α_{s} from a tiny heavy universe

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Introduction

Motivation

The strong coupling $\alpha_s \equiv \alpha_{\overline{\rm MS}}^{(5)}(m_Z)$

- Fundamental parameter of the SM
- Enters all pQCD processes @ LHC
- Impacts vacuum stability, unification arguments, searches of new coloured sectors, ...

What is the situation?

see e.g. (d'Enterria *et al.* '19)

- $\delta\alpha_s/\alpha_s \approx 1\%$: least-precisely known SM coupling
- Leads to **relevant** uncertainties in key processes

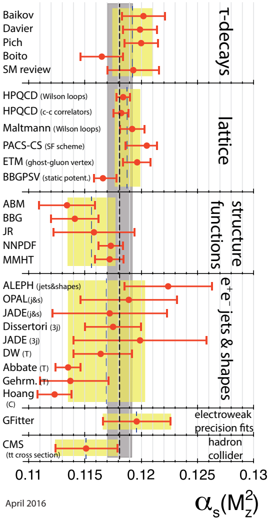
$$H \to b\bar{b}, \quad H \to c\bar{c}, \quad H \leftrightarrow gg, \quad \dots$$

- **Limiting** factor for precision determinations at future colliders, e.g., top-quark mass
- Many current determinations are limited by **systematic** uncertainties

Problem

We need **accurate** measurements in order to reach **sub-percent** precision!

PDG (2016/18)



$$\alpha_s = 0.1181(11)$$

Introduction

How do we determine α_s?

Phenomenology

$$\mathcal{O}_{\mathrm{exp}}(q) \overset{q\to\infty}{\approx} \sum_{n=0}^{N} c_n\, \alpha_{\mathrm{s}}^{n}(q) + \mathrm{O}(\alpha_{\mathrm{s}}^{N+1}) + \mathrm{O}\left(\frac{\Lambda^p}{q^p}\right) \qquad \left[\alpha_{\mathcal{O}}(q) \equiv \frac{\mathcal{O}(q) - c_0}{c_1}\right]$$

Sources of uncertainty

see e.g. (Salam '18)

- Experimental errors on $\mathcal{O}_{\mathrm{exp}}$
- Impact of the unknown c_n
- Size (and form) of "non-perturbative corrections"
- Uncertainties on other fundamental parameters
- Missing higher-order electroweak contributions
- …and (why not) new physics contributions

What about lattice QCD?

Fix $g_0, m_{0,u}, m_{0,d}, \ldots$ using low-energy inputs, replace $\mathcal{O}_{\mathrm{exp}}$ with $\mathcal{O}_{\mathrm{lat}}$, and extract α_{s}!

Why is this a good idea?

- Lots of freedom in choosing $\mathcal{O}_{\mathrm{lat}}$; no need to be experimentally measurable
- $\mathcal{O}_{\mathrm{lat}}$ is defined within QCD alone; Euclidean signature helps too
- $\mathcal{O}_{\mathrm{lat}}(q)$ can be measured non-perturbatively up to arbitrary large q
- Does not rely on any hadronization model

Lattice QCD determinations of $\alpha_{\rm s}$

Finite-volume schemes: couplings from finite-volume effects

Caveat: Typically to have systematic errors under control for $\mathcal{O}_{\rm lat}(q)$ requires, e.g.,

$$m_\pi L \sim 4 \quad \textbf{and} \quad \Lambda \ll q \ll a^{-1} \sim 10\,\mathrm{GeV} \;\Rightarrow\; \boldsymbol{L/a > 100} \;\to\; \textbf{HARD!}$$

Solution: Take for $\mathcal{O}_{\rm lat}$ a **finite-volume effect** (Wilson; ...; Lüscher, Weisz, Wolff '92)

$$\mathcal{O}_{\rm lat}(L) \;\propto\; \bar{g}^2_{\mathcal{O}}(\mu = L^{-1}) \;\Rightarrow\; \mu \ll a^{-1} \;\Rightarrow\; L/a \gg 1 \;\to\; \textbf{EASY!}$$

