Quark Masses (from Lattice QCD)

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Fundamental Parameters

- In SM (and various extensions), the product of a fundamental parameter (eigenvalue of a Yukawa matrix) and the vev of the (a) Higgs field.
- Isolating QCD from the (boring, because perturbative) weak and BSM interactions, they are fundamental parameters of the Lagrangian:
  - a change in the value of m_q changes measurable observables, *e.g.*, hadron masses or jet properties.
- Must deal with confining nature of the strong interactions:
  - make it smaller than other uncertainties (e.g., $t\bar{t}$ production at colliders);
  - calculate the relationship between quark masses and hadron masses $\Leftarrow$ lattice QCD: adjust lattice bare mass m_0 to a hadron mass.

Conversion aka Renormalization

- Any Lagrangian bare mass depends, by definition, on the UV regulator, in particular on the formulation of lattice fermions on the level of Symanzik improvement of the action:
  - different formulations—different chiral symmetry from different ways of coping with the fermion doubling problem;
  - Symanzik improvement—different discretization errors at finite UV cutoff.
- Need a standard definition to make the results portable:
  - regulator independent definitions (would be best);
  - $\overline{\mathrm{MS}}$ because dimensional regularization is so popular.

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never nonperturbative, hence always incomplete

Four Methods

- Mass renormalization [$\bar{m} = m_{\overline{\rm MS}}(m_{\overline{\rm MS}})$, $m_{\rm cr} \neq 0$ for Wilson fermions]:
  - $\bar{m} = Z_m(m_0 - m_{\rm cr})$, with Z_m either ≤ 2 loops or nonperturbatively.
- Ward identities:
  - $m_{\rm AW} Z_P \langle P \rangle = Z_A \langle \partial \cdot A \rangle$, run $m_{\rm AW}$ to high scale μ & convert to $m_{\overline{\rm MS}}(\mu)$.
- Continuum limit ⊗ continuum pQCD:
  - $\lim_{a\to 0} \mathscr{G}_n(a) = G_n + \Lambda^{n+1}/m_Q^n$, $G_n/\bar{m}_Q = 1 + \sum_k^{K\geq 2} G_{n,k} \alpha_s^{k+1}$.
- Continuum limit ⊗ HQET ⊗ continuum pQCD:
  - $M = m_{\rm MRS} + \bar{\Lambda}_{\rm MRS} + \cdots$, *e.g.*, heavy-light meson masses.

Mass Renormalization (Nonperturbative)

Review

- In perturbative quantum field theory, one fixes a gauge.
- Then quark correlation functions don't vanish by gauge symmetry.
- Renormalization can be carried out by subtractions in momentum space, namely

$$Z_q = -\frac{ip_\mu}{12p^2} \operatorname*{tr}_{\text{spin,color}} \left[\partial S^{-1}(p)\gamma^\mu\right]_{p^2=\mu^2}$$

$$Z_S^{-1} Z_q = \frac{1}{12} \operatorname*{tr}_{\text{spin,color}} \langle q(p_s)|\bar{q}q|q(p_1)\rangle_{p_1^2=p_2^2=\mu^2}$$

$$Z_m = Z_S^{-1}$$

- These formulas transcend perturbation theory!

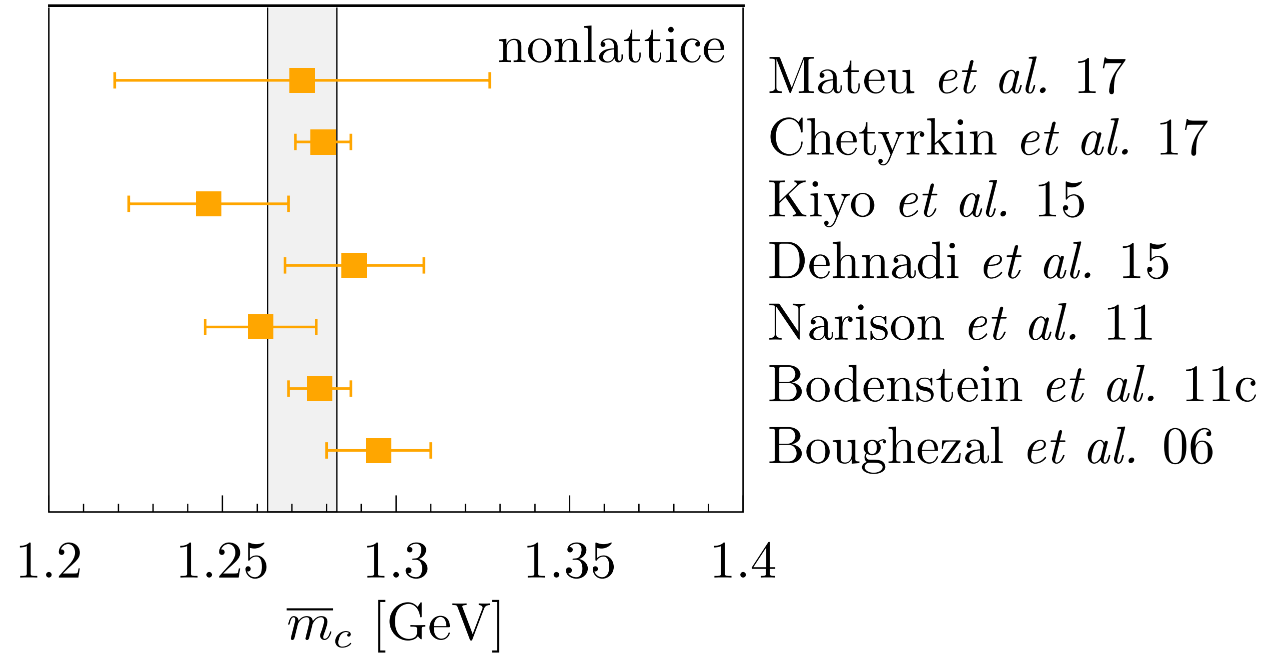
RI-MOM and RI-SMOM Methods

- If your calculation includes nonperturbative effects, the RHS are "polluted".
- It is necessary to specify p_1 and p_2 more:
  - MOM scheme $p_1 = p_2$, so $q = p_1 - p_2 = 0$: "exceptional momenta" that Weinberg warns against;
  - SMOM scheme: $p_1^2 = p_2^2 = q^2$.
- In the latter case, the OPE restricts the nonperturbative effects, which start with the “gluon-mass condensate” ~$\langle A^2 \rangle / \mu^2$.
- Last, $m^{\text{SMOM}}(\mu) = Z_m^{\text{SMOM}}(\mu) m_0$ to a high scale and convert to $m_{\overline{\text{MS}}}(\mu)$ with perturbation theory.

