Nucleon σ terms

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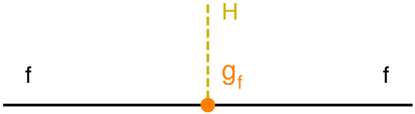
Standard model fermion masses

Elementary fermions:
mass proportional Higgs coupling

$$m_f = \sqrt{2} g_f v \quad \longrightarrow \quad m_f = g_f \frac{\partial m_f}{\partial g_f}$$

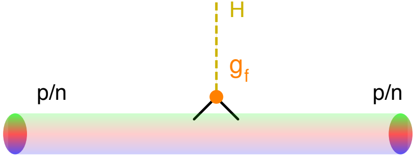


All of an elementary fermions mass comes from its Higgs coupling

$$\frac{\partial \ln m_f}{\partial \ln g_{f'}} = \delta_{ff'}$$

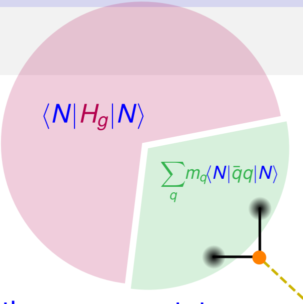
What fraction of the nucleon mass couples to the Higgs via g_f?

$$f_f^N = \frac{\partial \ln M_N}{\partial \ln g_f} = \frac{\partial \ln M_N}{\partial \ln m_f}$$



$\sum_f f_f^N \simeq \sum_q f_q^N < 1$: large part of nucleon mass from scale anomaly

Nucleon mass in QCD



The QCD Hamiltonian is given by (Ji, 1995)

$$H = \sum_q \underbrace{m_q\bar{q}q}_{\text{quark mass}} + \underbrace{H_g(U)}_{\text{anomaly}} \quad \text{with} \quad \frac{\partial H_g}{\partial m_q} = 0$$

We extract masses as energy eigenvalues above the vacuum state

$$M_N = \langle N|H|N\rangle - \langle 0|H|0\rangle$$

Varying the quark mass we thus find (Hellmann 33; Feynman 39)

$$\frac{\partial M_N}{\partial m_q} = \langle N|\frac{\partial H}{\partial m_q}|N\rangle - \langle 0|\frac{\partial H}{\partial m_q}|0\rangle = \langle N|\bar{q}q|N\rangle - \langle 0|\bar{q}q|0\rangle$$

relating nucleon mass variation to scalar quark content

Nucleon mass decomposition

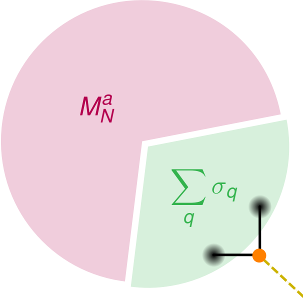
One possible nucleon mass decomposition:

$$M_N = \sum_q \sigma_q + M_N^a$$

with Higgs coupling contribution and anomaly contribution

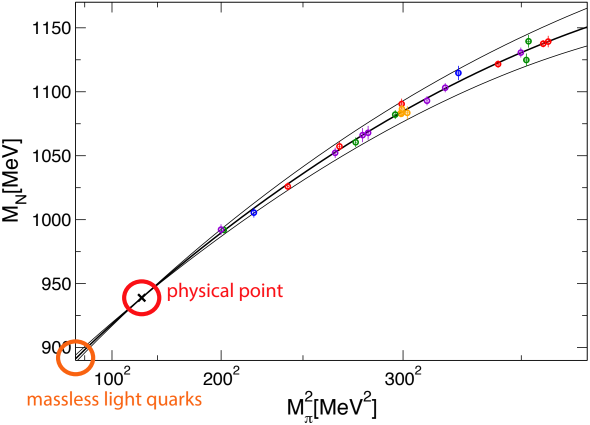
$$\sigma_q = M_N f_q^N = m_q \frac{\partial M_N}{\partial m_q} \qquad M_N^a = \langle N | H_g(U) | N \rangle$$

- ✓ Sum of positive contributions
- ✓ All contributions scale and scheme independent observables
- ✓ All contributions have clear physical meaning
- ✓ No reference to unphysical thery



Definition ambiguity

Not an unambiguous definition of m_q contribution to M_N



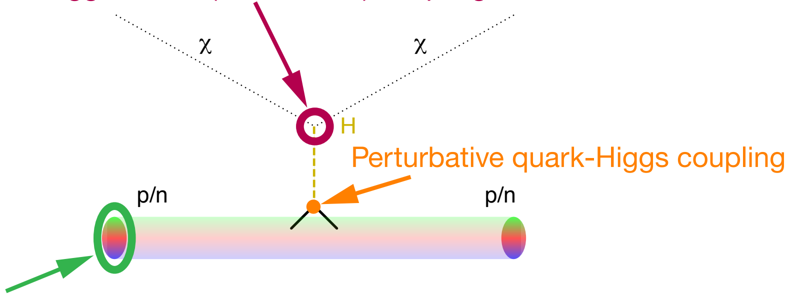
Other possibility:

- Extrapolate M_N to $m_q = 0$
- Sensible for u, d, s
- $O(m_q^2)$ different (numerically tiny!)
- Integrate out heavy quarks

Relevance for DM searches

Spin independent WIMP cross section (E.g. Higgs portal models):