Step-scaling function

$$\sigma(u) = \bar{g}^2_{\mathcal{O}}(\mu/2)\big|^{u=\bar{g}^2_{\mathcal{O}}(\mu)}_{\overline{m}(\mu)=0} \qquad\qquad \sigma(u) = \lim_{a/L\to 0} \Sigma(a/L, u)$$

β-function

$$\log 2 = \int_{\sqrt{\sigma(u)}}^{\sqrt{u}} \frac{\mathrm{d}x}{\beta_{\mathcal{O}}(x)} \qquad\qquad \frac{\mathrm{d}\bar{g}_{\mathcal{O}}(\mu)}{\mathrm{d}\ln\mu} = \beta_{\mathcal{O}}(\bar{g}) \overset{\bar{g}\to 0}{\approx} -b_0\bar{g}^3 - b_1\bar{g}^5 - b_2^{\mathcal{O}}\bar{g}^7 + \ldots$$

Λ-parameter

$$\Lambda^{(N_{\rm f})}_{\mathcal{O}} = \mu\,\varphi^{(N_{\rm f})}_{\mathcal{O}}(\bar{g}(\mu)) \qquad\qquad \varphi^{(N_{\rm f})}_{\mathcal{O}}(\bar{g}(\mu)) = \ldots \times \exp\left\{ -\int_0^{\bar{g}(\mu)} \frac{\mathrm{d}x}{\beta^{(N_{\rm f})}_{\mathcal{O}}(x)} + \ldots \right\}$$

- Non-perturbatively defined **RGI** $[\,\mathrm{d}\Lambda_{\mathcal{O}}/\mathrm{d}\mu = 0\,]$
- $\Lambda_{\rm X}/\Lambda_{\rm Y} = e^{c_{\rm XY}/2b_0}$ **exactly**, where $\bar{g}^2_{\rm X}(\mu) = \bar{g}^2_{\rm Y}(\mu) + c_{\rm XY}\,\bar{g}^4_{\rm Y}(\mu) + \mathrm{O}(\bar{g}^6_{\rm Y})$
- Defined for a **fixed** number $N_{\rm f}$ of **massless** quarks. **For $\alpha_{\rm s}$ we need $\Lambda^{(5)}_{\overline{\rm MS}}$!**

Lattice QCD determinations of α_{s}

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Λ-parameter

$$\Lambda^{(N_\mathrm{f})}_{\mathcal{O}} = \mu_{\mathrm{had}}\,\varphi^{(N_\mathrm{f})}_{\mathcal{O}}(\bar{g}(\mu_{\mathrm{had}})) \qquad \mu_{\mathrm{had}} = \mathrm{O}(100\,\mathrm{MeV}) \qquad \beta^{\mathrm{PT}}_{\mathcal{O}}(\bar{g}) = -\bar{g}^3 \sum_{k=0}^{N-1} b_k \bar{g}^{2k}$$

The challenge is …

$$\varphi_{\mathcal{O}}(\bar{g}(\mu_{\mathrm{had}})) \overset{\mu_{\mathrm{PT}}\to\infty}{\approx} \ldots \times \exp\left\{ -\int_{\bar{g}(\mu_{\mathrm{PT}})}^{\bar{g}(\mu_{\mathrm{had}})} \frac{\mathrm{d}x}{\beta_{\mathcal{O}}(x)} - \int_0^{\bar{g}(\mu_{\mathrm{PT}})} \frac{\mathrm{d}x}{\beta^{\mathrm{PT}}_{\mathcal{O}}(x)} \right\} + \mathrm{O}(\bar{g}^{2N-2}(\mu_{\mathrm{PT}}))$$

α_s from the femto universe

The QCD coupling from a non-perturbative determination of $\Lambda^{(3)}_{\overline{\rm MS}}$