Current-current Correlators

Quarkonium Correlators

- Non-lattice results at right often proceed as follows:
  - use a dispersion relation to relate the momentum-space quarkonium correlator to the discontinuity along the cut, $s \geq (2m_c)^2$;
  - determine the integral along the cut from e^+e^- data (as in muon g–2);
  - compute derivatives w.r.t. q^2 near $q^2 = 0$ in perturbation theory in α_s, as function of m_Q.



- These moments can be computed in lattice QCD: same PT.

Charmonium Correlators

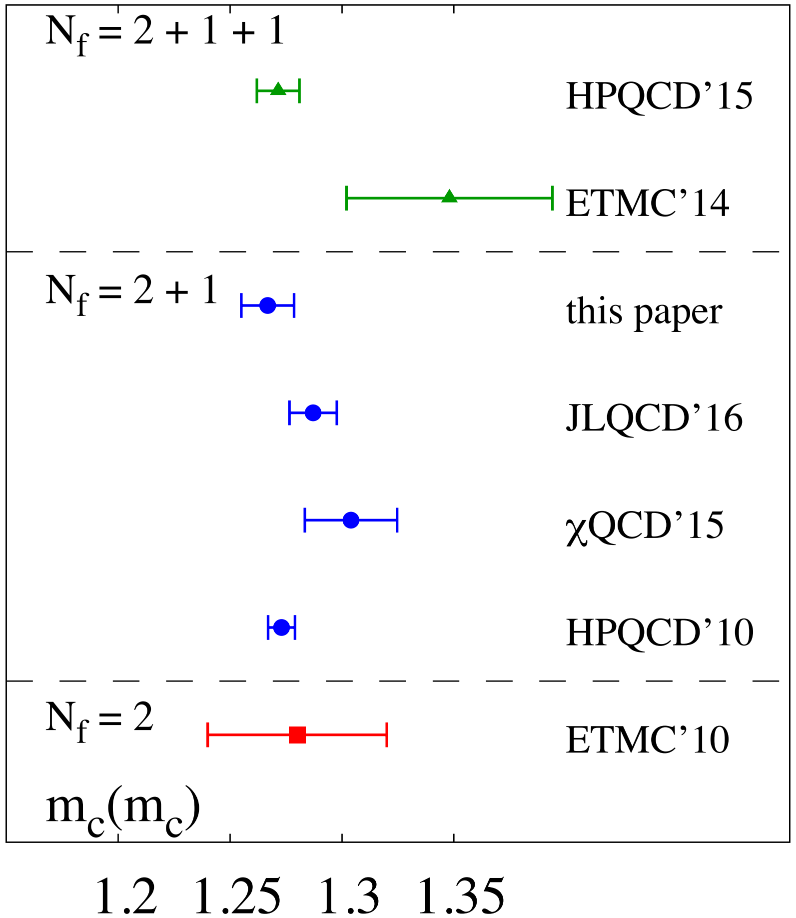
- Lately, the most precise determinations of heavy-quarks come from:

$$\lim_{a\to 0}\frac{G_n}{G_n^{\text{tree}}} = g_n(\alpha_s(\mu), m_{c,\overline{\text{MS}}}/\mu)$$

$$\mathcal{G}_n = \sum_t t^n m_0^2 \left\langle \bar{c}\gamma^5 c(t)\,\bar{c}\gamma^5 c(0)\right\rangle, \quad \overset{\text{even}}{n \geq 4}$$

Bochkarev & de Forcrand [hep-lat/9505025], Allison et al. [HPQCD, arXiv:0805.2999].

- Recent analysis on 11 ensembles from hotQCD ($n_f = 2{+}1$ HISQ) by Maezawa & Petreczky [arXiv:1606.08798] compatible with earlier HPQCD results [arXiv: 1408.4169].



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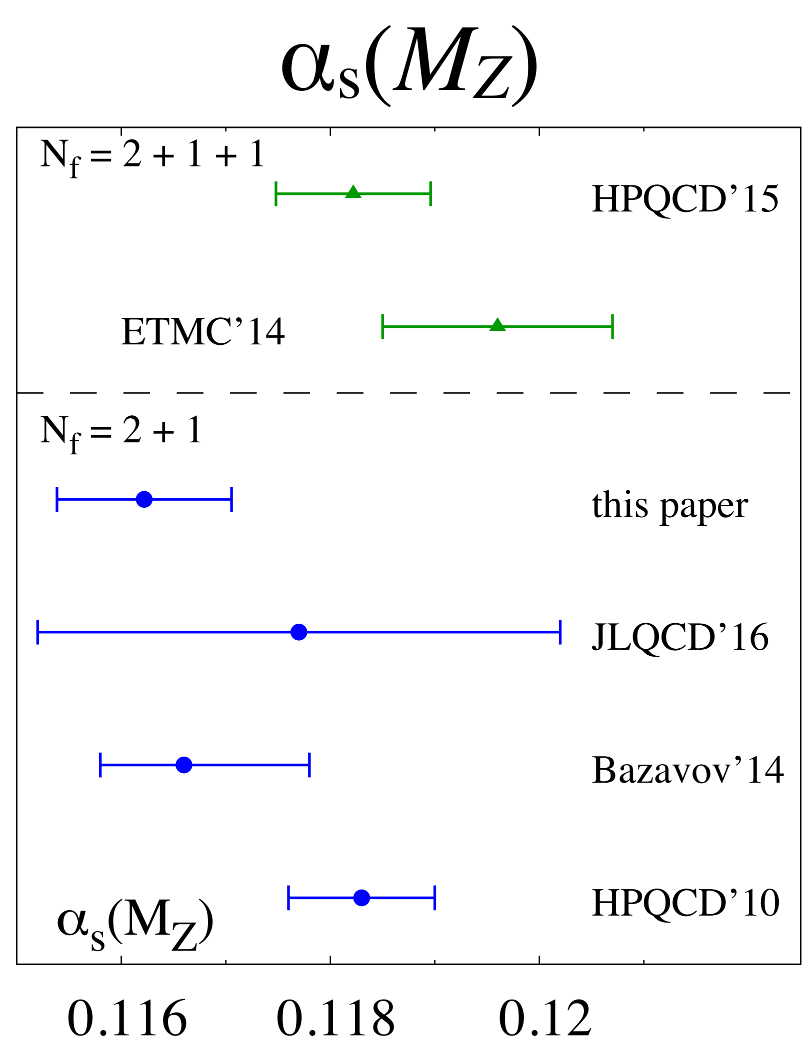


Mass in Quantum Field Theory

What’s a Quark Mass?

