Lattice QCD calculation on $0\nu2\beta$ decays: $\pi^-\pi^- \to ee$ and $\pi^- \to \pi^+ ee$

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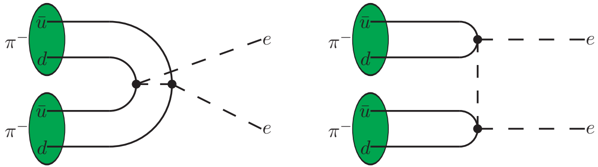
$\pi^-\pi^- \to ee$: XF, L. Jin, X. Tuo, S. Xia, PRL122, 2019

$\pi^- \to \pi^+ ee$: XF, L. Jin, X. Tuo, in preparation

$\pi^-\pi^- \to ee$: standard procedure



Construct the correlation function

$$C(t_x, t_y, t_{\pi\pi}) = \frac{1}{2!}\langle e_1 e_2|\mathcal{L}_{eff}(t_x)\,\mathcal{L}_{eff}(t_y)\,\phi_{\pi\pi}(t_{\pi\pi})|0\rangle$$

Massless neutrino propagator is implemented stochastically

Define the amplitude $\mathcal{M}(t)$ with $t = t_x - t_y$:

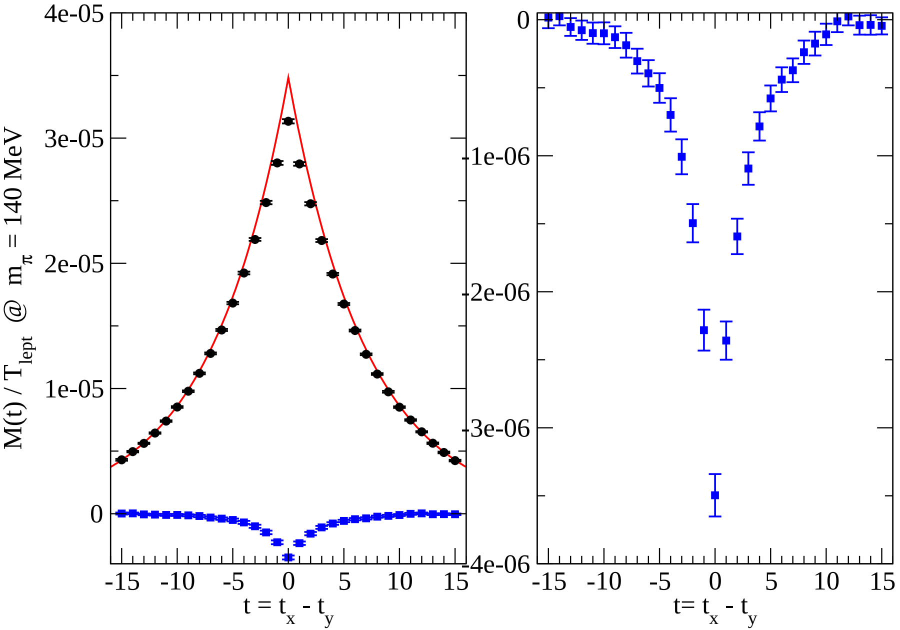
$$\mathcal{M}(t) = C(t_x, t_y, t_{\pi\pi}) \Big/ \left(V \frac{N_{\pi\pi}}{2E_{\pi\pi}} e^{E_{\pi\pi} t_{\pi\pi}}\right)$$

At large $|t|$, $\mathcal{M}(t)$ is saturated by ground intermediate state - $e\bar{\nu}\pi$

$$\mathcal{M}(t) \xrightarrow{|t|\gg 0} -T_{\mathrm{lept}} \frac{1}{V} \frac{2\langle 0|J_{\mu L}|\pi\rangle_{\mathrm{V}}\langle \pi|J_{\mu L}|\pi\pi\rangle_{\mathrm{V}}}{(2m_\pi)(2E_\nu)} e^{-m_\pi|t|}$$

Lellouch-Lüscher factor is multiplied to relate $|\pi\pi\rangle_{\mathrm{V}}$ to $|\pi\pi\rangle_\infty$

$\pi\pi \to ee$ decay amplitude @ $m_\pi = 140$ MeV

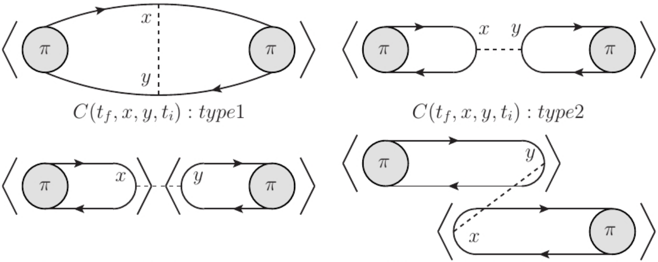


$\pi^- \to \pi^+ ee$: infinite volume reconstruction

$$\mathcal{A} = -2T_{lept} \int d^4x\, H(x) S_0(x)$$

$H(x)$ can be calculated on lattice

$$H(x-y) = V \frac{C(t_f, x, y, t_i) - C_0(t_f, x, y, t_i)}{C_\pi(t_f, ti)}$$



$C(t_f, x, y, t_i)$: *type1*

$C(t_f, x, y, t_i)$: *type2*

$C_0(t_f, x, y, t_i)$: *vaccum*

$C_0(t_f, x, y, t_i)$: 2π

Subtraction removes exp. growing contamination from vacuum state

What does subtraction terms mean?

