EDMs of nucleons and nuclei: EFT and the lattice

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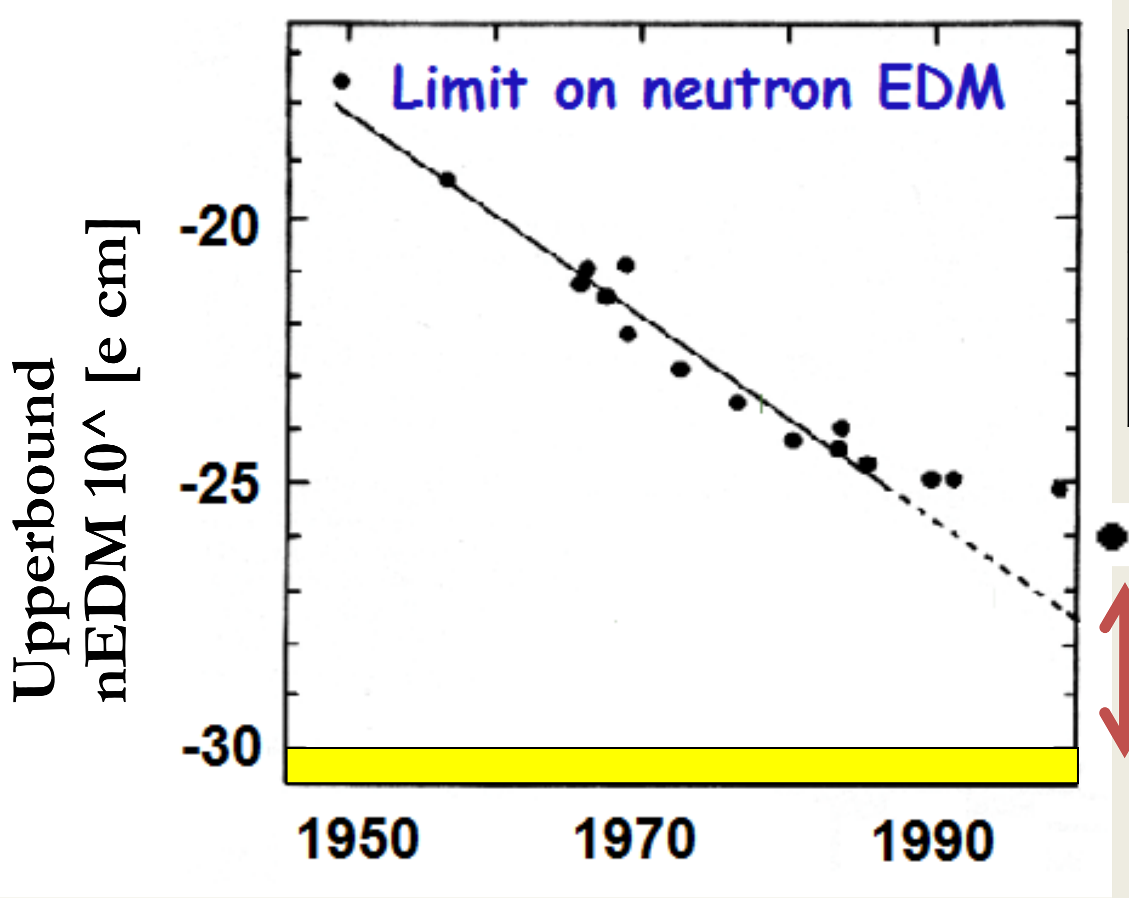
Standard Model suppression



Quarks	$10^{-33,-34}$ e cm
Neutron/Pro ton	$10^{-31,-32}$ e cm
^{199}Hg	$10^{-32,-34}$ e cm
Electron	$10^{-37,-38}$ e cm

Baker et al '06 '15

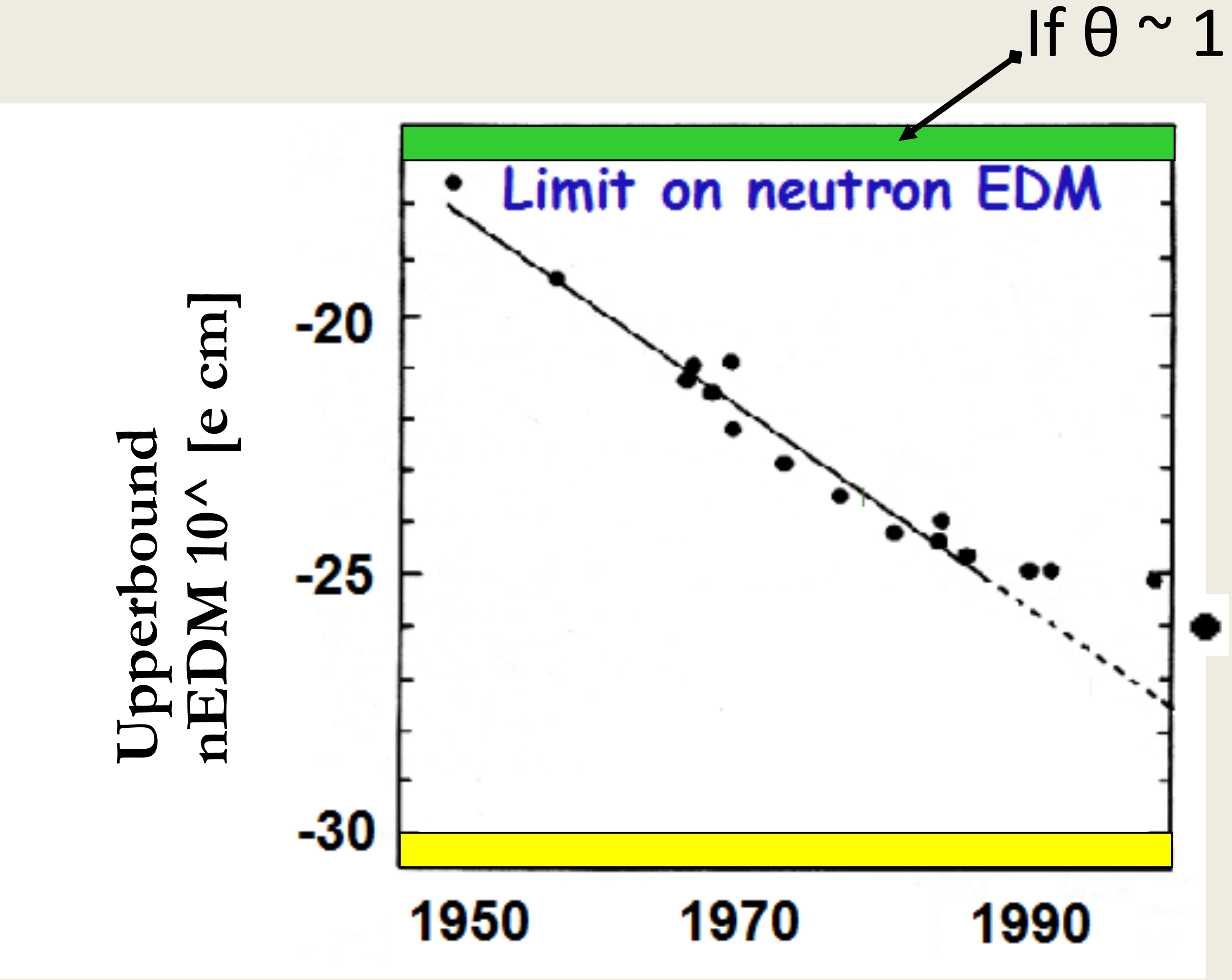
"Here be dragons"

5 to 6 orders **below** upper bound ⟷ **Out of reach!**

Extrapolate: CKM neutron EDM in 2075….

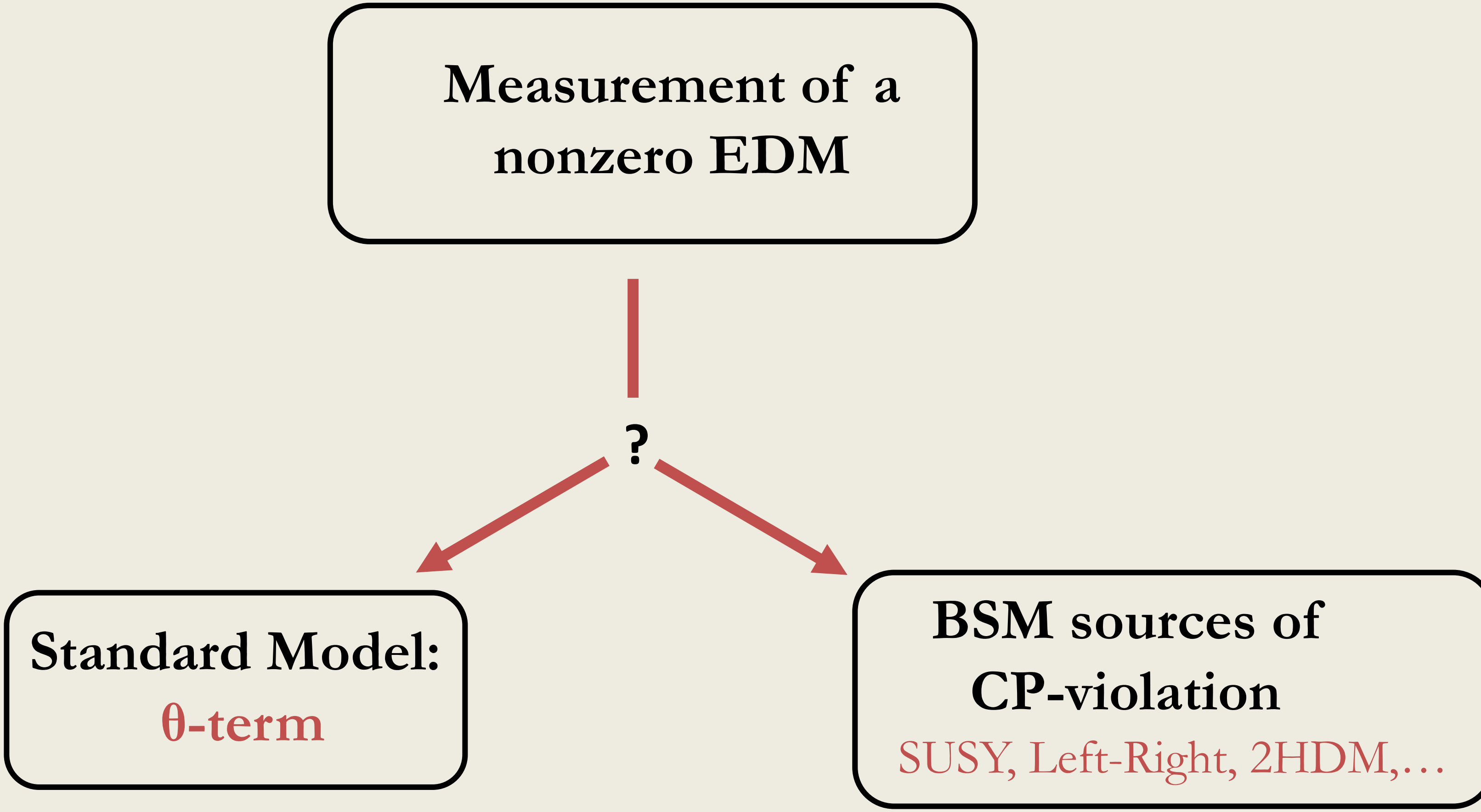
Note: actual size of SM nEDM is not very well determined