Perturbative Higgs-WIMP (dark matter) coupling

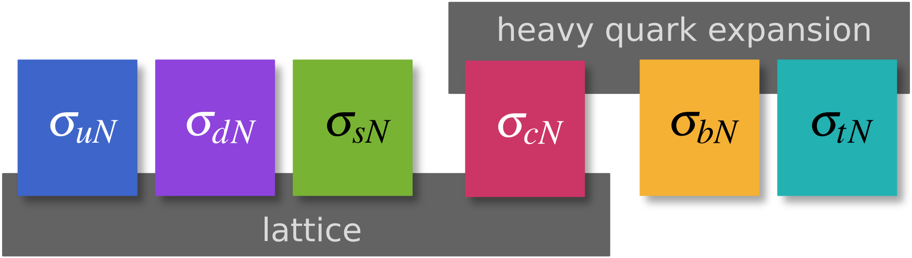


Scalar quark content from lattice QCD

$$f_N = \frac{1}{M_N} \sum_q \sigma_q$$

Relate spin independent DM coupling to nuclear recoil cross section

Strategy



- Light sigma terms σ_u^N, σ_d^N, σ_s^N: lattice (via Feynman-Hellmann)
- Heavy sigma terms σ_b^N, σ_t^N: HQET
- Charm sigma term σ_c^N: lattice and in HQET as crosscheck

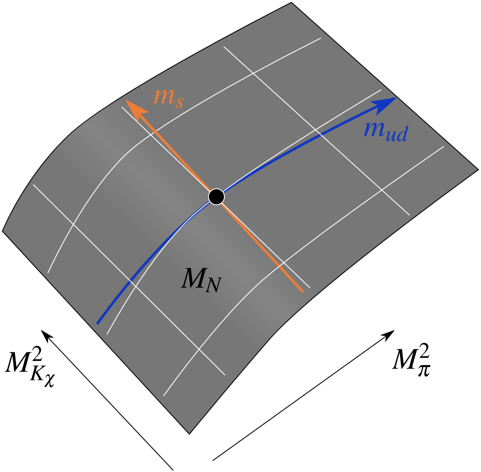
Light quark σ terms

σ_{uN} σ_{dN} σ_{sN}

Strategy:

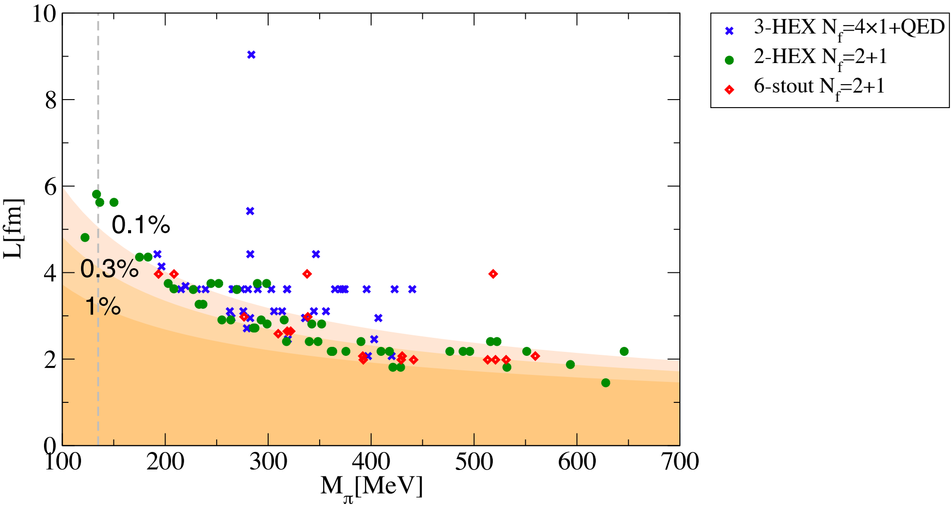
$$\frac{\partial \ln M_N}{\partial \ln m_q} = \frac{\partial \ln M_N}{\partial \ln M_P^2} \frac{\partial \ln M_P^2}{\partial \ln m_q}$$

- $\partial \ln M_P^2 / \partial \ln m_q$ with physical point staggered data
- $\partial \ln M_N / \partial \ln M_P^2$ with 3-HEX clover



