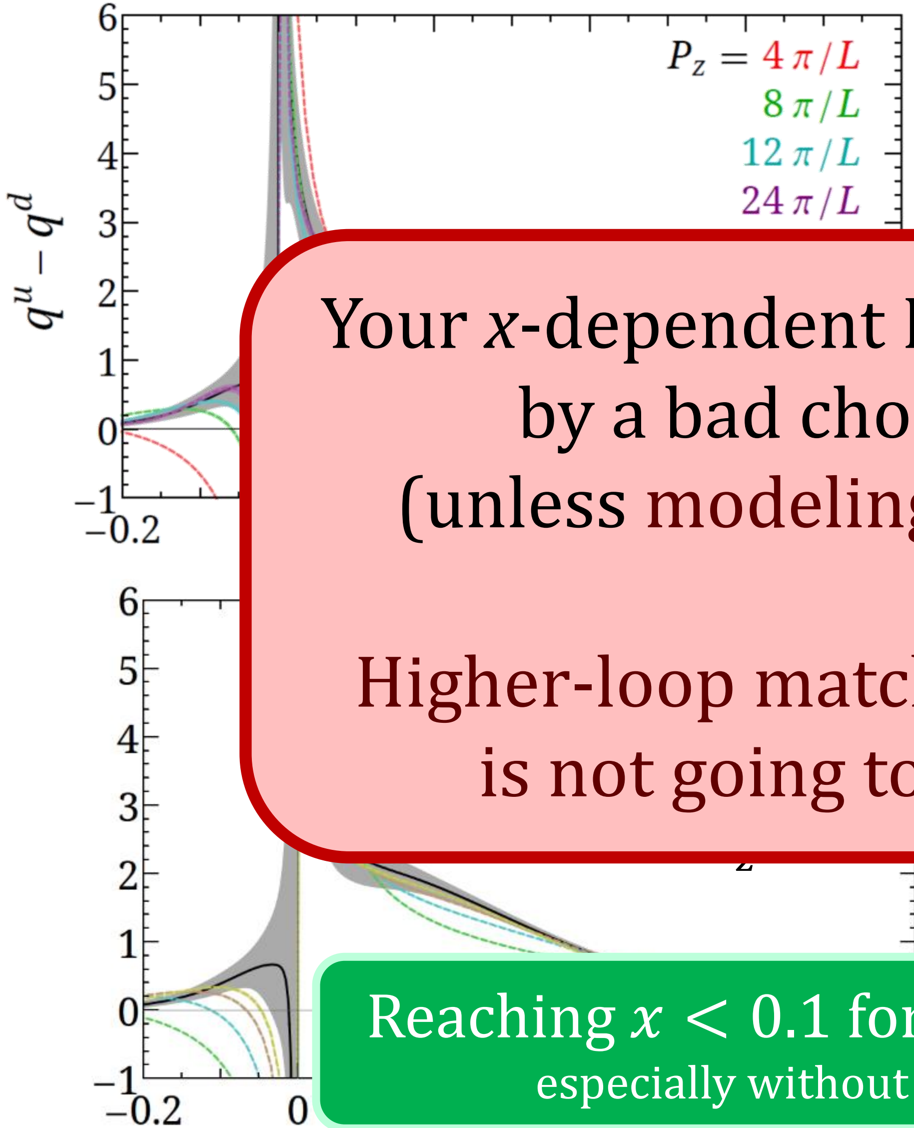


§ What lessons learned here?

- Given $L_z \approx 15$, you need large momentum to just get the sign of the antiquark correct!
- With small zP_z, you will miss over the majority of x

- Not just a quasi-PDF problem
- Going for large P_z is an unavoidable direction for any method that requires steps similar to Fourier transformation

Continuum



§ What lessons learned here?

- Given $L_z \approx 15$, you need large momentum to just get … uark … ill miss … problem … n … tion for any method that requires steps

Your x-dependent PDFs will be doomed by a bad choice of max zP_z!
(unless modeling zP_z dependence)

Higher-loop matching in LaMET later is not going to do much for it!