- You can’t put a quark on a scale and weigh it.
- Need definition, preferably regularization-independent, in QFT.
- Natural candidate is the “perturbative pole mass.” Alas, ambiguous:
  - physics—infrared gluons need to find a sink:

  - numbers—$m_{\text{b,pole}}/\bar{m}_b = (1, 1.093, 1.143, 1.183, 1.224)$.

Short-Distance Definitions

- Usual work-around is to use a “short-distance” mass.
- The $\overline{\text{MS}}$ mass in dimensional regularization, $m_{h,\overline{\text{MS}}}(\mu)$; $\bar{m}_h \equiv m_{h,\overline{\text{MS}}}(\bar{m}_h)$:
    - spoils HQET power counting: $m_\text{pole} - \bar{m}_h \propto \alpha_s(\bar{m}_h)\bar{m}_h$.
- Other definitions subtract out infrared part at a new scale ν_f:
    - “kinetic mass” (Uraltsev) via a Wilsonian renormalization;
    - “renormalon subtracted mass” (Pineda) subtracts out renormalon at ν_f;
    - “MSR mass” (Hoang, Jain, Scimemi, Stewart) similarly, at $\nu_f = \bar{m}_h$.
- The new scale satisfies $1\ \text{GeV} < \nu_f < m_h$; often need yet another for $\alpha_s(\mu)$.

What is the Pole Mass?

- Consider quark propagator FT $[q(x)\bar{q}(0)]$.
- The quark field is a $\mathbf{3}$, so have to choose a (covariant) gauge. Then,

$$\text{FT}[q(x)\bar{q}(0)] = \frac{i}{\not{p} - m_0 - \Sigma(p;m_0)} = \frac{i}{\not{p}[1 - A(p^2;m_0)] - m_0[1 - B(p^2;m_0)]}$$

$$m_{\text{pole}} = \lim_{p^2 \to m_{\text{pole}}^2} m_0 \frac{1 - B(p^2;m_0)}{1 - A(p^2;m_0)}$$

where m_0 is chosen to absorb UV divergences, an asymptotic expansion for A and B is developed, and m_{pole} is obtained order-by-order using iteration.

- (Use dimensional regularization and $\overline{\text{MS}}$ UV renormalization.)

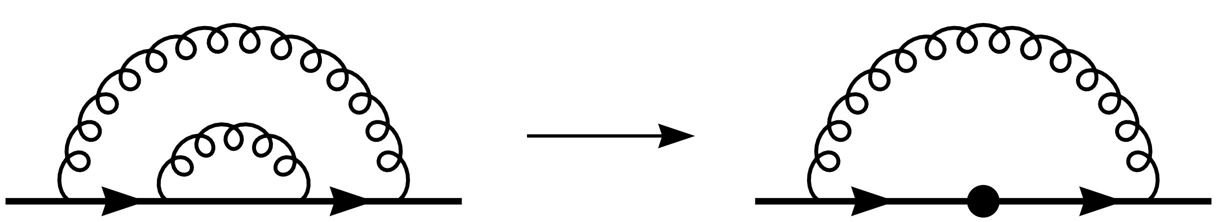
Pole Mass vs. $\overline{\text{MS}}$ Mass

- Consider the relation between the pole mass and the $\overline{\text{MS}}$ mass:

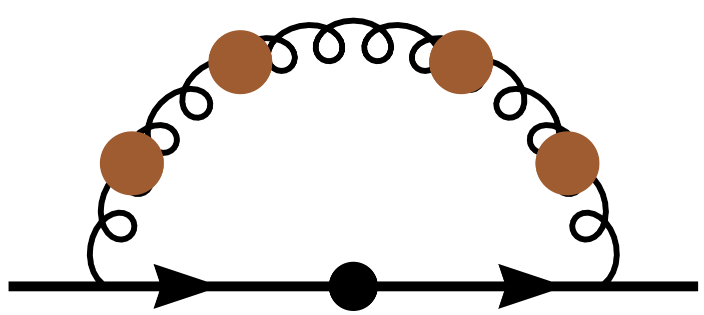
$$m_{\text{pole}} = \bar{m}\left(1 + \sum_{n=0}^{N} r_n \alpha_{\text{g}}^{n+1}(\bar{m}) + \mathrm{O}(\alpha_{\text{g}}^{N+2})\right)$$

  where α_{g} is a scheme for α_{s} that simplifies the algebra (next slide).

- The r_n are infrared finite and gauge-independent [hep-ph/9805215].



- Low loop-momentum parts of self-energy diagrams cause the n^{th} coefficient to grows like $n!$



Infrared Renormalons

- Borel summation:

$$\sum_{n=0}^{\infty} r_n \alpha_g^{n+1} = \int_0^{\infty} dt\, e^{-t/\alpha_g} \sum_{n=0}^{\infty} \frac{r_n}{n!} t^n$$

which sums the original series, if the integral on the RHS exists.

- Analysis of large-orders of our r_n uncovers a series of poles in the t-plane, at the poles of $\Gamma(1-2\beta_0 t)$ [hep-ph/9402360, hep-ph/9402364].
- Integral does not exist, unless ones specifies what to do at the poles.
- The asymptotic series has ambiguities, of order Λ, Λ^2/m_h, ….