The strong CP problem

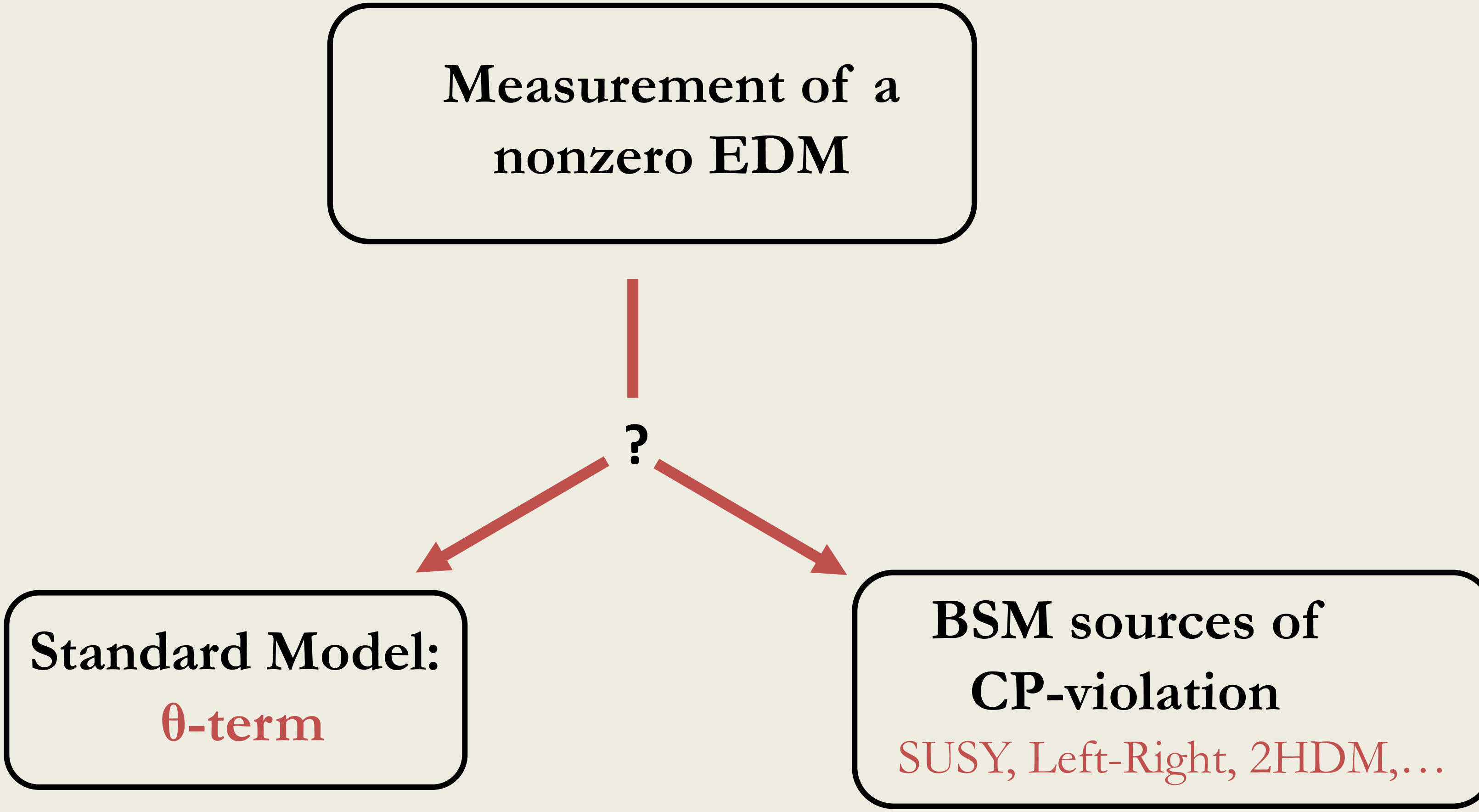


More details on calculation later

Sets θ upper bound: $\boldsymbol{\theta < 10^{-10}}$



Forseeable future: EDMs are **'background-free'** searches for new physics



Forseeable future: EDMs are **'background-free'** searches for new physics

1. How can we parametrize BSM CP violation at low energy ?
2. What lattice-QCD input do we need to interpret EDMs ?
3. What is the interplay between lattice + chiral EFT ?

Very active experimental field

System	Group	Limit	C.L.	Value	Year
^{205}Tl	Berkeley	1.6×10^{-27}	90%	$6.9(7.4) \times 10^{-28}$	2002
YbF	Imperial	10.5×10^{-28}	90	$-2.4(5.7)(1.5) \times 10^{-28}$	2011
ThO	ACME	1.1×10^{-29}	90	$4.3(3.1)(2.6) \times 10^{-30}$	2018
HfF^{+}	Boulder	1.3×10^{-28}	90	$0.9(7.7)(1.7) \times 10^{-29}$	2017
n	Sussex-RAL-ILL	3.0×10^{-26}	90	$0.2(1.5)(0.7) \times 10^{-26}$	2006
^{129}Xe	UMich	4.8×10^{-27}	95	$0.26(2.3)(0.7) \times 10^{-27}$	2019
^{199}Hg	UWash	7.4×10^{-30}	95	$-2.2(2.8)(1.5) \times 10^{-30}$	2016
^{225}Ra	Argonne	1.4×10^{-23}	95	$4(6.0)(0.2) \times 10^{-24}$	2016
muon	E821 BNL g−2	1.8×10^{-19}	95	$0.0(0.2)(0.9) \times 10^{-19}$	2009

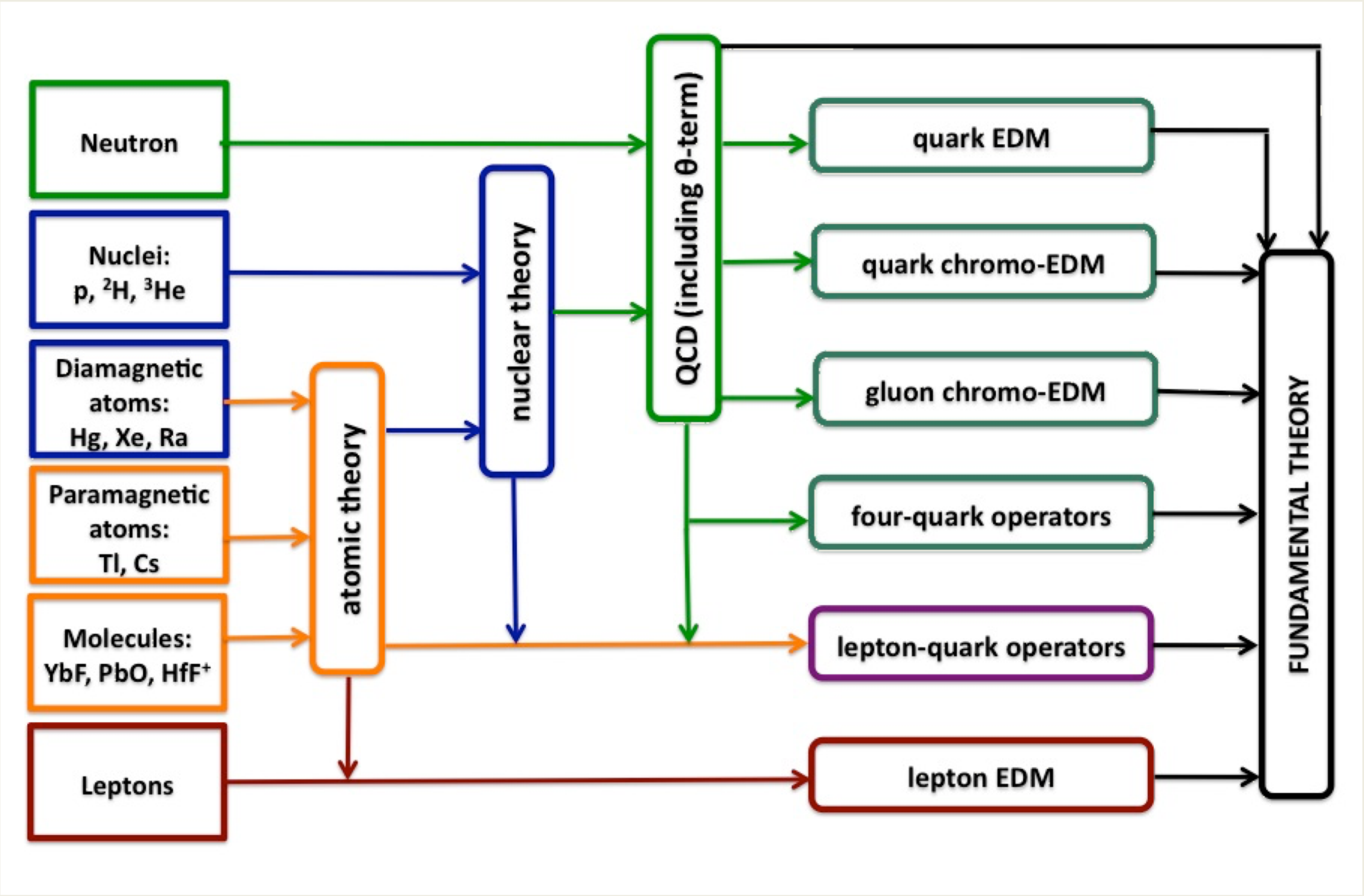
e (first three rows)

+ new electron, muon, neutron, proton, Xe, Ra, Rn ….. experiments

$$d_e \sim \left(\frac{\alpha_{em}}{\pi}\right)^n \frac{m_e}{\Lambda^2} \sin\phi$$

If phase = O(1): $\Lambda > 60$ TeV (n=1)

The EDM metromap



Preliminaries

- To separate theta from 'whatever' we need a 'whatever' description
  - Consider specific (class of) Beyond-the-SM models:
    - *Minimal supersymetric model (MSSM, cMSSM, pMSSM, …)*
    - *Multi-Higgs or composite Higgs models*
    - *Left-right symmetric models*
    - ……......

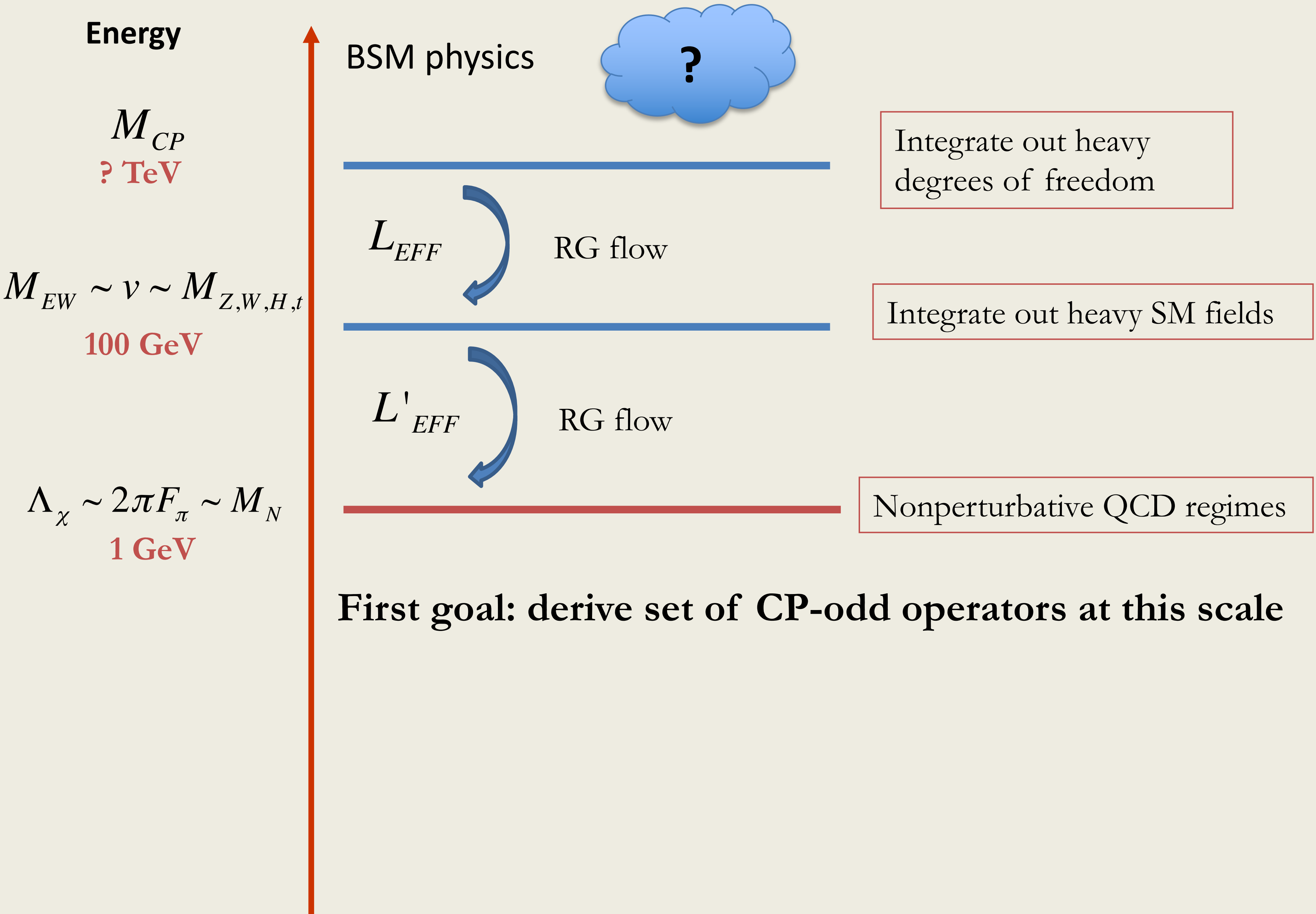
  - EDMs are low-energy experiments → insensitive to many UV details
  - EDMs unlikely to arise from 'light BSM' fields
  - Suggests an EFT approach can be useful