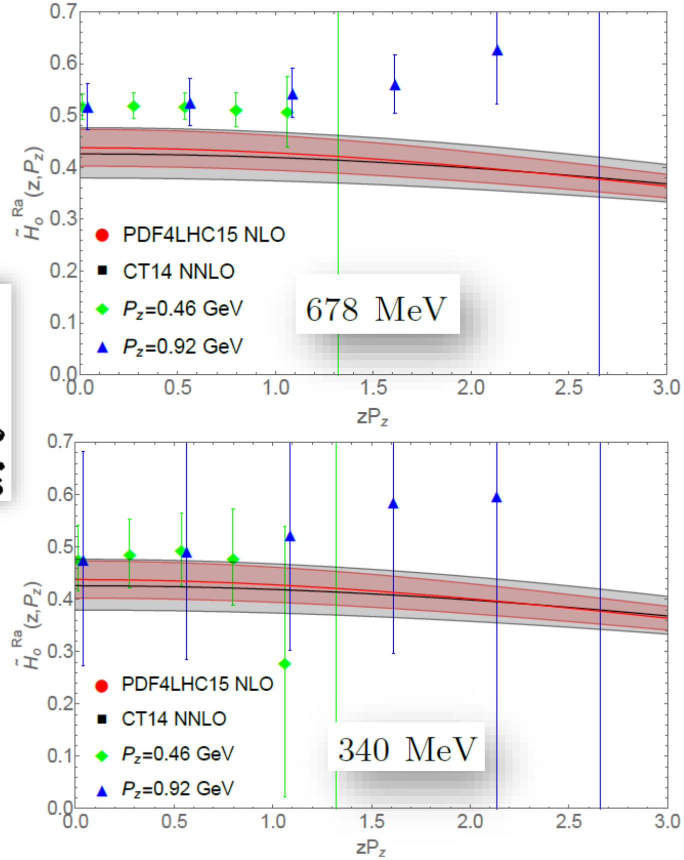
Reaching $x < 0.1$ for (anti)quark remains challenging
especially without replying on an assumed parametrization

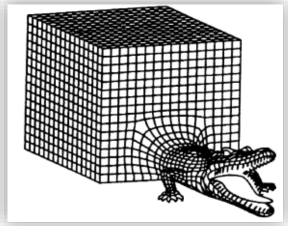
Motivation #2: Gluon PDF

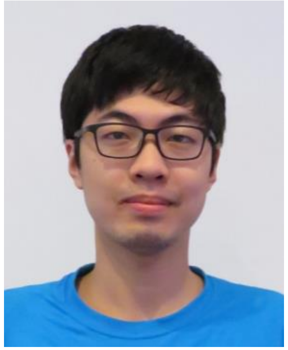
§ Pioneering first glimpse into gluon PDF using LaMET

- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Study strange/light-quark
- Promising results using coordinate-space comparison,
  but signal does not go far in z
- Hard numerical problem to be solved

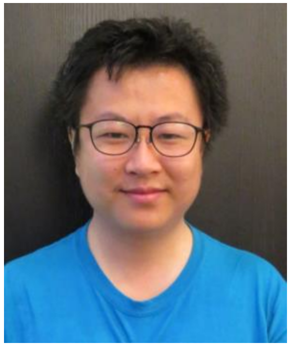
Fan. et al, Phys.Rev.Lett. 121, 242001 (2018)







