A dynamical mechanism for elementary particle mass generation
Towards a beyond-the-Standard-Model model

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Bibliography

- The talk is based on the papers

R. Frezzotti and G. C. Rossi
Phys. Rev. D **92** (2015) no.5, 054505

S. Capitani, P. Dimopoulos, R. Frezzotti, M. Garofalo, B. Kostrzewa, F. Pittler, G. C. Rossi and C. Urbach
PRL 123 (2019) 061802

- Preliminary simulation results can be found in

S. Capitani *et al.*
EPJ Web Conf. **175** (2018) 08009
S. Capitani *et al.*
EPJ Web Conf. **175** (2018) 08008

- See also

R. Frezzotti, M. Garofalo and G. C. Rossi
Phys. Rev. D **93** (2016) no.10, 105030

Outline of the talk

1. Motivation & Introduction
   - SM and its limitations
2. A toy-model
   - endowed with “naturally” light NP fermion masses
3. Lattice formulation
   - numerical simulations & results
4. Dynamical fermion mass generation
   - unconventional alternative to the Higgs mechanism
   - mass hierarchy made natural
5. Introducing electroweak interactions
   - the W mass
   - understanding the electroweak scale
   - the need for a superstrongly interacting (Tera-)sector
6. Towards a beyond-the-Standard-Model-model
   - adding leptons and hypercharge
7. A bit of phenomenology
   - the 125 GeV resonance: a WW/ZZ bound state?
   - comparing with the SM
8. Comments, conclusions & outlook

Part 0
Take home message

Take home message

- We identify & numerically confirm the existence of a field-theoretical dynamical mechanism for elementary particle mass generation - giving $m \sim c(g^2)\Lambda_{RGI}$ - in models where chirality is broken at the UV cutoff by some irrelevant operator
- NP-ly generated masses display peculiar gauge coupling dependence allowing
  - an understanding of the EW scale & mass hierarchy
  - leading to "predict" the existence of a super-strongly interacting sector with $\Lambda_{RGI} \sim$ a few TeV's (if the top-quark has to have its physical mass)
- A "natural" bSMm can be envisaged/constructed
  - incorporating all the above features
  - and including EW interactions
  - that seems to be able to offer a solution of some of the SM problems
  - and lead to gauge coupling unification (without SUSY)

Open issues

- Lack of well established, reliable analytical tools to deal with NP effects
- No detailed phenomenological study of the proposed bSMm
- At the moment we have little understanding on the
  - origin of weak-isospin splitting
  - differences among generations

Part I
Motivation & Introduction

Motivation & Introduction

- Discovery of a 125 GeV resonance: spectacular confirmation of SM
- SM can't be the ultimate theory of fundamental interactions ...
- ... even ignoring gravitation and the lack of dark matter candidates

1) it does not explain dominance of matter over anti-matter
2) it gives no clue of why we have three fermion families
3) electroweak and strong interactions are not really “unified”
4) the Higgs mechanism trades masses for Yukawa couplings
5) mass hierarchy $m_e \sim 3 \cdot 10^{-6} m_{top}$ (m_ν even smaller) is unexplained
6) EW scale is “unnatural”
7) it has O(20) parameters (excluding the neutrino sector)

- The key issue is the origin of elementary particle masses

Part II
A Toy Model

Theoretical background - I

Consider a model – where an SU(2) fermion doublet, subjected to non-abelian gauge interactions (of the QCD type), is coupled to a complex scalar doublet via a $d = 4$ Yukawa and an "irrelevant" $d = 6$ Wilson-like chiral breaking terms – described by the Lagrangian

$$\mathcal{L}_{\rm toy}(q, A, \Phi) = \mathcal{L}_{kin}(q, A, \Phi) + \mathcal{V}(\Phi) + \mathcal{L}_{Yuk}(q, \Phi) + \mathcal{L}_{Wil}(q, A, \Phi)$$

$$\mathcal{L}_{kin}(q, A, \Phi) = \frac{1}{4}(F \cdot F) + \bar{q}_L \not{D} q_L + \bar{q}_R \not{D} q_R + \frac{1}{2}\mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right]$$

$$\mathcal{V}(\Phi) = \frac{\mu_0^2}{2}\mathrm{Tr}\left[\Phi^\dagger \Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{Tr}\left[\Phi^\dagger \Phi\right]\right)^2$$

$$\mathcal{L}_{Yuk}(q, \Phi) = \eta\left(\bar{q}_L \Phi q_R + \bar{q}_R \Phi^\dagger q_L\right)$$

$$\mathcal{L}_{Wil}(q, A, \Phi) = \frac{b^2}{2}\rho\left(\bar{q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu q_R + \bar{q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \mathcal{D}_\mu q_L\right)$$

- $\Phi = \varphi_0 1\!\!1 + i\varphi_j \tau^j = [-i\tau_2 \varphi^\star | \varphi]$ with $\varphi = (\varphi_2 - i\varphi_1, \varphi_0 - i\varphi_3)^T$
- $b^{-1} \sim \Lambda_{UV} =$ UV cutoff, $\eta =$ Yukawa coupling
- ρ constrained if EW interactions are introduced (see below)

Theoretical background - II

- $\mathcal{L}_{\rm toy}$ is formally power-counting renormalizable
- and exactly invariant under the (global) transformations

$$\chi_L \times \chi_R = [\tilde\chi_L \times (\Phi \to \Omega_L \Phi)] \times [\tilde\chi_R \times (\Phi \to \Phi \Omega_R^\dagger)]$$

$$\tilde\chi_{L/R} : \begin{cases} q_{L/R} \to \Omega_{L/R} q_{L/R} \\ \bar q_{L/R} \to \bar q_{L/R} \Omega_{L/R}^\dagger \end{cases} \qquad \Omega_{L/R} \in \mathrm{SU}(2)$$

- standard (possibly linearly divergent) masses forbidden because the operator $\bar q_L q_R + \bar q_R q_L$ is not invariant under $\chi_L \times \chi_R$
- Interplay of $\chi_L \times \chi_R$ and S$\tilde\chi_L \times \tilde\chi_R$SB solves naturalness problem
  - NP mass-like terms invariant under $\chi_L \times \chi_R$ dynamically generated
  - masses kept small by $\tilde\chi_L \times \tilde\chi_R$ symmetry enhancement ['t Hooft]

Theoretical background - III

- $\mathcal{L}_{\rm toy}$ is not invariant under the purely fermionic $\tilde{\chi}_R \times \tilde{\chi}_L$ chiral transformations because of the presence of $\mathcal{L}_{Yuk}$ and $\mathcal{L}_{Wil}$
- There is a critical value of η where the effective Yukawa term vanishes - up to $O(b^2)$ - and $\tilde{\chi}_R \times \tilde{\chi}_L$ transformations become symmetries of $\mathcal{L}_{\rm toy}$ (barring NP obstructions) Frezzotti & Rossi, 2015
- For generic η one gets the renormalized SDE Bochicchio *et al.*,1985

$$\partial_\mu \langle Z_{\tilde{J}} \widetilde{J}^{L\,i}_\mu(x)\, \hat{O}(0)\rangle = \qquad \text{(similarly for } \widetilde{J}^{R\,i}_\mu) \tag{1}$$

$$= [\eta - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0)] \langle (\bar{q}_L \frac{\tau^i}{2} \Phi q_R - \bar{q}_R \Phi^\dagger \frac{\tau^i}{2} q_L)(x) \hat{O}(0)\rangle + O(b^2) + \dots$$

- Chiral ($\widetilde{J}^{L\,i}, \widetilde{J}^{R\,i}$) currents are conserved - up to $O(b^2)$ - if

$$\eta - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0) = 0 \quad \to \quad \eta_{cr} = O(g_s^2)$$

- In the critical theory, i.e. at $\eta = \eta_{cr}$
  - $\mathcal{L}_{Yuk}$ and $\mathcal{L}_{Wil}$ “compensate”
  - much like at m_{cr}, mass and Wilson term do in Wilson lattice QCD
- Physics depends on whether $\mathcal{L}_{\rm toy}$ lives in Wigner or NG phase

Proof of (1)

- Bare $\tilde{\chi}_L \times \tilde{\chi}_R$ SDEs read [Bochicchio *et al.* 1985]
  - $\partial_\mu \langle \tilde{J}^{L,i}_\mu(x)\, \hat{O}(0)\rangle = \langle \tilde{\Delta}^i_L \hat{O}(0)\rangle \delta(x) - \eta\, \langle O^{L,i}_{Yuk}(x)\, \hat{O}(0)\rangle - b^2 \langle \underline{O^{L,i}_{Wil}}(x)\, \hat{O}(0)\rangle$
    - $\tilde{J}^{L,i}_\mu = \bar{q}_L \gamma_\mu \frac{\tau^i}{2} q_L - \frac{b^2}{2}\rho\Big(\bar{q}_L \frac{\tau^i}{2}\Phi \mathcal{D}_\mu q_R - \bar{q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} q_L\Big)$
    - $O^{L,i}_{Yuk} = \Big[\bar{q}_L \frac{\tau^i}{2}\Phi q_R - \text{h.c.}\Big]$ • $\underline{O^{L,i}_{Wil}} = \frac{\rho}{2}\Big[\bar{q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi \mathcal{D}_\mu q_R - \text{h.c.}\Big]$
- Mixing & Renormalization
- $b^2 \underline{O^{L,i}_{Wil}} = (Z_{\tilde{J}} - 1)\partial_\mu \tilde{J}^{L,i}_\mu - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0)\, O^{L,i}_{Yuk} + \ldots + \mathrm{O}(b^2)$
- $\partial_\mu \langle Z_{\tilde{J}} \tilde{J}^{L,i}_\mu(x)\, \hat{O}(0)\rangle = \langle \tilde{\Delta}^i_L \hat{O}(0)\rangle \delta(x) - (\eta - \bar{\eta}(\eta))\, \langle O^{L,i}_{Yuk}(x)\, \hat{O}(0)\rangle + \ldots + \mathrm{O}(b^2)$
- Critical theory $\to \eta - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0) = 0 \implies \eta_{cr}(g_s^2, \rho, \lambda_0) = \mathrm{O}(g_s^2)$
  - $\partial_\mu \langle Z_{\tilde{J}} \tilde{J}^{L,i}_\mu(x)\, \hat{O}(0)\rangle_{\eta_{cr}} = \langle \tilde{\Delta}^i_L \hat{O}(0)\rangle_{\eta_{cr}} \delta(x) + \mathrm{O}(b^2) + \ldots$
- All the same with $[L \leftrightarrow R \;\&\; \Phi \leftrightarrow \Phi^\dagger]$

Wigner phase

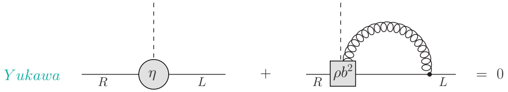
- Wigner phase: $\hat{\mu}_\Phi^2 > 0$

  - $\mathcal{V}(\Phi)$ has a single minimum at the origin with $\langle \Phi^\dagger \Phi \rangle = 0$
- No spontaneous $\chi_L \times \chi_R$ symmetry breaking
- $d=4$ EL (as determined by symmetries) takes the obvious form

$$\Gamma_4^{Wig}\Big|_{\hat{\mu}_\Phi^2>0} = \frac{1}{4}(F \cdot F) + \bar{q}_L \mathcal{D}\!\!\!/\, q_L + \bar{q}_R \mathcal{D}\!\!\!/\, q_R + \frac{1}{2}\mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] +$$

$$+[\eta - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0)]\left(\bar{q}_L \Phi q_R + \bar{q}_R \Phi^\dagger q_L\right) + \frac{\hat{\mu}_\phi^2}{2}\mathrm{Tr}\left[\Phi^\dagger \Phi\right] + \frac{\hat{\lambda}}{4}\left(\mathrm{Tr}\left[\Phi^\dagger \Phi\right]\right)^2$$