Le Dall, Pospelov, Ritz '15

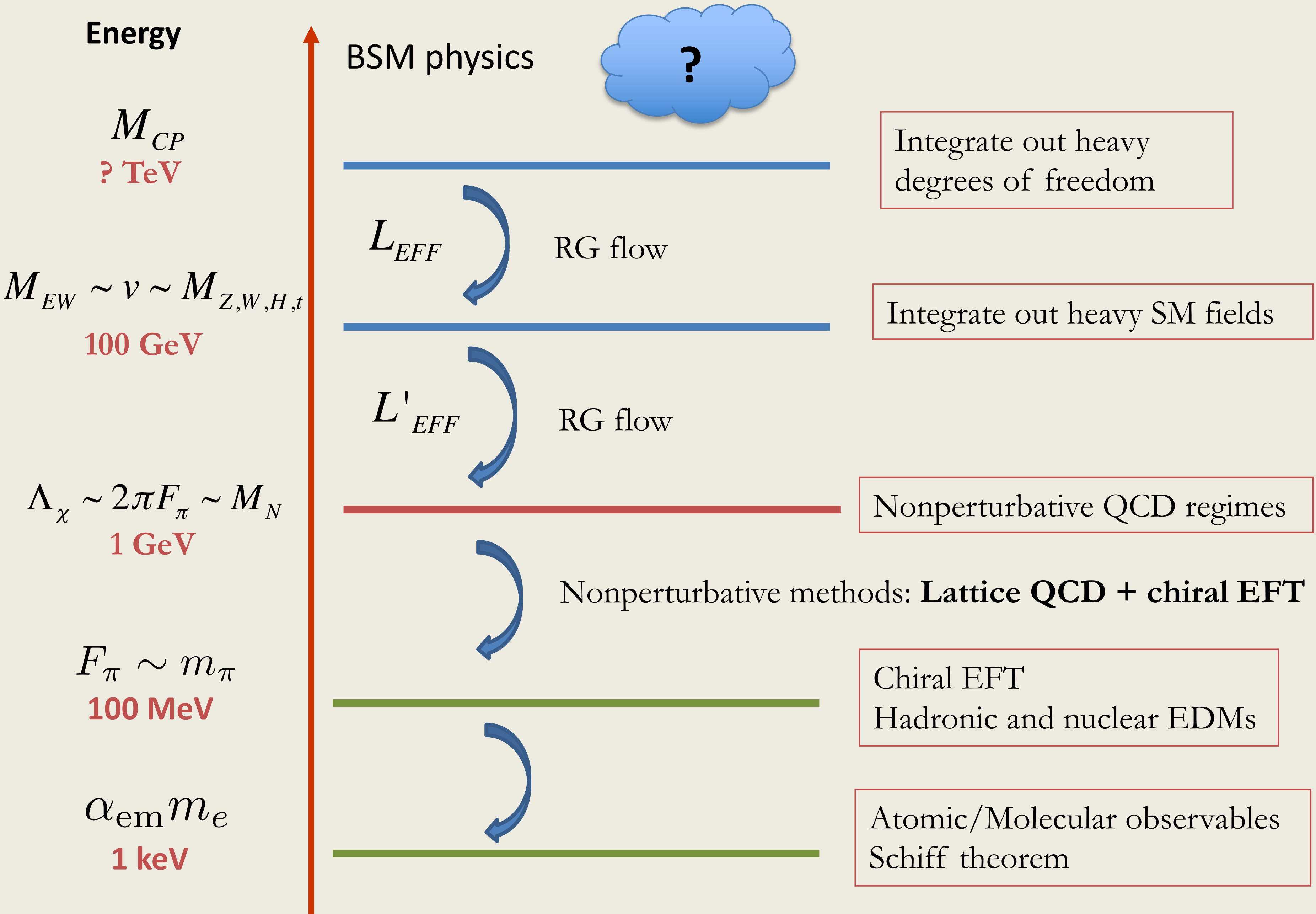
$$M_{CP} > v >> m_N > m_\pi >> m_e$$

- Require **(semi-)-precise EDM** predictions to separate theta from BSM sources, and to interpret limits.
  - Not easy since EDM experiments involve horrible objects

Separation of scales



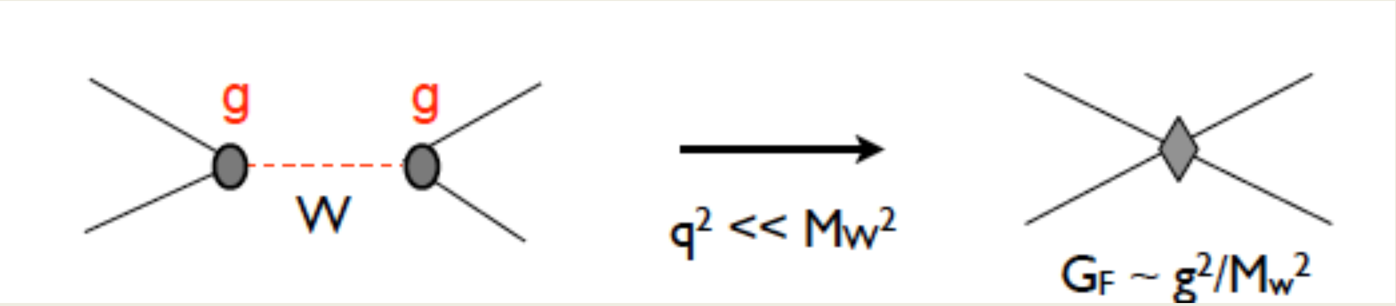
Separation of scales



Heavy BSM physics and the SM EFT

- Assume BSM fields exists but are heavy → **Integrate them out**

Fermi's theory:



- The SM might **just** be the dim-4 part of an effective field theory

$$L_{new} = L_{SM} + \frac{1}{\Lambda} L_5 + \frac{1}{\Lambda^2} L_6 + \cdots$$

Buchmuller & Wyler '86
Gradzkowski et al '10
Many others

- Lorentz- and gauge-invariant operators from all SM fields
- For a given BSM model, we can calculate $L_{5,6,7....}$ Explicitly
- EFT approximation good at scales << Λ

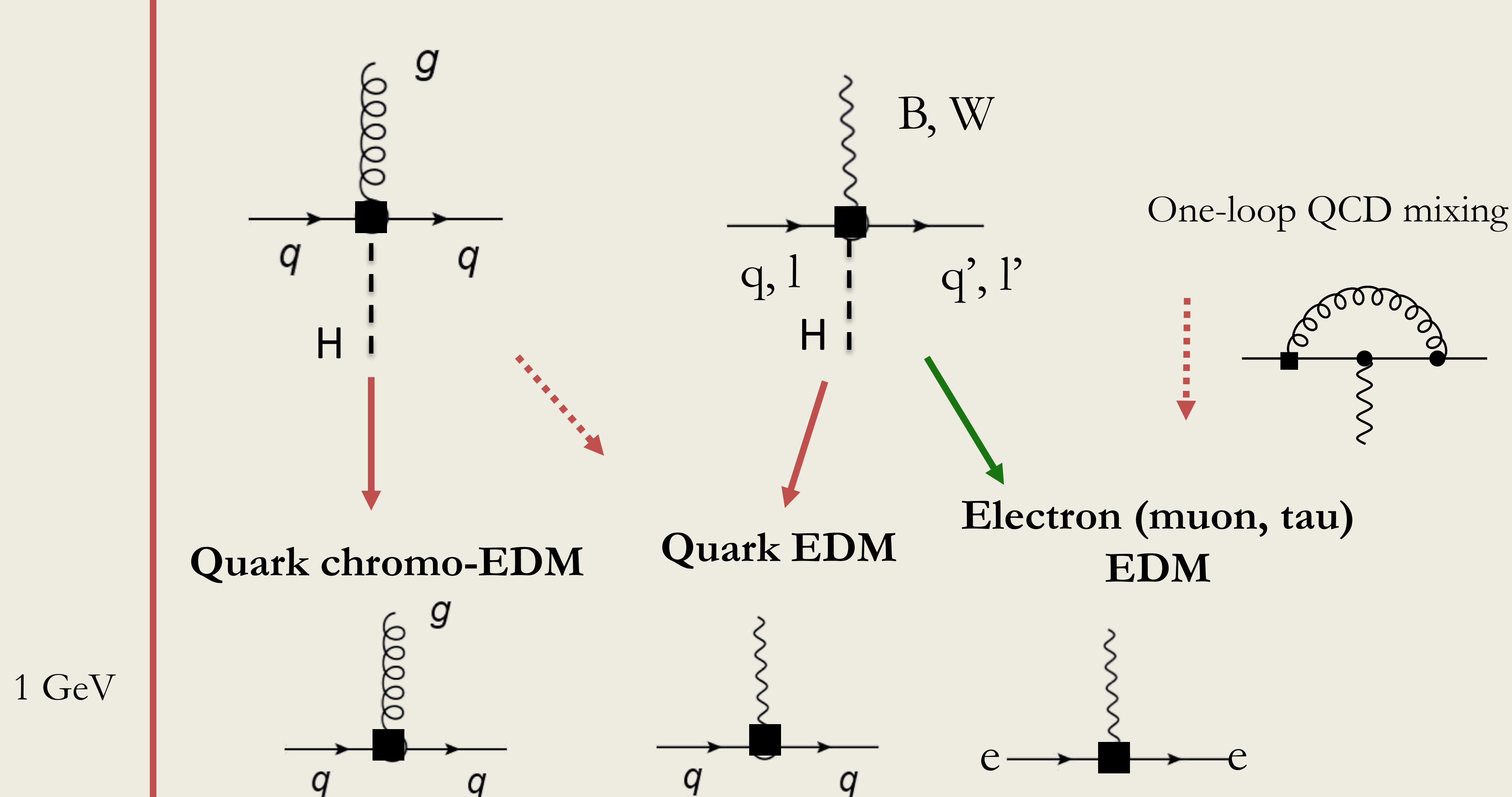
Examples of EFT operators: dipoles

EDMs and MDMs appear in the SMEFT Lagrangian at dimension-six

M_{CP}

? TeV

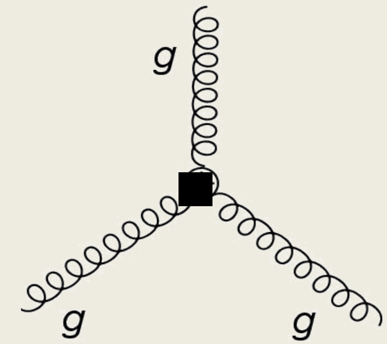
$$\frac{1}{\Lambda^2}\,\tilde{\varphi}\,\bar{\psi}_L\sigma^{\mu\upsilon}\psi_R X_{\mu\upsilon} + h.c. \quad \rightarrow \frac{v}{\Lambda^2}\,\overline{\psi}_L\sigma^{\mu\upsilon}\psi_R X_{\mu\upsilon} + h.c.$$



Gluon chromo-EDM

Weinberg PRL '89
Braaten et al PRL '90

Weinberg operator

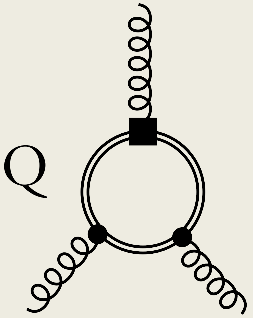


$$d_w \, f^{abc} \varepsilon^{\mu\nu\alpha\beta} \, G^a_{\alpha\beta} G^b_{\mu\lambda} \, G^{c\,\lambda}_{\nu}$$

M_{CP}
? TeV

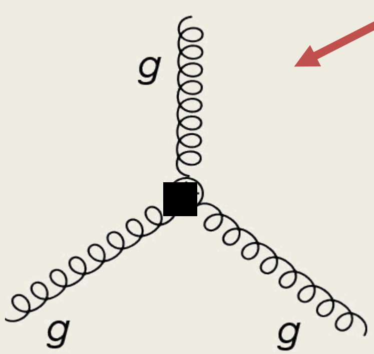
m_Q

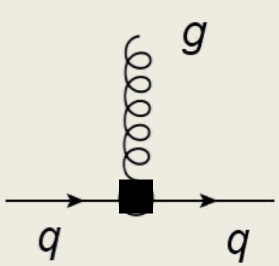
1 GeV

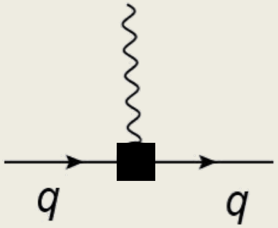


Threshold contributions
from heavy CEDMs

$$d_w \, f^{abc} \varepsilon^{\mu\nu\alpha\beta} \, G^a_{\alpha\beta} G^b_{\mu\lambda} \, G^{c\,\lambda}_{\nu}$$





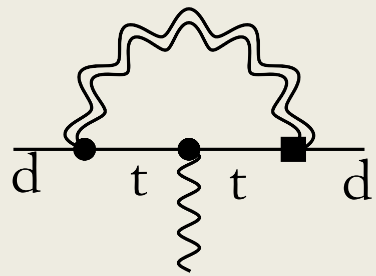


Third-generation CP violation

- What if the BSM physics couples mainly to third generation ?
- Top **CEDM** generate Weinberg operator
- What about top EDM ?