Our Ensembles

σ_{uN} σ_{dN} σ_{sN}



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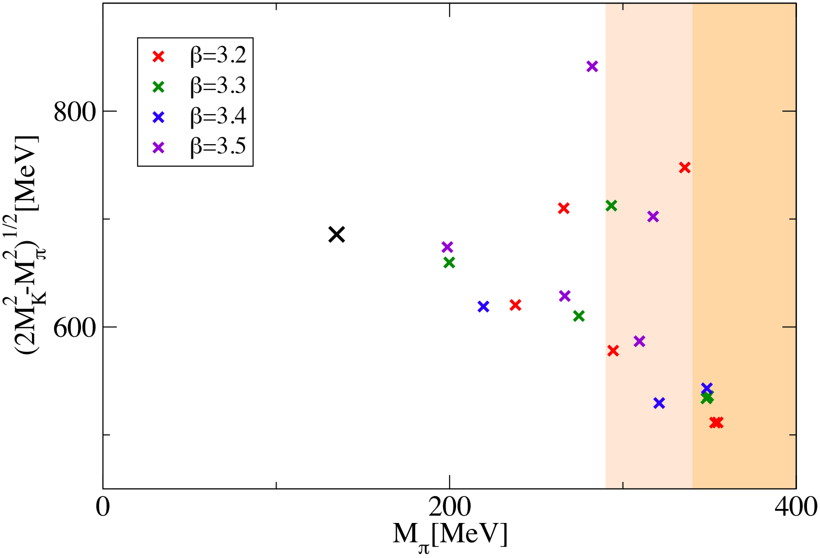
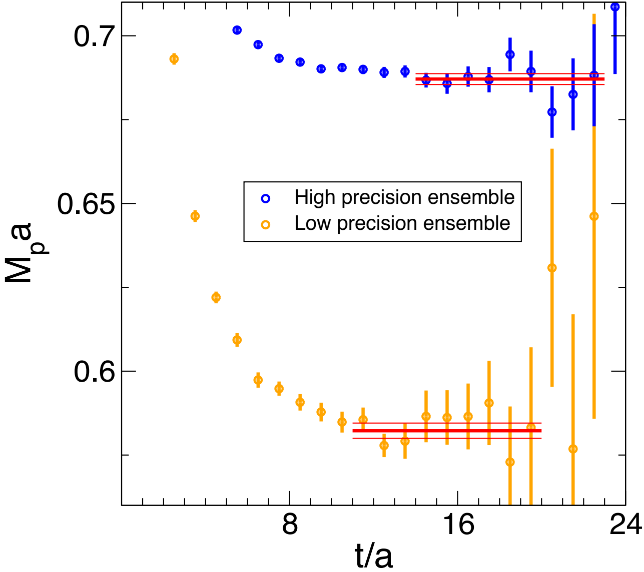
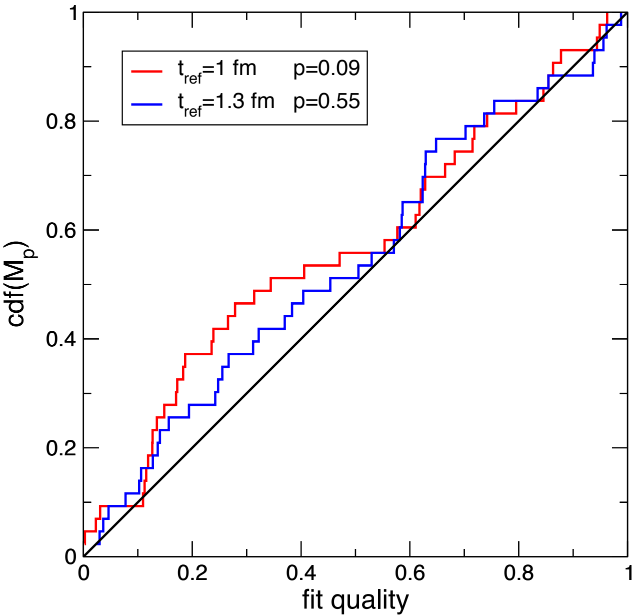


Excited state contributions

σ_{uN} σ_{dN} σ_{sN}



- Multiple fit ranges
- Per range, keep excited state error constant relative to statistical (Assume $\Delta M = 500\text{MeV}$)
- Crosschecked for consistency with excited state fits



σ_{uN} σ_{dN} σ_{sN}

- Check for random distribution of ensemble fit qualities
- KS test of quality of fit cdf
- 4 plateaux ranges in final analysis

Analysis strategy

σ_{uN} σ_{dN} σ_{sN}

Problem:

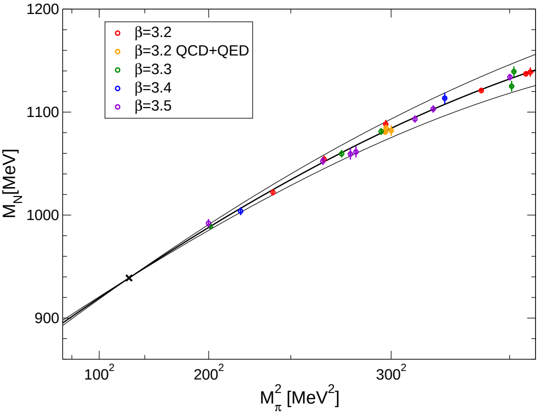
- Determine $M_P^2 = M_\pi^2, M_{K_\chi}^2 (= M_K^2 - M_\pi^2/2)$ dependence of M_N at physical point

Method:

- Fit $M_N(M_\pi, M_{K_\chi}, L, a)$
  - Added dedicated FV configs from QCD+QCD ensembles (neutral mesons and baryons extracted)
- Set scale with M_N
  - Crosscheck with M_Ω scale setting
  - No discretization terms at physical point ϕ: either αa or a^2 times $(M_\pi^2 - (M_\pi^\phi)^2)$ and $(M_{K_\chi}^2 - (M_{K_\chi}^\phi)^2)$
- Estimate systematic error