Zhouyou Fan



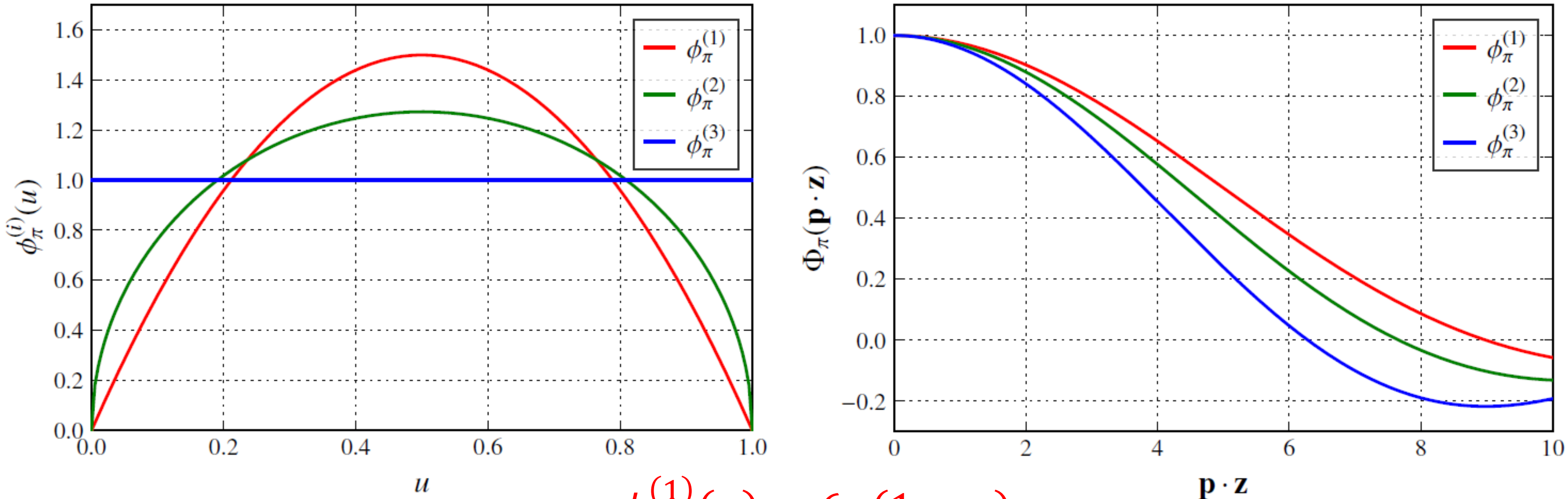
Yi-Bo Yang

Alternative Solution?

§ Fitting the coordinate-space matrix elements?

- Lose sensitivity in constraining PDF forms

1709.04325(RQCD)



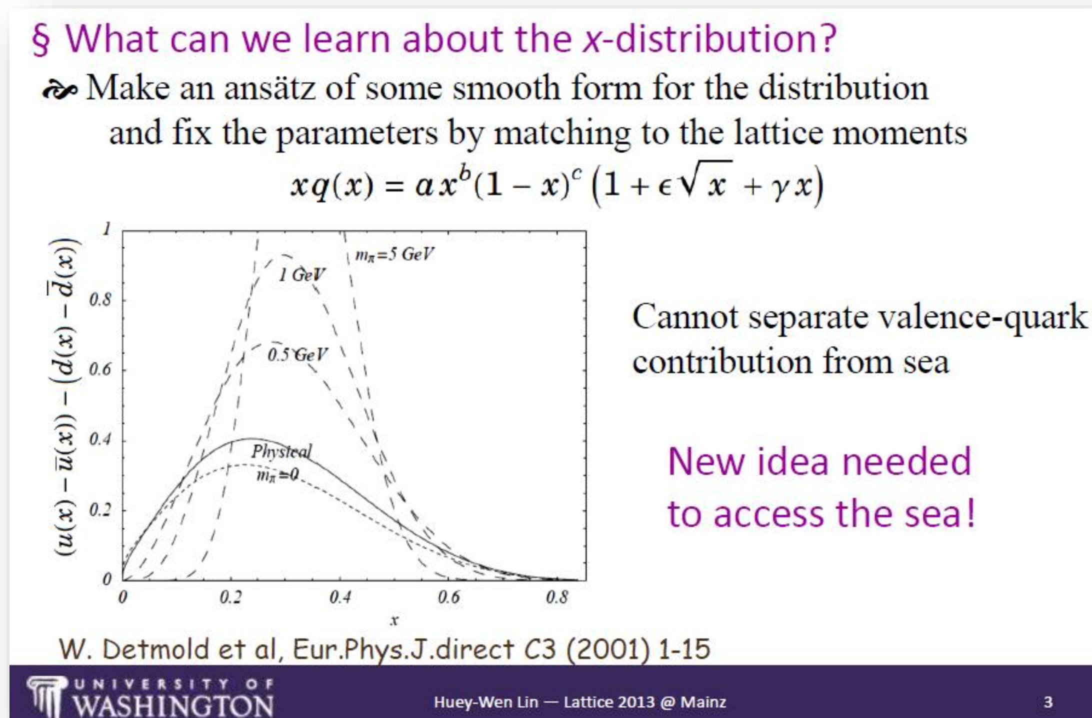
$$\phi_\pi^{(1)}(u) = 6u(1-u)$$

$$\phi_\pi^{(2)}(u) = \frac{8}{\pi}\sqrt{u(1-u)}$$

$$\phi_\pi^{(3)} = 1$$

Alternative Solution?