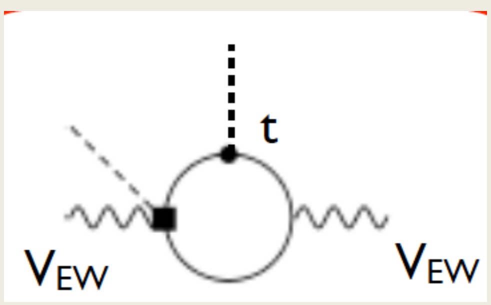
Concero-Cid et al '08

- 1-loop suppressed by $|V_{td}|^2 \sim 10^{-5}$

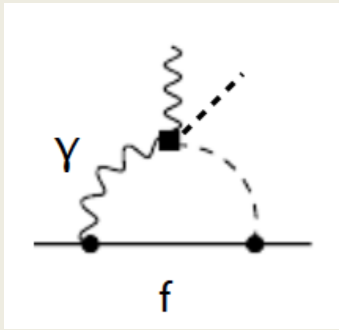


- Two-loop path to electron EDM

JdV et al '16, Fuyuto,Ramsey-Musolf '17



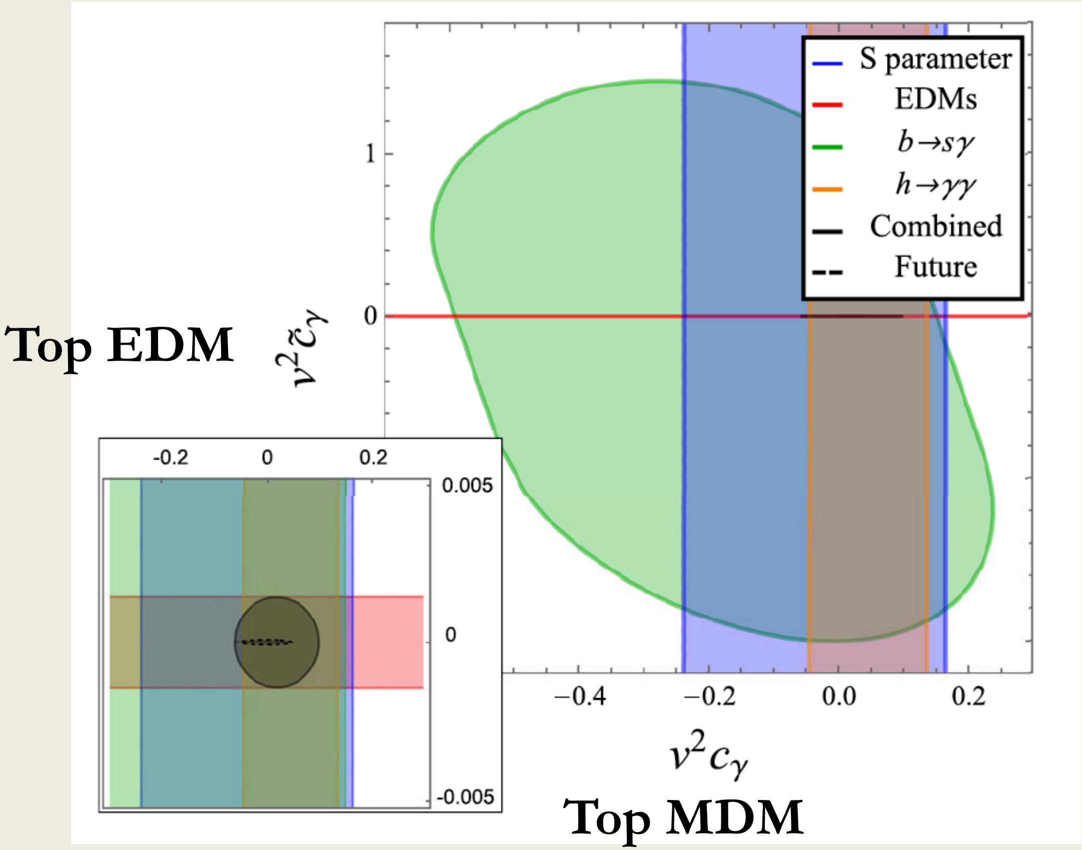
$$(\varphi^\dagger\varphi) F\tilde{F}$$



- Despite loop suppression still very stringent
- Strong interplay with LHC and flavor physics

Top electromagnetic dipoles

JdV et al ‘16



- EDM experiments indirectly set strong limits on ‘heavy’ CP violation
- Limit on **top EDM 100x stronger** than limit on magnetic dipole moment

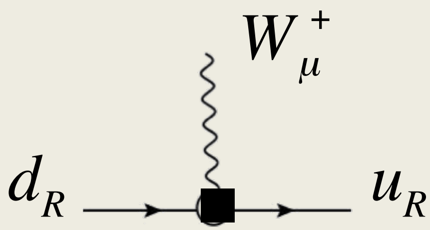
Four-quark operators

Fermion-Scalar interactions **(appears in left-right models)**

Energy

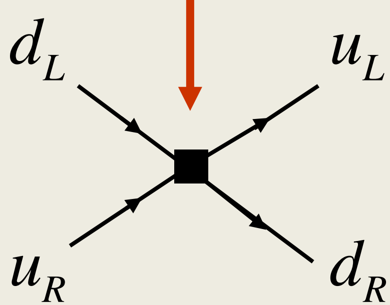
$$\Xi\,\bar{u}_R\gamma^\mu d_R\,(\tilde{\varphi}^\dagger i D_\mu \varphi) + \text{h.c.} \longrightarrow \Xi v^2 g\,(\bar{u}_R\gamma^\mu d_R\, W^\pm_\mu + \text{h.c.})$$

M_{CP}



A right-handed quark-W coupling

$< M_W$



$$L = i\Xi(\bar{u}_R\gamma_\mu d_R)(\bar{u}_L\gamma_\mu d_L) + \text{h.c.}$$

Λ_χ

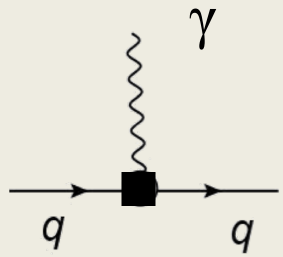
Two four-quarks terms (FQLR operators)

Ng & Tulin '12
Mereghetti et al '12

Plus others… But when the dust settles…..

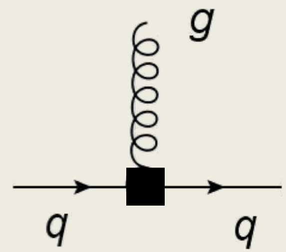
Few GeV

QCD (θ-term) +



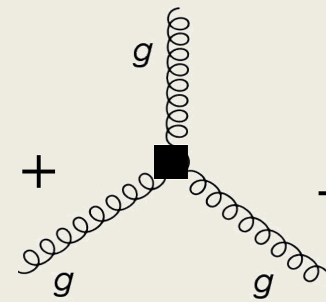
Quark EDM

+



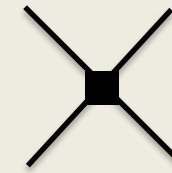
Quark C-EDM

+



Gluon C-EDM

+

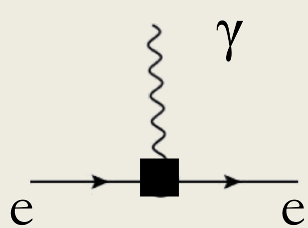


FQ operators

Just u,d quarks: 10 operators (without $SU_L(2)$ would be ~20)

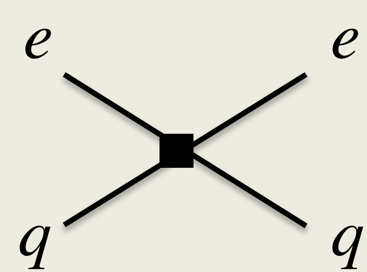
Handful more with strange quarks (more with charm)

(semi-)leptonic interactions (1 + 3)



Electron EDM

(muon + tau)



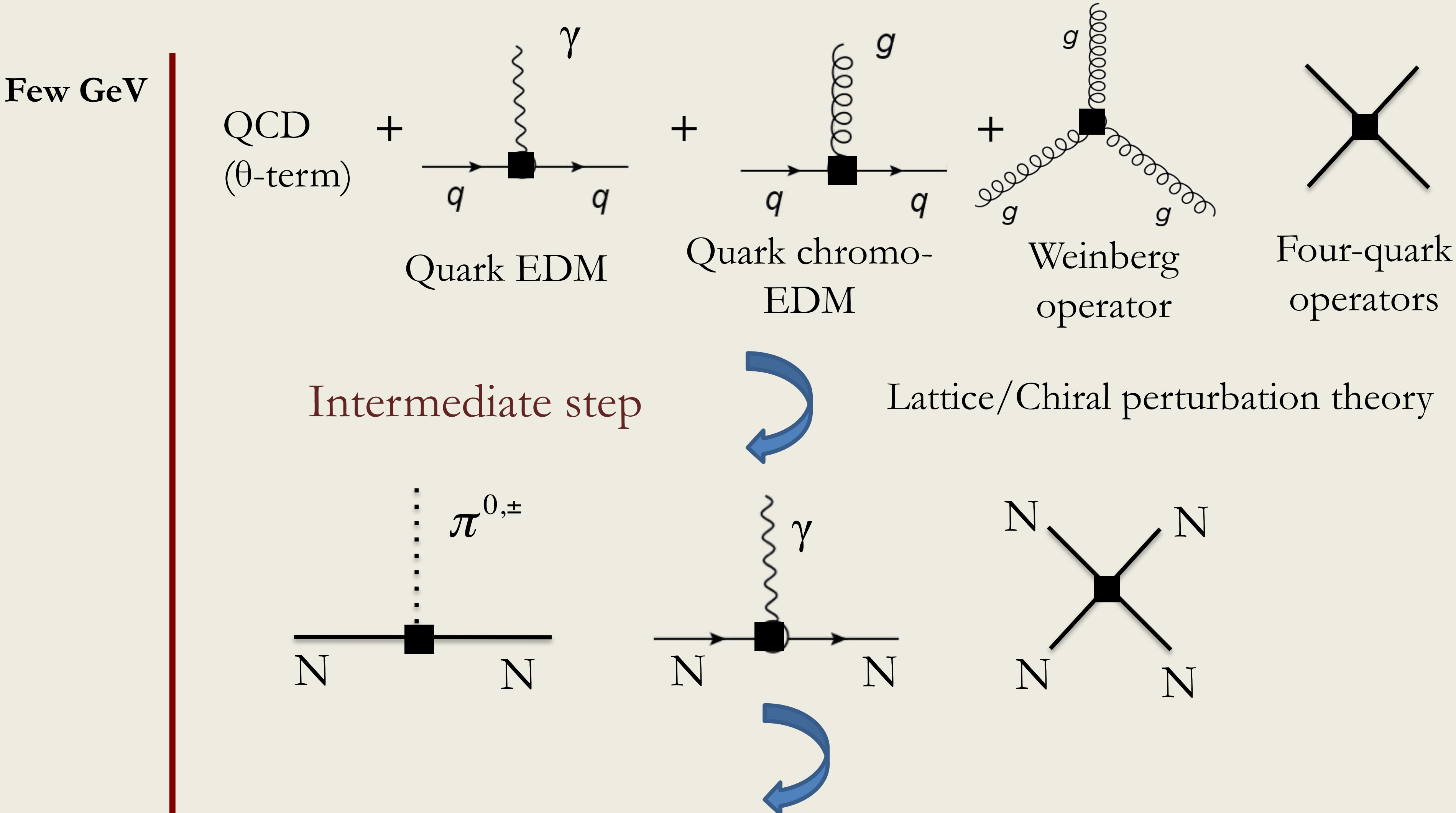
Intermediate summary

- Parametrized BSM CP violation in terms of **dim6** operators
- 1 GeV ~O(10) operators left: theta, (C)EDMs, Weinberg, Four-fermion
- **Important:** different BSM models → different EFT operators

1. Standard Model: only **theta** has a chance to be measured
2. 2-Higgs doublet model: **quark+electron EDM, CEDMs, Weinberg** (exact hierarchy depends on detail of models)
3. Split SUSY: only **electron + quark EDMs** (ratio fixed)
4. Left-right symmetric models: **FQ operators**, way smaller (C)EDMs
5. Leptoquark: **FQ + semi-leptonic** operators

Can‘t say which CP-odd operator will be the most important

Onwards to hadronic CPV



Goal: **Electric dipole moments of nucleons, nuclei, and diamagnetic atoms**

An ultrashort intro to Chiral EFT

- Use the symmetries of QCD to obtain **chiral Lagrangian**

$$L_{QCD} \rightarrow L_{chiPT} = L_{\pi\pi} + L_{\pi N} + L_{NN} + \cdots$$

- Quark masses = 0 → SU(2)$_L$xSU(2)$_R$ symmetry
  - Spontaneously broken to SU(2)-isospin (pions = Goldstone)
  - Explicit breaking (quark mass) → pion mass

- ChPT has systematic expansion in $Q/\Lambda_\chi \sim m_\pi/\Lambda_\chi$ $\Lambda_\chi \cong 1\,GeV$
  - **Form of interactions fixed by symmetries**
  - Each interactions comes with an unknown constant (LEC)

- **Extended to include CP violation**

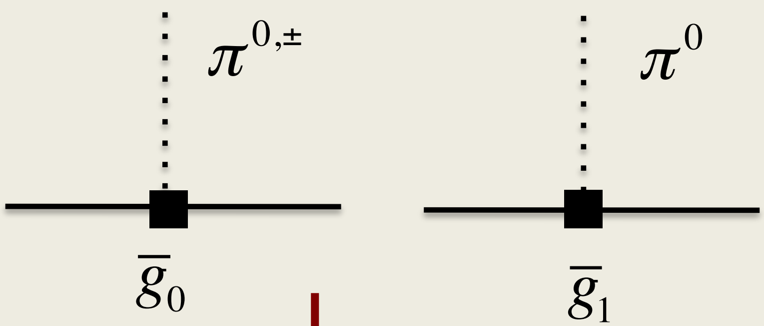
Mereghetti et al' 10, JdV et al '12, Bsaisou et al '14