Subtraction term contributes

$$2T_{lept}|\langle 0|J_{\mu L}|\pi\rangle|^2 \lim_{m_\nu \to 0} \underbrace{\int dt \left[e^{m_\pi |t|} + e^{-m_\pi |t|}\right] \frac{e^{m_\nu |t|}}{2m_\nu}}_{\Downarrow \ \frac{1}{2m_\nu}\left[\frac{-1}{m_\nu + m_\pi} + \frac{-1}{m_\nu - m_\pi}\right]} = 2T_{lept}F_\pi^2$$

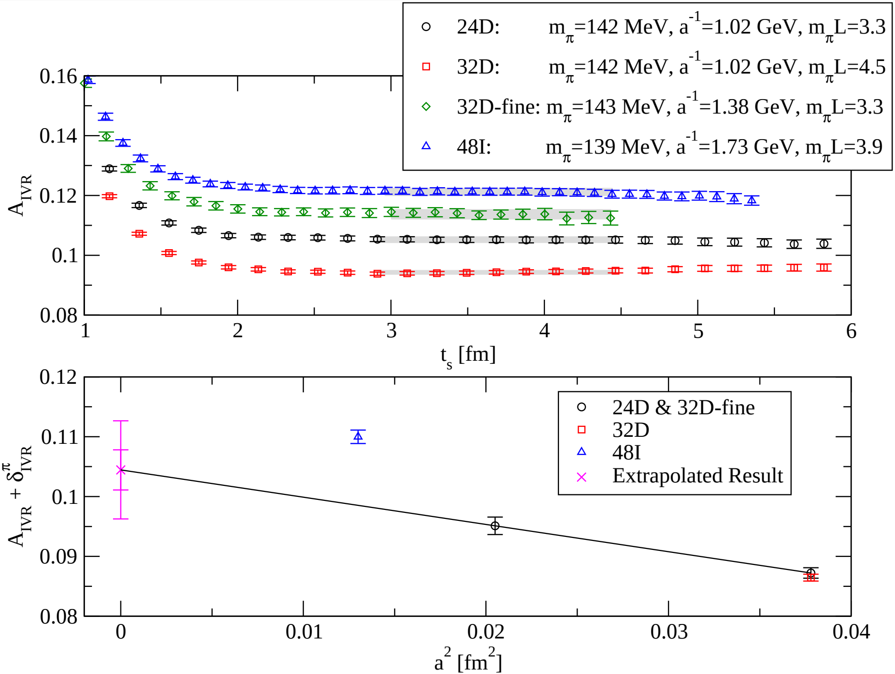
Subtract vacuum and 2π pieces simultaneously to cancel singularity at $m_\nu \to 0$

Subtraction term is equivalent to leading order term in χPT

[Cirigliano, Dekens, Mereghetti, Walker-Loud, PRC97 (2018) 065501]

$$\mathcal{A}_{\chi PT} = 2T_{lept}F_\pi^2\left[1 + \frac{m_\pi^2}{4\pi F_\pi^2}\left(6 + 3\log\frac{\mu^2}{m_\pi^2} + \frac{5}{6}g_\nu^{\pi\pi}(\mu)\right)\right]$$

Results for $\pi^- \to \pi^+ ee$ decay amplitudes



Summary of $\pi^-\pi^- \to ee$ and $\pi^- \to \pi^+ ee$

Chiral perturbation theory for $\pi^-\pi^- \to ee$

[Cirigliano, Dekens, Mereghetti, Walker-Loud, PRC97 (2018) 065501]

$$\frac{\mathcal{A}(\pi^-\pi^- \to ee)}{2F_\pi^2\, T_{\rm lept}} = 1 - \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(3\log\frac{\mu^2}{m_\pi^2} + \frac{7}{2} + \frac{\pi^2}{4} + \frac{5}{6} g_\nu^{\pi\pi}(\mu)\right)$$

Lattice calculation yields (statistical error only)

$$\frac{\mathcal{A}(\pi\pi \to ee)}{2F_\pi^2\, T_{\rm lept}} = 0.910(3) \quad \Rightarrow \quad g_\nu^{\pi\pi}(m_\rho) = -12.0(3)$$

Chiral perturbation theory for $\pi^- \to \pi^+ ee$

$$\frac{\mathcal{A}(\pi^- \to \pi^+ ee)}{2F_\pi^2\, T_{\rm lept}} = 1 + \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(3\log\frac{\mu^2}{m_\pi^2} + 6 + \frac{5}{6} g_\nu^{\pi\pi}(\mu)\right)$$

Lattice calculation yields (statistical + systematical errors)

$$\frac{\mathcal{A}(\pi^- \to \pi^+ ee)}{2F_\pi^2\, T_{\rm lept}} = 1.105(3)(7) \quad \Rightarrow \quad g_\nu^{\pi\pi}(m_\rho) = -10.9(3)(7)$$