- Inspired by global fits?
  - No obvious advantage for direct-*x* approach from moments (*already did this in 2001!*) with 2-parameter fit
  - Nor guide the global PDF with correct *x*-forms or improve that fit-form dependence



Reaching for Large P_z

Inspired by

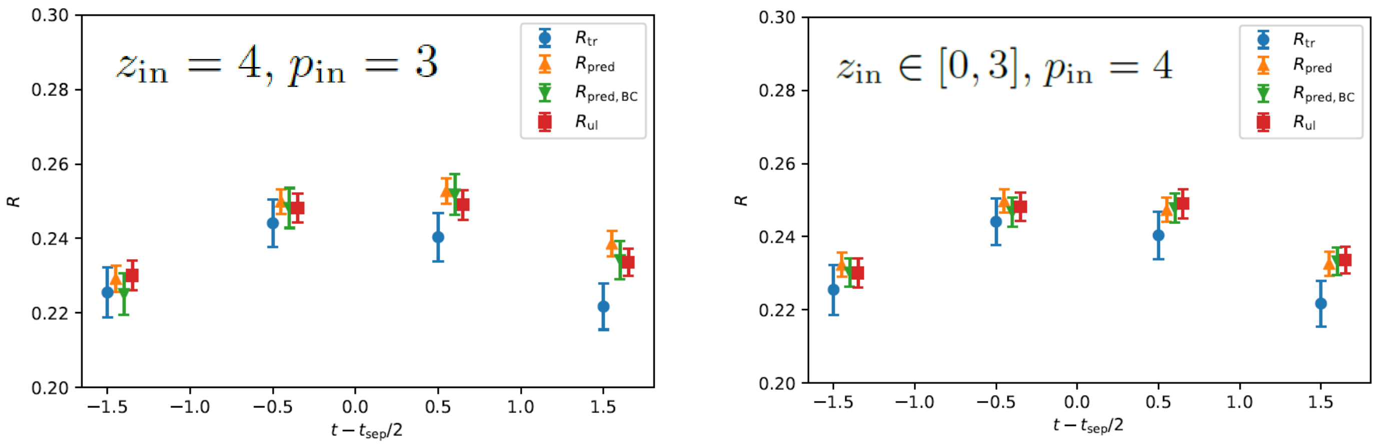
Machine learning estimators for lattice QCD observables

Boram Yoon, Tanmoy Bhattacharya, and Rajan Gupta
Phys. Rev. D **100**, 014504 – Published 9 July 2019

§ P_z and z prediction

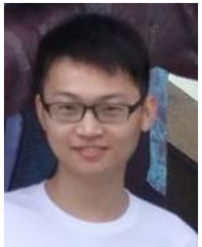
- Preliminary work using kaon PDF at 220-MeV ensemble using gradient boosting tree (See Boram's talk earlier this workshop)

$$z_{\rm pred} = 4,\ p_{\rm pred} = 4$$



$$N_{\rm tr} = N_{\rm bc} = 400 \quad N_{\rm ul} = 1160$$

Plots by Rui Zhang



Reaching for Large P_z

Are there lessons we can learn from heavy-quark physics?

§ Is anyone interested in generating $a^{-1}\approx 10$ GeV lattices?

§ Someone should try step scaling?

§ Effective large to infinite-P_z action to reach small-x PDFs?