Leading Infrared Renormalon

- The leading renormalon is independent of m_h, so

$$\frac{d}{d\bar{m}}\bar{m}\left(1+\sum_{n=0}^{\infty} r_n \alpha_g^{n+1}(\bar{m})\right) = 1+\sum_{n=0}^{\infty} r'_n \alpha_g^{n+1}(\bar{m})$$

$$r'_k = r_k - 2\left[\beta_0 k r_{k-1} + \beta_1 (k-1) r_{k-2} + \cdots + \beta_{k-1} r_0\right]$$

no longer suffers from the factorial growth.

- In arXiv:1701.00347, Javad Komijani derived a recurrence relation based on the r'_k, reproducing known results for the asymptotic behavior of the r_n.

- He also found an asymptotic solution to this differential equation, yielding a formula for the overall normalization.

Factorial Growth

- Remarkably, the β function tells us almost everything about this growth:

$$r_n \sim R_n = R_0(2\beta_0)^n \frac{\Gamma(n+1+b)}{\Gamma(1+b)}, \quad n \geq 0$$

$$b = \frac{\beta_1}{2\beta_0^2} = \frac{231}{645} \text{ for } (n_f = 4)$$

  Hence name "renormalon." Normalization R_0 depends on the r'_k.

- Formula for R_n is exact in the α_g coupling scheme; in other UV schemes, terms suppressed by powers of $1/n$ appear on RHS, still multiplied by $R_0(2\beta_0)^n$.

“Geometric” Scheme for α_s

- Scheme defined by the sum of a geometric series for the beta function:

$$\beta\left(\alpha_{\mathrm{g}}(\mu)\right) = -\frac{\beta_0 \alpha_{\mathrm{g}}^2(\mu)}{1-(\beta_1/\beta_0)\alpha_{\mathrm{g}}(\mu)}$$

supplemented with

$$\frac{1}{\alpha_{\mathrm{g}}(\mu)} = \frac{1}{\alpha_{\overline{\mathrm{MS}}}(\mu)} + b_1 + b_2 \alpha_{\overline{\mathrm{MS}}}(\mu) + \cdots$$

- Must choose b_1, which is proportional to $\ln\left(\Lambda_{\mathrm{g}}/\Lambda_{\overline{\mathrm{MS}}}\right)$.
- One finds $b_2 = \beta_2/\beta_0 - (\beta_1/\beta_0)^2$, $b_3 = \frac{1}{2}[\beta_3/\beta_0 - (\beta_1/\beta_0)^3]$, ….
- Note that α_{g} is regularization independent.

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Leading Renormalon Normalization

- Newly discovered formula [arXiv:1701.00347]:

$$R_0 = \sum_{k=0}^{\infty} r'_k \frac{\Gamma(1+b)}{\Gamma(2+k+b)} \frac{1+k}{(2\beta_0)^k}$$

$$r'_k = r_k - 2\left[\beta_0 k r_{k-1} + \beta_1 (k-1) r_{k-2} + \cdots + \beta_{k-1} r_0\right]$$

- We re-write the relation between the pole mass and the $\overline{\text{MS}}$ mass:

$$m_{\text{pole}} = \bar{m} + \bar{m} \sum_{n=0}^{\infty} [r_n - R_n] \alpha_{\text{g}}^{n+1}(\bar{m}) + \bar{m} \sum_{n=0}^{\infty} R_n \alpha_{\text{g}}^{n+1}(\bar{m})$$

and truncate the first sum, as usual, but carry out the second sum analytically.

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$$R_0 = \sum_{k=0}^{\infty} r'_k \frac{\Gamma(1+b)}{\Gamma(2+k+b)} \frac{1+k}{(2\beta_0)^k} = 0.535 \pm 0.010 \ (n_f = 3)$$

$$r'_k = r_k - 2\left[\beta_0 k r_{k-1} + \beta_1 (k-1) r_{k-2} + \cdots + \beta_{k-1} r_0\right]$$

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Renormalon-a-Ding-Dong

- Use the technique of Borel resummation, one finds ($z = 2\beta_0 t$):

$$\mu \sum_{n=0}^{\infty} R_n \alpha_g^{n+1}(\mu) = \frac{R_0}{2\beta_0}\mu \int_0^{\infty} dz \, \frac{e^{-z/(2\beta_0 \alpha_g(\mu))}}{(1-z)^{1+b}}$$
$$\equiv \mathscr{I}(\mu)$$

- The integrand has a branch point at $z = 1$. That's the (leading) ambiguity!
- Our suggestion:
  - break the integral into an unambiguous part $z \in [0,1]$ and a totally ambiguous part $z \in [1,\infty]$.

Minimal Renormalon Subtraction

arXiv:1712.04983

- Splitting the integral (Brambilla, **Komijani**, ASK, Vairo):

$$\mathcal{J}(\mu) = \mathcal{J}_{\mathrm{MRS}}(\mu) + \delta m$$

$$\mathcal{J}_{\mathrm{MRS}}(\mu) = \frac{R_0}{2\beta_0}\mu \int_0^1 dz\, \frac{e^{-z/[2\beta_0 \alpha_{\mathrm{g}}(\mu)]}}{(1-z)^{1+b}}$$

$$\delta m = \frac{R_0}{2\beta_0}\mu \int_1^\infty dz\, \frac{e^{-z/[2\beta_0 \alpha_{\mathrm{g}}(\mu)]}}{(1-z)^{1+b}}$$

$$= -(-1)^b \frac{R_0}{2\beta_0}\Gamma(-b)\,\mu\, \frac{e^{-1/[2\beta_0 \alpha_{\mathrm{g}}(\mu)]}}{[2\beta_0 \alpha_{\mathrm{g}}(\mu)]^b}$$

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convergent (asymptotic) expansion numerator (denominator)

$$\delta m = \frac{R_0}{2\beta_0}\mu \int_1^\infty dz\, \frac{e^{-z/[2\beta_0 \alpha_g(\mu)]}}{(1-z)^{1+b}}$$

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Minimal Renormalon Subtraction

arXiv:1712.04983

- Minimal renormalon-subtracted (MRS) mass (scheme independent):

$$m_{\rm MRS} \equiv m_{\rm pole} - \delta m$$
$$= \bar{m}\left(1 + \sum_{n=0}^{\infty}\left[r_n - R_n\right]\alpha_{\rm g}^{n+1}(\bar{m})\right) + \mathscr{J}_{\rm MRS}(\bar{m})$$

$$\mathscr{J}_{\rm MRS}(\bar{m}) = \frac{R_0}{2\beta_0}\bar{m}e^{-1/[2\beta_0\alpha_{\rm g}(\bar{m})]}\Gamma(-b)\gamma^{\star}\left(-b, -[2\beta_0\alpha_{\rm g}(\bar{m})]^{-1}\right)$$

- This function is easy enough to evaluate: convergent $1/\alpha_{\rm g}$ expansion.
- NB: MRS mass has same asymptotic series as the pole mass!
- Just as good a solution of the pole condition, without as bad behavior.

