Δm_K and Long-Distance Processes

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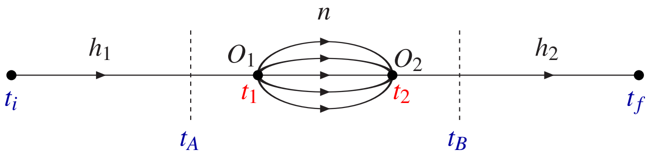


1. Fiducial volume: the integration over $t_{1,2}$ is performed in a large, but finite, interval ($t_A \leq t_{1,2} \leq t_B$). This is required to allow sufficiently large intervals $t_A - t_i$ and $t_f - t_B$ to ensure that it is indeed the hadrons $h_{1,2}$ in the initial and final states.
2. Growing exponentials: If there are intermediate states n with lower energies that those of the external states, then unphysical terms of relative size $e^{(E_{i,f}-E_n)T}$ (where $T = t_B - t_A$) are generated.
   • For kaon physics the number of such terms is small and can be handled. For heavy mesons this is much more challenging.
3. Renormalisation: New UV divergences may be generated as $x_1 \to x_2$.
   • For Δm_K and $K \to \pi \ell^+ \ell^-$ decays this doesn't happen with $N_f = 4$.
   • The additional renormalisation, necessary for ε_K and $K \to \pi \nu \bar{\nu}$ decays, has been developed and implemented. N.H.Christ, X.Feng, CTS, A.Portelli, arXiv:1605.04442
4. Finite-volume effects: Power-like FV effects can be calculated by developing an extension of the Lüscher formalism. N.H.Christ, X.Feng, G.Martinelli, CTS, arXiv:1504.01170

- Following the development of the theoretical background and exploratory numerical studies, we presented the first numerical results at physical masses at Lattice 2017 and updated them at Lattice 2018 and Lattice 2019.

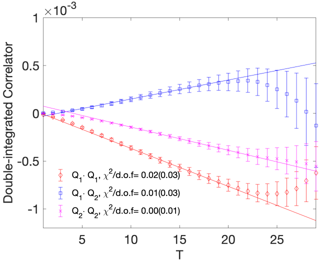
Bigeng Wang, Lattice 2019; results are still preliminary

$$C^D = N_K^2\, e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar K^0|H_W|n\rangle\,\langle n|H_W|K^0\rangle}{E_n-m_K}\left\{T+\frac{e^{-(E_n-m_K)T}-1}{E_n-m_K}\right\}$$

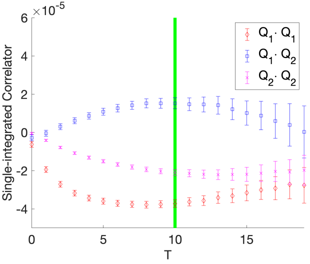
$$C^S = N_K^2\, e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar K^0|H_W|n\rangle\,\langle n|H_W|K^0\rangle}{E_n-m_K}\left\{1-e^{-(E_n-m_K)T}\right\}$$

T is the range of integration.

- The calculation is performed on a $64^3\times 128\times 12$ lattice with Möbius DWF and the Iwasaki gauge action. a^{-1}=2.359(7) GeV, $m_\pi = 135.5(2)$ MeV and $m_K = 496.5(2)$ MeV. T.Blum et al., RBC-UKQCD Collabs., arXiv:1411.7017
- Charm-physics studies $\Rightarrow am_c \simeq 0.32-0.33$. We have used $am_c \simeq 0.31$ and studied the dependence on $m_c \Rightarrow$ largest source of systematic error.
- After completion of the present analysis, the priority is to reduce these artefacts. To this end, a project is beginning on finer $(96^3\times 128, a^{-1}\simeq 2.8\,GeV)$ lattices at SUMMIT.



Double Integrated Correlator



Single Integrated Correlator

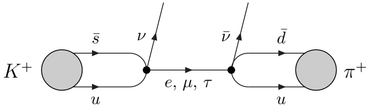
- Current preliminary results are

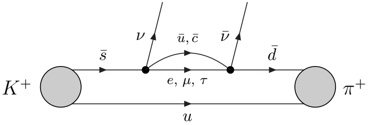
$$\Delta m_K = 7.9(1.2)_{\rm stat}(2.0)_{\rm sys} \times 10^{-12}\,\text{MeV}, \qquad \text{Double Integration}$$

$$\Delta m_K = 6.7(0.6)_{\rm stat}(2.0)_{\rm sys} \times 10^{-12}\,\text{MeV}, \qquad \text{Single Integration}$$

to be compared to the physical value $\Delta m_K^{\rm phys} = 3.483(6) \times 10^{-12}\,\text{MeV}$.

