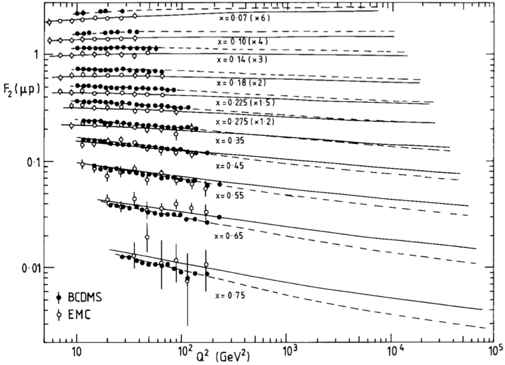
Quasi PDF as observables

L Del Debbio

Higgs Centre for Theoretical Physics
University of Edinburgh

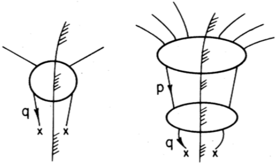


in progress/collaboration with K Cichy, T Giani

based on Collins 1980

bilocal operators

$$\mathcal{F}(k^+) = k^+ \int \frac{dy^-}{2\pi}\, e^{-ik^+y^-}\, \phi(y)\phi(0)$$



(a) (b)

$$\mathcal{F}_{R,\overline{\mathrm{MS}}}(k^+) = \int_1^\infty \frac{d\eta}{\eta}\, K(\eta, g_R, \epsilon)\, \mathcal{F}(\eta k^+, g_R, m_R, \mu, \epsilon)$$
$$= \int_0^1 \frac{d\eta}{\eta}\, \mathcal{K}(\eta, g_R, \epsilon)\, \mathcal{F}(k^+/\eta, g_R, m_R, \mu, \epsilon)$$

parton distribution functions

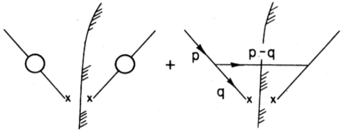
$$\boxed{f(x) = \langle P|\mathcal{F}(xP^+)|P\rangle}$$

$$\begin{aligned} f_R(x) = \langle P|\mathcal{F}_R(xP^+)|P\rangle &= \int_1^{1/x} \frac{d\eta}{\eta}\, K(\eta)\, f(\eta x) \\ &= \int_x^1 \frac{d\eta}{\eta}\, \mathcal{K}(\eta)\, f(x/\eta) = \mathcal{K} \otimes f(x) \end{aligned}$$

renormalization at 1-loop

$$\hat{f}_R(x) = \int_x^1 d\xi\, \xi^{-1}\, \hat{f}_0(\xi)\, K(\xi/x, \alpha)$$
$$K(\eta, \alpha) = (1 + \alpha\,\kappa)\,\delta(1-\eta) + \alpha\, K^{(1)}(\eta) + \mathcal{O}\left(\alpha^2\right)$$

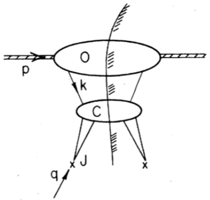
$$\hat{f}_0\left(x, \mu^2, \epsilon\right) = z\left(\mu^2, \epsilon\right)\, \delta(1-x) + \alpha_R(\mu)\, \hat{f}_0^{(1)}\left(x, \mu^2, \epsilon\right) + \mathcal{O}\left(\alpha^2\right)$$



$$K(\eta, \alpha) = \left(1 - \frac{\alpha}{12}\frac{1}{\epsilon}\right)\delta(1-\eta) - \alpha\,\frac{1}{\epsilon}\frac{\eta - 1}{\eta^2} + \mathcal{O}\left(\alpha^2\right).$$

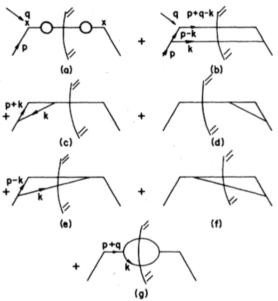
deep-inelastic scattering

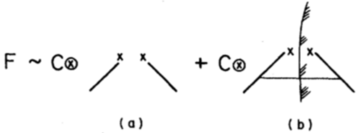
$$\boxed{F(x,Q^2) = \int dy\, e^{iqy} \langle P | j_R(x) j_R(0) | P \rangle}$$



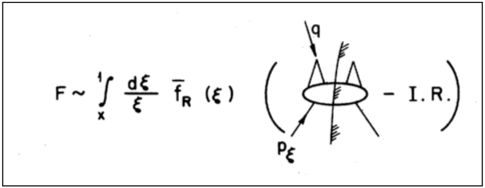
$$F(x,Q^2) \sim \int_1^{1/x} \frac{d\eta}{\eta}\, C(\eta, Q^2/\mu^2, g_R)\, f_R(\eta x, g_R, m_R, \mu)$$
$$= \mathcal{C} \otimes f_R(x) + \ldots$$