Weinberg, Gasser, Leutwyler, and many many others

Nucleon and nuclear EDMs up to NLO

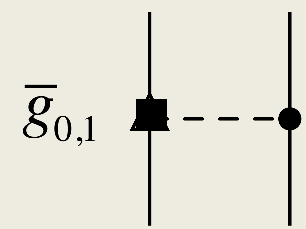
Lowest-order CP-odd interactions

$$L = \bar{g}_0 \, \bar{N} \pi \cdot \tau N + \bar{g}_1 \, \bar{N} \pi^0 N$$

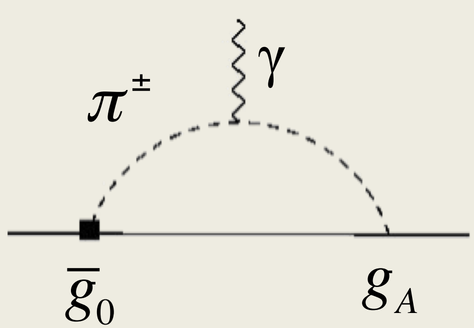


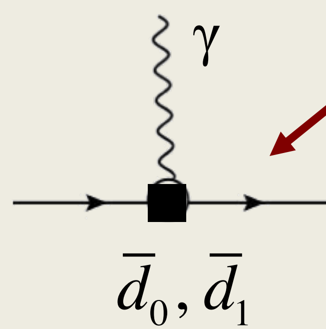
Tree

CP-odd nuclear force



1-Loop
Neutron EDM



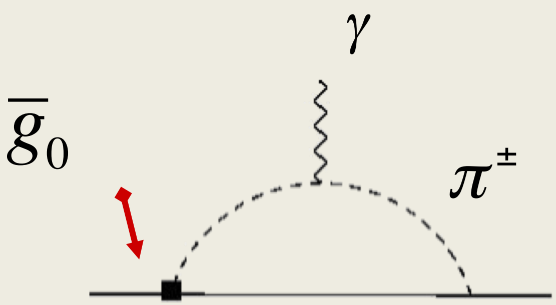


2 new low-energy constants ☹

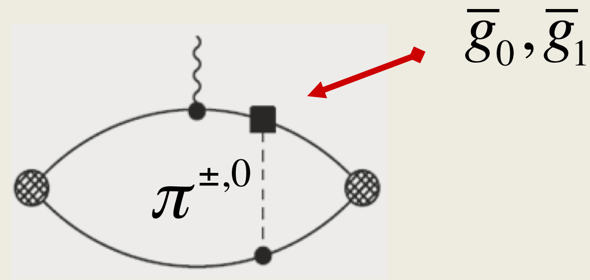
nucleon

pion

The CPV NN force and nuclear EDMs



Nucleon EDM



Nuclear EDM

- Tree-level: **no loop** suppression → EDM predictions
- Orthogonal to nucleon EDMs, sensitive to different CPV structures

$$d_A = <\Psi_A \| \vec{J}_{\cancel{CP}} \| \Psi_A> + 2 <\Psi_A \| \vec{J}_{CP} \| \tilde{\Psi}_A>$$

$$(E - H_{PT}) | \Psi_A > = 0 \qquad (E - H_{PT}) | \tilde{\Psi}_A > = V_{\cancel{CP}} | \Psi_A >$$

- **Pion-exchange contribution can be larger than nucleon EDMs !**
- Goal : calculate nuclear EDMs in terms of LECs
- Note I only consider subset of CP-odd LECs

EDMs of light nuclei

Farley *et al* PRL '04

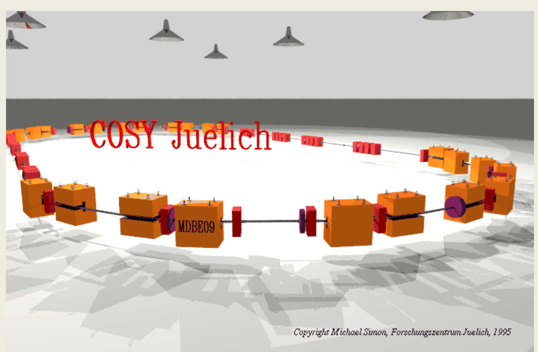
Anomalous magnetic moment

Electric dipole moment

$$\frac{d\vec{S}}{dt} = \vec{S} \times \vec{\Omega} \qquad \vec{\Omega} = \frac{q}{m}\left[a\vec{B} + \left(\frac{1}{v^2} - a\right)\vec{v} \times \vec{E}\right] + 2d\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

All-purpose ring (^{1}H, ^{2}H, ^{3}He, …) $\sim 10^{-28,29}\ e\,cm$

100-1000 x current neutron EDM sensitivity! (takes a while tough….)



Already used for muon EDM

$$d_\mu \leq 1.8 \cdot 10^{-19}\ e\,cm \quad (95\%\ C.L.)$$

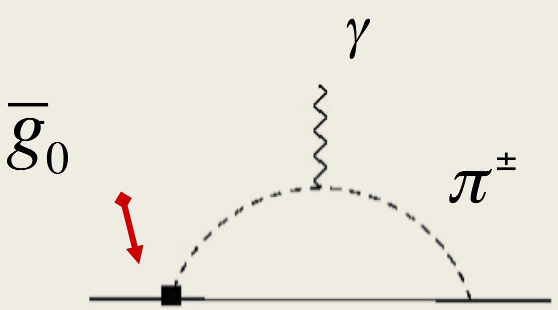
Bennett *et al* (BNL g-2) PRL '09

Major progress in:

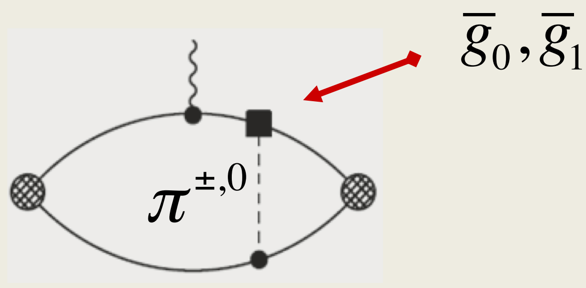
JEDI collaboration, '15, '16

Test d_D measurement in 2019

The CPV NN force and nuclear EDMs



Nucleon EDM



Nuclear EDM

$$d_D = 0.9(d_n + d_p) + \left[(0.18 \pm 0.02)\,\bar{g}_1 + (0.0028 \pm 0.0003)\,\bar{g}_0\right] e\ fm$$

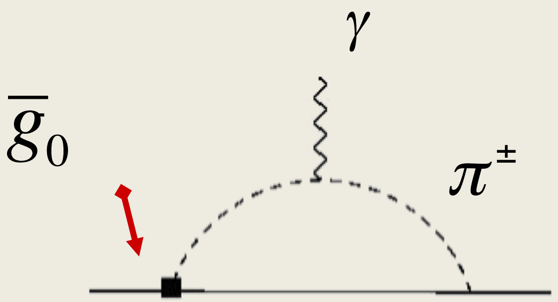
$$d_{3He} = 0.9\ d_n - 0.05\ d_p + \left[(0.14 \pm 0.04)\,\bar{g}_1 + (0.10 \pm 0.03)\,\bar{g}_0\right] e\ fm + ...$$

Stetcu et al '08, JdV et al '11 '12, Bsaisou et al '14, Viviani et al '19

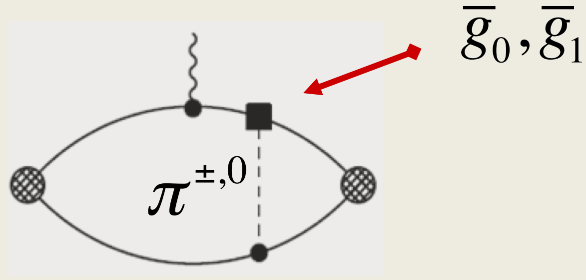
- Calculations from chiral EFT potentials (CP-even + CP-odd)
- Most CP-odd sources: pion exchange ~5-10x bigger then nucleon EDMs
- d_D/d_n ratio would point towards underlying CPV source JdV et al '11 '14

- **But need nonperturbative calculations for the LECs to be sure**

The CPV NN force and nuclear EDMs



Nucleon EDM



Nuclear EDM

Graner et al, '16

Strongest bound on atomic EDM: $d_{^{199}Hg} < 8.7 \cdot 10^{-30}\ e\ cm$

- Similar for diamagnetic atoms, but no first-principle calculations
- Plus a well-known atomic screening factor (Schiff screening)
- Large nuclear uncertainty but pions dominate over nucleon EDMs

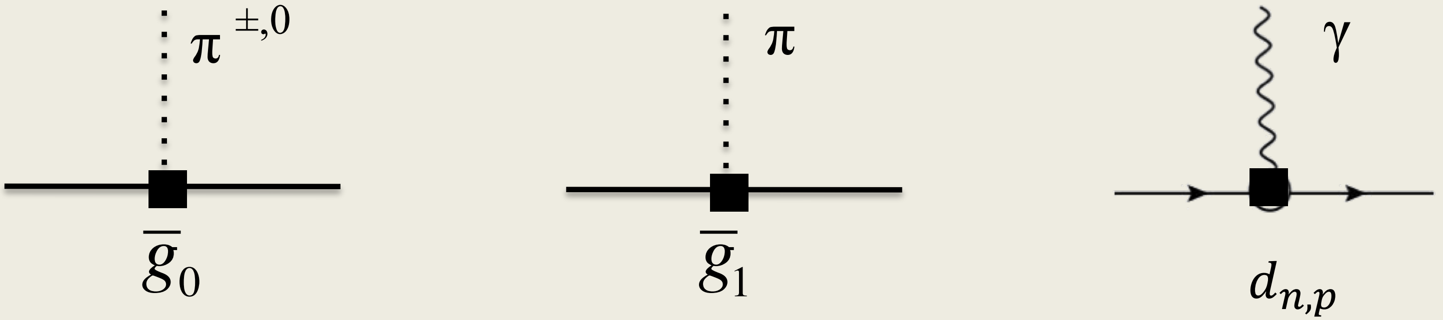
$$d_{199\,Hg} \propto 1.9\, d_n + 0.2\, d_p + \left[(0.25^{+0.9}_{-0.6})\, \bar{g}_1 + (0.13^{+0.5}_{-0.07})\, \bar{g}_0 \right] e\ fm + ...$$

$$d_{225\,Ra} \propto \left[(76^{+227}_{-25})\, \bar{g}_1 - (19^{+7}_{-55})\, \bar{g}_0 \right] e\ fm + ...$$

Engel et al '13 '18

- **Still: need LECs to interpret limits in terms of particle physics**

Goals



- Goal: get $g_{0,1}$ + nucleon EDMs from quark-gluon CP-odd source
- Even 25-50% uncertainty would be very welcome

- Let's start with QCD theta term

Theta and chiral perturbation theory

After axial U(1) and SU(2) rotations, **complex CP-odd quark mass**:

Crewther et al' 79
Baluni '79

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{kin}} - \bar{m}\bar{q}q - \varepsilon\bar{m}\,\bar{q}\tau^3 q + m_\star\,\bar{\theta}\,\bar{q}i\gamma^5 q$$

$$\varepsilon = \frac{m_u - m_d}{m_u + m_d}$$

$$\mathcal{L}'_\chi = \mathcal{L}_\chi - \frac{m_\pi^2}{2}\pi^2 - \delta m_N\,\bar{N}\tau^3 N + \bar{g}_0\,\bar{N}\tau\cdot\pi N$$

Strong proton-neutron mass splitting

Theta and chiral perturbation theory

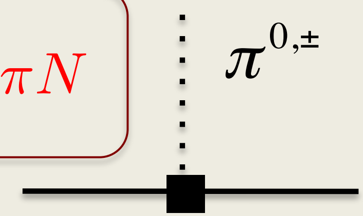
After axial U(1) and SU(2) rotations, **complex CP-odd quark mass**:

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm kin} - \bar{m}\bar{q}q - \varepsilon\bar{m}\,\bar{q}\tau^3 q \;\; + m_\star\,\bar{\theta}\,\bar{q}i\gamma^5 q$$

Crewther et al' 79
Baluni '79

$$m_\star = \frac{m_u m_d}{m_u + m_d}$$

$$\mathcal{L}'_\chi = \mathcal{L}_\chi - \frac{m_\pi^2}{2}\pi^2 - \delta m_N\,\bar{N}\tau^3 N \;\; + \bar{g}_0\,\bar{N}\tau\cdot\pi N$$



$\boldsymbol{\pi}^{0,\pm}$

$\bar{g}_0$

CP-odd pion-nucleon interaction

Theta and chiral perturbation theory

After axial U(1) and SU(2) rotations, **complex CP-odd quark mass:**

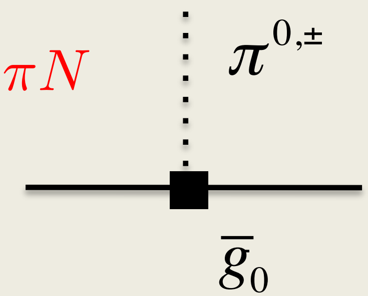
Crewther et al' 79
Baluni '79

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{kin}} - \bar{m}\bar{q}q - \varepsilon\bar{m}\,\bar{q}\tau^3 q \;+ m_\star\,\bar{\theta}\,\bar{q}i\gamma^5 q$$

$$m_\star = \frac{m_u m_d}{m_u + m_d}$$

Linked via $\mathrm{SU_A(2)}$ rotation

$$\mathcal{L}'_\chi = \mathcal{L}_\chi - \frac{m_\pi^2}{2}\pi^2 - \delta m_N\,\bar{N}\tau^3 N \;+ \bar{g}_0\,\bar{N}\tau\cdot\pi N$$



Nucleon mass splitting (strong part, no EM!) ⟷ **CP-odd pion-nucleon interaction**

Use **lattice** for mass splitting

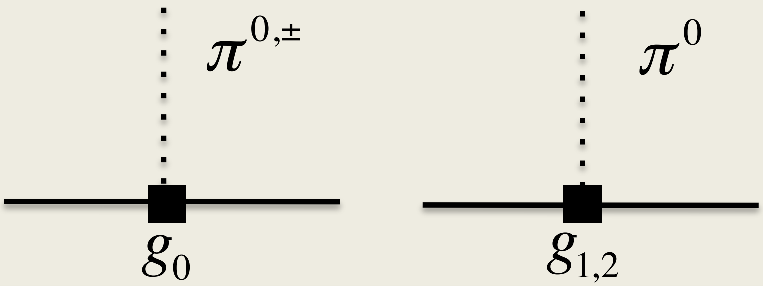
$$g_0 = \delta m_N \frac{1-\varepsilon^2}{2\varepsilon}\bar{\theta} = (15.5 \pm 2.5)\cdot 10^{-3}\,\bar{\theta}$$

Walker-Loud et al '16, Borsanyi et al '14

JdV, Mereghetti, Walker-Loud '15

Pion-nucleon couplings

- 2 relevant CP-odd structures



$$L = g_0 \, \bar{N} \pi \cdot \tau N + g_1 \, \bar{N} \pi^0 N$$

- θ-term conserves isospin! So g_1 is **suppressed**.