Nucleon fit

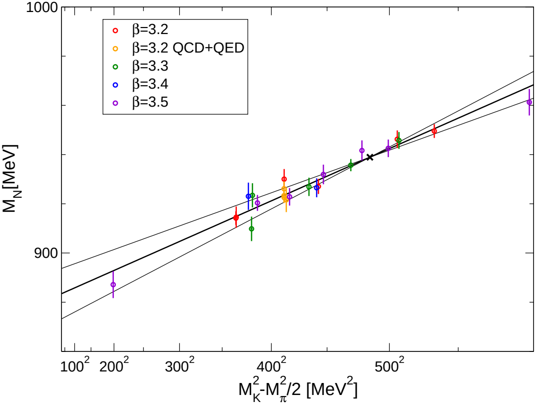
σ_{uN} σ_{dN} σ_{sN}



- $\frac{M_\pi}{\text{MeV}} < \{360, 420\}$
- Various Polynomial, Padé and χPT ansätze
- Spread into systematic error
- $M_N \propto M_0 + cM_\pi$ bad Q and wrong M_Ω

Nucleon fit

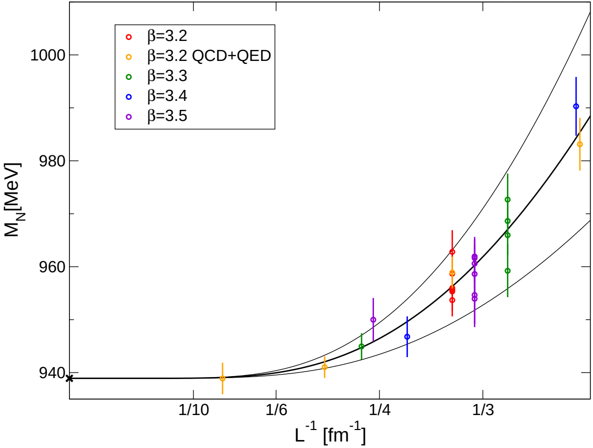
σ_{uN} σ_{dN} σ_{sN}



- $\frac{M_\pi}{\text{MeV}} < \{360, 420\}$
- Various Polynomial, Padé and χPT ansätze
- Spread into systematic error
- $M_N \propto M_0 + cM_\pi$
  bad Q and wrong M_Ω

Finite volume effects

σ_{uN} σ_{dN} σ_{sN}



- We fit leading effects $\frac{M_X(L)-M_X}{M_X} = cM_\pi^{1/2}L^{-3/2}e^{-M_\pi L}$
- Compatible with χPT expectation (Colangelo et. al., 2010)

Mixing matrix

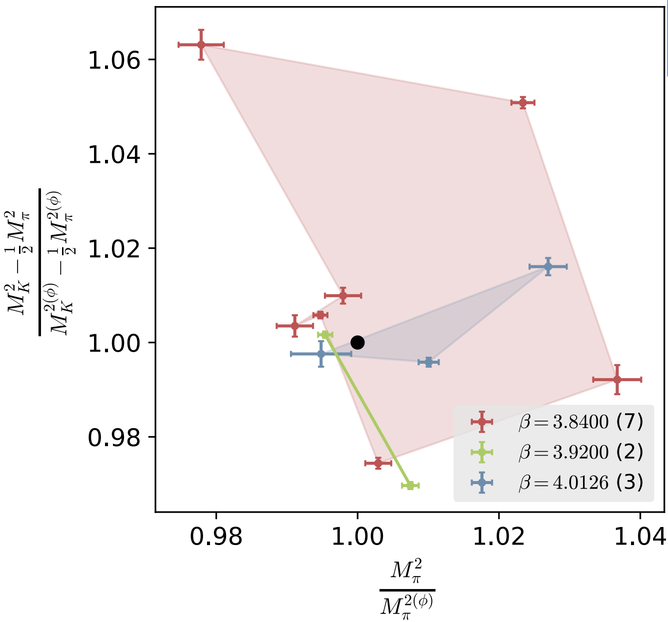
σ_{uN} σ_{dN} σ_{sN}

Transforming from mesonic to quark basis:

$$\begin{pmatrix} \sigma_{ud}^N \\ \sigma_s^N \end{pmatrix} = \underbrace{\begin{pmatrix} \left.\frac{\partial \ln M_\pi^2}{\partial \ln m_{ud}}\right|_{m_s,a} & \left.\frac{\partial \ln M_{K_\chi}^2}{\partial \ln m_{ud}}\right|_{m_s,a} \\ \left.\frac{\partial \ln M_\pi^2}{\partial \ln m_s}\right|_{m_{ud},a} & \left.\frac{\partial \ln M_{K_\chi}^2}{\partial \ln m_s}\right|_{m_{ud},a} \end{pmatrix}}_{J} \begin{pmatrix} \sigma_\pi^N \\ \sigma_{K_\chi}^N \end{pmatrix}$$

Mixing matrix J best determined with staggered ensembles:

- ✓ No additive quark mass renormalization
- ✓ Only pseudoscalar meson masses need to be extracted
- ✓ Available configs bracket physical point



σ_{uN} σ_{dN} σ_{sN}

4 stout smeared
$N_f = 2 + 1 + 1$
staggered
ensembles

Meson mass extraction

σ_{uN} σ_{dN} σ_{sN}

Extracting staggered meson masses:

- Multi-state fit
- Time-shifted propagator

Basic idea: staggered propagator for $m(T/2 - t) \ll 1$

$$c_t = e^{-mt}(c_0 + (-1)^t c_1 e^{-\Delta t})$$

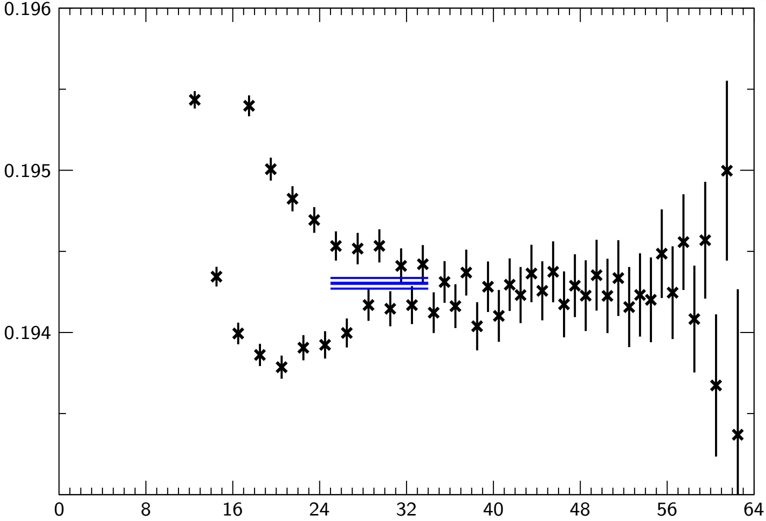
define time shifted propagator

$$d_t := c_t + e^{m+\Delta} c_{t+1}$$

Determine Δ by minimizing effective mass fluctuations

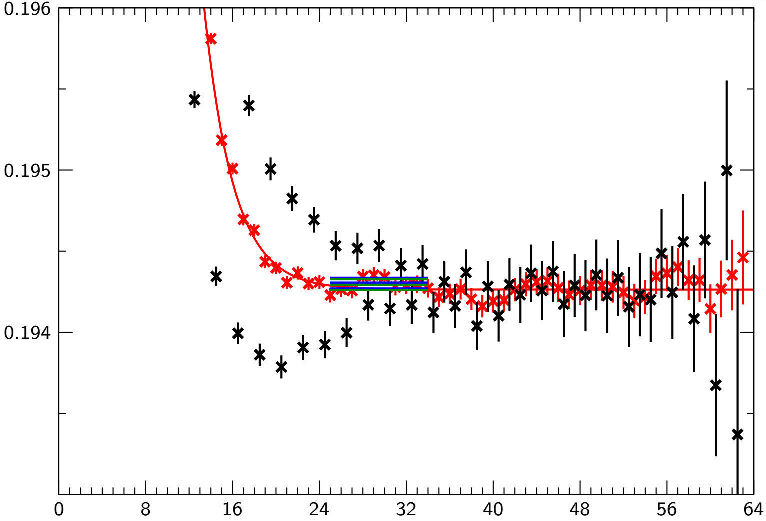
- Cross-checked with variational multi-state fit