- Diagrammatically the criticality condition ($\eta = \bar{\eta}$) means



  - explicit b^2 factor compensated by the loop quadratic divergency
  - we are effectively killing $\tilde{\chi}$ breaking effective Yukawa vertex
  - no seed for S$\tilde{\chi}$SB
- How do we enforce this and compute η_{cr}? $\rightarrow$ Go to the lattice

Part III
Lattice simulations

Lattice formulation

$$S^{lat}_{\rm toy} = b^4 \sum_x \Big\{ \mathcal{L}^{YM}_{kin}[U] + \mathcal{L}^{scal}_{kin}(\Phi) + \mathcal{V}(\Phi) + \bar{q} D_{lat}[U;\Phi] q \Big\}$$

$\mathcal{L}^{YM}_{kin}[U]$: SU(3) plaquette action

$$\mathcal{L}^{scal}_{kin}(\Phi) + \mathcal{V}(\Phi) = \frac{1}{2}\mathrm{Tr}\,[\Phi^\dagger(-\partial^*_\mu \partial_\mu)\Phi] + \frac{m_0^2}{2}\mathrm{Tr}\,[\Phi^\dagger \Phi] + \frac{\lambda_0}{4}(\mathrm{Tr}\,(\Phi^\dagger\Phi])^2$$

where in terms of the 2×2 matrix-field Φ and the 8×8 matrix-field F

$$\Phi = \varphi_0 1\!\!1 + i\varphi_j \tau^j \qquad \text{and} \qquad F(x) \equiv [\varphi_0 1\!\!1 + i\gamma_5 \tau^j \varphi_j](x)$$

we have

- $(D_{lat}[U,\Phi]q)(x) = \gamma_\mu \widetilde{\nabla}_\mu q(x) + \eta F(x) q(x) - b^2 \rho \frac{1}{2} F(x) \widetilde{\nabla}_\mu \widetilde{\nabla}_\mu q(x) +$

$-b^2\rho\frac{1}{4}\Big[(\partial_\mu F)(x) U_\mu(x) \widetilde{\nabla}_\mu q(x+\hat{\mu}) + (\partial^*_\mu F)(x) U^\dagger_\mu(x-\hat{\mu}) \widetilde{\nabla}_\mu q(x-\hat{\mu})\Big]$

- $\widetilde{\nabla}_\mu f(x) \equiv \frac{1}{2}(\nabla_\mu + \nabla^*_\mu) f(x)$
- $b\nabla_\mu f(x) \equiv U_\mu(x) f(x+\hat{\mu}) - f(x)\,, \quad b\nabla^*_\mu f(x) \equiv f(x) - U^\dagger_\mu(x-\hat{\mu}) f(x-\hat{\mu})$

Technical remarks - I

- $S^{lat}_{\rm toy}$ describes 2 (flavours) $\times$ 16 (doublers) fermion dof's.
  - the $d=6$ "Wilson-like term" does not remove doublers
  - it makes no harm in current quenched studies aimed at testing whether dynamical mass generation does occur at all or not
- Global $\chi_L \times \chi_R$ transformations are exact symmetries of $S^{lat}_{\rm toy}$
  - only O(b^2) discretization terms expected
  - no linearly divergent fermionic mass terms
- Lattice covariant derivatives are such that $S^{lat}_{\rm toy}$ is invariant under the "spectrum doubling symmetry" [Montvay & Münster] implying
  - at tree level $\to$ "Wilson-like term" contributes only O(b^2) effects
  - beyond tree level $\to$ power counting is as in the formal continuum
  - these nice renormalization properties imply that η_{cr} (determined from the vanishing of r_{AWI} - see slide 16) is well defined and unique for all fermion doubler modes
  - η_{cr} is a dimensionless quantity independent of the renormalized scalar squared mass $\hat{\mu}^2_\Phi$ (up to negligible O($b^2\hat{\mu}^2_\Phi$) artefacts)
  - $\eta_{cr} = \eta_{cr}(g_0^2, \rho, \lambda_0)$ is the same in Wigner and NG phase

Technical remarks - II

- In numerical simulations (*finite volume*) the scalar vev is always zero, even if $\hat{\mu}_\Phi^2 = m_0^2 - m_{cr}^2 < 0$. Hence an "axial fixing" of χ is a convenient way to get $\langle \Phi^\dagger \Phi \rangle = v^2 1\!\!1$, $v \neq 0$ in NG phase
- FSE (from massless NG exchanges) are suppressed in the fermionic correlators that have no scalars in the valence
- In the Wigner phase, in the quenched approximation, spurious fermion zero modes (exceptional "gauge-scalar" configurations) occur. To cope with them we introduce a soft IR cutoff in the form of a "twisted mass" $i\mu b^4 \sum_x \bar{q}(x)\gamma_5(\tau^3/2)q(x)$
- Owing to the quenched approximation, gauge and scalar configurations can be generated independently
- Parameter renormalization can be carried out separately for gauge and scalar fields
- The gauge coupling is renormalized by keeping the Sommer scale r_0 fixed in physical units. If for *mere orientation* we make reference to QCD, we can take $r_0 = 0.5\ fm \sim (394\ MeV)^{-1}$

Determining η_{cr} - I

- Consider the ratio from Eq. (1) in slide 9 (no sum over $i = 1, 2, 3$)

$$r_{AWI}(\eta) = \frac{\sum_{\vec{x}} \partial_\mu \langle \tilde{A}^i_\mu(\vec{x}, x_0) \tilde{D}^i_P(0) \rangle}{\sum_{\vec{x}} \langle \tilde{D}^i_P(\vec{x}, x_0) \tilde{D}^i_P(0) \rangle} \Big|^{W}, \qquad \tilde{A}^i_\mu = \tilde{J}^{L\,i}_\mu - \tilde{J}^{R\,i}_\mu,$$

$$\bullet\, \tilde{A}^i_\mu(x) = \frac{1}{2}\Big[\bar{q}(x-\hat{\mu})\gamma_\mu\gamma_5 \frac{\tau^i}{2} U_\mu(x-\hat{\mu}) q(x) + \bar{q}(x)\gamma_\mu\gamma_5 \frac{\tau^i}{2} U^\dagger_\mu(x-\hat{\mu}) q(x-\hat{\mu})\Big]$$

$$\bullet\, \tilde{D}^i_P(x) = \bar{q}(x)\left\{\Phi, \frac{\tau^i}{2}\right\}\frac{1+\gamma_5}{2} q(x) - \bar{q}(x)\left\{\frac{\tau^i}{2}, \Phi^\dagger\right\}\frac{1-\gamma_5}{2} q(x)$$

- From the renormalized SDE ($x \neq 0$)

$$\partial_\mu \langle Z_A \widetilde{A}^i_\mu(x)\, \hat{O}(0) \rangle = [\eta - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0)] \langle \tilde{D}^i_P(x)\, \hat{O}(0) \rangle + \mathrm{O}(b^2) + \ldots$$

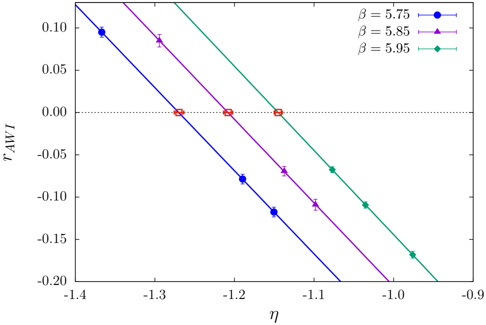
- we expect, inserting $\hat{O} = \tilde{D}_P$

$$r_{AWI}(\eta) \propto [\eta - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0)] \sim \left(1 - \frac{\partial \bar{\eta}(\eta)}{\partial \eta}\Big|_{\eta_{cr}}\right)[\eta - \eta_{cr}(g_s^2, \rho, \lambda_0)] + \ldots$$

Determining η_{cr} - II

Indeed we find

β	ρ	λ_0	η_{cr}
5.75	1.96	0.5807	-1.271(10)
5.85	1.96	0.5917	-1.207(8)
5.95	1.96	0.6022	-1.145(6)

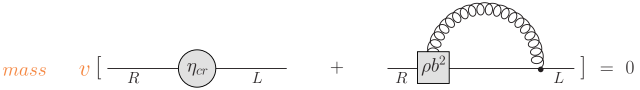


- We plot r_{AWI} as a function of η at $\beta = 5.75$, 5.85 and 5.95
- Red squares denote the values, η_{cr}, at which $r_{AWI} = 0$
- r_0/b ranges from 3.3 to 4.9, $V = L^3 \times 2L$ with $L \sim 2$ fm (r_0 Sommer scale)

Part IV
Dynamical mass generation occurs in the NG phase

Nambu–Goldstone phase - I

- Nambu–Goldstone phase: $\hat{\mu}^2_\Phi < 0$
  - $\mathcal{V}(\Phi)$ displays a double-well shape with $\langle \Phi^\dagger \Phi \rangle = v^2 \neq 0$
- Spontaneous $\chi_L \times \chi_R$ symmetry breaking
- Diagrammatically the criticality condition (of slide 11) implies



$\rightarrow$ yielding the vanishing of the "Higgs-like" fermion mass

- Standard $S(\tilde{\chi}_L \times \tilde{\chi}_R)SB$ occurs due to strong interactions ...
- ... triggered by residual $O(b^2 v)$ terms ($v \neq 0$)
- similarly to what happens in (massless) LQCD at m_{cr}, identifying

$$\frac{b^2}{2}\rho\left(\bar{q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu q_R + \bar{q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \mathcal{D}_\mu q_L\right) \xrightarrow{\langle \Phi^\dagger \Phi \rangle = v^2} \left(\frac{b^2 v}{2}\right)\rho\left(\bar{q}\mathcal{D}^2_\mu q\right)$$

$$\eta\left(\bar{q}_L \Phi q_R + \bar{q}_R \Phi^\dagger q_L\right) \xrightarrow{\langle \Phi^\dagger \Phi \rangle = v^2} \eta v\left(\bar{q}q\right)$$

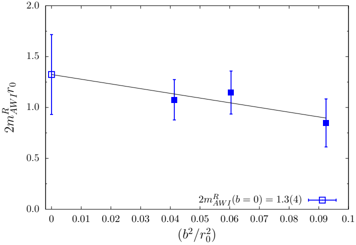
where still the standard phenomenon of $S\chi SB$ takes place

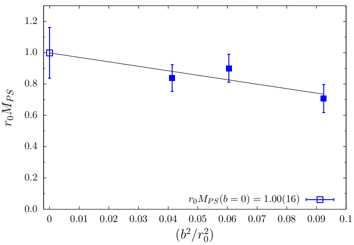
Nambu–Goldstone phase - II

- Compute the "PCAC quark mass" in NG phase (no sum over i)

$$m_{AWI}(\eta) = \frac{\sum_{\vec{x}} \partial_\mu \langle \tilde{A}^i_\mu(\vec{x}, x_0) P^i(0) \rangle}{\sum_{\vec{x}} \langle P^i(\vec{x}, x_0) P^i(0) \rangle} \Big|^{NG}, \qquad P^i = \bar{q}\gamma_5 \frac{\tau^i}{2} q$$