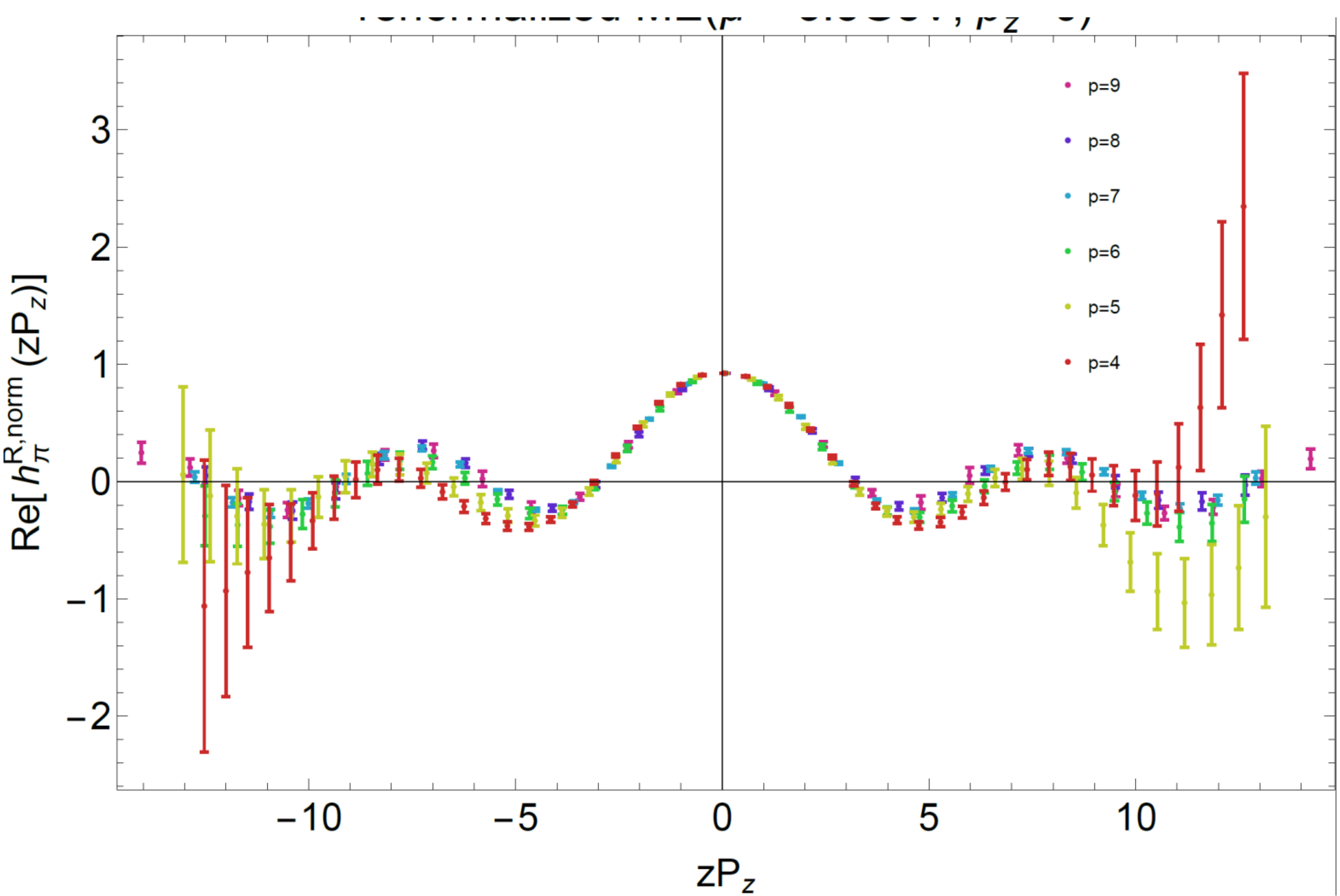
Reaching for Large P_z

Are there lessons we can learn from heavy-quark physics?

- Is anyone interested in generating a^{-1}≈10 GeV lattices?
- Someone should try step scaling?
- Effective large to infinite-P_z action to reach small-x PDFs?

- Took more than half a century of experimental data to get to where we are on PDFs
  - Lattice PDF calculations are only 5–6 years old
  - Need more young people to think ahead and in time for EIC prediction in 20 years

