#### Towards electroweak reactions with quantum computers

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figure credit:  $\mu$ BooNE collab.

figure credit: IBM



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## What is a Quantum computer?



### Quantum Simulations with qubits

"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical."

— R.Feynman (1982)

• in 1996 S.Lloyd shows this intuition is correct for local interactions

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- in 1996 S.Lloyd shows this intuition is correct for local interactions
- choose a finite basis to discretize system  $\longrightarrow dim(\mathcal{H}) = \Omega \propto e^A$
- physical states can be mapped in states of  $\sim log_2(\Omega)$  qubits

$$|\Psi(t)\rangle = U(t) |\Psi(0)\rangle$$



## Exclusive cross sections in neutrino oscillation experiments





$$P(\nu_{\alpha} \to \nu_{\alpha}) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E_{\nu}}\right)$$

• need to use measured reaction products to constrain  $E_{\nu}$  of the event

DUNE, MiniBooNE, T2K, Miner $\nu$ a, NO $\nu$ A,...





Quantum algorithms for the nuclear response

$$R(\omega) = \int dt e^{i\omega t} C(t) \quad \text{with} \quad C(t) = \langle \Psi_0 | O(t) O(0) | \Psi_0 \rangle$$

Blueprint of quantum algorithms

state preparation  $\rightarrow$  unitary evolution  $\rightarrow$  measurement

$$|0\rangle \not - W_{GS} - O - U(t) - O - U^{\dagger}(t) - W_{GS}^{\dagger} - \swarrow$$

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strategy A

[Ortiz, Somma et al (2001-2003)]

- compute C(t) on quantum computer for different times
- perform Fourier transform classically
- strategy B

[Roggero & Carlson (2018)]

• sample directly final states from approximate response function

$$\ket{\Phi_B} = \sum_{\omega} \sqrt{R_{\Delta}(\omega)} \ket{\omega} \otimes \ket{\Psi_{\omega}}$$

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 $\circ$  both algorithms are poly in A and target energy resolution !

### Part I: baby steps on current machines



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credit: Atari Inc.

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- interaction Hamiltonian from pion-less EFT becomes

$$H = -t \sum_{k} X_{k} + U_{1} \sum_{k} Z_{k}$$
$$+ \sum_{i,j} U_{ij} Z_{i} Z_{j} + \sum_{i,j,k} V_{ijk} Z_{i} Z_{j} Z_{k}$$
$$+ \sum_{i,j,k,l} W_{ijkl} Z_{i} Z_{j} Z_{k} Z_{l}$$

## Preparation of an approximate ground state

#### Variational Quantum Eigensolver

Perruzzo(2014), McClean(2015), ...

Use Rayleigh-Ritz variational principle to find the lowest energy state

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• up to 36 parameters  $\longrightarrow$  could be reduced using symmetries

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• first problem: latency  $\longrightarrow$  more compact trial states, better optimizers



## Preparation of an approximate ground state III

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ullet second problem: persistence  $\longrightarrow$  track changes and reoptimize



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for perturbative noise:  $R(\epsilon) = R_0 + \epsilon R_1 + \epsilon^2 R_2 + \cdots$ 

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 $\circ~$  currently we are working hard on different mitigation techniques

#### Part II: back to neutrino scattering off $^{40}Ar$



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- need to repeat for many values of momentum transfer  $O \equiv O(q)$

#### Where are we right now?

figure adapted from Google AI

## Need Both Quality and Quantity



Threshold Theorem(s)Ben-Or, Aharonov, Kitaev, Knill, Gottesman,...When rate below threshold can extend  $\tau_{coh}^{eff} \rightarrow \infty$  with polylog(N) effortAlessandro RoggeroSanta Fe - 30 Aug 2019 12/14

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we need a quantum device with  $\approx 4000$  qubits (current record is 72)



# $\begin{array}{l} \mbox{coherence time for } {}^{40}\mbox{Ar} \\ \mbox{naive } \approx 9 \mbox{ years} \\ \mbox{optimized } \approx 3 \mbox{ minutes} \end{array}$

- algorithm efficiency is critical
- there is still a long way to go
- find new algorithms and/or approximations for near term

## Summary

- understanding low-energy dynamics of nuclear many-body systems is important for current and planned neutrino oscillation experiments
- QC is an emerging technology with the potential of revolutionarize the way theory calculations are done
- we already know how to simulate efficiently the time-evolution of non relativistic systems and how to study exclusive scattering
- more work has to be done to make all this viable in the near term

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- J. Carlson & R. Gupta (LANL)
- Andy Li & G. Perdue (FNAL)
- $\circ~\ensuremath{\mathsf{QPU}}$  access thanks to ORNL



#### Quantum Phase Estimation

Kitaev (1996), Brassard et al. (2002), Svore et. al (2013), Weibe & Granade (2016),...

QPE is a general algorithm to estimate eigenvalues of a unitary operator $U|\xi_k\rangle = \lambda_k |\xi_k\rangle \ , \lambda_k = e^{2\pi i \phi_k} \quad \Leftarrow \quad U = e^{-itH}$ 

- starting vector  $|\psi\rangle = \sum_k c_k |\xi_k\rangle$
- store time evolution  $|\psi(t)\rangle$  in auxiliary register of M qubits
- perform (Quantum) Fourier transform on the auxiliary register
- measures will return  $\lambda_n$  with probability  $P(\lambda_n) \approx |c_n|^2$



BONUS: final state after measurement is  $|\psi_{fin}\rangle \approx \sum_k \delta(\lambda_k - \lambda_n)c_k |\xi_k\rangle$