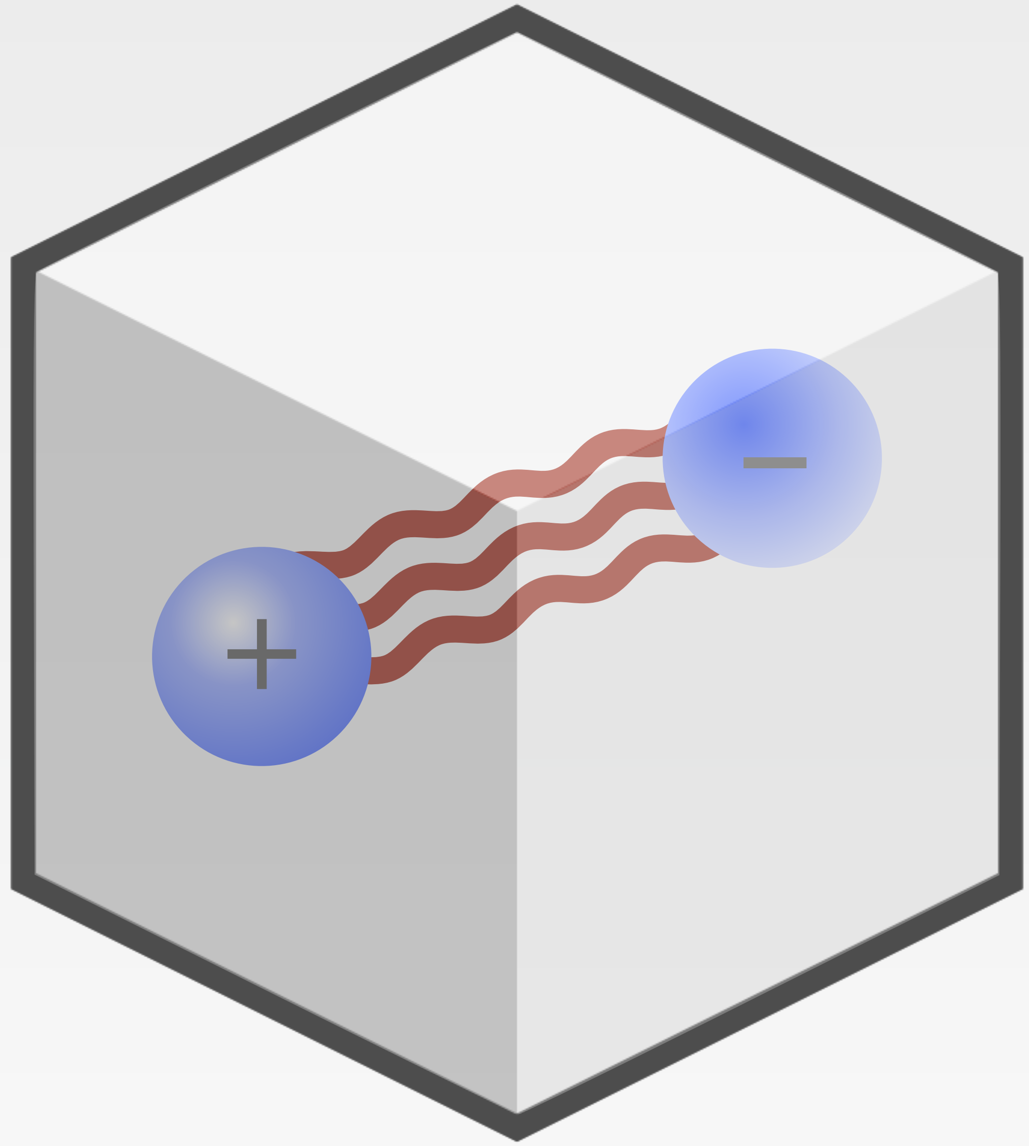


Progress on analytical calculations of EM finite-size effects

Antonin Portelli
27th of August 2019
Santa Fe, NM, USA

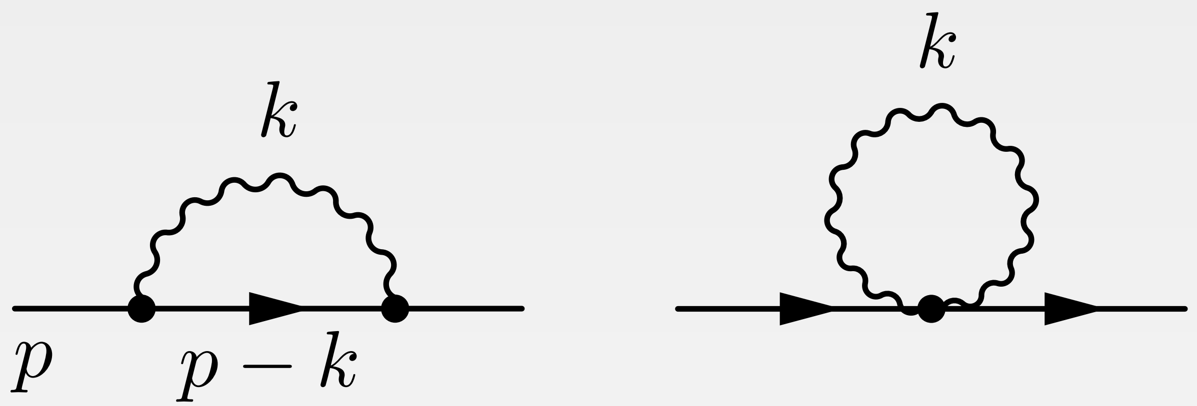
In collaboration with

Johan Bijnens (Lund U.)
Zohreh Davoudi (UMD)
James Harrison (Southampton U.)
Nils Hermansson Truedsson (Lund U./ITP Bern)
Andreas Jüttner (Southampton U.)
Martin Savage (UW)

[Davoudi *et al.*, PRD 99(3) 034510, 2019]
[Bijnens *et al.*, PRD 100(1) 014508, 2019]

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 757646.