Pospelov et al '01,'04
Mereghetti et al '10, '12,
Bsaisou et al '12

$$g_0 = (15.5 \pm 2.5) \cdot 10^{-3} \, \bar{\theta}$$

$$g_1 = -(3 \pm 2) \cdot 10^{-3} \, \bar{\theta}$$

$$\frac{\bar{g}_1}{\bar{g}_0} = -(0.2 \pm 0.1)$$

- Large uncertainty for g_1 due to pion mass splitting and unknown LEC
- g_0 relation **protected** from higher-order SU(2) and SU(3) corrections

JdV, Mereghetti, Walker-Loud '15

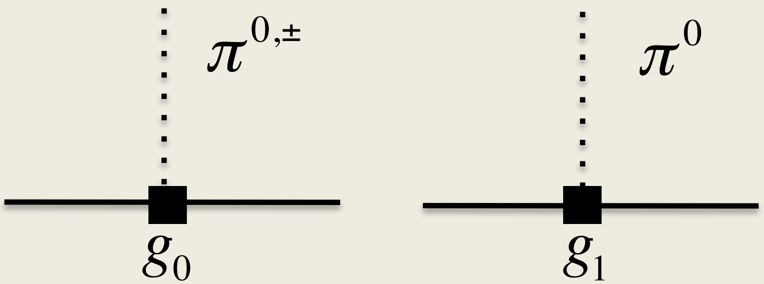
Chromo-EDM and lattice spectroscopy

- Quark chromo-EDM in many BSM scenarios (SUSY, 2HDM, leptoquarks..)

$$\tilde{d}_{CE}\ \bar{q}\sigma^{\mu\nu} i\gamma^5 \lambda^a q\ G^a_{\mu\nu}$$

- Induces both g_0 and g_1 at leading order. ChPT gives **no info** about sizes…

$$L = g_0\ \bar{N}\pi\cdot\tau N + g_1\ \bar{N}\pi^0 N$$



- QCD sum rules estimate uncertain

Pospelov '02

$$\bar{g}_0 = (5\pm 10)(\tilde{d}_u + \tilde{d}_d)\,\mathrm{fm}^{-1} \qquad \bar{g}_1 = (20^{+20}_{-10})(\tilde{d}_u - \tilde{d}_d)\,\mathrm{fm}^{-1}$$

$$|\bar{g}_1| \geq |\bar{g}_0|$$

Chromo-EDM and lattice spectroscopy

- Repeat the same trick as for theta term

JdV, Mereghetti, Seng, Walker-Loud '16

$$\tilde{d}_{CE}\ \bar{q}\sigma^{\mu\nu}i\gamma^5\lambda^a q\ G^a_{\mu\nu} \quad \overset{SU_A(2)}{\longleftrightarrow} \quad \tilde{d}_{CM}\ \bar{q}\sigma^{\mu\nu}\lambda^a\tau^3 q\ G^a_{\mu\nu}$$

- Add **CP-even** quark chromo-magnetic dipole moments

- Relations between $g_{0,1}$ and the shift in nucleon and pion masses

$$\bar{g}_0 = \tilde{d}_0\left(\frac{d}{d\tilde{c}_3} + r\frac{d}{d(\bar{m}\varepsilon)}\right)\delta m_N$$

$$\bar{g}_1 = -2\tilde{d}_3\left(\frac{d}{d\tilde{c}_0} - r\frac{d}{d\bar{m}}\right)\Delta m_N + 4\frac{\phi}{\sqrt{3}}\left[\tilde{d}_s\left(\frac{d}{d\tilde{c}_s} - r\frac{d}{dm_s}\right)\right]\Delta m_N$$

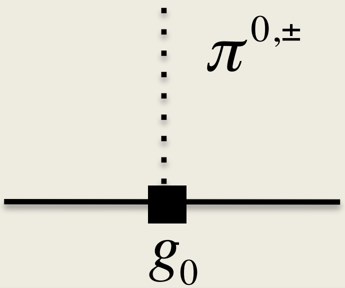
- All relations **stable** under higher-order and SU(3) corrections
- No NNpi calculation or CPV on the lattice needed

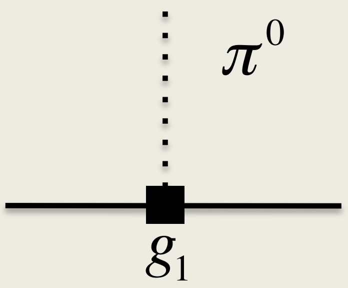
- CalLat is attempting a calculation with this strategy

Back to pion-nucleon couplings

- 2 CP-odd structures

$$L = g_0\, \bar{N}\pi\cdot\tau N + g_1\, \bar{N}\pi^0 N$$





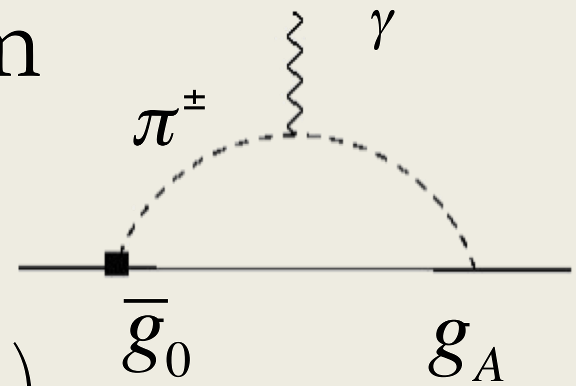
	Theta term	Quark CEDMs	Four-quark operators	Weinberg	Quark EDM
g_0	🟢	🟠	🔴	🔴	Don't matter
g_1	🟠	🟠	🟠	🔴	Don't matter

🟢 <25% uncertainty

🟠 Some estimate (~100% uncertainty) and/or lattice-QCD in progress

🔴 A long way to go ….

The strong CP problem



Nucleon EDM

$$d_n = \bar{d}_0(\mu) - \bar{d}_1(\mu) - \frac{e g_A \bar{g}_0}{4\pi^2 F_\pi}\left(\ln\frac{m_\pi^2}{\mu^2} - \frac{\pi}{2}\frac{m_\pi}{m_N}\right)$$

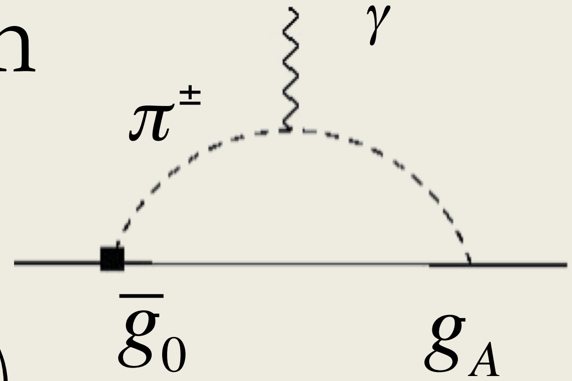
$$d_p = \bar{d}_0(\mu) + \bar{d}_1(\mu) + \frac{e g_A \bar{g}_0}{4\pi^2 F_\pi}\left(\ln\frac{m_\pi^2}{\mu^2} - 2\pi\frac{m_\pi}{m_N}\right) - \frac{e g_A \bar{g}_1}{8\pi F_\pi}\frac{m_\pi}{m_N}$$

Crewther '79 Borasoy '02
Guo et al, '10 '12 '14,
JdV et al '10 '11 '14

- Loop **enhanced** by chiral logarithm (long-range physics)
- But divergent and depends on renormalization-scale **μ**

- Counter terms absorb **μ**: no direct link between EDMs and CPV potential **at the hadronic level**

The strong CP problem



Nucleon EDM

$$d_n = \bar{d}_0 - \bar{d}_1 - \frac{e g_A \bar{g}_0}{4\pi^2 F_\pi}\left(\ln\frac{m_\pi^2}{m_N^2} - \frac{\pi}{2}\frac{m_\pi}{m_N}\right)$$

$$d_p = \bar{d}_0 + \bar{d}_1 + \frac{e g_A \bar{g}_0}{4\pi^2 F_\pi}\left(\ln\frac{m_\pi^2}{m_N^2} - 2\pi\frac{m_\pi}{m_N}\right) - \frac{e g_A \bar{g}_1}{8\pi F_\pi}\frac{m_\pi}{m_N}$$

Crewther '79 Borasoy '02
Guo et al, '10 '12 '14,
JdV et al '10 '11

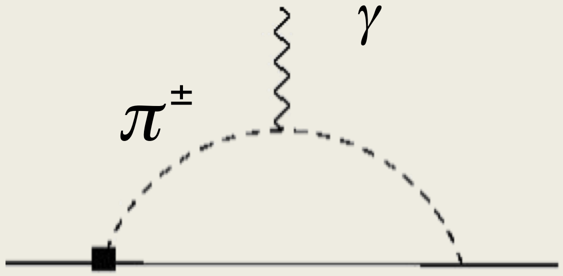
- Typical approach: set $\mu = m_N$

$$\bar{g}_0 = -(15.5 \pm 2.5)\cdot 10^{-3}\,\bar{\theta} \longrightarrow d_n \simeq -2.5\cdot 10^{-16}\,\bar{\theta}\, e\,\mathrm{cm}$$

- Experimental constraint: $\longrightarrow$ $\bar{\theta} < 10^{-10}$

- But this is not really consistent nor precise: **need lattice**
- Also affects axion experiments (e.g. Casper)

ChPT is of some use



Nucleon EDM

- The EDM is a divergent quantity, but the Q^2 dependence is not

$$F(Q^2) = d + Q^2 S + Q^4 H + \ldots$$

$$S_n = -S_p = -\frac{e g_A \bar{g}_0}{48\pi^2 F_\pi} \frac{1}{m_\pi^2}\left(1 - \frac{5\pi}{4}\frac{m_\pi}{m_N}\right) \cong 7 \cdot 10^{-5}\, \bar{\theta}\, e\, fm^3$$

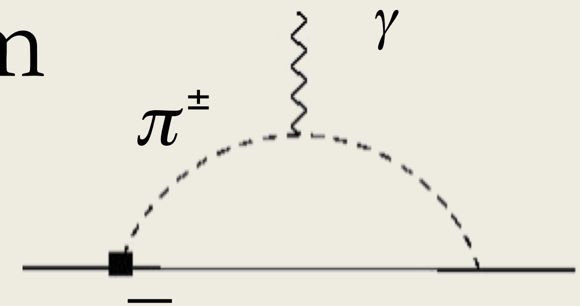
- H are complicated but known functions

$$H_1(Q^2) = \frac{4 e g_A \bar{g}_0}{15(2\pi F_\pi)^2}\left[h_1^{(0)}\left(\frac{Q^2}{4m_\pi^2}\right) - \frac{7\pi}{8}\frac{m_\pi}{m_N} h_1^{(1)}\left(\frac{Q^2}{4m_\pi^2}\right) - \frac{2\breve{\delta} m_\pi^2}{m_\pi^2}\breve{h}_1^{(1)}\left(\frac{Q^2}{4m_\pi^2}\right)\right].$$

$$h_1^{(0)}(x) = -\frac{15}{4}\left[\sqrt{1+\frac{1}{x}}\ln\left(\frac{\sqrt{1+1/x}+1}{\sqrt{1+1/x}-1}\right) - 2\left(1+\frac{x}{3}\right)\right]$$