- Surprisingly we find that neither m_{AWI} nor M_{PS} vanish at η_{cr}
  $\rightarrow$ a NP fermion mass is getting dynamically generated
  $\rightarrow$ together with a non-vanishing PS-meson mass

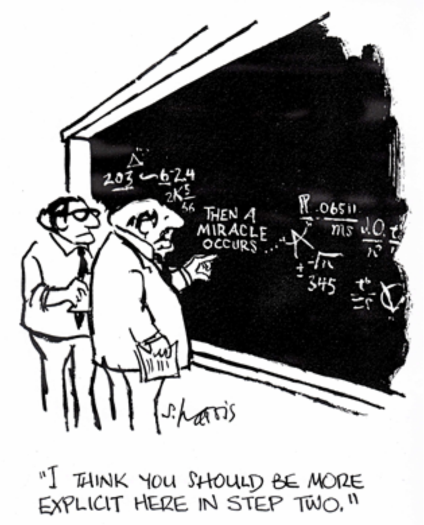




- $2m^R_{AWI} r_0 \equiv 2r_0 m_{AWI} Z_{\tilde{A}} Z_P^{-1}$ (left) and $r_0 M_{PS}$ (right) vs. $(b/r_0)^2$
- straight lines are linear extrapolations to the continuum limit

Nambu–Goldstone phase - Dealing with “step two”

- Compute the “PCAC quark mass” in NG phase



Nambu–Goldstone phase - III

- How can we interpret these results?
- $d=4$ EL, Γ_4^{NG}, may & should have extra NP $\chi_L \times \chi_R$ invariant terms

$$\Gamma_4^{NG} = \Gamma_4^{Wig}\Big|_{\hat{\mu}_\Phi^2<0} + \underline{c_1\Lambda_s[\bar{q}_L U q_R + \bar{q}_R U^\dagger q_L]} + \text{“}\ldots\text{”}$$

$$\Gamma_4^{Wig}\Big|_{\hat{\mu}_\Phi^2<0} = \frac{1}{4}(F\cdot F) + \bar{q}_L \not{D} q_L + \bar{q}_R \not{D} q_R + \frac{1}{2}\mathrm{Tr}\left[\partial_\mu\Phi^\dagger\partial_\mu\Phi\right] +$$

$$+\underline{[\eta-\bar{\eta}(\eta;g_s^2,\rho,\lambda_0)]\left(\bar{q}_L\Phi q_R + \bar{q}_R\Phi^\dagger q_L\right)} + \frac{\hat{\mu}_\phi^2}{2}\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \frac{\hat{\lambda}}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2$$

  - with $U=\frac{\Phi}{\sqrt{\Phi^\dagger\Phi}} = \exp(i\vec{\tau}\vec{\zeta}/v) = 1\!\!1 + i\vec{\tau}\vec{\zeta}/v + \ldots$ transforming like Φ
  - Γ_4^{NG} is invariant under $\chi_L \times \chi_R$
  - Λ_s is the RGI scale of the theory
  - $c_1(\alpha_s)$ was argued in [Frezzotti & Rossi, 2015] to be $O(\alpha_s^2)$
  - “…” are further terms allowed by symmetries (see below)
- Expanding $U = 1\!\!1 + i\vec{\tau}\vec{\zeta}/v + \ldots$ we get
  - a mass for the fermion
  - plus a wealth of non-linear interaction terms

Nambu–Goldstone phase - IV

- In the NG phase things go as if

$$\partial_\mu \langle Z_{\tilde{J}} \tilde{J}^{L\,i}_\mu(x)\,\hat{O}(0)\rangle\Big|^{NG} = c_1(\alpha_s)\Lambda_s\,\langle(\bar{q}_L\frac{\tau^i}{2}Uq_R - \bar{q}_R U^\dagger\frac{\tau^i}{2}q_L)(x)\,\hat{O}(0)\rangle +$$

$$+[\eta - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0)]\langle(\bar{q}_L\frac{\tau^i}{2}\Phi q_R - \bar{q}_R\Phi^\dagger\frac{\tau^i}{2}q_L)(x)\,\hat{O}(0)\rangle + \mathrm{O}(b^2)$$

- We find a NP obstruction to $\tilde{\chi}_L \times \tilde{\chi}_R$ symmetry restoration

- From the above equation one infers the formula

$$m_{AWI}(\eta) \propto c_1\Lambda_s + (\eta - \eta_{cr})v\,, \quad c_1 \neq 0$$

- and understands why $m_{AWI}(\eta_{cr}) \neq 0$
- A similar phenomenon occurs in LQCD where

$$m_{cr} = \frac{c_0(g_s^2)}{a} + c_1(g_s^2)\Lambda_{QCD} + \mathrm{O}(a)$$

The emergence of a mass term

- In NG phase the $d=4$ EL of the critical theory takes the form

$$\Gamma^{NG}_{4\,cr} = \frac{1}{4}(F\cdot F) + \bar q_L \not{\mathcal{D}} q_L + \bar q_R \not{\mathcal{D}} q_R + \frac{1}{2}\mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \mathcal{V}(\Phi)$$
$$+\underline{c_1\Lambda_s[\bar q_L U q_R + \bar q_R U^\dagger q_L]} + c_2\Lambda_s^2 \mathrm{Tr}\left[\partial_\mu U^\dagger \partial_\mu U\right] + \tilde c \Lambda_s |\Phi| \mathrm{Tr}\left[\partial_\mu U^\dagger \partial_\mu U\right]$$

  - The underlined operator breaks $\tilde\chi_L \times \tilde\chi_R$
  - **It gives rise to a fermion mass term upon expanding U**

- Observations
  - In NP effects U has a role like the non-analytic $r/|r|$ ratio in LQCD
  - as if we had promoted the LQCD Wilson r parameter to a field, Φ
  - non-linear $\chi_L \times \chi_R$ realization implied by fermion mass of NP origin

- Two issues
  - Symmetries can't exclude awkward "form" of scalar kinetic terms
    - Problem solved gauging χ_L, i.e. introducing EW interactions
  - Can we "understand" the emergence of a NP mass?
    - Yes, by making reference to the Symanzik expansion

An interlude

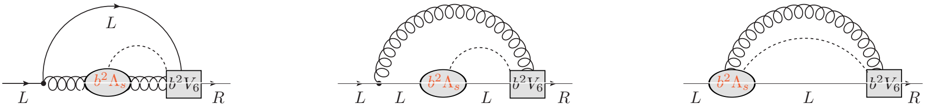
Understanding the origin of NP effects

See “back-up slides package” for details

- Symanzik expansion allows identifying NP vertex corrections
- Bookkeeping of these NP effects is obtained by including new diagrams generated by the “ad hoc modified” Feynman rules derived by adding to $\mathcal{L}_{\rm toy}$ $\chi_L \times \chi_R$ invariant terms like

$$\Delta\mathcal{L} \propto b^2\Lambda_s\alpha_s|\Phi|\Big[\frac{1}{4}FF + \bar{q}\not{D}q\Big]$$

- NP fermion masses arise from diagrams like (dotted line is a scalar)



- yielding $m_q = c_1\Lambda_s$ with $c_1 = O(\alpha_s^2)$

Origin of NP effects - Symanzik language

- Consider the small-b^2 expansion of a formally $\tilde{\chi}_L \times \tilde{\chi}_R$ inv. correlator

$$\langle O(x,x',\ldots)\rangle\Big|_{cr}^{R} = \langle O(x,x',\ldots)\rangle\Big|_{cr}^{F} - b^2 \langle O(x,x',\ldots)\int d^4z\,[L_6^{\tilde{\chi}br} + L_6^{\tilde{\chi}co}](z)\rangle\Big|_{cr}^{F} + \mathrm{O}(b^4)$$

$$O(x,x',\ldots) \Leftrightarrow A_\mu^b A_\nu^c\,\sigma\,,\;\; Q_{L/R}\bar{Q}_{L/R}\,\sigma\,,\;\; Q_{L/R}\bar{Q}_{L/R}A_\mu^b\,\sigma$$

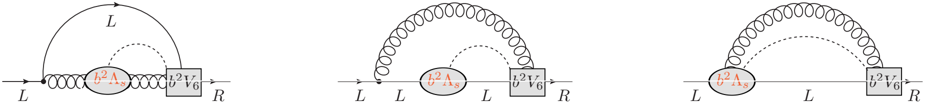
- $\langle\ldots\rangle|^R$ = UV-Regulated $\quad\langle\ldots\rangle|^F$ = Formal correlator
- $\mathcal{L}_{Yuk} + \mathcal{L}_{Wil} \implies L_6^{\tilde{\chi}br} \to \tilde{\chi}$-violating, d = 6 Symanzik operators
- $\tilde{\chi}$-even $O \to b^2\langle O \int L_6^{\tilde{\chi}br}\rangle|_{cr}^F \neq 0$ only due to spontaneous $\tilde{\chi}$SB
  - would be zero by $\tilde{R}_5 \equiv [Q \to \gamma_5 Q\,,\,\bar{Q} \to -\bar{Q}\gamma_5] \in \tilde{\chi}_L \times \tilde{\chi}_R$
- dimensional arguments $\to$ NP $\quad b^2\,\mathrm{O}(\alpha_s\Lambda_s)$ terms get generated
  - that add up to perturbative propagators and vertices
- Observation
  - The $\chi_L \times \chi_R$ symmetry constrains dynamical $\tilde{\chi}$SB effects

From NP vertex corrections to fermion mass

$\Delta\Gamma_{AA\Phi,q\bar{q}\Phi,\dots} = b^2\Lambda_s \mathrm{O}(|\rho|\alpha_s) w(\mathrm{mom}) F_{AA\Phi,q\bar{q}\Phi,\dots}(\frac{\Lambda_s^2}{\mathrm{mom}^2})$ occur for $p^2 \ll b^{-2}$

We conjecture they persist up to $p^2 \sim b^{-2} \to \infty \iff F_{\dots}(0) = \mathrm{O}(1)$

- self-energy diagrams like



- give (e.g. central panel – surviving in quenched approximation)

$$\underline{\underline{m_q}} \propto g_s^2\rho|\rho|\alpha_s(\Lambda_s)\int^{1/b}\frac{d^4k}{k^2}\frac{\gamma_\mu k_\mu}{k^2}\int^{1/b}\frac{d^4\ell}{\ell^2+m_\sigma^2}\frac{\gamma_\nu(k+\ell)_\nu}{(k+\ell)^2}\,\cdot$$

$$\cdot b^2\gamma_\rho(k+\ell)_\rho\, b^2\Lambda_s\gamma_\lambda(2k+\ell)_\lambda \sim \underline{\underline{g_s^2\rho|\rho|\alpha_s(\Lambda_s)\Lambda_s}}$$

with the b^4 factor compensated by the two-loop quartic divergency

- yielding in $\Gamma^{NG}_{4\,cr}$ the NP mass term $c_1\Lambda_s[\bar{q}_L U q_R + \mathrm{h.c.}]_{U=1\!\!1}$, $c_1|_{LO} = k_{LO}\rho|\rho|\,\alpha_s^2$