Meson mass extraction



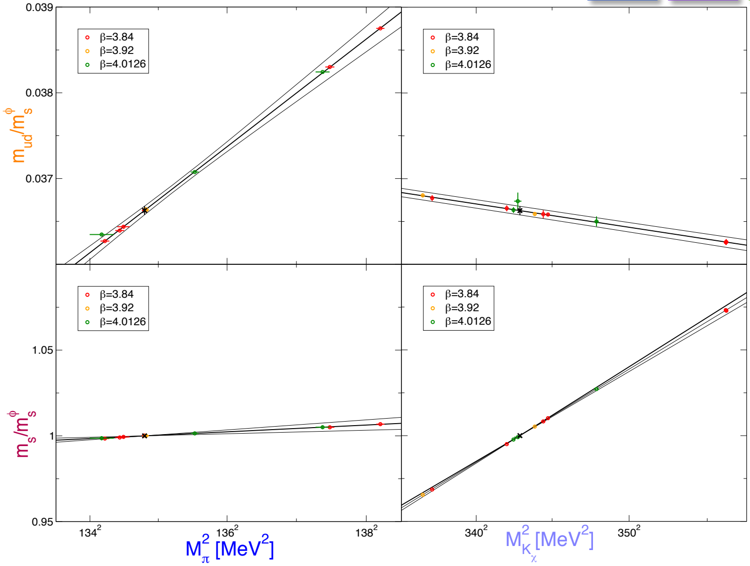
Meson mass extraction

σ_{uN} σ_{dN} σ_{sN}



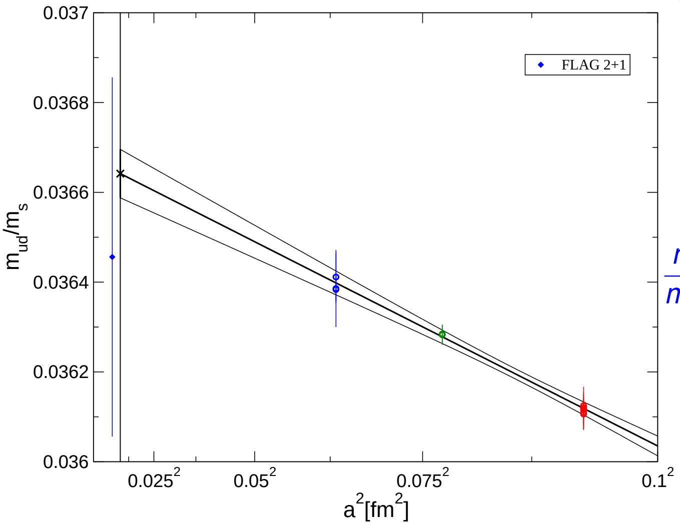
Mixing matrix *J*

σ_{uN} σ_{dN} σ_{sN}



Crosscheck: quark mass ratio

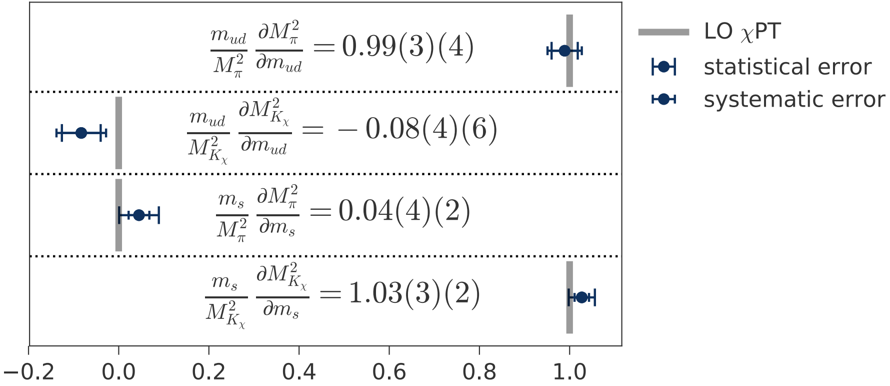
σ_{uN} σ_{dN} σ_{sN}



$$\frac{m_s}{m_{ud}} = 27.29(33)(8)$$

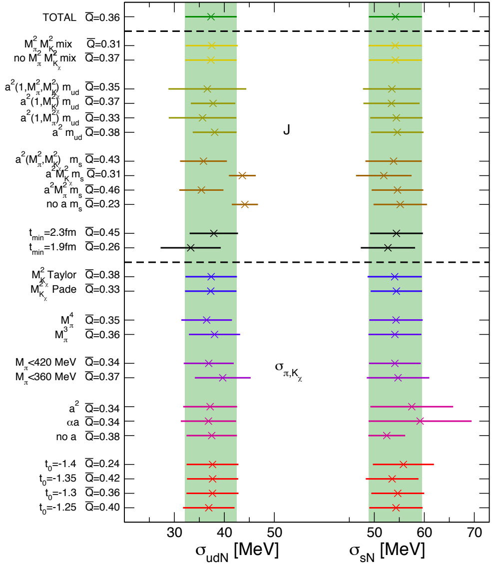
Mixing Matrix result

σ_{uN} σ_{dN} σ_{sN}



Systematic error

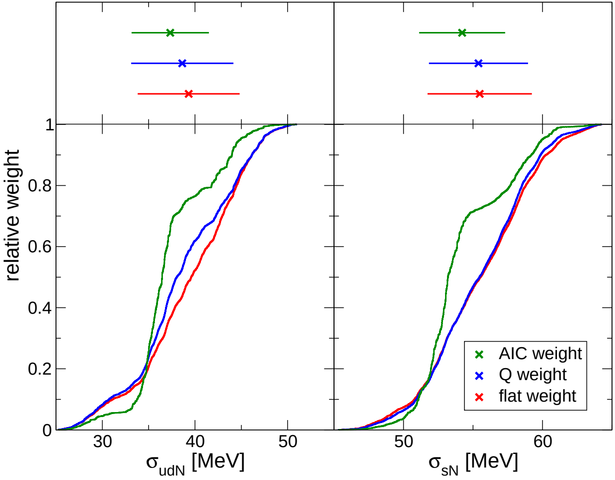
σ_{uN} σ_{dN} σ_{sN}



- Total 6144 analyses:
- 64 variations of matrix J:
  - 4 m_{ud} continuum terms
  - 4 m_s continuum terms
  - 2 plateaux ranges
- 96 variations of σ_{π,K_χ}
  - 2 M_{K_χ} fit forms
  - 2 M_π fit forms
  - 2 M_π cuts
  - 3 continuum terms
  - 4 plateaux ranges
- Other variations crosschecked: no further relevant terms found

Systematic error

σ_{uN} σ_{dN} σ_{sN}



- Total 6144 analyses
- Difference: higher order effects
- Draw cdf of results
- Different weights possible
- Crosscheck agreement

From the effective Hamiltonean

σ_uN σ_dN σ_sN

$$H = H_{\text{iso}} + \frac{\delta m}{2} \int d^3x (\bar{d}d - \bar{u}u)$$

we obtain (with $\delta m = m_d - m_u$ and normalization $\langle N|N\rangle = 2M_N$)

$$\Delta_{QCD} M_N = \frac{\delta m}{2M_p} \langle p|\bar{u}u - \bar{d}d|p\rangle$$

which, together with

$$\sigma^p_{u/d} = \left(\frac{1}{2} \mp \frac{\delta m}{4m_{ud}}\right) \sigma^p_{ud} + \left(\frac{1}{4} \mp \frac{m_{ud}}{2\delta m}\right) \frac{\delta m}{2M_p} \langle p|\bar{d}d - \bar{u}u|p\rangle$$

gives ($r = m_u/m_d$)

$$\sigma^{p/n}_u = \left(\frac{r}{1+r}\right) \sigma^N_{ud} \pm \frac{1}{2} \left(\frac{r}{1-r}\right) \Delta_{QCD} M_N + O(\delta m^2, m_{ud}\delta m)$$

$$\sigma^{p/n}_d = \left(\frac{1}{1+r}\right) \sigma^N_{ud} \mp \frac{1}{2} \left(\frac{1}{1-r}\right) \Delta_{QCD} M_N + O(\delta m^2, m_{ud}\delta m)$$