QED$_L$ finite-volume effects on masses



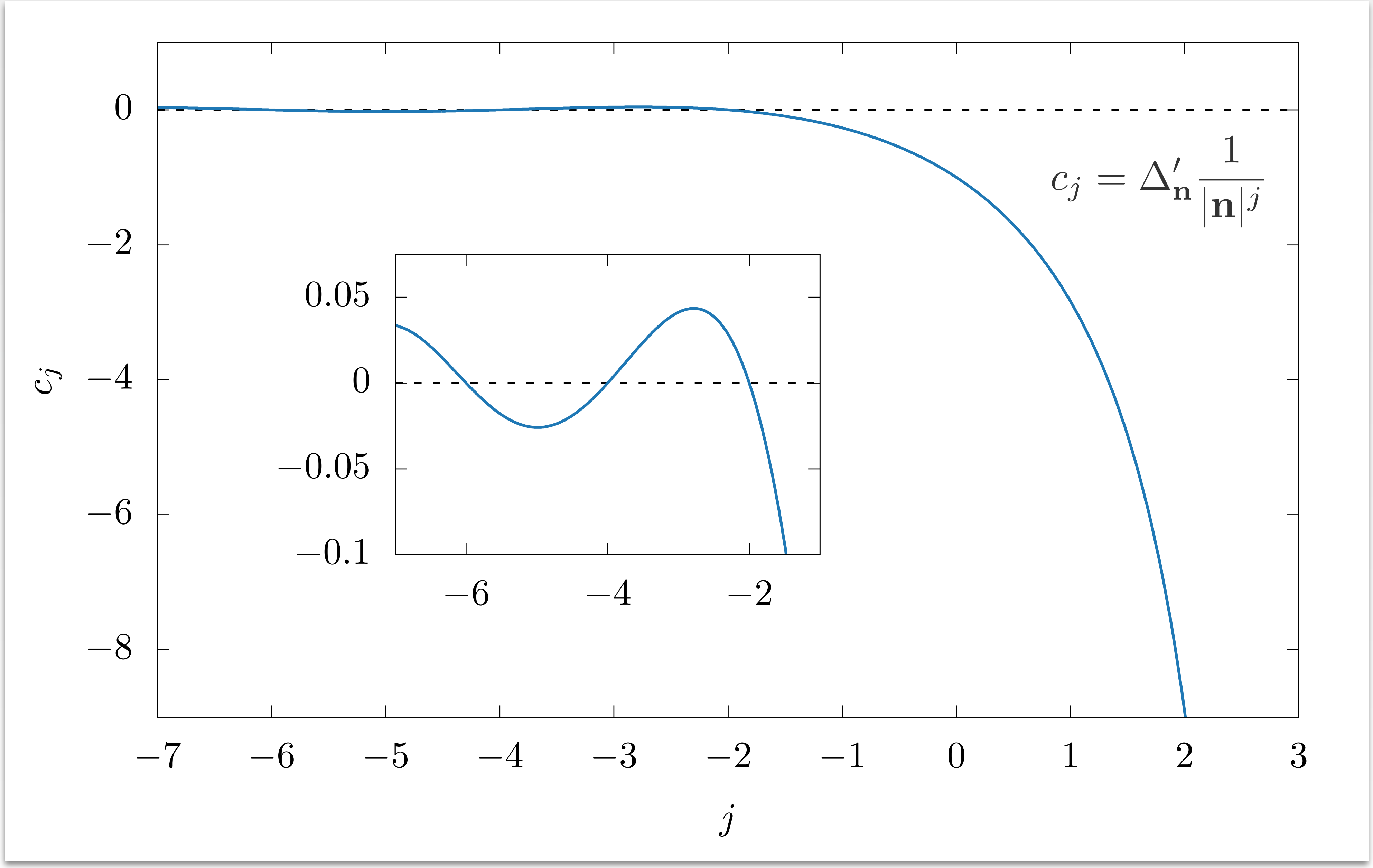
$$f(k) = \frac{4}{k^2} - \frac{(2p-k)^2}{k^2[(p-k)^2+m^2]}$$

$$\int \frac{\mathrm{d}k_0}{2\pi} f(k) = \frac{m}{|\mathbf{k}|^2} + \frac{1}{|\mathbf{k}|} + R(\mathbf{k})$$