Master formula

(Bruno, MDB, Fritzsch, Korzec, Ramos, Schaefer, Simma, Sint, Sommer '17)

$$\Lambda^{(5)}_{\overline{\rm MS}} = \left[\frac{\Lambda^{(5)}_{\overline{\rm MS}}}{\Lambda^{(3)}_{\overline{\rm MS}}}\right]_{\rm PT} \times \Lambda^{(3)}_{\overline{\rm MS}} \qquad \Lambda^{(3)}_{\overline{\rm MS}} = \underbrace{\frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mu_{\rm had}}}_{\text{RUNNING}} \times \underbrace{\mu_{\rm had}\sqrt{8t_0} \times \frac{1}{\sqrt{8t_0}f_{\pi K}} \times f^{(\rm PDG)}_{\pi K}}_{\text{SCALE SETTING}}$$

Strategy

- **Running:** Through the NP running from $\mu_{\rm had} \approx 0.2\,{\rm GeV}$ to $\mu_{\rm PT} \approx 120\,{\rm GeV}$ of two finite-volume couplings, $\bar{g}^2_{\rm SF}(\mu)$ and $\bar{g}^2_{\rm GF}(\mu)$, we obtain

$$\frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mu_{\rm had}} = 1.729(57) \approx 3.3\% \qquad [\,\bar{g}^2_{\rm GF}(\mu_{\rm had}) = 11.31\,]$$

- **Scale-setting:** We first relate $\mu_{\rm had}$ to $\sqrt{8t_0}$ (large volume) and express the latter in physical units through f_π, f_K. This yields

$$1/\sqrt{8t_0} = 478(7)\,{\rm MeV} \approx 1.5\% \quad \Rightarrow \quad \Lambda^{(3)}_{\overline{\rm MS}} = 341(12)\,{\rm MeV} \approx 3.5\%$$

- **Matching:** Using PT to compute the ratios $\Lambda^{(4,5)}_{\overline{\rm MS}}/\Lambda^{(3)}_{\overline{\rm MS}}$ we find

$$\Lambda^{(3)}_{\overline{\rm MS}} \to \Lambda^{(4)}_{\overline{\rm MS}} \to \Lambda^{(5)}_{\overline{\rm MS}} = 215(10)(3)_{\rm PT}\,{\rm MeV} + {\rm O}(M_c^{-2}) + {\rm O}(M_b^{-2})$$

$$\alpha^{(5)}_{\overline{\rm MS}}(m_Z) = 0.1185(8)(3)_{\rm PT} \approx 0.7\%$$

Heavy-quark effects

Perturbative decoupling

Fundamental theory

$$\mathcal{L}_{\mathrm{QCD}_{\mathrm{N_f}}} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{f=1}^{N_\mathrm{l}} \overline{\psi}_f \not{D} \psi_f + \sum_{f=N_\mathrm{l}+1}^{N_\mathrm{f}} \overline{\psi}_f (\not{D} + M) \psi_f$$

Effective theory

(Weinberg '80; ...)

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{QCD}_{\mathrm{N_l}}} + \frac{1}{M^2} \sum_i \omega_i \Phi_i + \ldots \qquad \Rightarrow \qquad \mathbf{LO:} \quad \mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{QCD}_{\mathrm{N_l}}}$$

Matching the couplings

(Bernreuther, Wetzel '82; ...; Chetyrkin, Kühn, Sturm '06; Schröder, Steinhauser '06)

$$\bar{g}^{(N_\mathrm{l})}_{\overline{\mathrm{MS}}}(m^*) = \bar{g}^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(m^*)\, \widetilde{C}\big(\bar{g}^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(m^*)\big) \qquad \overline{m}_{\overline{\mathrm{MS}}}(m^*) = m^* \qquad \Big[g_* \equiv \bar{g}^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(m^*) \Big]$$

In terms of RGI quantities

$$\Lambda^{(N_\mathrm{l})}_{\overline{\mathrm{MS}}}(M, \Lambda^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}) = P_{\mathrm{l,f}}(M/\Lambda^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}})\, \Lambda^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}} \qquad \Rightarrow \qquad P_{\mathrm{l,f}}(M/\Lambda^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}) = \frac{\varphi^{(N_\mathrm{l})}_{\overline{\mathrm{MS}}}\big(g_* \widetilde{C}(g_*)\big)}{\varphi^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(g_*)}$$

where

$$M = \overline{m}_{\overline{\mathrm{MS}}}(\mu)\, \varepsilon^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(\bar{g}(\mu)) \qquad \varepsilon^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(\bar{g}(\mu)) = \ldots \times \exp\left\{ -\int_0^{\bar{g}(\mu)} \mathrm{d}x \frac{\tau^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(x)}{\beta^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(x)} + \ldots \right\}$$