DIS at 1-loop





IR picture



$$F(x,Q^2) \sim \int_x^1 \frac{d\xi}{\xi}\, f(\xi)\, \hat{F}(x/\xi, Q^2)$$
$$\sim \int_x^1 \frac{d\xi}{\xi}\, f_R(\xi)\, \mathcal{C}(x/\xi)$$

where

$$\int_x^1 \frac{d\xi}{\xi}\, \mathcal{K}(x/\xi)\, \mathcal{C}(\xi) = \hat{F}(x, Q^2)$$

$\mathcal{C}$ is IR-finite

IR picture at 1-loop

$$F\left(x, Q^2\right) = \int_x^1 \frac{d\xi}{\xi} f_R\left(\xi\right) \left(\hat{F}\left(\frac{x}{\xi}, Q^2\right) - \mathsf{IR}\right)$$

$$f_R\left(x\right) = \int_x^1 d\xi\, \xi^{-1}\, f_0\left(\xi\right) \bar{K}\left(\xi/x, \alpha\right)$$

$$F\left(x, Q^2\right) = \int_x^1 \frac{d\xi}{\xi}\, f_R\left(\xi\right)\, \left(1 - \alpha\, \bar{\kappa}\right)\, \hat{F}\left(\frac{x}{\xi}, Q^2\right) - \\ - \alpha \int_x^1 \frac{d\xi}{\xi} \int_1^{1/\xi} \frac{d\eta}{\eta}\, \bar{K}^{(1)}\left(\eta\right)\, f_R\left(\eta\xi\right)\, \hat{F}\left(\frac{x}{\xi}, Q^2\right)$$

$$\bar{K}\left(\eta, \alpha\right) = \left(1 - \frac{\alpha}{12}\frac{1}{\epsilon}\right) \delta\left(1 - \eta\right) - \alpha\, \frac{1}{\epsilon}\frac{\eta - 1}{\eta^2} = K(\eta, \alpha)$$

quasi-PDF

$$q\left(x, P_z\right) = \int_x^1 \frac{d\xi}{\xi} f_0\left(\xi\right) \hat{q}\left(\frac{x}{\xi}, P_z\right) = \int_x^1 \frac{d\xi}{\xi} f_R\left(\xi\right) \left(\hat{q}\left(\frac{x}{\xi}, P_z\right) - \mathsf{IR}\right)$$

$$\begin{aligned} q\left(x, P_z\right) = & \int_x^1 \frac{d\xi}{\xi} f_R\left(\xi\right) \left(1 - \alpha\,\kappa\right) \hat{q}\left(\frac{x}{\xi}, Q^2\right) - \\ & - \alpha \int_x^1 \frac{d\xi}{\xi} \int_1^{1/\xi} \frac{d\eta}{\eta} K^{(1)}\left(\eta\right) f_R\left(\eta\xi\right) \hat{q}\left(\frac{x}{\xi}, P_z\right) \\ = & \int_x^1 \frac{d\xi}{\xi} f_R\left(\xi\right) \left[\left(1 - \alpha\,\kappa\right) \hat{q}\left(\frac{x}{\xi}, P_z\right) - \alpha\, K^{(1)}\left(\frac{\xi}{x}\right)\right] \end{aligned}$$

QCD matrix elements

$$\boxed{\mathcal{M}_{\Gamma,A}(\zeta) = \bar{\psi}(\zeta)\Gamma\lambda_A \, \mathrm{P}\exp\left(-ig\int_0^\zeta d\eta\, A(\eta)\right)\psi(0)}$$

Ioffe time distributions

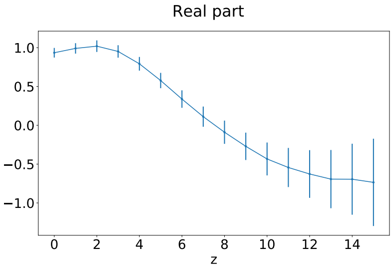
$$M_{\gamma^\mu,A}(\zeta, P) = \langle P|\mathcal{M}_{\gamma^\mu,A}(\zeta)|P\rangle$$

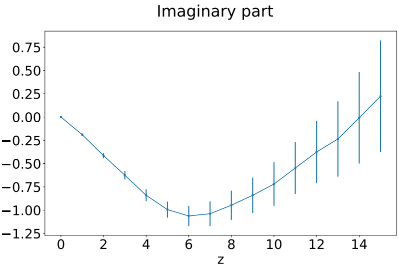
Lorentz covariance

$$M_{\gamma^\mu,A}(\zeta, P) = P^\mu h_{\gamma^\mu,A}(\zeta\cdot P, z^2) + \zeta^\mu h'_{\gamma^\mu,A}(\zeta\cdot P, z^2)$$

lattice data as observables

$$\mathcal{O}^{\mathsf{Re}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \mathsf{Re}\left[\mathsf{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right] \qquad \mathcal{O}^{\mathsf{Im}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \mathsf{Im}\left[\mathsf{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right]$$