Part V
Introducing weak interactions

Extending the model

- $\mathcal{L}_{\rm toy}$ (slide 7) is readily extended to encompass weak interactions
- It's enough to gauge the exact $\chi_L = SU(2)_L$ symmetry thus getting

$$\mathcal{L}^W_{\rm toy}(q;\Phi;A,W) = \mathcal{L}_{kin}(q,\Phi;A,W) + \mathcal{V}(\Phi) + \\ + \mathcal{L}_{Yuk}(q,\Phi) + \mathcal{L}_{Wil}(q,\Phi;A,W)$$

$$\mathcal{L}_{kin}(q;\Phi;A,W) = \frac{1}{4}\Big(F^A\cdot F^A + F^W\cdot F^W\Big) + \\ + \Big[\bar{q}_L \not{\mathcal{D}}^{A,W} q_L + \bar{q}_R \not{\mathcal{D}}^{A} q_R\Big] + \frac{1}{2}\mathrm{Tr}\left[\Phi^\dagger \overleftarrow{\mathcal{D}}^W_\mu \mathcal{D}^W_\mu \Phi\right]$$

$$\mathcal{V}(\Phi) = \frac{\mu_0^2}{2}\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2$$

$$\mathcal{L}_{Yuk}(q;\Phi) = \eta\,\left(\bar{q}_L\Phi\, q_R + \bar{q}_R\Phi^\dagger q_L\right)$$

$$\mathcal{L}_{Wil}(q;\Phi;A,W) = \frac{b^2}{2}\rho\,\left(\bar{q}_L \overleftarrow{\mathcal{D}}^{A,W}_\mu \Phi \mathcal{D}^A_\mu q_R + \bar{q}_R \overleftarrow{\mathcal{D}}^{A}_\mu \Phi^\dagger \mathcal{D}^{A,W}_\mu q_L\right)$$

Covariant derivatives & symmetries

- Covariant derivatives now read

$$\begin{cases} \mathcal{D}^A_\mu = \partial_\mu - ig_s\lambda^a A^a_\mu \\ \mathcal{D}^W_\mu = \partial_\mu - ig_w\frac{\tau^i}{2}W^i_\mu \\ \mathcal{D}^{A,W}_\mu = \partial_\mu - ig_s\lambda^a A^a_\mu - ig_w\frac{\tau^i}{2}W^i_\mu \end{cases} \qquad \begin{matrix} \overleftarrow{\mathcal{D}}^A_\mu = \overleftarrow{\partial}_\mu + ig_s\lambda^a A^a_\mu \\ \overleftarrow{\mathcal{D}}^W_\mu = \overleftarrow{\partial}_\mu + ig_w\frac{\tau^i}{2}W^i_\mu \\ \overleftarrow{\mathcal{D}}^{A,W}_\mu = \overleftarrow{\partial}_\mu + ig_s\lambda^a A^a_\mu + ig_w\frac{\tau^i}{2}W^i_\mu \end{matrix}$$

- $\mathcal{L}^W_{\rm toy}$ is invariant under $\chi_L \times \chi_R$ (acting on all fields)
  - χ_L : $\tilde{\chi}_L \times (\Phi \to \Omega_L\Phi)$

$$\tilde{\chi}_L : \begin{cases} q_L \to \Omega_L q_L \\ \bar{q}_L \to \bar{q}_L\Omega_L^\dagger \\ W_\mu \to \Omega_L W_\mu \Omega_L^\dagger \end{cases} \qquad \Omega_L \in \mathrm{SU}_L(2)$$

  - χ_R : $\tilde{\chi}_R \times (\Phi \to \Phi\Omega_R^\dagger)$

$$\tilde{\chi}_R : \begin{cases} q_R \to \Omega_R q_R \\ \bar{q}_R \to \bar{q}_R\Omega_R^\dagger \end{cases} \qquad \Omega_R \in SU_R(2)$$

- but, generically, not under $\tilde{\chi}_L \times \tilde{\chi}_R$ (acting only on quarks & W's)

Currents & bare SDEs

- $\tilde\chi_L \times \tilde\chi_R$ currents with $K^i_\mu = g_w \mathrm{Tr}\left([W_\nu, F^W_{\mu\nu}]\frac{\tau^i}{2}\right)$

$$\tilde J^{L,i}_\mu = K^i_\mu + \bar q_L \gamma_\mu \frac{\tau^i}{2} q_L - \frac{b^2}{2}\rho\Big(\bar q_L \frac{\tau^i}{2}\Phi \mathcal{D}^A_\mu q_R - \bar q_R \overleftarrow{\mathcal{D}}^A_\mu \Phi^\dagger \frac{\tau^i}{2} q_L\Big)$$

$$\tilde J^{R,i}_\mu = \bar q_R \gamma_\mu \frac{\tau^i}{2} q_R - \frac{b^2}{2}\rho\Big(\bar q_R \frac{\tau^i}{2}\Phi^\dagger \mathcal{D}^{A,W}_\mu q_L - \bar q_L \overleftarrow{\mathcal{D}}^{A,W}_\mu \Phi \frac{\tau^i}{2} q_R\Big)$$

- Bare SDE's of $\tilde\chi_L \times \tilde\chi_R$

$$\partial_\mu \langle \tilde J^{L,i}_\mu(x)\,\hat O(0)\rangle = \langle \tilde\Delta^i_L \hat O(0)\rangle\delta(x) - \eta\,\langle\Big(\bar q_L \frac{\tau^i}{2}\Phi q_R - \bar q_R \Phi^\dagger \frac{\tau^i}{2} q_L\Big)(x)\,\hat O(0)\rangle +$$
$$-\frac{b^2}{2}\rho\,\langle\Big(\bar q_L \overleftarrow{\mathcal{D}}^{A,W}_\mu \frac{\tau^i}{2}\Phi \mathcal{D}^A_\mu q_R - \bar q_R \overleftarrow{\mathcal{D}}^A_\mu \Phi^\dagger \frac{\tau^i}{2}\mathcal{D}^{A,W}_\mu q_L\Big)(x)\,\hat O(0)\rangle +$$
$$+\frac{i}{2} g_w \langle \mathrm{Tr}\left(\Phi^\dagger[\frac{\tau^i}{2}, W_\mu]\mathcal{D}^W_\mu \Phi + \Phi^\dagger \overleftarrow{\mathcal{D}}^W_\mu [W_\mu, \frac{\tau^i}{2}]\Phi\right)(x)\hat O(0)\rangle$$

$$\partial_\mu \langle \tilde J^{R,i}_\mu(x)\,\hat O(0)\rangle = \langle \tilde\Delta^i_R \hat O(0)\rangle\delta(x) - \eta\,\langle\Big(\bar q_R \frac{\tau^i}{2}\Phi^\dagger q_L - \bar q_L \Phi \frac{\tau^i}{2} q_R\Big)(x)\,\hat O(0)\rangle +$$
$$-\frac{b^2}{2}\rho\,\langle\Big(\bar q_R \overleftarrow{\mathcal{D}}^{A}_\mu \frac{\tau^i}{2}\Phi^\dagger \mathcal{D}^{A,W}_\mu q_L - \bar q_L \overleftarrow{\mathcal{D}}^{A,W}_\mu \Phi \frac{\tau^i}{2}\mathcal{D}^{A}_\mu q_R\Big)(x)\,\hat O(0)\rangle$$

Mixing

- We generalize the argument in slide 10
  - Bare $d = 6$ operators: $\tilde{\chi}_L \times \tilde{\chi}_R$ rotations of Wilson-like operators

$$O_6^{L,\,i} = \frac{1}{2}\rho\Big(\bar{q}_L \overleftarrow{\mathcal{D}}^{A,W}_\mu \frac{\tau^i}{2}\Phi \mathcal{D}^A_\mu q_R - \bar{q}_R \overleftarrow{\mathcal{D}}^{A}_\mu \Phi^\dagger \frac{\tau^i}{2}\mathcal{D}^{A,W}_\mu q_L\Big)$$

$$O_6^{R,\,i} = \frac{1}{2}\rho\Big(\bar{q}_R \overleftarrow{\mathcal{D}}^{A}_\mu \frac{\tau^i}{2}\Phi^\dagger \mathcal{D}^{A,W}_\mu q_L - \bar{q}_L \overleftarrow{\mathcal{D}}^{A,W}_\mu \Phi \frac{\tau^i}{2}\mathcal{D}^{A}_\mu q_R\Big)$$

  - Renormalized $d = 6$ operators

$$O_6^{L,\,i} = \Big[O_6^{L,\,i}\Big]_{sub} + \frac{Z_{\tilde{J}}-1}{b^2}\partial_\mu \tilde{J}^{L\,i}_\mu - \frac{\bar{\eta}^L}{b^2}\Big(\bar{q}_L \frac{\tau^i}{2}\Phi q_R - \bar{q}_R \Phi^\dagger \frac{\tau^i}{2} q_L\Big) +$$
$$+\frac{\bar{\gamma}}{b^2}\frac{i}{2}g_w\langle \mathrm{Tr}\Big(\Phi^\dagger[\frac{\tau^i}{2}, W_\mu]\mathcal{D}^W_\mu \Phi + \Phi^\dagger \overleftarrow{\mathcal{D}}^{W}_\mu [W_\mu, \frac{\tau^i}{2}]\Phi\Big) + \dots$$

$$O_6^{R,\,i} = \Big[O_6^{R,\,i}\Big]_{sub} + \frac{Z_{\tilde{J}}-1}{b^2}\partial_\mu \tilde{J}^{R\,i}_\mu - \frac{\bar{\eta}^R}{b^2}\Big(\bar{q}_R \frac{\tau^i}{2}\Phi^\dagger q_L - \bar{q}_L \Phi \frac{\tau^i}{2} q_R\Big) + \dots$$

- There is an important new feature
  - $O_6^{L,\,i}$ mixes with the $\tilde{\chi}_L$ rotation of the Φ kinetic term

Renormalized SDE & tuning

- Renormalized SDEs take the form

- $\partial_\mu \langle Z_{\tilde J} \tilde J^{L,\,i}_\mu(x)\,\hat O(0)\rangle = \langle \tilde\Delta^i_L \hat O(0)\rangle \delta(x) +$

  $-(\eta - \bar\eta^L)\,\langle\Big(\bar q_L \frac{\tau^i}{2}\Phi q_R - \bar q_R \Phi^\dagger \frac{\tau^i}{2} q_L\Big)(x)\,\hat O(0)\rangle +$

  $+(1-\bar\gamma)\frac{i}{2} g_w \langle \mathrm{Tr}\Big(\Phi^\dagger[\frac{\tau^i}{2}, W_\mu]\mathcal{D}^W_\mu \Phi + \Phi^\dagger \overleftarrow{\mathcal{D}}^W_\mu [W_\mu, \frac{\tau^i}{2}]\Phi\Big)(x)\hat O(0)\rangle + O(b^2) + \ldots$

- $\partial_\mu \langle Z_{\tilde J} \tilde J^{R,\,i}_\mu(x)\,\hat O(0)\rangle = \langle \tilde\Delta^i_R \hat O(0)\rangle \delta(x) +$

  $-(\eta - \bar\eta^R)\,\langle\Big(\bar q_R \frac{\tau^i}{2}\Phi^\dagger q_L - \bar q_L \Phi \frac{\tau^i}{2} q_R\Big)(x)\,\hat O(0)\rangle + O(b^2) + \ldots$