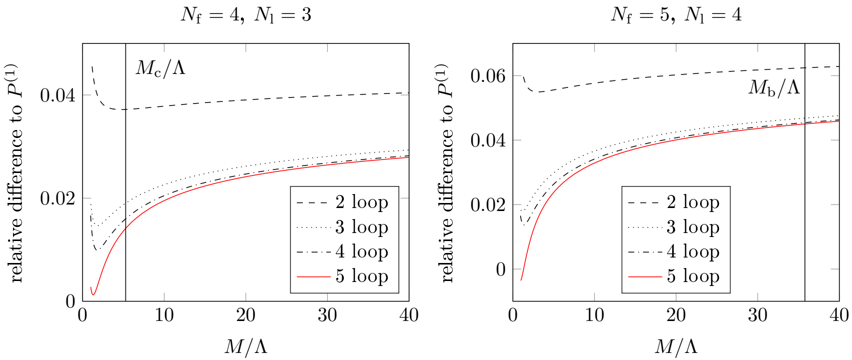
and $g_* \equiv g_*(M/\Lambda^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}})$ is solution of

$$\frac{\Lambda^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}}{M} = \frac{\varphi^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(g_*)}{\varepsilon^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}}(g_*)} \qquad \Rightarrow \qquad g_* \xrightarrow{M/\Lambda \to \infty} 0 \qquad \Rightarrow \text{ We can use PT for } P_{\mathrm{l,f}}(M/\Lambda \gg 1)!$$

Heavy-quark effects

Perturbative decoupling (cont.)

(Athenodorou *et al.* '18)



- PT expansion shows very **good** "convergence"
- PT uncertainties are therefore quite **small**

Q: But can perturbative decoupling really be trusted at M_c/Λ?

A: Yes!

n-loop	$\alpha^{(5)}_{\overline{\rm MS}}(m_Z)$	$\alpha_n - \alpha_{n-1}$
2	0.11699	
3	0.11827	0.00128
4	0.11846	0.00019
5	0.11852	0.00006

$$\alpha^{(5)}_{\overline{\rm MS}}(m_Z) = 0.1185(8)(3)_{\rm PT}$$

How perturbative are heavy sea quarks?

Charm decoupling: perturbative or non-perturbative?

Non-perturbative matching

$$\frac{\Lambda^{(N_\mathrm{l})}}{\mathcal{S}_{N_\mathrm{l}}} = P^{\mathcal{S}}_{\mathrm{l,f}}\left(M/\Lambda^{(N_\mathrm{f})}\right)\frac{\Lambda^{(N_\mathrm{f})}}{\mathcal{S}_{N_\mathrm{f}}(M)} \quad \Rightarrow \quad \mathcal{S}_{N_\mathrm{l}} = \mathcal{S}_{N_\mathrm{f}}(M) + \mathrm{O}\left(\frac{\Lambda^2}{M^2}\right)$$

Ratios of scales

$$\frac{\mathcal{S}_{N_\mathrm{f}}(M)}{\mathcal{S}'_{N_\mathrm{f}}(M)} = \frac{\mathcal{S}_{N_\mathrm{l}}}{\mathcal{S}'_{N_\mathrm{l}}} + \mathrm{O}\left(\frac{\Lambda^2}{M^2}\right) \quad \text{e.g.} \quad \mathcal{S} = \left\{(8t_0)^{-\frac{1}{2}}, r_0, w_0, \dots\right\} \equiv \text{low-energy scale}$$

Result: Typical $\mathrm{O}(\Lambda^2/M_c^2)$ corrections to such ratios are $< 0.5\%$ effects (Knechtli *et al.* '17)

$\Rightarrow N_\mathrm{f} = 3$ QCD is good enough for a **per-cent** precision **scale setting**!