Preliminary results

σ_{uN} σ_{dN} σ_{sN}

Mesonic σ terms:

$$\sigma_\pi^N = 42.0(1.3)(1.4)\text{MeV} \qquad \sigma_{K_\chi}^N = 50.9(3.3)(2.8)\text{MeV}$$

Nucleon mass in $SU(2)$ and $SU(3)$ chiral limit:

$$M_{N\chi}^{SU(2)} = 895.7(1.4)(1.9)\text{MeV} \qquad M_{N\chi}^{SU(3)} = 848.1(3.5)(3.3)\text{MeV}$$

Quark σ terms with staggered mixing matrix:

$$\sigma_{ud}^N = 37.3(3.0)(4.2)\text{MeV} \qquad \sigma_s^N = 54.2(4.3)(3.1)\text{MeV}$$

With $\Delta_{QCD}M_N = 2.52(17)(24)\text{MeV}$ from (BMWc 2014)

$$\sigma_u^p = 13.4(1.0)(1.4)\text{MeV} \qquad \sigma_d^p = 22.7(2.1)(2.8)\text{MeV}$$

$$\sigma_u^n = 11.0(1.0)(1.4)\text{MeV} \qquad \sigma_d^n = 27.6(2.0)(2.8)\text{MeV}$$

Heavy quark contributions?

σ_{cN} σ_{bN} σ_{tN}

$M_N = \langle N|H_{N_f}|N\rangle$ from QCD N_f effective Hamiltonian at $O(\alpha_s)$

$$H_{N_f} = \sum_{q=1}^{N_f} m_q \bar{\Psi}_q \Psi_q + \frac{11 - \frac{2}{3}N_f}{2\alpha_s} G^2 + \text{regulator dep.}$$

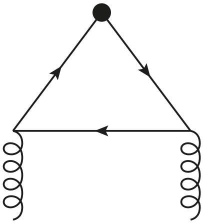
Integrating heaviest quark, from $M_N = \langle N|H_{N_f}|N\rangle = \langle N|H_{N_f-1}|N\rangle$:

$$\sigma_Q = m_Q \langle N|\bar{\Psi}_Q \Psi_Q|N\rangle = \frac{\alpha_s}{12\pi}\langle N|G^2|N\rangle + O((\Lambda/m_Q)^2)$$

$m_Q \bar{Q} Q$

Heavy quark relation (Shifman et.al. 78) to $O(\alpha_s^4)$ (Hill, Solon 17)

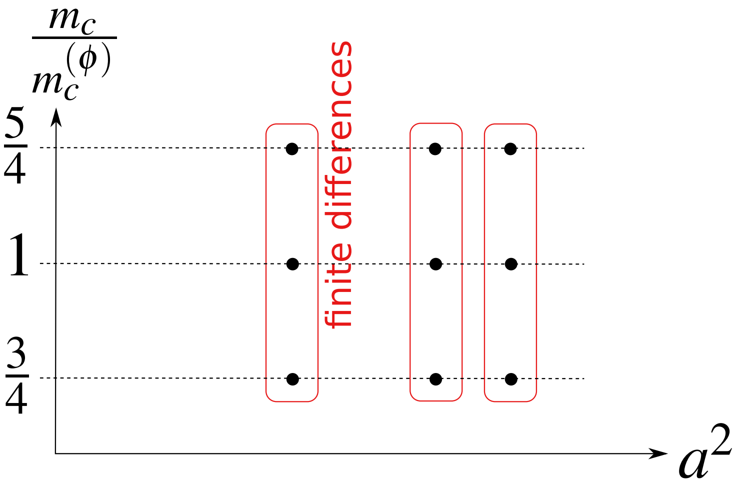
$$\sigma_Q = \frac{2M_N}{33 - 2(N_f - 1)} \left(1 - \sum_{q=1}^{N_f-1} \sigma_q\right)(1 + O(\alpha_s))$$



Charm sigma term strategy

σ_{cN}

Nucleon mass on 9 4-stout ensembles with physical m_{ud}, m_s.



Computing charm sigma term

σ_{cN}

- Finite difference approximation:

$$\Delta^+ M_N = M_N(m_c = \frac{5}{4} m_c^{(\phi)}) - M_N(m_c = m_c^{(\phi)})$$
$$\Delta^- M_N = M_N(m_c = m_c^{(\phi)}) - M_N(m_c = \frac{5}{4} m_c^{(\phi)})$$