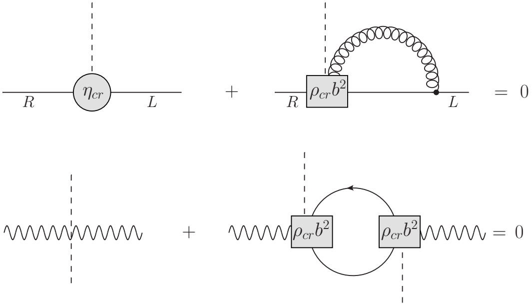
- Conditions of $\tilde\chi_L \times \tilde\chi_R$ enhancement read

$$\eta - \bar\eta^L(g_s, g_w; \mu_0, \lambda_0; \eta, \rho) = 0$$
$$\eta - \bar\eta^R(g_s, g_w; \mu_0, \lambda_0; \eta, \rho) = 0$$
$$1 - \bar\gamma(g_s, g_w; \mu_0, \lambda_0; \eta, \rho) = 0$$

- These equations determine the critical values, η_{cr} and ρ_{cr}

Tuning conditions in the Wigner phase

- Here are the lowest order diagrams corresponding to the tuning of the parameters η and ρ in the Wigner phase



- Explicit b^2 factors are compensated by the power divergencies of the loop diagrams

EL in the Wigner phase

At generic values of η and ρ the (local) $d = 4$ EL of the theory in the Wigner phase takes the $\chi_L \times \chi_R$ invariant form

$$\Gamma_4^{Wig} = \frac{1}{4}\Big(F^A \cdot F^A + F^W \cdot F^W\Big) + \Big[\bar{q}_L \, \mathcal{D}\!\!\!/^{A,W} q_L + \bar{q}_R \, \mathcal{D}\!\!\!/^{A} q_R\Big] +$$
$$+\frac{k_{eff}}{2}\mathrm{Tr}\left[\Phi^\dagger \overleftarrow{\mathcal{D}}_\mu^{\,W} \mathcal{D}_\mu^W \Phi\right] + y_{eff}\left(\bar{q}_L \Phi \, q_R + \bar{q}_R \Phi^\dagger q_L\right) + \mathcal{V}(\Phi)$$

with

$$y_{eff} = \eta - \bar{\eta} \overset{\eta \to \eta_{cr},\, \rho \to \rho_{crit}}{\longrightarrow} 0$$
$$k_{eff} = 1 - \bar{\gamma} \overset{\eta \to \eta_{cr},\, \rho \to \rho_{crit}}{\longrightarrow} 0$$

In the critical limit the effective Yukawa coupling vanishes and the kinetic term of the scalar disappears, so the EL simply becomes

$$\Gamma_{4\,cr}^{Wig} = \frac{1}{4}\Big(F^A \cdot F^A + F^W \cdot F^W\Big) + \Big[\bar{q}_L \, \mathcal{D}\!\!\!/^{AW} q_L + \bar{q}_R \, \mathcal{D}\!\!\!/^{A} q_R\Big]$$

where Φ is completely decoupled from fermions

Implications of critical tuning in the NG phase

In NG phase, $\langle \Phi^\dagger \Phi \rangle = v^2 \neq 0$ and tuning conditions imply vanishing

- of fermion and weak boson "Higgs-like" masses
- and by gauge invariance of the whole scalar kinetic term
  Bardeen, Hill and Lindner, 1990

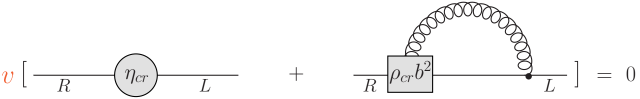


Figure: The mechanism behind the "Higgs-like" fermion mass term -$v\bar{q}q$ - cancellation in the NG phase of the critical theory. The circle represents the critical Yukawa coupling and the square box the insertion of the critical $d = 6$ Wilson vertex

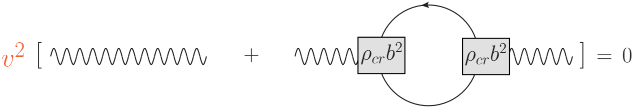
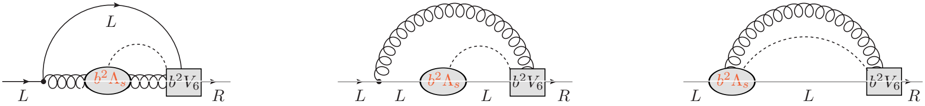


Figure: The mechanism behind the "Higgs-like" W mass term - $g_w^2 v^2 W_\mu W_\mu$ - cancellation in the NG phase of the critical theory

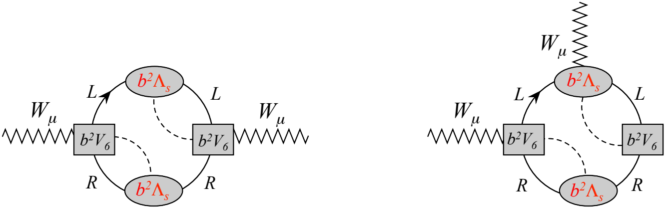
Elementary particle masses: fermions & weak bosons

From the same building blocks (vertices)

- NP fermion mass is generated by diagrams like (see Interlude)



- NP weak boson mass is generated by diagrams like



- from which we get (see back-up slides for details)

$$m_q = C_q \Lambda_s \,, \qquad C_q = \mathrm{O}(\alpha_s^2)$$
$$M_W = C_w \Lambda_s \,, \qquad C_w = \mathrm{O}(\alpha_s \sqrt{\alpha_w})$$

The critical EL in the NG phase

$$
\begin{aligned}
&\Gamma_{4\,cr}^{NG}(q;\Phi;A,W) = \\
&\quad = \frac{1}{4}\Big(F^A\cdot F^A + F^W\cdot F^W\Big) + \\
&\quad + \Big[\bar q_L \mathcal{D}\!\!\!/^{WA} q_L + \bar q_R \mathcal{D}\!\!\!/^{A} q_R\Big] + C_q\Lambda_s\Big(\bar q_L U q_R + \bar q_R U^\dagger q_L\Big) + \\
&\quad + \frac{1}{2} C_w^2 \Lambda_s^2 \mathrm{Tr}\,\big[U^\dagger \overleftarrow{\mathcal{D}}{}^W_\mu \mathcal{D}^W_\mu U\big] + \dots
\end{aligned}
$$

with

$$
U = \frac{\Phi}{\sqrt{\Phi^\dagger\Phi}} = \exp\Big(i\frac{\vec\tau\cdot\vec\zeta}{\sqrt{C_w}\Lambda_s}\Big) = 1\!\!1 + i\frac{\vec\tau\cdot\vec\zeta}{\sqrt{C_w}\Lambda_s} + \dots
$$

then

$$
\begin{aligned}
m_q &= C_q\Lambda_s\,, \qquad & C_q &= \mathrm{O}(\alpha_s^2) \\
M_W &= C_w\Lambda_s\,, \qquad & C_w &= \mathrm{O}(\sqrt{\alpha_w}\alpha_s)
\end{aligned}
$$

Notes

- rescaling factor in U chosen to get canonically normalized $\vec\zeta$ fields
- ρ_{cr} dependence discussed below

The need for superstrongly interacting (Tera-)particles

Some phenomenology

- In the mass formulae above Λ_s is the RGI scale of the theory
- Can we make the mass formulae

$$m_q = C_q \Lambda_{RGI}\,, \qquad C_q = \mathrm{O}(\alpha_s^2)$$
$$M_W = C_w \Lambda_{RGI}\,, \qquad C_w = \mathrm{O}(\sqrt{\alpha_w}\alpha_s)$$

compatible with the phenomenological top and W mass values?

- As an order of magnitude we clearly need to have

$$\Lambda_{QCD} \ll \Lambda_{RGI} = \mathrm{O}(\text{a few TeV's})$$

in order to get physical masses in the 10^2 GeV range
- $\rightarrow$ there must exist a superstrongly interacting sector with

$$\Lambda_{RGI} \equiv \Lambda_T = \mathrm{O}(\text{a few TeV's})$$

- This could be an explanation for the magnitude of the EW scale
- Warning - Partition function vanishes as one has an odd number ($N_c = 3$) of SU(2) doublets (Witten anomaly). The problem can be cured, for instance by taking $N_c = 4$, or having more doublets

Extending the model

- Superstrongly interacting (Tera-)particles are easily incorporated

$$\mathcal{L}(q,Q;\Phi;A,G,W) = \mathcal{L}_{kin}(q,Q;\Phi;A,G,W) + \\ +\mathcal{V}(\Phi) + \mathcal{L}_{Yuk}(q,Q;\Phi) + \mathcal{L}_{Wil}(q,Q;\Phi;A,G,W)$$

- $\mathcal{L}_{kin}(q,Q;\Phi;A,W) = \frac{1}{4}\Big(F^A\cdot F^A + F^G\cdot F^G + F^W\cdot F^W\Big) +$

$$+\Big[\bar q_L \not{\mathcal{D}}^{AW} q_L + \bar q_R \not{\mathcal{D}}^{A} q_R\Big] + \Big[\bar Q_L \not{\mathcal{D}}^{AGW} Q_L + \bar Q_R \not{\mathcal{D}}^{AG} Q_R\Big] + \frac{1}{2}\mathrm{Tr}\,\big[\Phi^\dagger \overleftarrow{\mathcal{D}}^W_\mu \mathcal{D}^W_\mu \Phi\big]$$

- $\mathcal{V}(\Phi) = \frac{\mu_0^2}{2}\mathrm{Tr}\,\big[\Phi^\dagger\Phi\big] + \frac{\lambda_0}{4}\big(\mathrm{Tr}\,\big[\Phi^\dagger\Phi\big]\big)^2$
- $\mathcal{L}_{Yuk}(q,Q;\Phi) = \eta_q\,(\bar q_L \Phi\, q_R + \bar q_R \Phi^\dagger q_L) + \eta_Q\,(\bar Q_L \Phi\, Q_R + \bar Q_R \Phi^\dagger Q_L)$
- $\mathcal{L}_{Wil}(q,Q;\Phi;A,G,W) = \frac{b^2}{2}\rho_q\,(\bar q_L \overleftarrow{\mathcal{D}}^{AW}_\mu \Phi \mathcal{D}^A_\mu q_R + \bar q_R \overleftarrow{\mathcal{D}}^{A}_\mu \Phi^\dagger \mathcal{D}^{AW}_\mu q_L) +$

$$+\frac{b^2}{2}\rho_Q\,(\bar Q_L \overleftarrow{\mathcal{D}}^{AGW}_\mu \Phi \mathcal{D}^{AG}_\mu Q_R + \bar Q_R \overleftarrow{\mathcal{D}}^{AG}_\mu \Phi^\dagger \mathcal{D}^{AGW}_\mu Q_L)$$

Covariant derivatives & Symmetries

$$\begin{cases} \mathcal{D}_\mu^{AGW} = \partial_\mu - ig_s\lambda^a A_\mu^a - ig_T T^\alpha G_\mu^\alpha - ig_w \frac{\tau^i}{2} W_\mu^i \\ \overleftarrow{\mathcal{D}}_\mu^{AGW} = \overleftarrow{\partial}_\mu + ig_s\lambda^a A_\mu^a + ig_T T^\alpha G_\mu^\alpha + ig_w \frac{\tau^i}{2} W_\mu^i \\ \mathcal{D}_\mu^{AG} = \partial_\mu - ig_s\lambda^a A_\mu^a - ig_T T^\alpha G_\mu^\alpha \\ \overleftarrow{\mathcal{D}}_\mu^{AG} = \overleftarrow{\partial}_\mu + ig_s\lambda^a A_\mu^a + ig_T T^\alpha G^\alpha \end{cases}$$