Factorization formula

(Bruno *et al.* '15; Athenodorou *et al.* '18)

$$\frac{\mathcal{S}_{N_\mathrm{f}}(M)}{\mathcal{S}_{N_\mathrm{f}}(0)} = \mathcal{Q}^{S}_{\mathrm{l,f}} \times P^{\mathcal{S}}_{\mathrm{l,f}}\left(M/\Lambda^{(N_\mathrm{f})}\right) \qquad \mathcal{Q}_{\mathrm{l,f}} = \frac{\mathcal{S}_{N_\mathrm{l}}/\Lambda^{(N_\mathrm{l})}}{\mathcal{S}_{N_\mathrm{f}}(0)/\Lambda^{(N_\mathrm{f})}}$$

$$= \underbrace{\mathcal{Q}^{S}_{\mathrm{l,f}}}_{\text{NP \& } M\text{-indep.}} \times \underbrace{P_{\mathrm{l,f}}\left(M/\Lambda^{(N_\mathrm{f})}\right)}_{\text{PT \& universal}} + \mathrm{O}\left(\frac{\Lambda^2}{M^2}\right) \qquad P^{\mathcal{S}}_{\mathrm{l,f}} = P^{\mathcal{S}'}_{\mathrm{l,f}} + \mathrm{O}\left(\frac{\Lambda^2}{M^2}\right) \sim P_{\mathrm{l,f}}$$

Result: Typical $\mathrm{O}(\Lambda^2/M_c^2)$ corrections to $P_{\mathrm{l,f}}(M_c/\Lambda)$ are $< 1\%$ effects (Athenodorou *et al.* '18)

$\Rightarrow \Lambda^{(3)}_{\overline{\mathrm{MS}}} \overset{\mathrm{PT}}{\to} \Lambda^{(4,5)}_{\overline{\mathrm{MS}}}$ is **precise** enough if $\delta\Lambda^{(3)}_{\overline{\mathrm{MS}}}/\Lambda^{(3)}_{\overline{\mathrm{MS}}} \gtrsim 2\%$!

$\Lambda^{(3)}_{\overline{\rm MS}}$ from the decoupling of heavy quarks

Decoupling as a tool for non-perturbative renormalization

Matching (again)

$$\frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mathcal{S}_3(M)} = \frac{\Lambda^{(0)}_{\overline{\rm MS}}}{\mathcal{S}_0} P_{0,3}\left(M/\Lambda^{(3)}_{\overline{\rm MS}}\right)^{-1} \quad \text{with} \quad \mathcal{S}_0 = \mathcal{S}_3(M) + \mathrm{O}\left(\frac{\Lambda^2}{M^2}\right) + \mathrm{O}\left(\frac{\mathcal{S}^2}{M^2}\right)$$

Basic ingredients

$$\mathcal{S} = \mu_{\rm dec} \quad \text{with} \quad \bar{g}^{(3)}_{\mathcal{O}}(\mu_{\rm dec}, z) = g_M \qquad z = M/\mu_{\rm dec}$$

Decoupling

$$\bar{g}^{(3)}_{\mathcal{O}}(\mu_{\rm dec}, z) = g_M = \bar{g}^{(0)}_{\mathcal{O}}(\mu'_{\rm dec}) \quad \Rightarrow \quad \mu'_{\rm dec} = \mu_{\rm dec} + \mathrm{O}\left(\frac{\Lambda^2}{M^2}\right) + \mathrm{O}\left(\frac{\mu^2_{\rm dec}}{M^2}\right)$$

Λ-parameters $\left[\text{using } \Lambda^{(0)}_{\overline{\rm MS}}/\mu_{\rm dec} = (\Lambda^{(0)}_{\overline{\rm MS}}/\Lambda^{(0)}_{\mathcal{O}})\varphi^{(0)}_{\mathcal{O}}(g_M)\right]$

$$\frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mu_{\rm dec}} = \frac{\Lambda^{(0)}_{\overline{\rm MS}}}{\Lambda^{(0)}_{\mathcal{O}}} \times \underbrace{\varphi^{(0)}_{\mathcal{O}}(g_M)}_{\text{NP}} \times \underbrace{P_{0,3}\left(M/\Lambda^{(3)}_{\overline{\rm MS}}\right)^{-1}}_{\text{High-order PT}} + \mathrm{O}\left(\frac{\Lambda^2}{M^2}\right) + \mathrm{O}\left(\frac{\mu^2_{\rm dec}}{M^2}\right)$$