- Either simple Taylor expansion (error $O((\delta m_c/m_c)^2) \sim 1/16$):

$$\sigma_c^N = 2(\Delta^+ M_N + \Delta^- M_N)$$

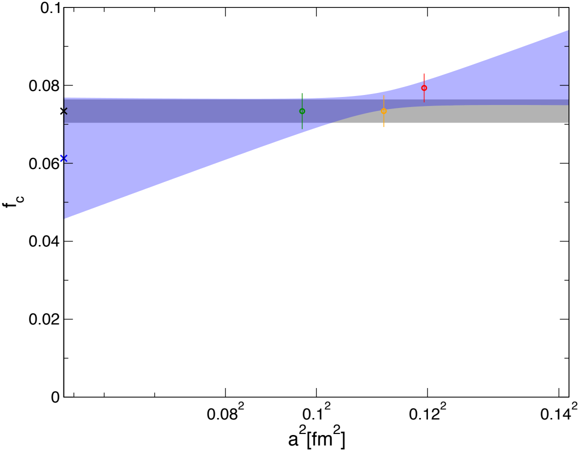
- Or HQ expansion based (error $O((\delta m_c/M_N)^3) \sim 3 \times 10^{-4}$):

$$\sigma_c^N = \frac{1}{\ln\frac{5}{4}\ln\frac{4}{3}\ln\frac{5}{3}}(\ln^2\frac{4}{3}\Delta^+ M_N + \ln^2\frac{5}{4}\Delta^- M_N)$$

Continuum extrapolation

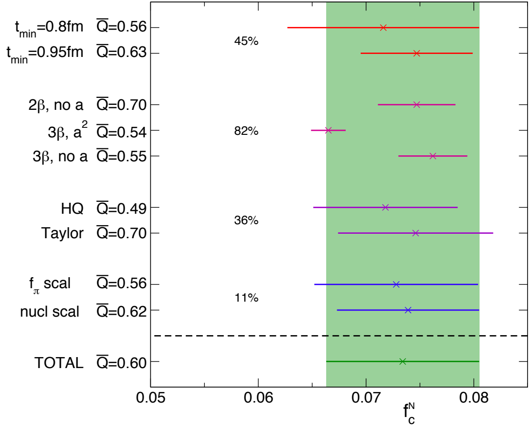
σ_{cN}

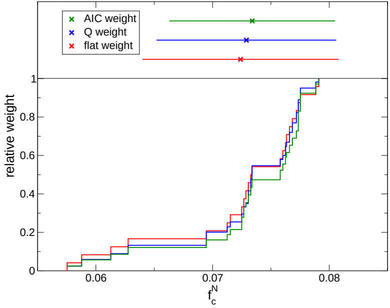
- 3 lattice spacings with $O(a^2)$ term
- 3 lattice spacings without $O(a^2)$ term
- 2 lattice spacings without $O(a^2)$ term



Systematic errors

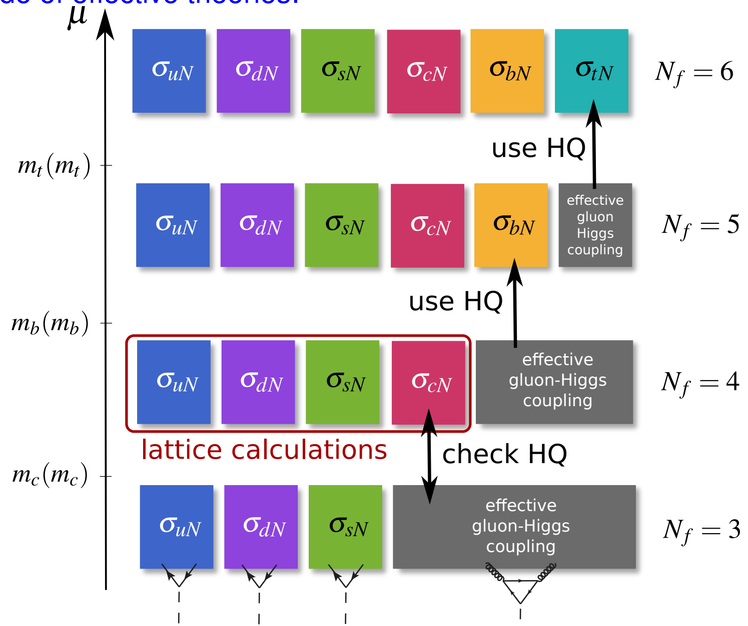
σ_{cN}



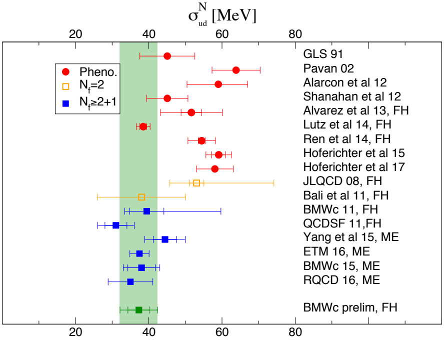


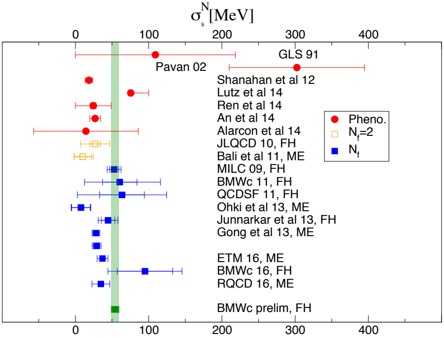
Continuum extrapolation dominates systematic error

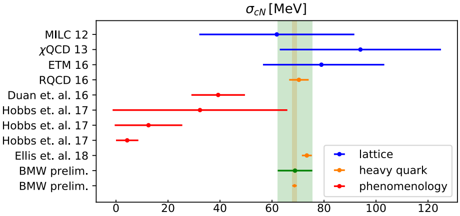
Cascade of effective theories:



Comparison







- ✓ Our lattice and HQ agree
- We take more conservative lattice result for σ_c

PRELIMINARY results