Backup Slides

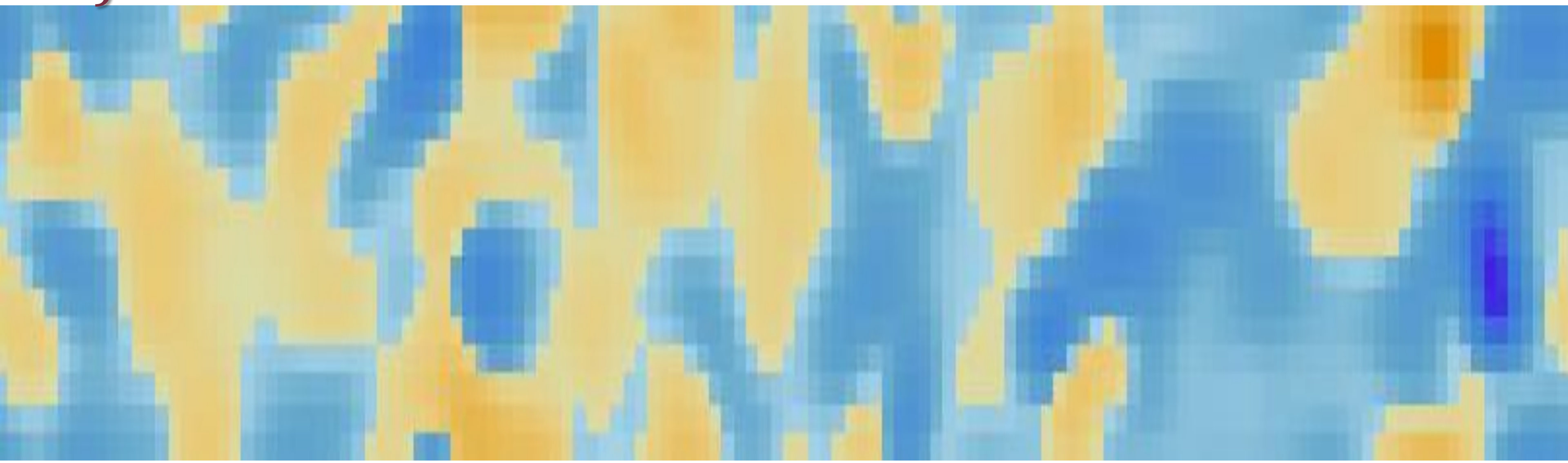

p=9
p=8
p=7
p=6
p=5
p=4
Re[$h_{\pi}^{R,norm}(zP_z)$]
zP_z
3
2
1
0
−1
−2
−10
−5
0
5
10

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice



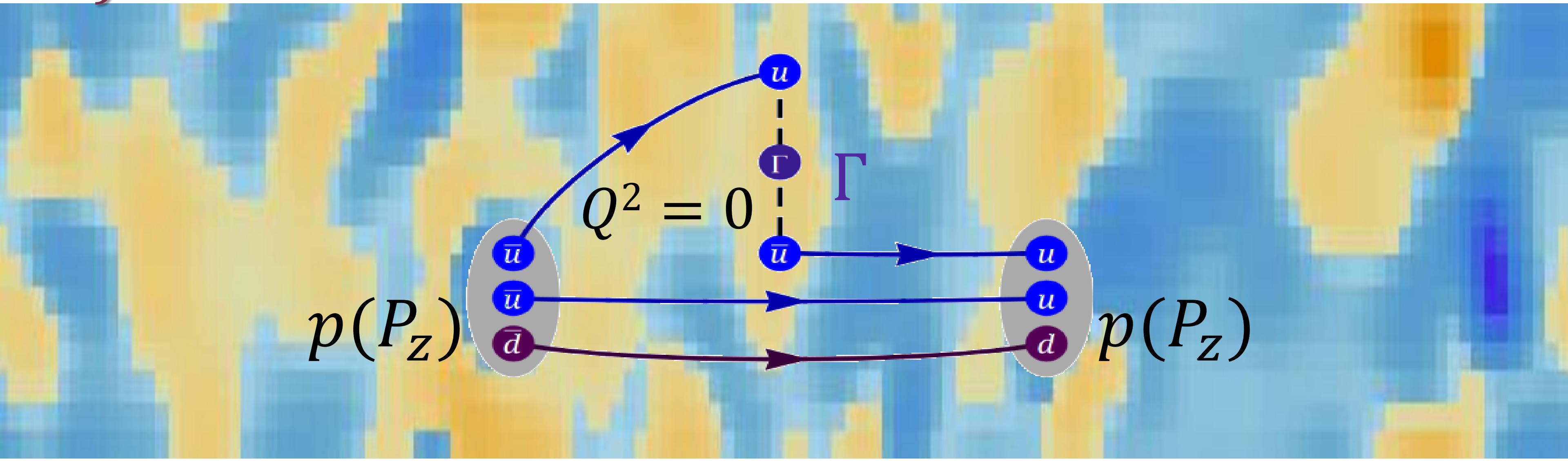
Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice



Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

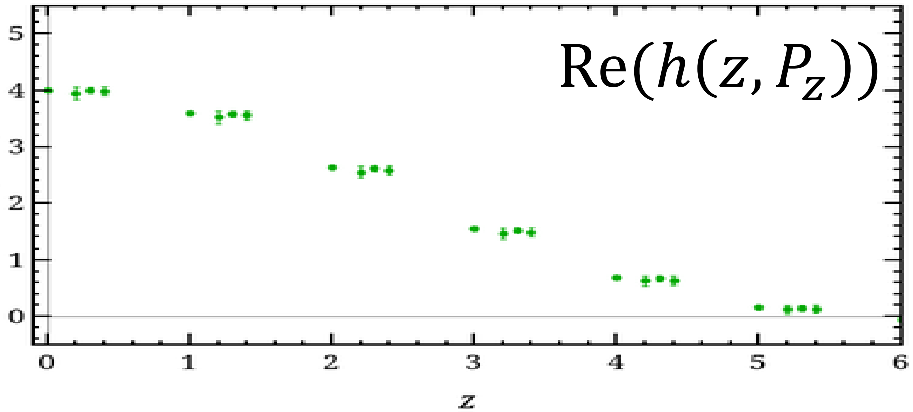
LaMET: Step-by-Step

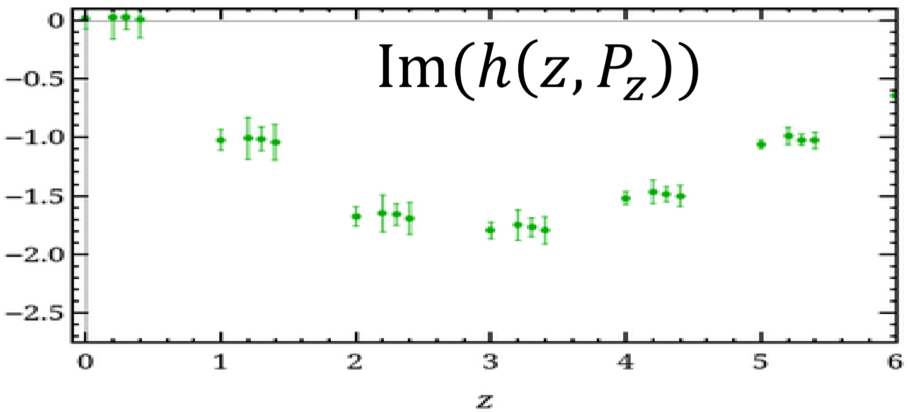
Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice

$$h(z, Pz) = \left\langle P \left| \overline{\psi}(z)\, \gamma_z \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$





$P_z =$ 2.6 GeV

$M_\pi \approx$ **135 MeV**, $a \approx 0.09$ fm

LP[3] 1804.01483

Blinded 3-state fits produced consistent results

Ruizi Li

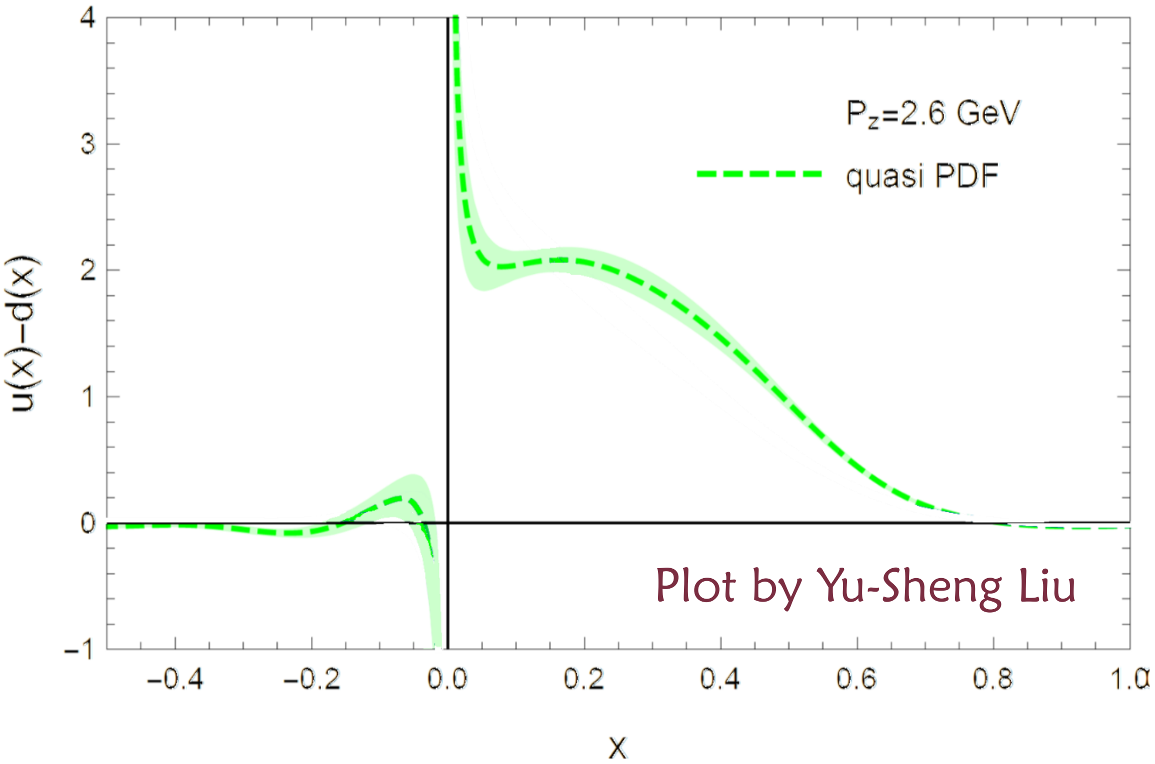
LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