How do we set the scale?

$$g_M = \Psi_M(g_\emptyset, z) \quad \text{with} \quad g_\emptyset = \bar{g}^{(3)}_{\mathcal{O}}(\mu_{\rm dec}, 0)$$

Remarks

- Using a finite-volume coupling allows for larger M; better PT for $P_{\rm l,f}(M/\Lambda)$
- $\mu_{\rm dec}$ is set in physical units within $\boldsymbol{N_{\rm f} = 3}$ **QCD**, e.g., computing $\mu_{\rm dec}\sqrt{8t_0}$
- The running from $\mu_{\rm dec}$ to $\mu_{\rm PT} = \mathrm{O}(100\,\mathrm{GeV})$ is done in **pure YM**!

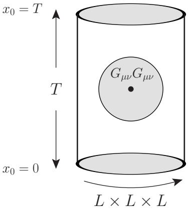
The gradient flow coupling

A new finite-volume coupling

(Lüscher '10; Fodor *et al.* '12; Fritzsch, Ramos '13)

Definition

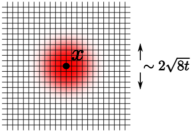
1. Finite volume with Schrödinger functional (SF) bc.'s
2. $\overline{g}^2_{\rm GF}(L^{-1}) \propto t^2 \langle \mathrm{tr}\{G_{\mu\nu}(t,x) G_{\mu\nu}(t,x)\}\rangle_{\rm SF}\big|_{\sqrt{8t}=0.3\times L}$
3. For $N_{\rm f}=3$, $\overline{m}_{u,d,s}(L^{-1})$ are fixed through M (s. later)



Gradient flow (GF)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) \qquad B_\mu(0,x) = A_\mu(x)$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu] \qquad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$



(Lüscher, Weisz '11)

- ✓ Composite fields are automatically renormalized
- ✓ Simple to evaluate in Monte Carlo simulations
- ✓ $\mathrm{var}(\overline{g}^2_{\rm GF})$ is small and finite as $a \to 0$
- ✗ Largish lattice artefacts
- ✗ PT is quite involved

The scale of SU(3) Yang-Mills theory

The GF coupling at high-energy

(MDB, Ramos '19)

Every trick in the book

- Different coupling definitions (Fritzsch, Ramos '13)

  Ele: $\bar{g}^2_{\mathrm{GF,ele}} \propto t^2 \langle G_{0k} G_{0k} \rangle$

  Mag: $\bar{g}^2_{\mathrm{GF,mag}} \propto t^2 \langle G_{kl} G_{kl} \rangle$

- Different lattice discretizations (Ramos, Sint '16)
- Projection to $Q = 0$ (Fritzsch, Ramos, Stollenwerk '14)
- NNLO β-function (MDB, Lüscher '17)
- NP matching to SF scheme (MDB *et al.* '16, '18)
- High-statistics and fine lattice resolutions

What did we learn?

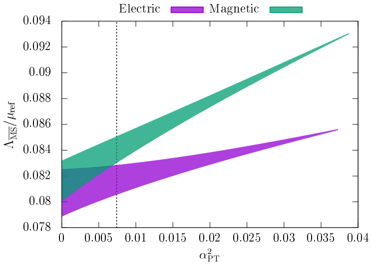
- Accurate NP determination of $\varphi^{(0)}_{\mathrm{GF}}(\bar{g})$

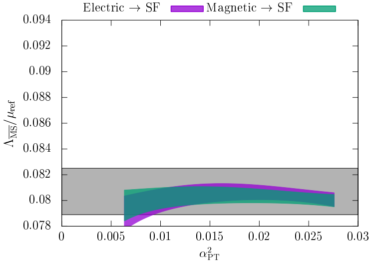
  $\bar{g}(\mu) \approx [1, 12] \Rightarrow \mu \approx 0.2 - 200\,\mathrm{GeV}$