Perturbation Theory

- The first four r_n are known:
  - one loop [NPB 183 (1981) 384]: $r_0 = \frac{C_F}{\pi} = 0.4244$
  - 2 loops [ZPC 48 (1990) 673]: $r_1 = 1.0351$ $(n_f = 3)$
  - 3 loops [2+1 papers]: $r_2 = 3.6932$ $(n_f = 3)$
  - 4 loops [arXiv:1606.06754]: $r_3 = 17.4358$ $(n_f = 3)$
- The 5-loop mass anomalous dimension is known [arXiv:1402.6611].
- The 5-loop Callan-Symanzik beta function is known [arXiv:1606.08659].

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Remarks

- MRS mass is a short-distance mass: subtract off long-range δm.
- No new scale: trim long-range field at $1/m_h$, not $1/\nu_f$.
- Numerically very stable: $m_{b,\text{MRS}}/\bar{m}_b = (1.157, 1.133, 1.131, 1.132, 1.132)$.

$$m_{t,\text{MRS}}/\bar{m}_t = (1.0687, 1.0576, 1.0573, 1.0574, 1.0574)$$

- Makes HQET formula unambiguous (to order $1/m_h$):

$$M_{H_J} = m_{h,\text{MRS}} + \bar{\Lambda}_{\text{MRS}} + \frac{\mu_\pi^2}{2m_h} - d_J \frac{\mu_G^2(m_h)}{2m_h}$$

- Next step: fit this formula to lattice-QCD data!

Results

Quark Masses

- We now fit the (augmented) HQET formula:

$$M_{H_x} = m_{h,\mathrm{MRS}} + \bar{\Lambda}_{\mathrm{MRS}} + \frac{\mu_\pi^2}{2m_h} - 3\,\frac{\mu_G^2(m_h)}{2m_h}$$

$$\begin{aligned} m_{h,\mathrm{MRS}} &= \frac{m_{r,\overline{\mathrm{MS}}}(\mu)\, am_h}{m_{h,\overline{\mathrm{MS}}}(\mu)\, am_r} m_{h,\mathrm{MRS}} \\ &= m_{r,\overline{\mathrm{MS}}}(\mu) \frac{\bar{m}_h}{m_{h,\overline{\mathrm{MS}}}(\mu)} \frac{m_{h,\mathrm{MRS}}}{\bar{m}_h} \frac{am_h}{am_r}, \end{aligned}$$