2) Compute "quasi-distribution" via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-i\,x\,zP_z}\, hR(z,\mu,P_z)$$



Yu-Sheng Liu

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q}(\boldsymbol{y},\boldsymbol{\mu}) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

Finite $P_z \leftrightarrow \infty$ perturbative matching

$$Z(x,\mu/P_z) = C\delta(x-1) - \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P_z)$$

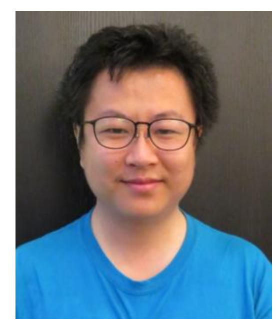
Non-singlet case only

Stewart, Zhao: 1709.04933

LP^3, 1807.06566, 1810.05043

Yong Zhao

Yu-Sheng Liu



Yi-Bo Yang

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x, \mu, P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q}(\boldsymbol{y}, \boldsymbol{\mu}) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

Dominant correction (for nucleon);
known scaling form

LP3 , 1402.1462, 1603.06664

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q}(\boldsymbol{y},\boldsymbol{\mu}) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

complicated higher-twist operator;
smaller P_z correction for nucleon
LP[3] , 1603.06664 and references within
(extrapolate it away)

§ Some similarity to more broadly-studied HQET…

$$O\left(\frac{m_b}{\Lambda}\right) = Z\left(\frac{m_b}{\Lambda},\frac{\Lambda}{\mu}\right) o(\mu) + \mathcal{O}\left(\frac{1}{m_b}\right) + \cdots$$

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

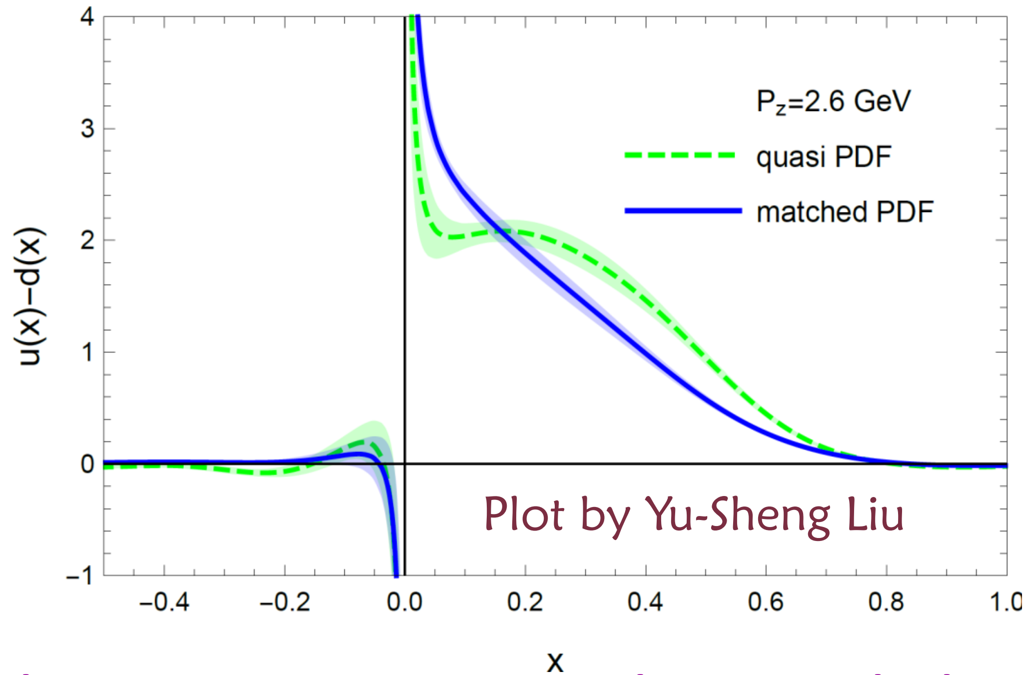
3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x, \mu, P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y, \mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2/P_z^2\right)$$

$M_\pi \approx$ **135 MeV**

$a \approx 0.09$ fm

LP[3] 1804.01483



§ Matching is a crucial step in recovering the true lightcone distribution