- Precise determination of

  $\sqrt{8t_0}\Lambda^{(0)}_{\overline{\mathrm{MS}}} = 0.6227(98) \approx 1.6\%$

- Large higher-order PT corrections for the GF couplings: we change to $\bar{g}^2_{\mathrm{SF}}$ at high-energy





$\mu_{\mathrm{ref}} \approx 3.3\,\mathrm{GeV}$

$\Lambda_{\overline{\rm MS}}^{(3)}$ from the decoupling of heavy quarks (PRELIMINARY)

Line of constant physics and massive couplings

Decoupling scale

$$[\bar{g}^{(3)}_{\rm GF}(\mu_{\rm dec},0)]^2 = 3.95 \quad \Rightarrow \quad \mu_{\rm dec} = L_{\rm dec}^{-1} \approx 0.8\,{\rm GeV}$$

Using the running of $\bar{g}^{(3)}_{\rm GF}(\mu_{\rm dec},0)$ we can relate $\mu_{\rm dec}$ to $\sqrt{8t_0}$ and fix its physical units

(MDB *et al.* '16)

$L_{\rm dec}/a$	$\beta = 6/g_0^2$	$[\bar{g}^{(3)}_{\rm GF}(\mu_{\rm dec},0)]^2$	$\mu_{\rm dec}$ (GeV)
12	4.3020	3.9533(59)	0.789(15)
16	4.4662	3.9496(77)	0.789(15)
20	4.5997	3.9648(97)	0.789(15)
24	4.7141	3.959(50)	0.789(15)
32	4.9000	3.949(11)	0.789(15)

Quark-masses

using the results of (Campos *et al.* '18)

$$z = M/\mu_{\rm dec} = Z_{\rm RGI}(g_0, a\mu_{\rm dec})\,[1 + b_{\rm m}^{1-\rm loop}(g_0)(am_0 - am_c)]\,(L_{\rm dec}/a)(am_0 - am_c)$$

Massive couplings

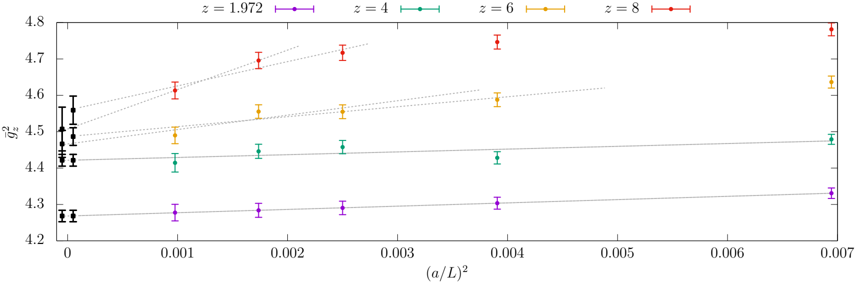
$$\left[\,\tilde{g}_0^2 = g_0^2\left(1 + b_{\rm g}^{1-\rm loop}(g_0)(am_0 - am_c)\right)\,\right]$$

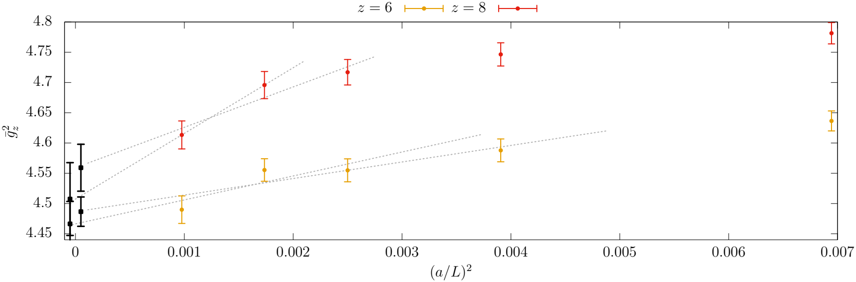
Example
$L/a = 20$

$\tilde{\beta} = 6/\tilde{g}_0^2$	$\kappa = (2m_0+8)^{-1}$	$z = M/\mu_{\rm dec}$	M (GeV)	$[\bar{g}^{(3)}_{\rm GF}(\mu_{\rm dec},z)]^2$
4.5997	0.135288900000	0	0	3.9648(97)
4.6083	0.133831710060	1.972(18)	1.6	4.290(15)
4.6172	0.132345249425	4.000(37)	3.2	4.458(14)
4.6266	0.130827894135	6.000(58)	4.7	4.555(14)
4.6364	0.129273827559	8.000(85)	6.3	4.717(14)