Analytic in $\mathbf{k}$, vanish at $|\mathbf{k}| = 0$

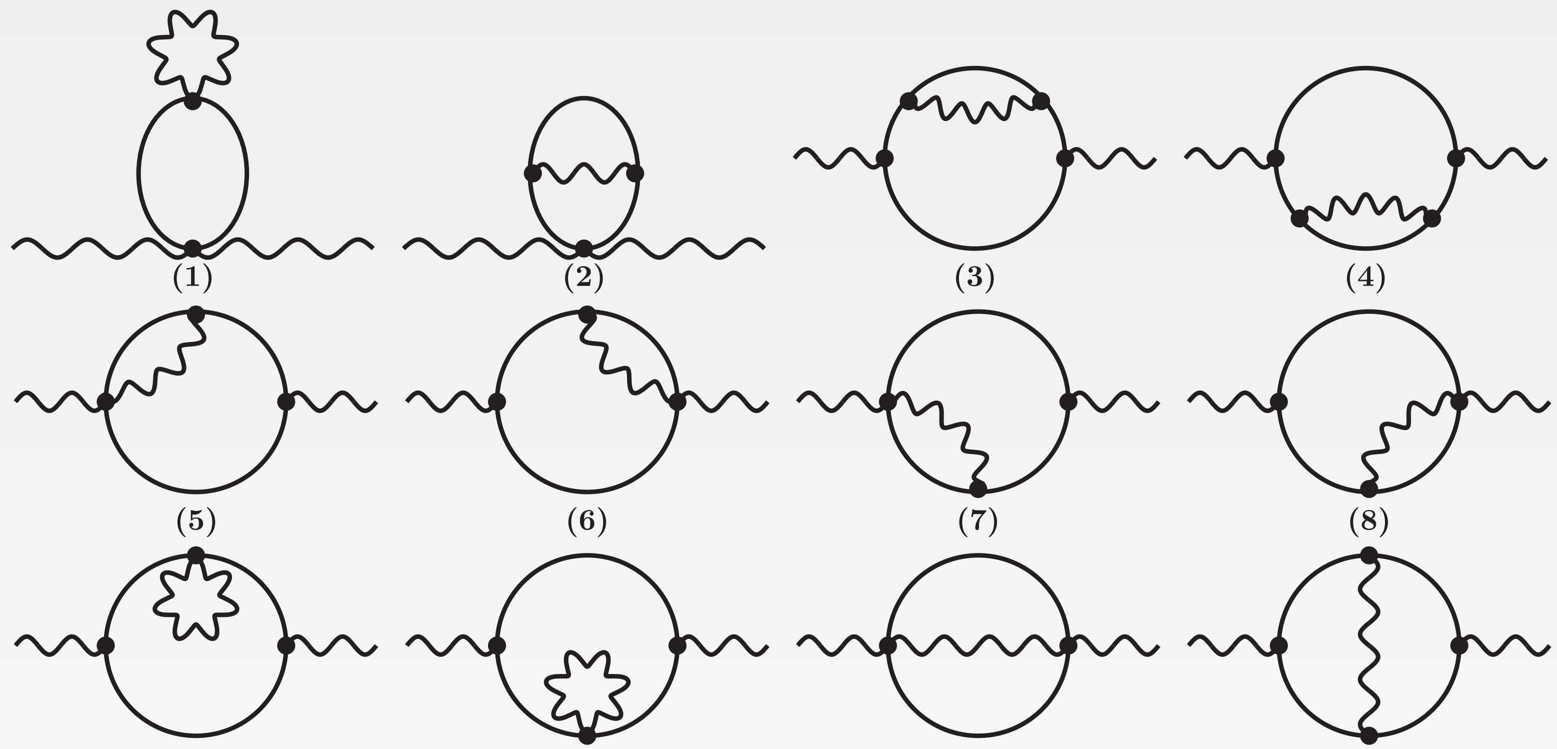
$$\Delta m^2 = \frac{c_2 m}{4\pi^2 L} + \frac{c_1}{2\pi L^2} + \cdots \qquad c_j = \Delta'_{\mathbf{n}} \frac{1}{|\mathbf{n}|^j}$$

Finite-volume coefficients



$$c_1 = -2.83730 \qquad c_2 = \pi c_1 = -8.91363$$