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convenient fit parameter

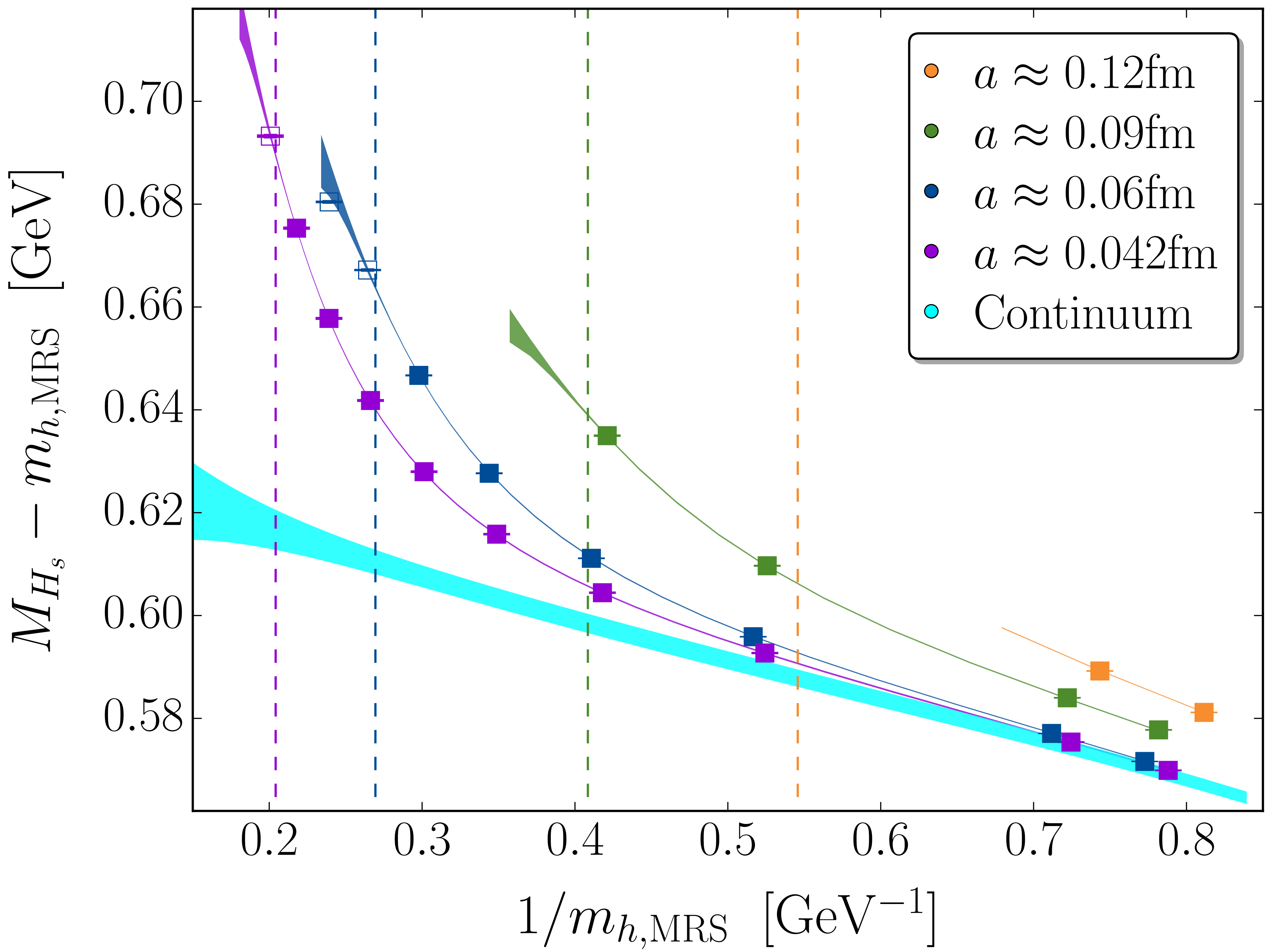
run with anomalous dimension

MRS definition

lattice input

HQET Fit ⊕ Symanzik EFT ⊕ χPT

- 384 data pts;
- 77 parameters;
- $\chi^2/\mathrm{dof} = 312/307$;
- $p = 0.3$;
- stable under fit variations;
- extra errors for FV, topology, EM.



Results & Comparisons 3

- Masses in numerical form:

$$m_{l,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 3.402(15)_{\mathrm{stat}}(05)_{\mathrm{syst}}(19)_{\alpha_s}(04)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$

$$m_{u,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 2.130(18)_{\mathrm{stat}}(35)_{\mathrm{syst}}(12)_{\alpha_s}(03)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$

$$m_{d,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 4.675(30)_{\mathrm{stat}}(39)_{\mathrm{syst}}(26)_{\alpha_s}(06)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$

$$m_{s,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 92.47(39)_{\mathrm{stat}}(18)_{\mathrm{syst}}(52)_{\alpha_s}(11)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$

$$m_{c,\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 983.7(4.3)_{\mathrm{stat}}(1.4)_{\mathrm{syst}}(3.3)_{\alpha_s}(0.5)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$

$$m_{b,\overline{\mathrm{MS}}}(\overline{m}_b) = 4201(12)_{\mathrm{stat}}(1)_{\mathrm{syst}}(8)_{\alpha_s}(1)_{f_{\pi,\mathrm{PDG}}}\ \mathrm{MeV}$$

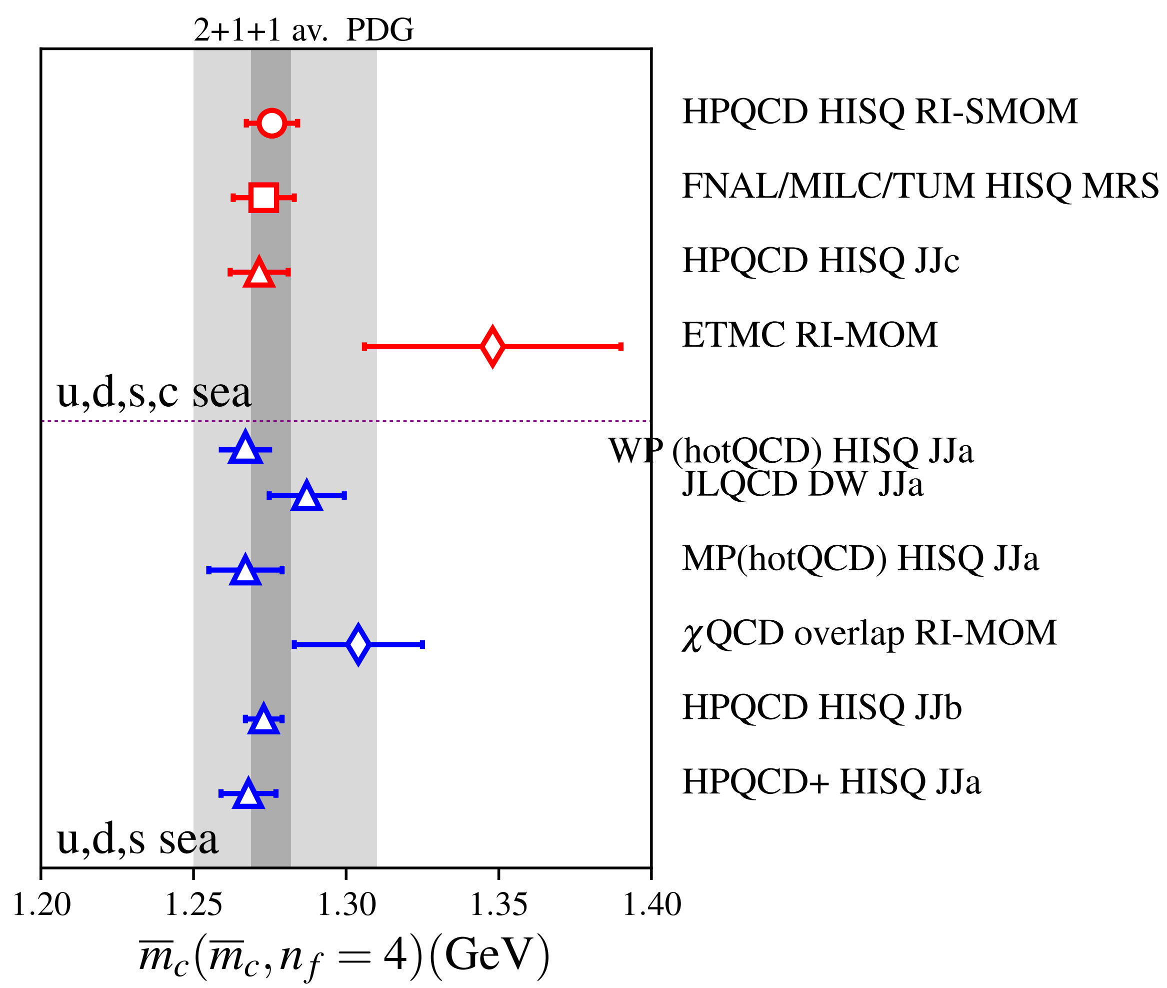
- Mass ratios:

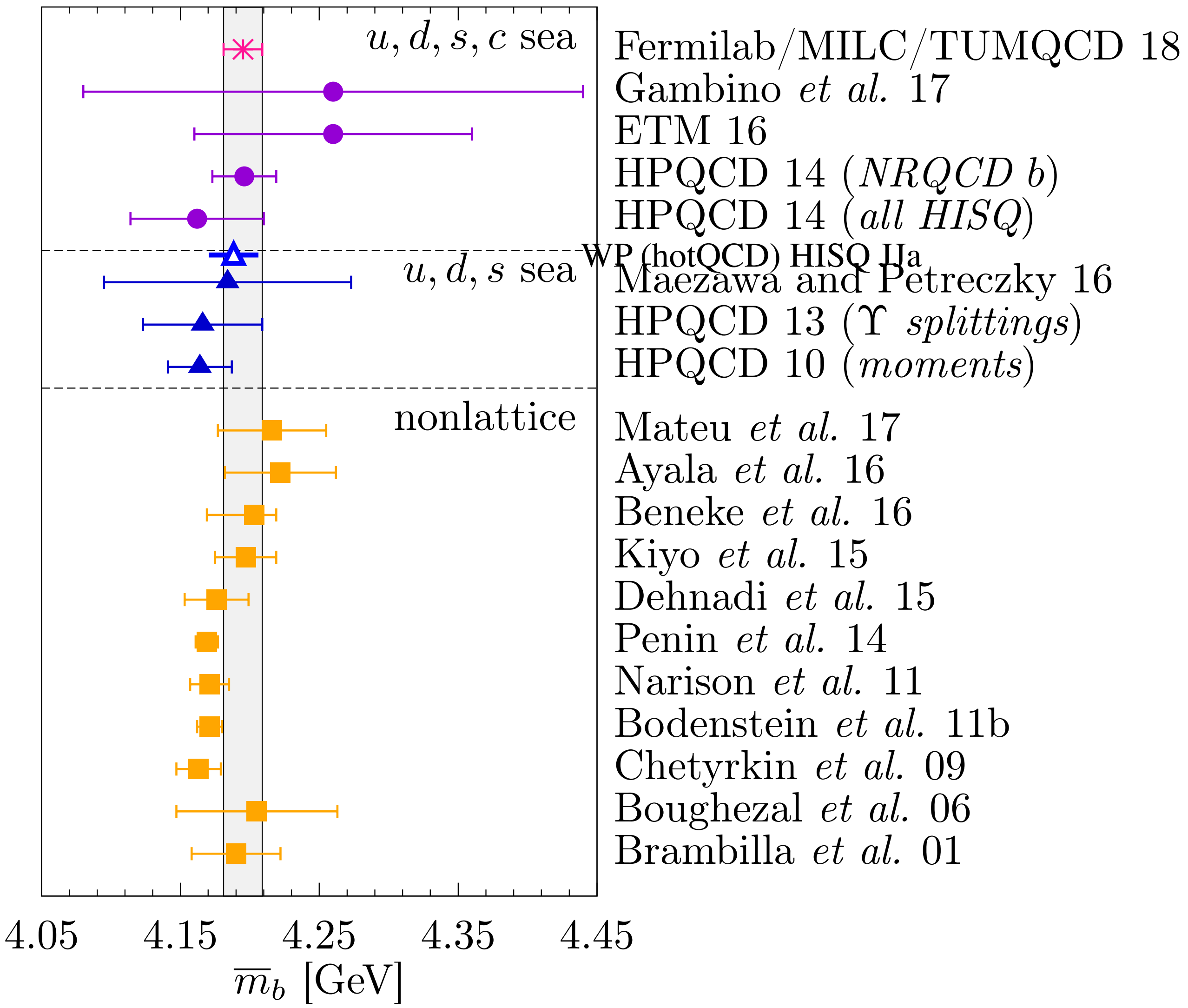
$$m_c/m_s = 11.783(11)_{\mathrm{stat}}(21)_{\mathrm{syst}}(00)_{\alpha_s}(08)_{f_{\pi,\mathrm{PDG}}}$$

$$m_b/m_s = 53.94(6)_{\mathrm{stat}}(10)_{\mathrm{syst}}(1)_{\alpha_s}(5)_{f_{\pi,\mathrm{PDG}}}$$

$$m_b/m_c = 4.578(5)_{\mathrm{stat}}(6)_{\mathrm{syst}}(0)_{\alpha_s}(1)_{f_{\pi,\mathrm{PDG}}}$$