Thomas '93, Hocking/van Kolck '06, JdV et al '10 '11, Guo et al '10

The strong CP problem



Nucleon EDM

		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	$-1.50(16)^a$	$-0.217(18)$	$-0.555(74)$	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	$-0.560(40)$	$-0.247(17)^b$	$-0.325(68)$	$-0.048(68)$
	p	530	1.334(8)	0.399(37)	$-0.247(17)^b$	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	$-1.715(46)$	$-0.070(20)$	$-1.39(1.52)$	$-1.15(1.52)$
	n	605	1.470(9)	$-1.698(68)$	$-0.160(20)$	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	$-1.491(22)^c$	$-0.079(27)^d$	$-0.375(48)$	$-0.130(76)^d$
	n	360	1.138(13)	$-1.473(37)^c$	$-0.092(14)^d$	$-0.248(29)$	$0.020(58)^d$

Abramczyk et al '17

- Many calculations of nEDM have been attempted
- **Results contaminated by spurious signal ~ nucleon phase α_N**

$$F_3(Q^2) = \cos(2\alpha_N)\tilde{F}_3(Q^2) + \sin(2\alpha_N)\tilde{F}_2(Q^2)$$

- Corrected EDM signal consistent with zero within errors …

A new attempt

Shindler et al '14

- Andrea Shindler suggested Gradient Flow for EDM calculations
- Attempt in '15 a , quenched and spurious…. Shindler et al '15
- 2+1+1 flavor calculation with GF, also spurious Alexandrou et al '15

- Assume theta is small: weigh operators by topological charge

Shintani et al '05
Aoki et al '15

$$\langle O\rangle_{\bar\theta} = \langle O\rangle + i\bar\theta\,\langle OQ\rangle + \mathcal{O}(\bar\theta^2)$$

$$Q = \int d^4x\, q$$

$$q = \frac{1}{32\pi^2} G\tilde{G}$$

- Make use of total-derivative-nature of theta term

$$\partial_{t_f} Q(t_f) = 0$$

Luscher '10, Giusti '15

- Take a → 0 limit at finite flow time.
- Signal-to-noise is a big issue. In particular for small pion masses

$$+\theta\frac{g_s^2}{32\pi^2}\varepsilon^{\alpha\beta\mu\nu}G_{\alpha\beta}G^{\mu\nu} \longleftrightarrow -\left(\frac{m_u m_d}{m_u+m_d}\right)\theta\,\bar{q}i\gamma^5 q$$

- Theta-induced EDMs scale as m_π^2

Numerical details

Dragos, Luu, A.S., de Vries, Yousif: 2019

NP improved Wilson + Iwasaki gauge

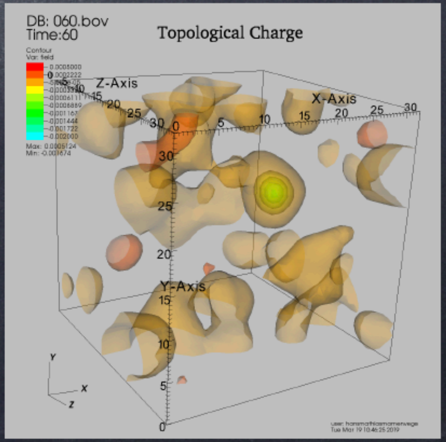
a=0.1-0.068 fm
mpi=400-700 MeV

O(L/2a) Stochastic source locations

3 Gaussian smearings

	β	κ_l	κ_s	L/a	T/a	c_{sw}	N_G	N_{corr}
M_1	1.90	0.13700	0.1364	32	64	1.715	322	30094
M_2	1.90	0.13727	0.1364	32	64	1.715	400	20000
M_3	1.90	0.13754	0.1364	32	64	1.715	444	17834
A_1	1.83	0.13825	0.1371	16	32	1.761	800	15220
A_2	1.90	0.13700	0.1364	20	40	1.715	789	15407
A_3	2.05	0.13560	0.1351	28	56	1.628	650	12867

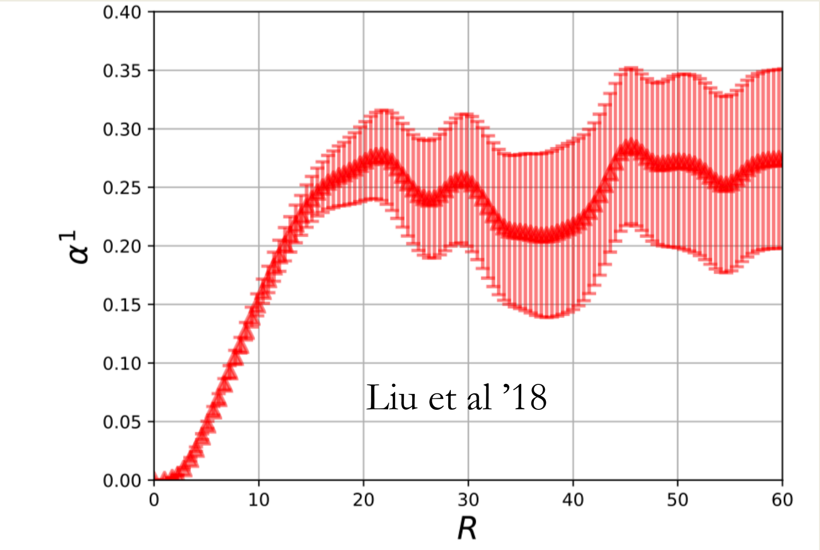
PACS-CS: 2009

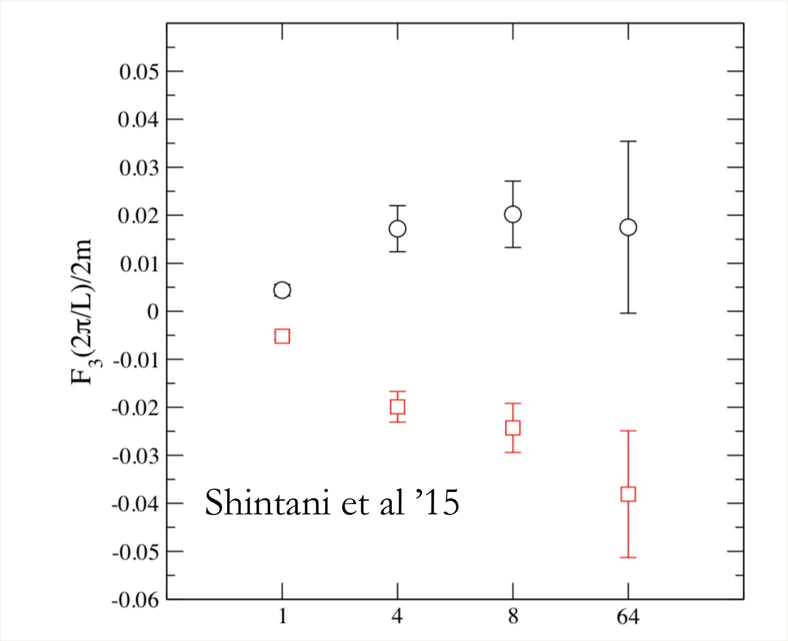


Slide stolen from A. Shindler

Improving signal to noise

- Insertion of topological charge is integrated over whole space-time box
- Liu et al '18 : signal dominated by space-time regions close to the source-sink
- Also found for CP-odd three-point function (N-N-photon) for just Euclidean time slices   Shintani et al '15



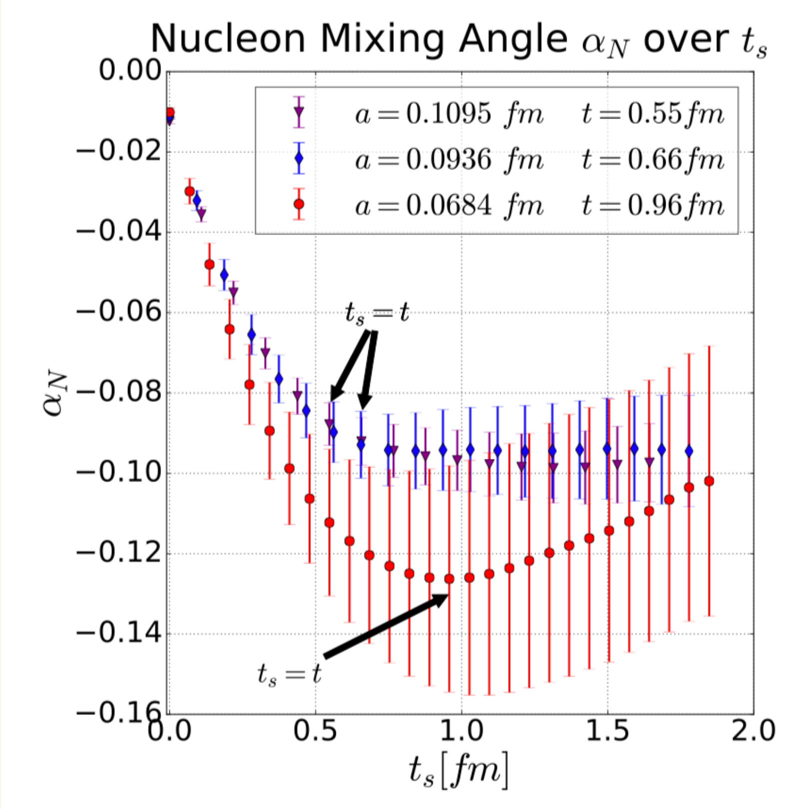


- We tried to improve S/N by not summing over the whole time-dimension of the box

Improving signal to noise

Shindler et al '19

- Example: two-point function used to extract the phase α_N
- Normally: $$G_2^{(Q)}(\boldsymbol{p}', t, \Pi, t_f) = a^3 \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}'\cdot\boldsymbol{x}} \,\mathrm{Tr}\left\{\Pi \left\langle \mathcal{N}(\boldsymbol{x}, t)\overline{\mathcal{N}}(\boldsymbol{0}, 0) Q(t_f)\right\rangle\right\}$$
- Instead: partially summed Q $$Q(t_s, t_f) = \frac{1}{32\pi^2} \sum_x \sum_{\tau_{Q=0}}^{t_s} q(x, \tau_Q, t_f)$$



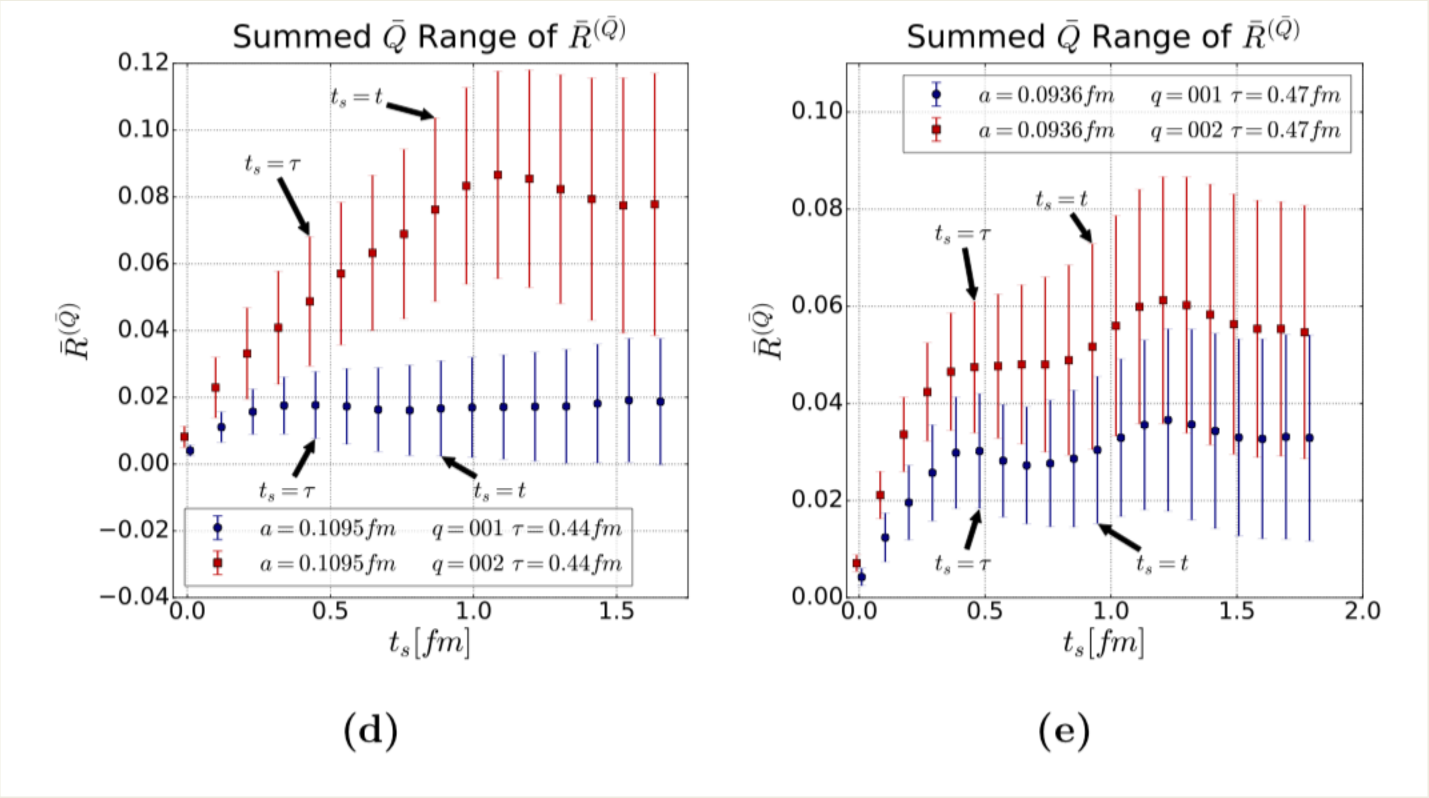
- Signal saturates at $t_s = t$ is source-sink separation
- Confirmed by spectral decomposition of correlator

$$G_2^{(Q)}(t_s \geq t, t, t_f) = G_2^{(Q)}(t, t_f) + O(e^{-Et_s})$$

Improving signal to noise

Shindler et al '19

- Example: two-point function used to extract the phase α_N
- Normally: $$G_2^{(Q)}(\boldsymbol{p}', t, \Pi, t_f) = a^3 \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}'\cdot\boldsymbol{x}}\, \mathrm{Tr}\left\{\Pi \left\langle \mathcal{N}(\boldsymbol{x}, t)\overline{\mathcal{N}}(\mathbf{0}, 0)Q(t_f)\right\rangle\right\}$$
- Similar but more complicated analysis for three-point function (NN-gamma)