QED corrections to the HVP



$= 0$

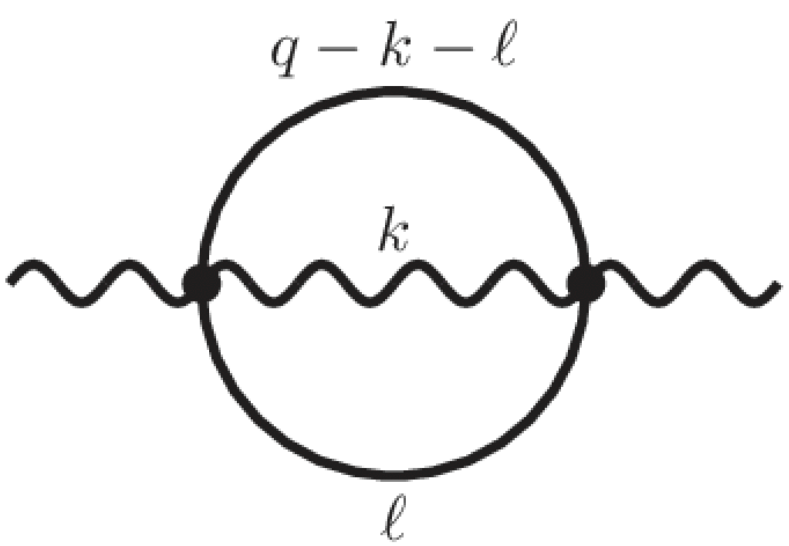
$$\Delta\Pi(z) = \frac{c_1}{\pi m^2 L^2}\left(\frac{16}{3}\Omega_{-1,3} - \frac{1}{3}\Omega_{1,2} - \frac{32}{3}\Omega_{1,3} - \frac{2}{3}\Omega_{3,2} + \frac{16}{3}\Omega_{3,3} - \frac{1}{8}\Omega_{5,1} + \Omega_{5,2}\right)$$