- χ_L : $\quad \tilde{\chi}_L \times (\Phi \to \Omega_L \Phi)$

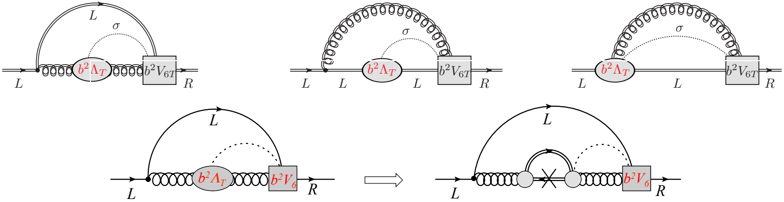
$$\tilde{\chi}_L : \begin{cases} q_L \to \Omega_L q_L \\ \bar{q}_L \to \bar{q}_L \Omega_L^\dagger \\ W_\mu \to \Omega_L W_\mu \Omega_L^\dagger \\ Q_L \to \Omega_L Q_L \\ \bar{Q}_L \to \bar{Q}_L \Omega_L^\dagger \end{cases} \qquad \Omega_L \in \mathrm{SU}_L(2)$$

- χ_R : $\quad \tilde{\chi}_R \times (\Phi \to \Phi \Omega_R^\dagger)$

$$\tilde{\chi}_R : \begin{cases} q_R \to \Omega_R q_R \\ \bar{q}_R \to \bar{q}_R \Omega_R^\dagger \\ \\ Q_R \to \Omega_R Q_R \\ \bar{Q}_R \to \bar{Q}_R \Omega_R^\dagger \end{cases} \qquad \Omega_R \in \mathrm{SU}_R(2)$$

Tera-strong & strong interactions: mass hierarchy

- Q's feel Tera-strong & strong interactions, q's only strong ones
  - $q \to N_g = 3$ families - gauge group $SU(N_c = 3)$
  - $Q \to 1$ family - gauge groups $SU(N_c = 3) \times SU(N_T = 3)$
  - $\beta^0_T / \beta^0_{QCD} = \frac{11N_T - 4N_c}{11N_c - 4N_g - 4N_T} = \frac{7}{3} \quad \Rightarrow \quad \Lambda_T \gg \Lambda_{QCD}$



- A crude leading order estimate gives
  - $m_Q(\Lambda_T) \sim \rho_{Q\,cr} |\rho_{Q\,cr}| \alpha^2_T(\Lambda_T)\,\Lambda_T \qquad m_q(\Lambda_T) \sim \rho_{q\,cr} |\rho_{Q\,cr}| \alpha^2_s(\Lambda_T)\,\Lambda_T$
  - $\frac{m_q}{m_Q}\Big|_{\Lambda_T} \sim \frac{\alpha^2_s(\Lambda_T)}{\alpha^2_T(\Lambda_T)} \sim \frac{1}{10} \div \frac{1}{100}$
  - $q = \text{top} \Rightarrow m_Q \sim$ few TeV's $\sim \underline{\underline{\Lambda_T \gg \Lambda_{QCD}}}$

The critical EL in the NG phase

With the same line of arguments as before, we end up with the EL

$$\Gamma^{NG}_{4\,cr}(q,Q;\Phi;A,G,W)=$$

$$=\frac{1}{4}\Big(F^A\cdot F^A+F^G\cdot F^G+F^W\cdot F^W\Big)+$$

$$+\Big[\bar q_L \not{\mathcal{D}}^{WA} q_L+\bar q_R \not{\mathcal{D}}^{A} q_R\Big]+C_q\Lambda_T\Big(\bar q_L U q_R+\bar q_R U^\dagger q_L\Big)+$$

$$+\Big[\bar Q_L \not{\mathcal{D}}^{WAG} Q_L+\bar Q_R \not{\mathcal{D}}^{AG} Q_R\Big]+C_Q\Lambda_T\Big(\bar Q_L U Q_R+\bar Q_R U^\dagger Q_L\Big)+$$

$$+\frac{1}{2}C_w^2\Lambda_T^2\,\mathrm{Tr}\left[U^\dagger \overleftarrow{\mathcal{D}}^W_\mu \mathcal{D}^W_\mu U\right]+\dots$$

$$U=\frac{\Phi}{\sqrt{\Phi^\dagger\Phi}}=\exp\left(i\frac{\vec\tau\cdot\vec\zeta}{C_w\Lambda_T}\right)=1\!\!1+i\frac{\vec\tau\cdot\vec\zeta}{C_w\Lambda_T}+\dots$$

then

$$M_W=C_w\Lambda_T\,,\qquad C_w=\mathrm{O}(\sqrt{\alpha_w}\alpha_s)$$

$$m_q=C_q\Lambda_T\,,\qquad C_q=\mathrm{O}(\alpha_s^2)$$

$$m_Q=C_Q\Lambda_T\,,\qquad C_Q=\mathrm{O}(\alpha_T^2)$$

Part VI
Towards a BSMM

Introducing leptons and hypercharge

$U_Y(1)$ gauge interactions, leptons & Tera-leptons - I

- One family
- Ready to break isospin degeneracy
- Assume invariance under $f(x) \to f(x) + \text{const}$, $\bar{f}(x) \to \bar{f}(x) + \text{const}$, as $g_{s,w,y} \to 0$ for all fermion species f [Goltermann & Petcher, 1990]

$$\mathcal{L}(q, \ell, Q, L; \Phi; A, G, W, B) = \mathcal{L}_{kin}(q, \ell, Q, L; \Phi; A, G, W, B) + \mathcal{V}(\Phi) + \\ + \mathcal{L}_{Yuk}(q, \ell, Q, L; \Phi) + \mathcal{L}_{Wil}(q, \ell, Q, L; \Phi; A, G, W, B)$$

- $\mathcal{L}_{kin}(q, \ell, Q, L; \Phi; A, G, W, B) =$

$$= \frac{1}{4}\Big(F^A \cdot F^A + F^G \cdot F^G + F^W \cdot F^W + F^B \cdot F^B\Big) + \\ + \Big[\bar{q}_L \not{\mathcal{D}}^{BWA} q_L + \bar{q}^u_R \not{\mathcal{D}}^{BA} q^u_R + \bar{q}^d_R \not{\mathcal{D}}^{BA} q^d_R + \bar{\ell}_L \not{\mathcal{D}}^{BW} \ell_L + \bar{\ell}^u_R \not{\partial} \ell^u_R + \bar{\ell}^d_R \not{\mathcal{D}}^{B} \ell^d_R\Big] + \\ + \Big[\bar{Q}_L \not{\mathcal{D}}^{BWAG} Q_L + \bar{Q}^u_R \not{\mathcal{D}}^{BAG} Q^u_R + \bar{Q}^d_R \not{\mathcal{D}}^{BAG} Q^d_R + \\ + \bar{L}_L \not{\mathcal{D}}^{BWG} L_L + \bar{L}^u_R \not{\mathcal{D}}^{BG} L^u_R + \bar{L}^d_R \not{\mathcal{D}}^{BG} L^d_R\Big] + \\ + \frac{1}{2}\mathrm{Tr}\left[\Phi^\dagger \overleftarrow{\mathcal{D}}^{WB}_\mu \mathcal{D}^{WB}_\mu \Phi\right]$$

- $\mathcal{V}(\Phi) = \frac{\mu_0^2}{2}\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2$ (2)

$U_Y(1)$ gauge interactions, leptons & Tera-leptons - II

- $\mathcal{L}_{Yuk}(q,\ell,Q,L;\tilde\phi,\phi) = \sum_{f=q,\ell,Q,L} \eta^{q,u}\,(\bar f_L \tilde\phi\, f^u_R + \text{h.c.}) + \eta^{q,d}\,(\bar f_L \phi\, f^d_R + \text{h.c.})$
- $\mathcal{L}_{Wil}(q,\ell,Q,L;\tilde\phi,\phi;A,G,W,B) =$

$$= \frac{b^2}{2}\rho^{q,u}\,(\bar q_L \overleftarrow{\mathcal{D}}{}^{BWA}_\mu \tilde\phi \mathcal{D}^{BA}_\mu q^u_R + \text{h.c.}) + \frac{b^2}{2}\rho^{q,d}\,(\bar q_L \overleftarrow{\mathcal{D}}{}^{BWA}_\mu \phi \mathcal{D}^{BA}_\mu q^d_R + \text{h.c.}) +$$

$$= \frac{b^2}{2}\rho^{\ell,u}\,(\bar \ell_L \overleftarrow{\mathcal{D}}{}^{BW}_\mu \tilde\phi\, \partial_\mu \ell^u_R + \text{h.c.}) + \frac{b^2}{2}\rho^{\ell,d}\,(\bar \ell_L \overleftarrow{\mathcal{D}}{}^{BW}_\mu \phi \mathcal{D}^{B}_\mu \ell^d_R + \text{h.c.}) +$$

$$= \frac{b^2}{2}\rho^{Q,u}\,(\bar q_L \overleftarrow{\mathcal{D}}{}^{BWAG}_\mu \tilde\phi \mathcal{D}^{BAG}_\mu Q^u_R + \text{h.c.}) + \frac{b^2}{2}\rho^{Q,d}\,(\bar Q_L \overleftarrow{\mathcal{D}}{}^{BWAG}_\mu \phi \mathcal{D}^{BAG}_\mu Q^d_R + \text{h.c.}) +$$

$$= \frac{b^2}{2}\rho^{L,u}\,(\bar L_L \overleftarrow{\mathcal{D}}{}^{BWG}_\mu \tilde\phi \mathcal{D}^{BG}_\mu L^u_R + \text{h.c.}) + \frac{b^2}{2}\rho^{L,d}\,(\bar L_L \overleftarrow{\mathcal{D}}{}^{BWG}_\mu \phi \mathcal{D}^{BG}_\mu L^d_R + \text{h.c.}) +$$

$$\mathcal{D}^{BWAG}_\mu = \partial_\mu - iYg_Y B_\mu - ig_w \frac{\tau^r}{2} W^r_\mu - ig_s \lambda^a A^a_\mu - ig_T \lambda^\alpha_T G^\alpha_\mu$$

Notes

- We have used $y_{\nu_R} = 0$ and Golterman Petcher symmetry at $g_w = 0$
- so neutrinos are massless

Our favourite hypercharge choice

q	ℓ
$y_{u_L} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$	$y_{\nu_L} = 0 - \frac{1}{2} = -\frac{1}{2}$
$y_{u_R} = \frac{2}{3} - 0 = \frac{2}{3}$	$y_{\nu_R} = 0$
$y_{d_L} = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$	$y_{el_L} = -1 + \frac{1}{2} = -\frac{1}{2}$
$y_{d_R} = -\frac{1}{3} - 0 = -\frac{1}{3}$	$y_{el_R} = -1 - 0 = -1$

Q	L
$y_{U_L} = \frac{1}{2} - \frac{1}{2} = 0$	$y_{N_L} = \frac{1}{2} - \frac{1}{2} = 0$
$y_{U_R} = \frac{1}{2} - 0 = \frac{1}{2}$	$y_{N_R} = \frac{1}{2} - 0 = \frac{1}{2}$
$y_{D_L} = -\frac{1}{2} + \frac{1}{2} = 0$	$y_{L_L} = -\frac{1}{2} + \frac{1}{2} = 0$
$y_{D_R} = -\frac{1}{2} - 0 = -\frac{1}{2}$	$y_{L_R} = -\frac{1}{2} - 0 = -\frac{1}{2}$