[C Alexandrou et al 18]

lattice observables

inverse Fourier transform

$$\mathcal{O}^{\mathsf{Re}}_{\gamma^0}\left(zP_z, z^2\right) = \int_{-\infty}^{\infty} dx\, \cos\left(xP_z z\right) \int_{-1}^{+1} \frac{dy}{|y|}\, C_3\left(\frac{x}{y}, \frac{\mu}{|y|P_z}\right)\, f_3\left(y, \mu^2\right)$$

$$\mathcal{O}^{\mathsf{Im}}_{\gamma^0}\left(zP_z, z^2\right) = \int_{-\infty}^{\infty} dx\, \sin\left(xP_z z\right) \int_{-1}^{+1} \frac{dy}{|y|}\, C_3\left(\frac{x}{y}, \frac{\mu}{|y|P_z}\right)\, f_3\left(y, \mu^2\right)$$

$$f_3\left(x, \mu^2\right) = \begin{cases} u\left(x, \mu^2\right) - d\left(x, \mu^2\right) & \text{if} \quad x > 0 \\ -\bar{u}\left(-x, \mu^2\right) + \bar{d}\left(-x, \mu^2\right) & \text{if} \quad x < 0 \end{cases}$$

factorization formula for ME

using the explicit expressions for C_3

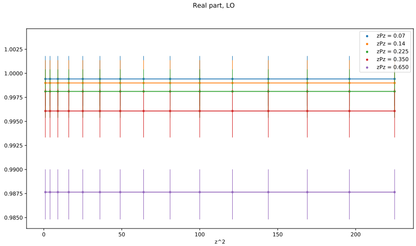
$$\mathcal{O}^{\mathsf{Re}}_{\gamma^0}(z,\mu) = \int_0^1 dx\ \mathcal{C}_3^{\mathsf{Re}}\left(x, z, \frac{\mu}{P_z}\right) V_3(x,\mu) = \mathcal{C}_3^{\mathsf{Re}}\left(z, \frac{\mu}{P_z}\right) \circledast V_3\left(\mu^2\right)$$
$$\mathcal{O}^{\mathsf{Im}}_{\gamma^0}(z,\mu) = \int_0^1 dx\ \mathcal{C}_3^{\mathsf{Im}}\left(x, z, \frac{\mu}{P_z}\right) T_3(x,\mu) = \mathcal{C}_3^{\mathsf{Im}}\left(z, \frac{\mu}{P_z}\right) \circledast T_3\left(\mu^2\right)$$

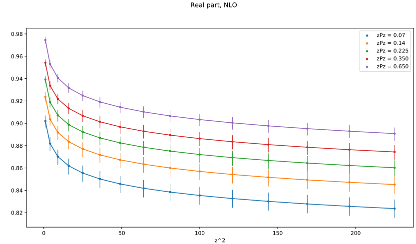
where V_3 and T_3 are the nonsinglet distributions defined by

$$V_3(x) = u(x) - \bar{u}(x) - \left[d(x) - \bar{d}(x)\right]$$
$$T_3(x) = u(x) + \bar{u}(x) - \left[d(x) + \bar{d}(x)\right]$$

$$\mathrm{LO}: \quad \mathcal{O}^{\mathsf{Re}}_{\gamma^0}\left(zP_z, z^2\right) = \int dx \cos(zP_z x) V_3(x,\mu^2)$$