Heavy Comparisons

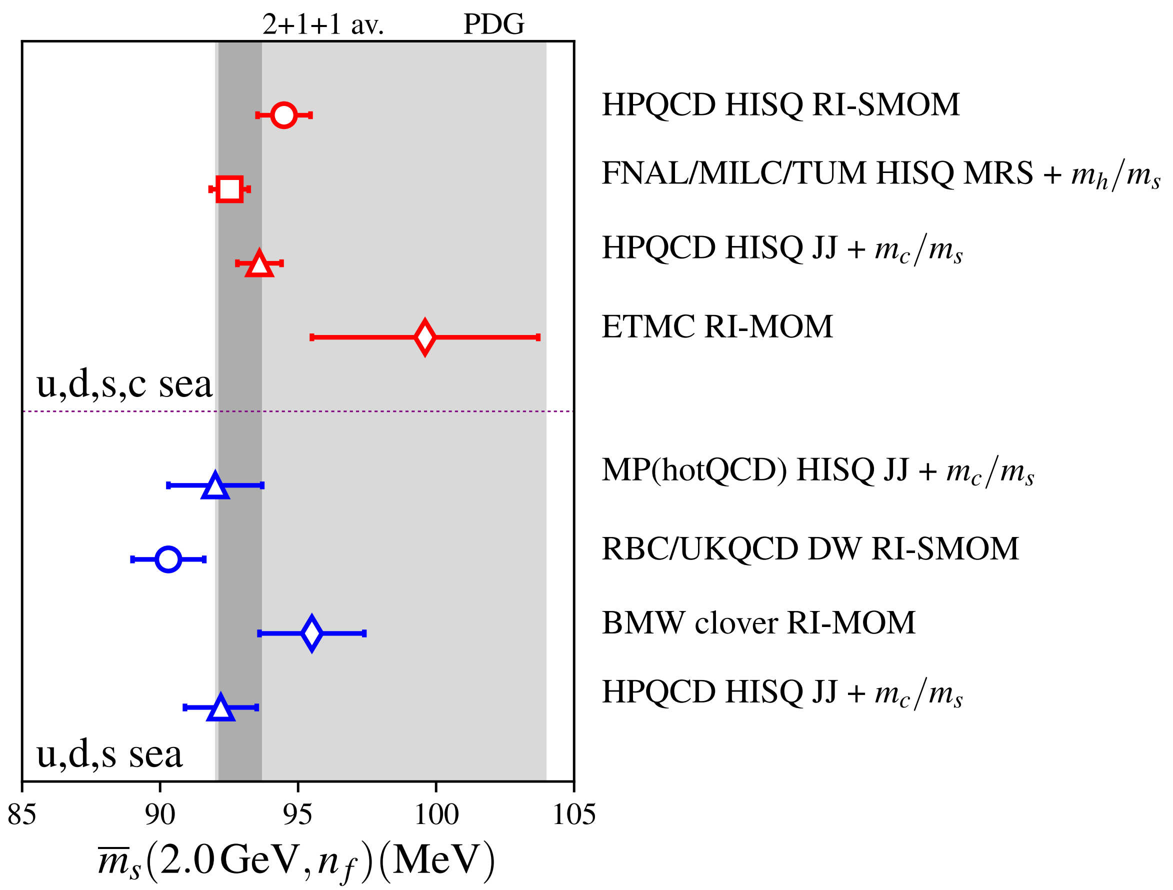


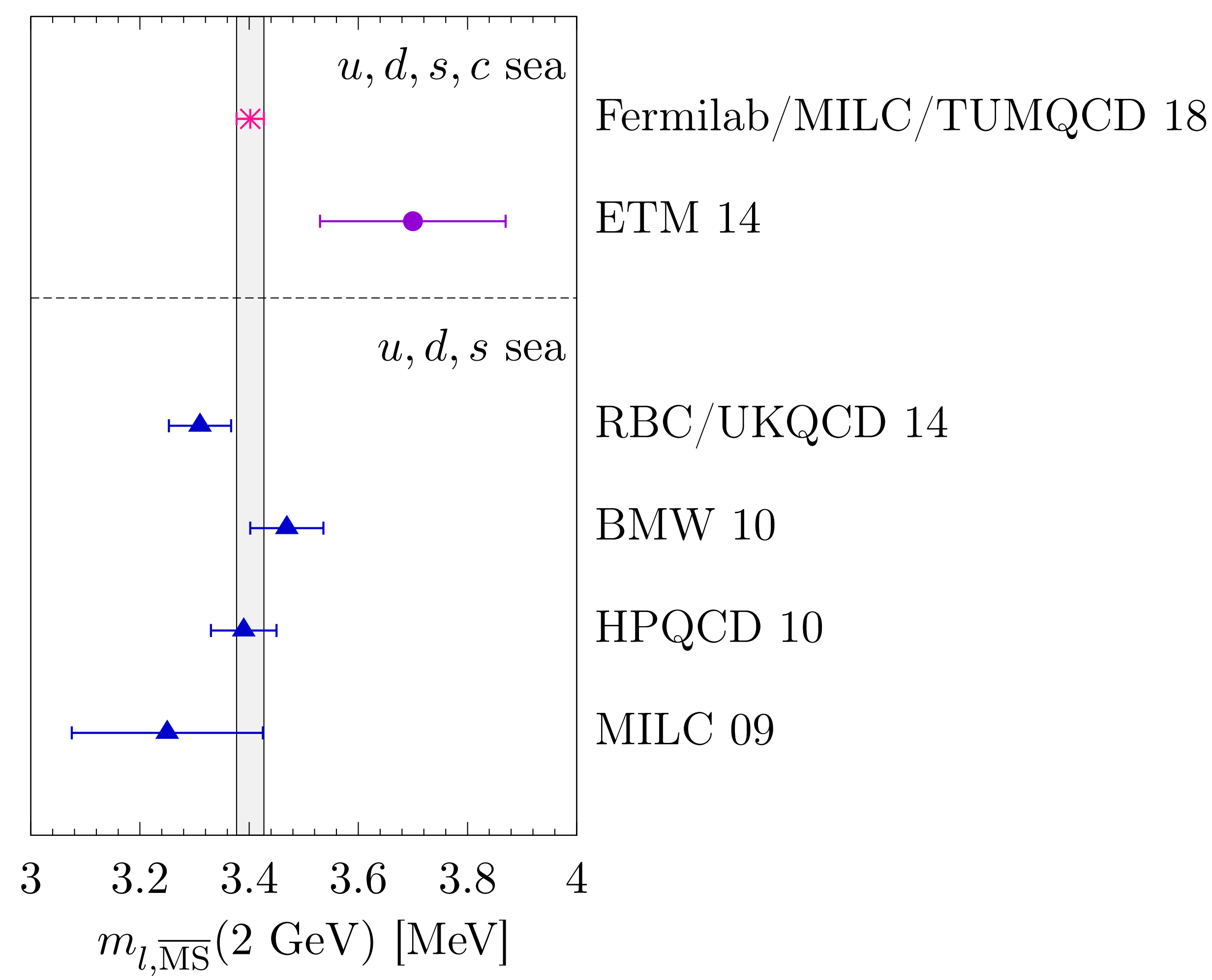


- Precision: 0.3% for bottom to 0.5% for charm.

plots from arXiv:1802.04248, arXiv:1805.06225; adding arXiv:1901.06424

Light Comparisons





plots from arXiv:1802.04248
arXiv:1805.06225

- Precision: 2% for up quark.

Consistent picture: all quarks but top

Outlook

Summary

- New approach to renormalons: may have wider applicability.
- MRS mass: a new version of the pole mass, with smaller IR sensitivity:
  - is there an analogous approach to the top mass (not with lattice QCD)?
- Consistent results from techniques with very different systematics and, in to some extent, different sets of ensembles (MILC, hotQCD, RBC/UKQCD, BMW, ETM):
  - desirable to achieve the precision of Fermilab/MILC/TUMQCD and of HPQCD on ensembles other than MILC's.
- PDG ranges seem unreasonable wide.

Thank you!