- SM fermions (top panel) & Tera-particles (bottom panel) hypercharges
- With the above assignments
  - "unification of gauge couplings" [Garofalo, Frezzotti, Rossi 2016]
  - Tera-particles have half-integer electric charges

The EL of the critical theory

$$\Gamma^{NG}_{4\,cr}(q,\ell,Q,L;\Phi;A,G,W,B) =$$

$$= \frac{1}{4}\Big(F^A\cdot F^A + F^G\cdot F^G + F^W\cdot F^W + F^B\cdot F^B\Big) +$$

$$+\Big[\bar q_L \not{\mathcal{D}}^{BWA} q_L + \bar q^u_R \not{\mathcal{D}}^{BA} q^u_R + \bar q^d_R \not{\mathcal{D}}^{BA} q^d_R\Big] + C_q\Lambda_T\left(\bar q_L U q_R + \bar q_R U^\dagger q_L\right) +$$

$$+\Big[\bar\ell_L \not{\mathcal{D}}^{BW}\ell_L + \bar\ell^u_R \not\partial\, \ell^u_R + \bar\ell^d_R \not{\mathcal{D}}^{B}\ell^d_R\Big] + C_\ell\Lambda_T\left(\bar\ell_L U\ell_R + \bar\ell_R U^\dagger \ell_L\right)$$

$$+\Big[\bar Q_L \not{\mathcal{D}}^{BWAG} Q_L + \bar Q_R \not{\mathcal{D}}^{BAG} Q_R\Big] + C_Q\Lambda_T\left(\bar Q_L U Q_R + \bar Q_R U^\dagger Q_L\right) +$$

$$+\Big[\bar L_L \not{\mathcal{D}}^{BWA} L_L + \bar L_R \not{\mathcal{D}}^{BA} L_R\Big] + C_L\Lambda_T\left(\bar L_L U L_R + \bar L_R U^\dagger L_L\right) +$$

$$+\frac{1}{2} C_w^2\Lambda_T^2 \mathrm{Tr}\left[U^\dagger \overleftarrow{\mathcal{D}}^{BW}_\mu \mathcal{D}^{BW}_\mu U\right] + \ldots$$

$$m_q = C_q\,\Lambda_T \qquad C_q = c_q O(\alpha_s^2) \qquad m_\ell = C_\ell\,\Lambda_T \qquad C_\ell = c_\ell O(\alpha_Y^2)$$

$$m_Q = C_Q\,\Lambda_T \qquad C_Q = c_Q O(\alpha_T^2) \qquad m_L = C_L\,\Lambda_T \qquad C_L = c_L O(\alpha_T^2)$$

$$M_W^2 = C_w^2\,\Lambda_T^2\,, \qquad C_w^2 = c_w O(\alpha_w \alpha_T^2) \qquad .$$

Note - No weak-isospin splitting

Part VII
A bit of phenomenology

A bit of phenomenology

- Enforcing $\tilde{\chi}$ symmetry gives the convex constraint on the ρ_{cr}'s

$$\sum_{f=q,\ell,Q,L} n_f(\rho^f_{cr})^2 = \mathrm{O}(1)\,, \quad n_q = N_c N_g\,,\ n_\ell = N_g\,,\ n_Q = N_c N_T\,,\ n_L = N_T$$

- The fermion multiplicity dependence of the mass coefficients is

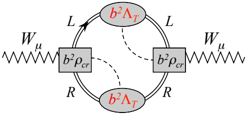
$$c_q = 2N_c N_T \rho^Q_{cr}\rho^q_{cr}\,, \qquad c_\ell = N_T(N_c\rho^Q_{cr} + \rho^L_{cr})\rho^\ell_{cr}$$
$$c_Q = 2N_T(N_c\rho^Q_{cr} + \rho^L_{cr})\rho^Q_{cr} \qquad c_L = 2N_T(N_c\rho^Q_{cr} + \rho^L_{cr})\rho^L_{cr}$$
$$c_w = N_T(N_c(\rho^Q_{cr})^4 + (\rho^L_{cr})^4)$$

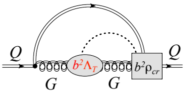
- If approximately $\forall f, \rho^f_{cr} = \rho_{cr} \Longrightarrow \rho^2_{cr}(N_c+1)(N_T+N_g) = \#$
  - $\rho^2_{cr} = \frac{\#}{(N_c+1)(N_T+N_g)} = \frac{\#}{24}$
  - $c_f \sim 1\,, \forall f$
  - $c_w \sim \rho^2_{cr}\frac{N_T}{N_T+N_g} = \frac{\#}{24}\frac{1}{2} = \frac{\#}{48}$
- Notes
  - $N_g =$ number of families
  - we have taken $N_g = N_c = N_T = 3$
  - and ignored $1/4\pi$ loop factors

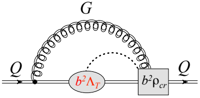
Computing $M_W/M_{T-meson} \sim M_W/\Lambda_T$: expected $\ll 1$

$$M_W = \sqrt{c_W} O(\sqrt{\alpha_W}\alpha_T)\Lambda_T = O(\sqrt{\alpha_W}\alpha_T)\rho_{cr}^2\sqrt{N_T(N_c+1)}\Lambda_T$$

$$M_{T-meson} \simeq 2M_Q = 2c_Q O(\alpha_T^2)\Lambda_T = \ 4O(\alpha_T^2)\rho_{cr}^2 N_T(N_c+1)\Lambda_T$$ expected from







$$\frac{M_W}{M_{T-meson}} \sim O(\frac{\sqrt{\alpha_W}}{\alpha_T})\frac{\#}{4\sqrt{N_T(N_c+1)}} \sim 10^{-2}$$

interesting & numerically computable with controlled O(20%) errors

i) $M_{T-meson}$: $\sum_{\vec{x}}\langle\bar{Q}\gamma_5 Q'(x)\bar{Q}'\gamma_5 Q(0)\rangle \overset{|x_0|\text{large}}{\propto} e^{-M_{T-meson}|x_0|}$

ii) M_W^2 : shifted above zero due to the double W-pole, with residue

computable from $\sum_y e^{ip\cdot y}\langle J^{\text{weak}}_{\mu,Q}(y)J^{\text{weak}}_{\lambda,Q}(0)\rangle$ at $g_w = g_Y = 0$

result depends/gives hints on N_T (e.g. 3,2) and N_{Q+L} (e.g. $N_c + 1 = 4$ doublets)

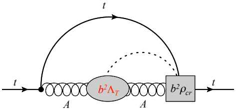
Hypercharge: M_Z/M_W, lepton masses

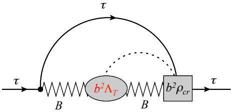
- $M_Z^2 = \dfrac{g_w^2 + g_Y^2}{g_w^2} M_W^2$, $\qquad M_\gamma = 0$

$$\Gamma^{NG}_{4\,cr} \supset c_w \Lambda_T^2 \tfrac{1}{2} \mathrm{Tr}\,[D_\mu^{WB} U^\dagger D_\mu^{WB} U] \supset c_w \Lambda_T^2 [g_w^2 \textstyle\sum_{j=1}^3 (W^j \cdot W^j) + g_Y^2 B \cdot B + 2 g_w g_Y W^3 \cdot B]$$

$\Rightarrow$ diagonalization in W^3–B sector gives massless γ and $M_Z^2 = (g_w^2 + g_Y^2) C_2 \Lambda_T^2$

owing to the custodial $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry in the $g_Y \to 0$ limit





- prediction $m_\tau / m_{\rm top} \;\sim\; \alpha_Y^2|_{\Lambda_T} / \alpha_s^2|_{\Lambda_T} \;\simeq\; 0.01$ from

$$\mathcal{L}^{\rm top}_{W+Y} = \tfrac{1}{2} b^2 \rho^t_{cr} [(\bar{q}_L \overleftarrow{\mathcal{D}}_\mu^{BWA} \tilde{\phi} \mathcal{D}_\mu^{BA} t_R) + \mathrm{h.c.}] + \eta^t_{cr} [\bar{q}_L \tilde{\phi} t_R]$$

$$\mathcal{L}^{\tau}_{W+Y} = \tfrac{1}{2} b^2 \rho^\tau_{cr} [(\bar{\ell}_L \overleftarrow{\mathcal{D}}_\mu^{BW} \phi \mathcal{D}_\mu^{B} \tau_R) + \mathrm{h.c.}] + \eta^\tau_{cr} [\bar{\ell}_L \phi \tau_R]$$

$\Rightarrow \quad m_{\rm top} = \mathrm{O}(g_s^4 \rho^t_{cr} \rho^Q_{cr} N_T N_c) \Lambda_T \qquad m_\tau = \mathrm{O}(g_Y^4 \rho^\tau_{cr} \rho^Q_{cr} N_T N^Y_{Q+L}) \Lambda_T$

EL at momenta $\ll \Lambda_T$

- Tera-dof's with masses O($\Lambda_T \sim$ a few TeV's) are integrated out
- Tera-forces bind a $WW/ZZ = h$ state
- h resonance, $m_h \sim 125 \ll \Lambda_T$ is left behind
- We need to include this "light" $\chi_L \times \chi_R$ singlet in the EL
- One gets an EL very similar to the SM

SM vs. our critical model EL (no leptons & $U_Y(1)$)

- SM Lagrangian [$I(f)$ = weak isospin of flavour f]

$$L_{SM} = \frac{1}{4}F_W F_W + \frac{1}{4}F_A F_A + \sum_f (\bar{f}_R \not{D}_f^A f_R + \bar{f}_L \not{D}_f^{W,A} f_L) +$$

$$+\mathrm{Tr}[D_\mu^W \Phi\, D_\mu^W \Phi^\dagger] + V(\Phi) + \sum_f y_f (\bar{f}_L \phi_{I(f)} f_R + \bar{f}_R \phi^\dagger_{I(f)} f_L)$$

$$y_f = m_f/v \qquad \Phi = (\phi_{1/2} = -i\tau_2\phi^* \,|\, \phi_{-1/2} = \phi) = (v + h)1\!\!1 + i\vec{\tau}\vec{\pi}$$

- Our model Effective Lagrangian at momenta $\ll \Lambda_T$

$$L_{cr} = \frac{1}{4}F_W F_W + \frac{1}{4}F_A F_A + \sum_f (\bar{f}_R \not{D}_f^A f_R + \bar{f}_L \not{D}_f^{W,A} f_L) +$$

$$+\frac{1}{2}\partial_\mu h \partial_\mu h + [\underline{\underline{c\Lambda_T^2}} + c'\Lambda_T h + c'' h^2]\mathrm{Tr}[D_\mu^W U^\dagger D_\mu^W U] + V(h) +$$

$$+\sum_f [\underline{\underline{x_f \Lambda_T}} + k_f h](\bar{f}_L u_{I(f)} f_R + \bar{f}_R u^\dagger_{I(f)} f_L) + \ldots$$

$$U = \exp(i\vec{\tau}\vec{\pi}/\sqrt{c}\Lambda_T) = (u_{1/2} = -i\tau_2 u^\star \,|\, u_{-1/2} = u)$$