Bjorken scaling of ME



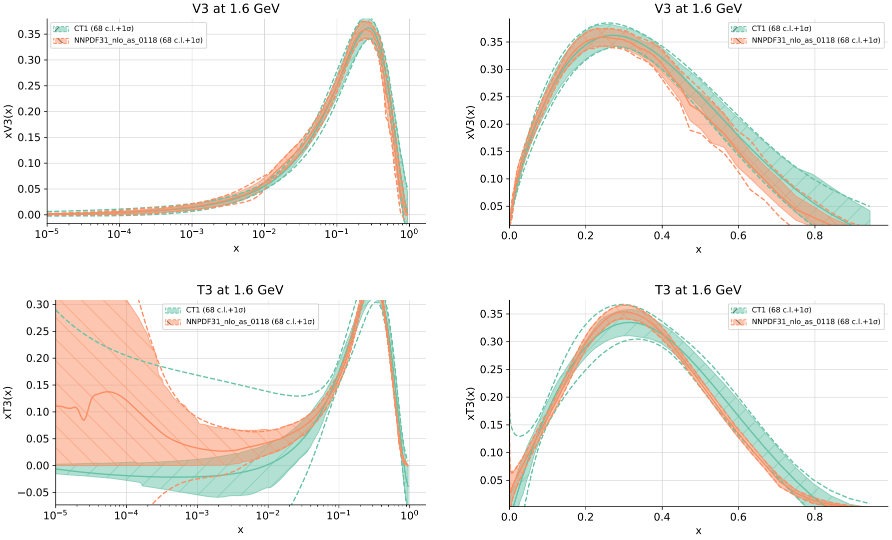


systematic errors

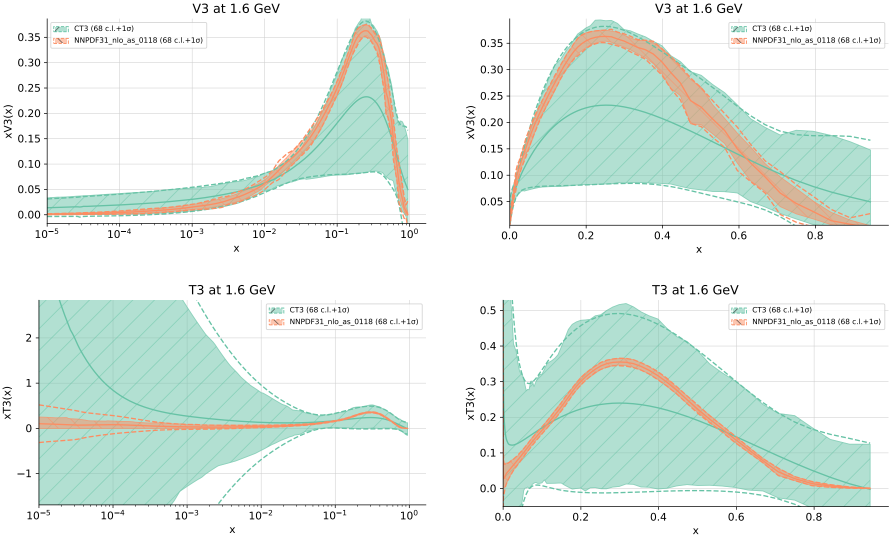
- cut-off effects
- finite volume effects
- excited states contamination
- truncation effects
- higher-twist terms
- isospin breaking

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

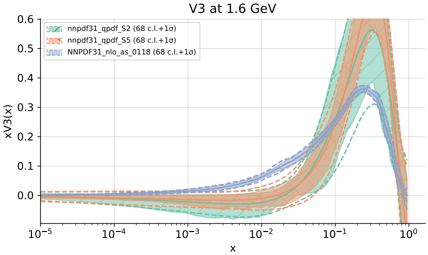
closure test – 1

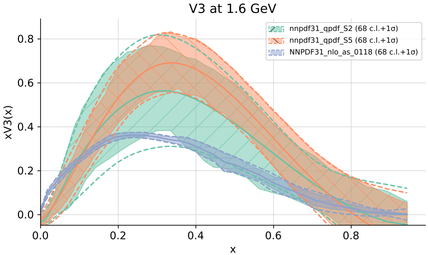


closure test – 2



fit results


V3 at 1.6 GeV
nnpdf31_qpdf_S2 (68 c.l.+1σ)
nnpdf31_qpdf_S5 (68 c.l.+1σ)
NNPDF31_nlo_as_0118 (68 c.l.+1σ)
xV3(x)
x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xV3(x)
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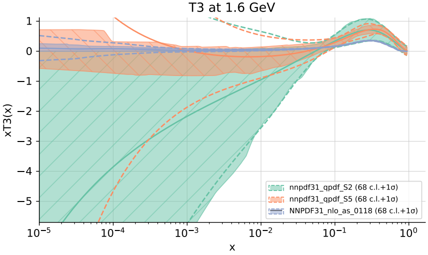
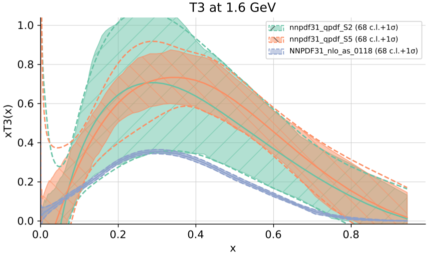

T3 at 1.6 GeV
nnpdf31_qpdf_S2 (68 c.l.+1σ)
nnpdf31_qpdf_S5 (68 c.l.+1σ)
NNPDF31_nlo_as_0118 (68 c.l.+1σ)
xT3(x)
x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NNPDF31_nlo_as_0118 (68 c.l.+1σ)
xT3(x)
x

outlook

- light-cone PDFs + factorization describe the structure of the proton
- necessary input for the exploitation of LHC, HL-LHC
- current extraction from data is very precise + improving
- lattice data provide complementary information, can be included in global fits like any other data
- identify the areas where a significant phenomenological impact from lattice QCD is possible