$\Lambda_{\overline{\mathrm{MS}}}^{(3)}$ from the decoupling of heavy quarks (PRELIMINARY)

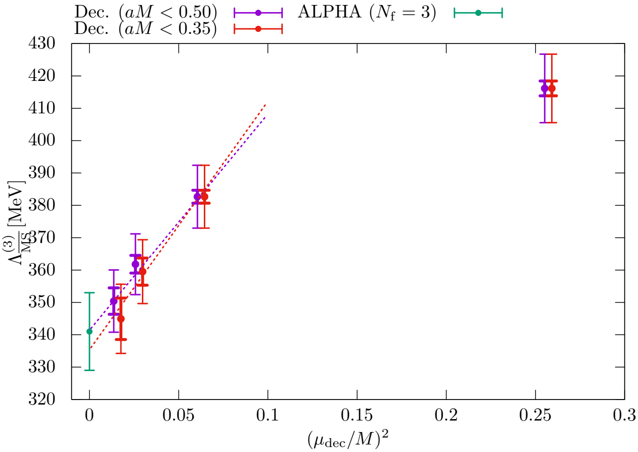
Continuum limit extrapolations of the massive couplings: $\bar{g}_z^2 \equiv [\bar{g}_{\mathrm{GF}}^{(3)}(\mu_{\mathrm{dec}}, z)]^2$





$\Lambda_{\overline{\mathrm{MS}}}^{(3)}$ from the decoupling of heavy quarks (PRELIMINARY)

M (GeV)	$[\bar{g}^{(3)}_{\mathrm{GF}}(\mu_{\mathrm{dec}}, z)]^2$	μ_{dec} (GeV)	$\times$	$\Lambda^{(0)}_{\overline{\mathrm{MS}}}/\mu_{\mathrm{dec}}$	$\times$	$[P_{0,3}(M/\Lambda)]^{-1}$	$=$	$\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ (MeV)
1.6	4.559(39)	0.789(15)		0.689(11)		0.7662(44)		416(11)
3.2	4.421(16)	0.789(15)		0.725(11)		0.6693(37)		382.7(96)
4.7	4.466(37)	0.789(15)		0.741(12)		0.6198(34)		362.0(92)
6.3	4.507(60)	0.789(15)		0.757(13)		0.5871(32)		350.3(92)



Conclusions and outlook

Conclusions

- Computing α_s precisely requires **non-perturbative** control over a wide range of energy scales
- LQCD in conjunction with **finite-volume** couplings and **step-scaling** is a powerful tool for this task
- **Perturbative decoupling** of heavy quarks **works** remarkably well and non-perturbative corrections are **below** the per-cent level in Λ
- α_s can be computed **precisely** by setting the scale in $N_\mathrm{f} = 3$ QCD and running to infinite energy in pure YM!

Outlook

- With this strategy we can reach a very **competitive** precision

$$\delta\Lambda^{(3)}_{\overline{\mathrm{MS}}}/\Lambda^{(3)}_{\overline{\mathrm{MS}}} \approx 2\% \;\Rightarrow\; \delta\alpha_\mathrm{s}/\alpha_\mathrm{s} \approx 0.5\,!$$

- To further halve this error several things need to be reconsidered: non-perturbative decoupling effects, isopisn breaking effects, …
- The idea is general and can be applied to other renormalization problems, e.g. quark-masses


Thanks

BACKUP

Non-perturbative running couplings in $N_\mathrm{f} = 3$ QCD

(MDB, Fritzsch, Korzec, Ramos, Sint, Sommer 16', 17', '18; Bruno, MDB, Fritzsch, Korzec, Ramos, Schaefer, Simma, Sint, Sommer '17)