- If $x_f : k_f = 1 : 1$ and $c : c' : c'' = 1 : 2 : 1 \implies L_{SM} = L_{cr}$

Part VIII
Conclusions & Outlook

Conclusions

- We have identified a NP mechanism for mass generation
  - successfully tested in lattice simulations
- We speculated that $m_{NP} \propto \alpha^2 \Lambda_{RGI}$, then
  - $m_{top} \sim 170$ GeV calls for a superstrong interaction at a few TeV's
  - leading to an understanding of the
    - EW scale magnitude
    - fermion mass hierarchy owing to

$$\alpha_W \ll \alpha_s \ll \alpha_T$$

- We got a NP solution to the "naturalness" problem
  - symmetry enhancement ($\sim$ recovery of $\tilde{\chi}$) keeps masses small
- Phenomenology
  - need a good&convincing interpretation of 125 GeV resonance
    - we suggest it is a WW/ZZ-bound state with $E_{bind} = \mathrm{O}(g_w^4 M_W)$
  - need to study how the "low energy theory" deviates from SM

Outlook

A road to a full beyond-the-Standard-Model model (bSMm)

1. Done
   - introduce electro-weak interactions by gauging $\chi_L \times U_Y(1)$
   - include Tera-quarks and Tera-leptons
     - no need for Extended Techni-Color
   - all masses $\propto$ ($\Lambda_T \times$ powers of coupling constants)
   - at this stage neutrinos are massless
2. Comparing with SM (issues in order of decreasing energy)
   - strong and electro-weak coupling unification: no fast proton decay
   - Tera-resonances with $M_{H_T} \sim$ a few TeV's predicted
   - low energy "critical theory" looks very similar to the SM
3. To be done
   - Introduce families and split quark & lepton weak isospin doublets
   - Phenomenology
     - EW precision tests - S-parameter bounds "ok" as $m_T \sim \mathrm{O}(\Lambda_T)$
     - FCNCs to a comfortably low level (no tree-level FCNC processes)
     - heavy dark matter candidates, mass $\sim$ O(a few TeV's)

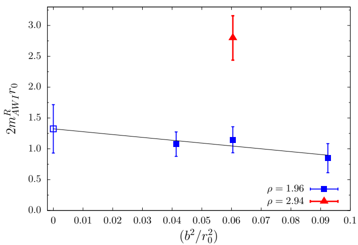
       $M_3 = \epsilon^{abc}(\bar{N}_\alpha U^\alpha_a)(\bar{N}_{\alpha'}\gamma_5 U^{\alpha'}_b)(\bar{L}_{\alpha''}\gamma_5 D^{\alpha''}_c) \leftarrow$ colour entanglement

       $\bar{B}_L B_Q = (\epsilon^{\alpha\beta\gamma}\bar{N}_\alpha \bar{L}_\beta \bar{L}_\gamma)\,(\epsilon_{abc}\epsilon^{\alpha'\beta'\gamma'} U^a_{\alpha'} D^b_{\beta'} D^c_{\gamma'}) \leftarrow$ e.m. binding

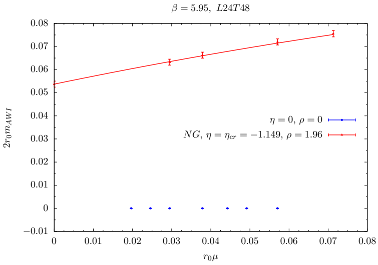
Thanks for your attention

Santa Fe' 1998

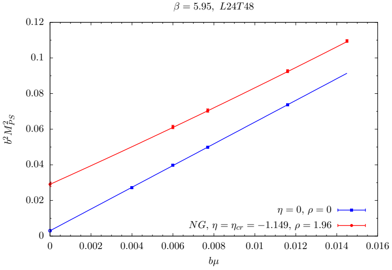
Back-up slides



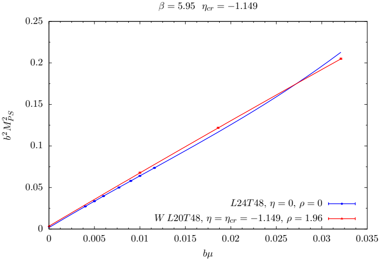
$2m^R_{AWI}r_0$
(b^2/r_0^2)
$\rho = 1.96$
$\rho = 2.94$



$\beta = 5.95,\ L24T48$
$2r_0 m_{AWI}$
$r_0\mu$
$\eta = 0,\ \rho = 0$
$NG,\ \eta = \eta_{cr} = -1.149,\ \rho = 1.96$

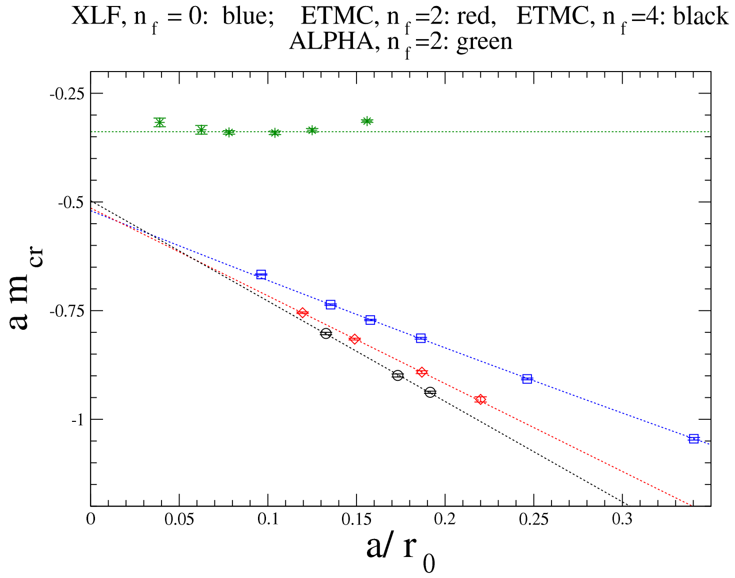


$\beta = 5.95,\ L24T48$
$b^2 M^2_{PS}$
$b\mu$
$\eta = 0,\ \rho = 0$
$NG,\ \eta = \eta_{cr} = -1.149,\ \rho = 1.96$



$\beta = 5.95\ \ \eta_{cr} = -1.149$
$b^2 M^2_{PS}$
$b\mu$
$L24T48,\ \eta = 0,\ \rho = 0$
$W\ L20T48,\ \eta = \eta_{cr} = -1.149,\ \rho = 1.96$

m_{cr} in L QCD


XLF, n f = 0: blue; ETMC, n f =2: red, ETMC, n f =4: black
ALPHA, n f =2: green
-0.25
-0.5
-0.75
-1
a m cr
0
0.05
0.1
0.15
0.2
0.25
0.3
a/ r 0

Towards a realistic bSMm: particle content

Hypothesis: effective mass of elementary particles stemming from our mechanism

Consistency of hypothesis with experimentally observed masses implies (hints at)

- new strong SU(N_T) interaction with RGI scale $\Lambda_T > M_W \gg \Lambda_{QCD}$
- new set of fermions subjected to the new force (besides to SM interactions)
  - $Q_L \in (N_T, 3, 2; Y_Q^L)$, $L_L \in (N_T, 1, 2; Y_L^L)$
  - $Q_R^u \in (N_T, 3, 1; Y_R^u)$, $L_R^u \in (N_T, 1, 1; Y_L^u)$
  - $Q_R^d \in (N_T, 3, 1; Y_R^d)$, $L_R^d \in (N_T, 1, 1; Y_L^d)$

with (irrep. of SU(N_T), SU(3)$_c$, SU(2)$_L$; $Y = Q_{em} - T_3$) ; besides SM fermions, e.g.

  - $q_L \in (1, 3, 2; 1/6)$, $\ell_L \in (1, 3, 2; -1/2)$
  - $t_R \in (1, 3, 1; 2/3)$, $\nu_R \in (1, 1, 1; 0)$
  - $b_R \in (1, 3, 1; -1/3)$, $\tau_R \in (1, 1, 1; -1)$

- composite higgs: a bound state in WW, ZZ, $Q\bar{Q}$, $L\bar{L}$ ... channel; new fermions & strong force crucial for binding; needed for unitarity of $WW \to WW$ scattering at LE

Isospin splitting from approximate symmetries

Impose further approximate symmetries to get $m_b < m_{top}$ and $m_\nu = 0$

1) Besides $\mathcal{L}$-invariance under $f(x) \to f(x) + \text{const}$, $\bar{f}(x) \to \bar{f}(x) + \text{const}$,
as $g_{s,w,y} \to 0$ for all fermion species f [Goltermann & Petcher, 1990]

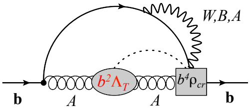
2) require invariance under $b_R(x) \to -b_R(x)$, $\bar{b}_R(x) \to -\bar{b}_R(x)$, as $g_{s,w,y} \to 0$

It forbids the "standard" $d = 6$ Wilson term of the b quark

1) + 2) allow

$$\mathcal{L}^{b}_{W+Y} = \frac{1}{2} b^4 \tilde{\rho}_{cr}^{q,d} \left(\bar{q}_L [\overleftarrow{\mathcal{D}}_\mu^{BWA}, \overleftarrow{\mathcal{D}}_\nu^{BWA}] \mathcal{D}_\mu^{BW} \phi \mathcal{D}_\nu^{BA} b_R + \text{h.c.} \right) + \eta_{cr}^{q,d} \bar{q}_L \phi b_R$$

$$\mathcal{L}^{\nu}_{W+Y} = \frac{1}{2} b^4 \tilde{\rho}_{cr}^{\ell,u} \left(\bar{\ell}_L [\overleftarrow{\mathcal{D}}_\mu^{BWA}, \overleftarrow{\mathcal{D}}_\nu^{BWA}] \mathcal{D}_\mu^{BW} \tilde{\phi}\, \partial_\nu \nu_R + \text{h.c.} \right) + \eta_{cr}^{\ell,u} \bar{\ell}_L \tilde{\phi} \nu_R$$



leading to $m_b = O(g_s^4 g_{s,w,y}^2 N_{s,w,y} \tilde{\rho}_{cr}^{q,d} \rho_{cr}^{Q} N_c N_T)\Lambda_T$ but still $m_\nu = 0$

125 GeV resonance (h): a $WW - ZZ$ bound state?

Loops of Tera-particles can bind $WW - ZZ$ yielding a "light" resonance with $m_h \ll \Lambda_T$

In $G_{V_3}(p) = \int dtd^3xd^3yd^3z\ e^{-ip_0t - i\vec{p}\vec{x} + i\vec{p}\vec{z}}\, V_3^{-2} \Big\langle W(\vec{x},t)W(\vec{y},t)W^\dagger(\vec{z},0)W^\dagger(\vec{0},0)\Big\rangle$

due to a large effective WW-WW coupling $\Delta_0^2 = \mathrm{O}(\Lambda_T^{2-2})\, g_W^4\, 4M_W^2$ one expects

$$G_{V_3}(p) \Rightarrow \frac{g_{analyt}(p^2)}{p^2 + 4M_W^2}\Big\{1 + \frac{\Delta_0^2(p^2)}{p^2 + 4M_W^2} + \dots\Big\} = \frac{g_{analyt}(p^2)}{p^2 + 4M_W^2 - \Delta_0^2(p^2)} \overset{p^2 \simeq -M_h^2}{\rightarrow} \frac{g_W^2 M_W^2}{-s + M_h^2}$$

$V_3 \to \infty$: besides a cut at $-p^2 > 4M_W^2$, sum over T-meson exchanges yields a pole

at $\quad p^2 = -M_h^2 = -4M_W^2 + \Delta_0^2(-M_h^2) \quad \leftrightarrow \quad M_h = 2M_W\Big(1 - \mathrm{O}\big(\frac{(m_{Q/L}^{eff})^2}{M^2_{\mathrm{T-meson}}}\big) g_W^4\Big)^{1/2}$