$$- \frac{1}{m^3 L^3}\left(-\frac{128}{3}\Omega_{-2,4} + \frac{256}{3}\Omega_{0,4} - \frac{5}{3}\Omega_{2,2} + \frac{8}{3}\Omega_{2,3} - \frac{128}{3}\Omega_{2,4}\right.$$

$$\left.- \frac{3}{8}\Omega_{4,1} + \frac{7}{6}\Omega_{4,2} - \frac{8}{3}\Omega_{4,3}\right) + \mathcal{O}\left(\frac{1}{L^4}, e^{-mL}\right)$$

Automatisation

Two-loop scalar sunset (S)



Expressions

```
dpiS = FullFV[4/((k0^2 + |k|^2) (l0^2 + ωl^2) ((k0 + l0 - q0)^2 + ωkl^2))]
```

-- computing k0 & l0 integrals...

$$-\frac{1}{\omega kl\,\omega l\,|k|\,(\omega kl+\omega l+|k|)\,\left(q0^2+(\omega kl+\omega l)^2+2\,(\omega kl+\omega l)\,|k|+|k|^2\right)}$$

-- computing small |k| expansion...

$$-\frac{1}{\left(2\,q0^2\,\omega l^3+8\,\omega l^5\right)\,|k|}+\frac{q0^2+12\,\omega l^2-\left(3\,q0^2+20\,\omega l^2\right)\,\hat{k}.v}{4\,\omega l^4\,\left(q0^2+4\,\omega l^2\right)^2}+O[|k|]^1$$

-- computing l integral...

$$-\frac{c_1\,\Omega_{3,1}[z]}{4\,(m^2\,\pi)\,L^2}+\frac{-\Omega_{4,1}[z]+8\,\Omega_{2,1}'[z]}{4\,m^3\,L^3}+O\left[\frac{1}{L}\right]^4$$

-- computing Ω substitution...

$$\frac{\left(\sqrt{z}-\sqrt{4+z}\,\mathrm{ArcCsch}\left[\frac{2}{\sqrt{z}}\right]\right)\,c_1}{8\,m^2\,\pi^3\,z^{3/2}\,L^2}-\frac{-8+z+\frac{16}{\sqrt{4+z}}}{32\,\left(m^3\,\pi\,z^2\right)\,L^3}+O\left[\frac{1}{L}\right]^4$$

QED corrections to $K_{\ell 2}$

$$\xi_3 = \Delta_{\mathrm{FV}} \left\{ \frac{1}{(k^2+\lambda^2)[(p_P-k)^2+M_P^2)][(p_\ell-k)^2+m_\ell^2)]} \right\}$$

- [Lubicz *et al.*, PRD 95(3) 034504, 2017]

$$\xi_3 = -\frac{1}{16\pi^2}\left\{ \frac{M_P}{3\omega_\ell} - \frac{\log(M_P^2/m_\ell^2)}{2(1-r_\ell^2)} \left[\log\left(\frac{L^2\lambda^2}{4\pi}\right) + \gamma_E \right] + K_{31} + K_{32} \right\}$$