- The dominant systematic error is due to discretisation effects because $am_c \simeq 0.31$. We have estimated these to be about 25%.
- Finite-volume effects are small ($-0.22(7) \times 10^{-12}$ MeV); are included in the above.





- The Infinite-Volume Reconstruction method has a number of important applications, particularly in QED corrections to hadronic processes.

  X.Feng, L.Jin, arXiv:1812.09817; talks by X.Feng and N.Christ

- Here we apply it to the rare kaon decay $K^+ \to \pi^+ \nu\bar{\nu}$, which includes contributions from the W-W diagrams shown above (Z-exchange diagrams also contribute but are not relevant for the present discussion). N.H.Christ, X.Feng, L.Jin & CTS, in preparation
- The presence of the almost massless electron $\Rightarrow$ large FV-effects.

$$\begin{aligned} A^M_{q\ell} &= i\int d^4x\,\langle \pi^+\nu\bar{\nu}\,|\,T\left\{O^{\Delta S=1}_{q\ell}(x)\,O^{\Delta S=0}_{q\ell}(0)\right\}|\,K^+\rangle \quad (q=u,c) \\ &\equiv \int d^4x\,H_{\alpha\beta}(x)\,L^{\alpha\beta}(x) \qquad \text{(schematic)} \end{aligned}$$

$$H_{\alpha\beta}(x) = \langle \pi^+\,|\,T\left\{O_{s,\alpha}(x)\,O_{d,\beta}(0)\right\}|\,K^+\rangle,\ O_{s,\alpha} = \bar{s}\gamma_\alpha(1-\gamma_5)q,\ O_{d,\beta} = \bar{q}\gamma_\beta(1-\gamma_5)d.$$

- The challenge is to organise the calculation so that H can be computed on a lattice and L be determined "analytically" in a way which reproduces the physical amplitude, up to exponentially small FV effects.

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- Inserting complete sets of eigenstates the physical amplitude $A^M_{q\ell}$ is

$$\int d\phi_n \frac{\langle\pi^+|O_{d,\beta}(0)|n\rangle\langle n|O_{s,\alpha}(0)|K^+\rangle}{E_n+E_{\ell^+}+E_\nu-E_K-i\varepsilon}\,\hat{L}_1^{\alpha\beta}(\vec{p}_n)+\int d\phi_{n_s}\frac{\langle\pi^+|O_{s,\alpha}(0)|n_s\rangle\langle n_s|O_{d,\beta}(0)|K^+\rangle}{E_{n_s}+E_{\ell^-}+E_{\bar\nu}-E_K-i\varepsilon}\,\hat{L}_2^{\alpha\beta}(\vec{p}_{n_s})$$

- $|n_s\rangle$ is a charge-2 hadronic state $\Rightarrow$ denominator in second term does not vanish $\Rightarrow$ can drop the $i\varepsilon$ and the Minkowski $\leftrightarrow$ Euclidean connection “straightforward".
- $|n\rangle=|0\rangle$ term given by $f_{\pi,K}$ (after subtraction of exponentially growing terms).
- For hadronic $|n\rangle$, assume that for $|t|>|t_s|$, $H^{\rm had}(t,\vec{x})$ is dominated by $|\pi^0\rangle$ and divide the integral into $(t_s,0)$ and $(-\frac{T}{2},t_s)$, giving $I^{(s)}$ and $I^{(l)}$ respectively.
- $I^{(s)}$ can be calculated on a lattice with only exponentially small FV effects:

$$I^{(s)}=\int d\phi_n \frac{\langle\pi^+|O_{d,\beta}(0)|n\rangle\langle n|O_{s,\alpha}(0)|K^+\rangle}{E_n+E_{\ell^+}+E_\nu-E_K}\,\hat{L}_1^{\alpha\beta}(\vec{p}_n)\left(1-e^{-(E_n+E_{l^+}+E_\nu-E_K)|t_s|}\right)$$

- Let $\tilde{I}^{(l)}$ be the remainder; it can be written as $\tilde{I}^{(l)}=\int d^3x\, H_{\alpha\beta}(t_s,\vec{x})\tilde{L}_1^{\alpha\beta}(t_s,\vec{x})$ where

$$\tilde{L}_1^{\alpha\beta}(t_s,\vec{x})=\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\pi,p}}e^{i(\vec{p}-\vec{p}_K)\cdot\vec{x}}\,\hat{L}_1^{\alpha\beta}(\vec{p})\,\frac{e^{-(E_{\ell^+}+E_\nu)|t_s|}}{E_{\pi^0}+E_{\ell^+}+E_\nu-E_K-i\varepsilon}$$

- Thus all components can be computed with only exponentially small FV effects, including the imaginary part.

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