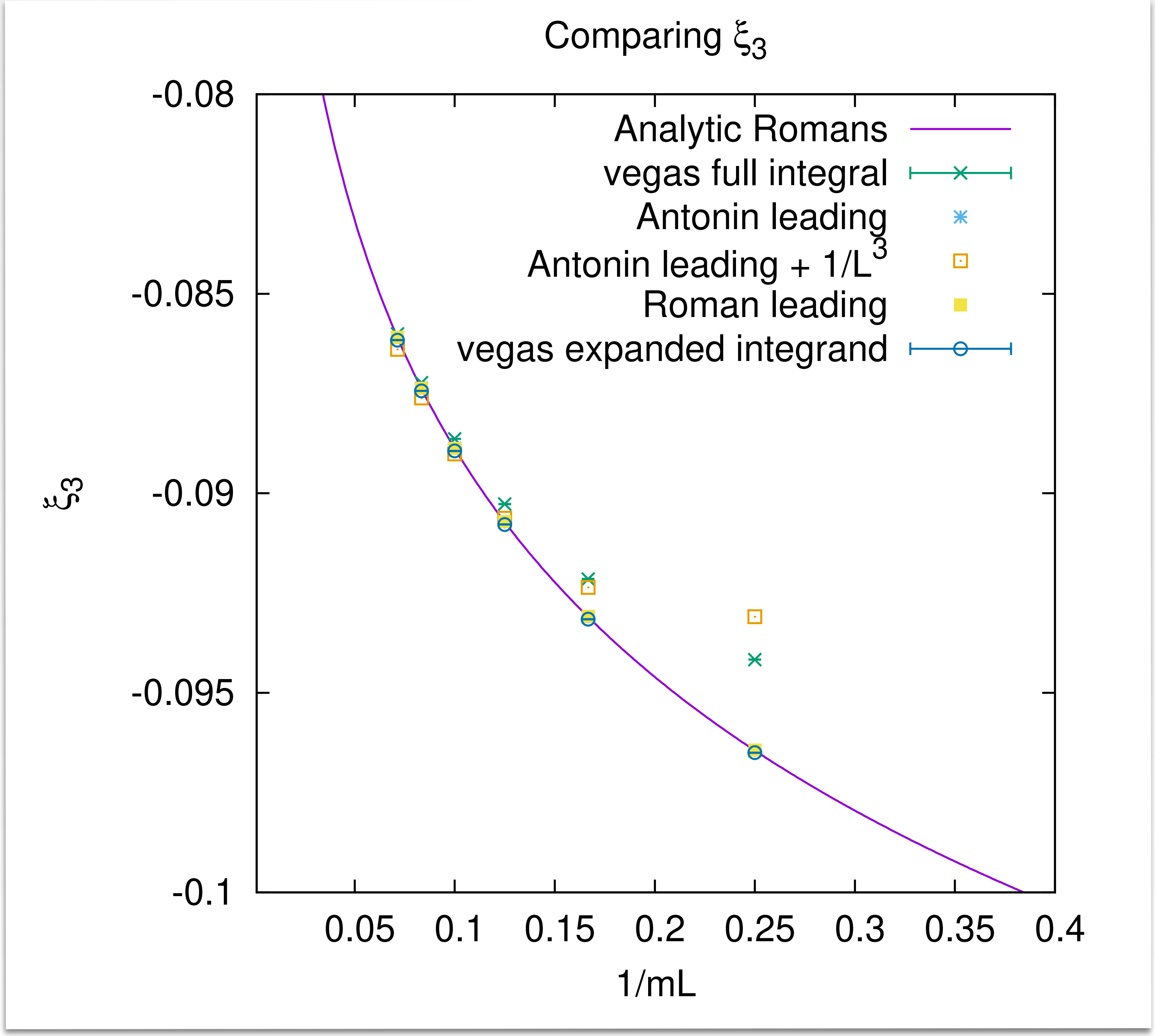
- Us

Equal

$$\xi_3 = \frac{M_P c_3(\mathbf{v})}{64\pi^3\omega_\ell} + \frac{M_P}{16\pi^2\omega_\ell}\frac{\eta}{|\mathbf{v}|}\left[1+\log\left(\frac{L\lambda}{4\pi}\right)\right] + \frac{(\omega_\ell+M_P)(\omega_\ell^2+M_P^2)}{32\omega_\ell^4 M_P^2 L^3}$$

?

QED corrections to $K_{\ell 2}$



Next steps

- Structure dependent FV corrections to $K_{\ell 2}$
- FV corrections to $K_{\ell 3}$
- Scattering with QED?

Finite-volume coefficients