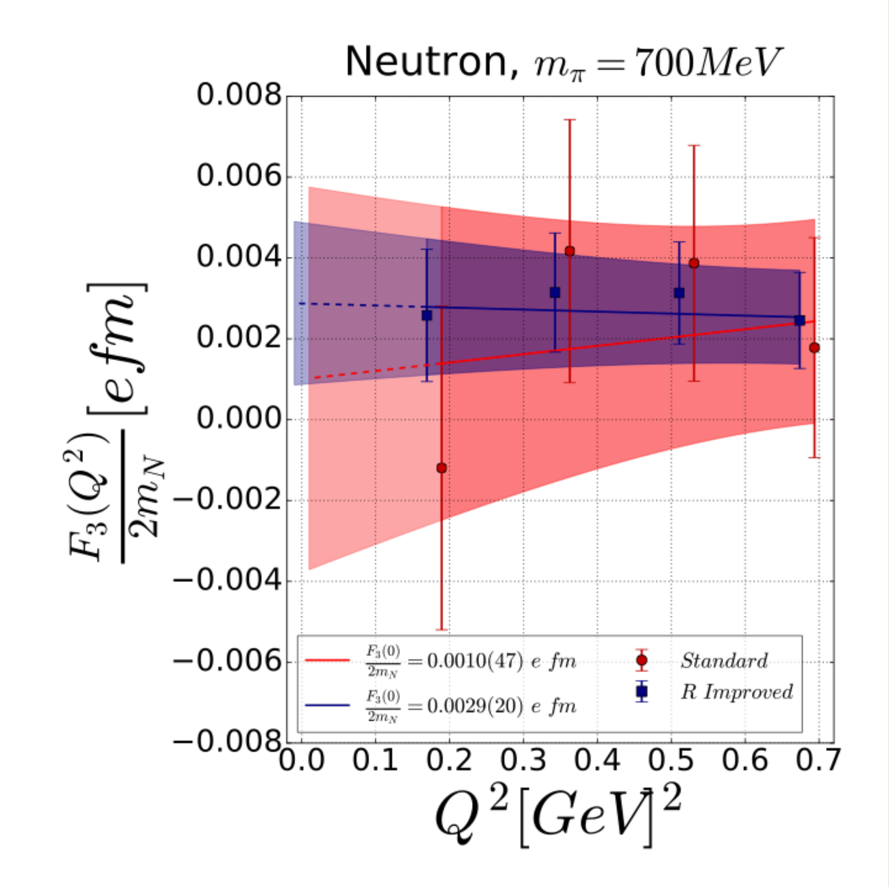


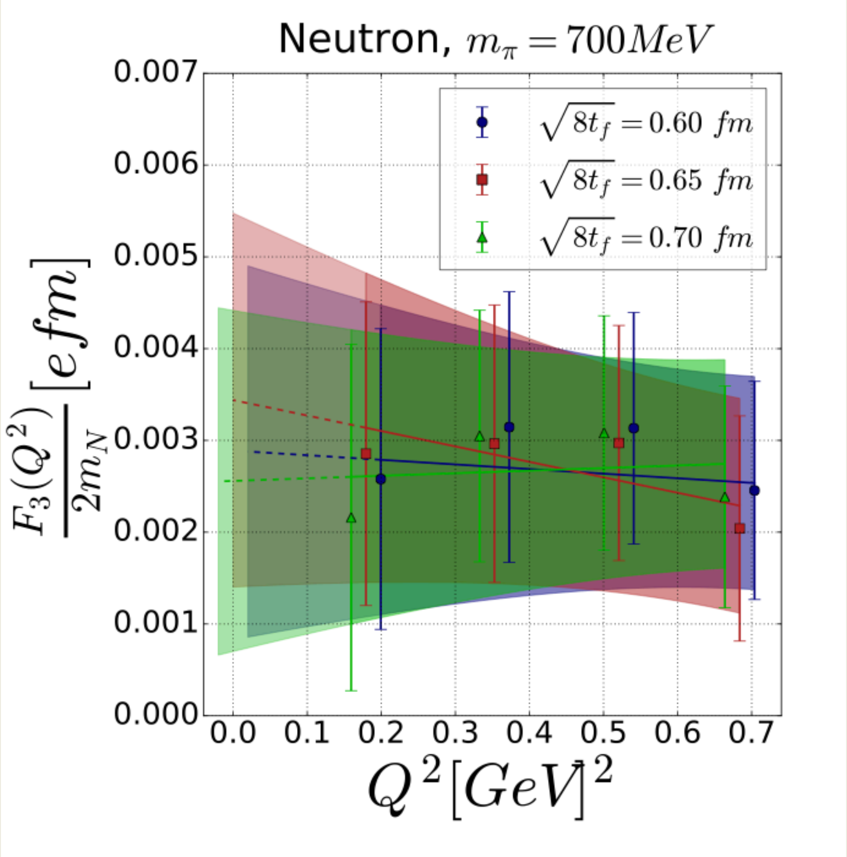
Form factor improvement + tf dependence

Shindler et al '19

- Then: extrapolate to zero momentum transfer using ChPT predictions
- Significantly improved results for partially summed topological charge

- Confirm flow-time independence





‘A less than convincing fit …’

- End up with EDMs at 3 pion masses and 3 lattice spacings
- Pion masses are large ... We nevertheless try a chiral fit …
- Note: we know in continuum+chiral limit that EDM should be zero :

$$d_{n,p} = C_1 m_\pi^2 + C_2\, m_\pi^2 \log m_\pi^2 + C_3 a^2$$

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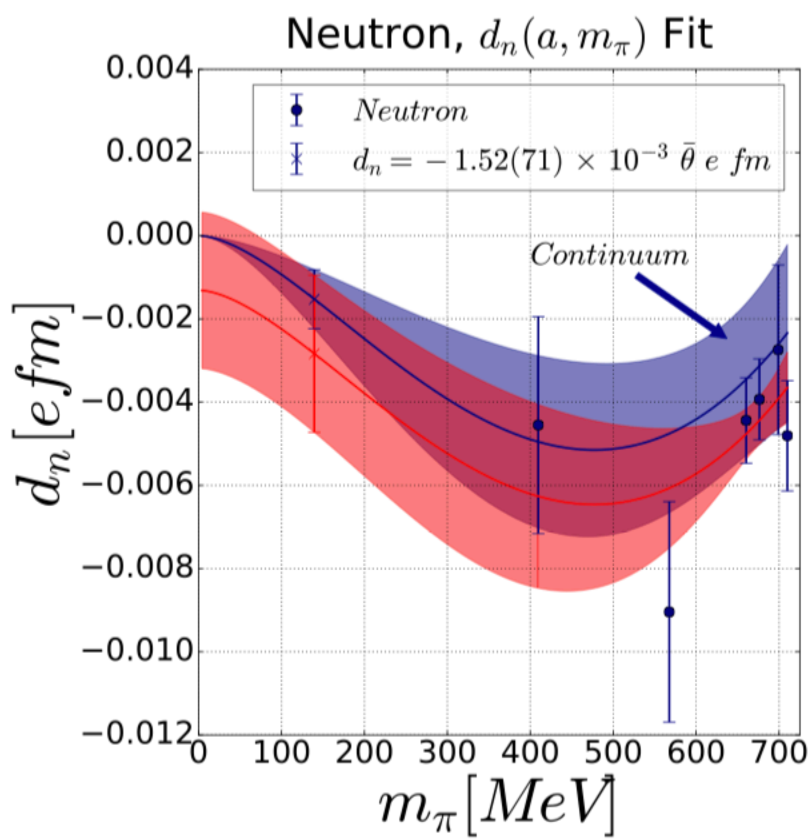


	$C_1\ [\bar{\theta}\, e\ \mathrm{fm}^3]$	$C_2\ [\bar{\theta}\, e\ \mathrm{fm}^3]$	$C_3\ \left[\frac{\bar{\theta}\, e\ \mathrm{fm}}{\mathrm{fm}^2}\right]$
proton	$-3.6(5.3) \times 10^{-4}$	$-6.8(6.6) \times 10^{-4}$	0.20(31)
neutron	$3.1(3.2) \times 10^{-4}$	$8.8(4.4) \times 10^{-4}$	−0.16(23)

- C_2 is related to g_0

$$\bar{g}_0 = -\frac{8\pi^2 f_\pi}{g_A} \frac{C_2\, m_\pi^2}{e} = -12.8(6.2) \cdot 10^{-3} \bar{\theta}$$

- Agrees with prediction from ChPT + np mass splitting

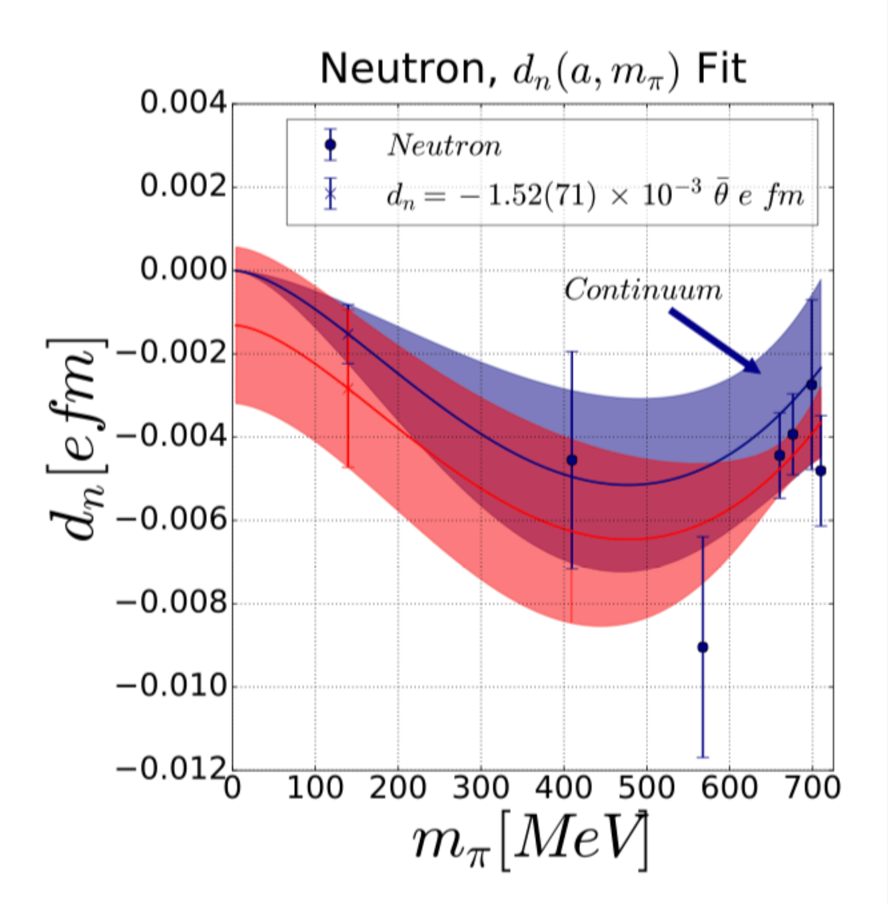
$$\bar{g}_0 = -15.5(2.5) \cdot 10^{-3} \bar{\theta}$$

- EDMs of ‘expected’ size

‘A less than convincing fit …’

- End up with EDMs at 3 pion masses and 3 lattice spacings
- Pion masses are large ... We nevertheless try a chiral fit …
- Note: we know in continuum+chiral limit that EDM should be zero :

$$d_{n,p} = C_1 m_\pi^2 + C_2\ m_\pi^2 \log m_\pi^2 + C_3 a^2$$



	$C_1\ [\bar{\theta}\, e\ \mathrm{fm}^3]$	$C_2\ [\bar{\theta}\, e\ \mathrm{fm}^3]$	$C_3\ \left[\frac{\bar{\theta}\, e\ \mathrm{fm}}{\mathrm{fm}^2}\right]$
proton	$-3.6(5.3) \times 10^{-4}$	$-6.8(6.6) \times 10^{-4}$	0.20(31)
neutron	$3.1(3.2) \times 10^{-4}$	$8.8(4.4) \times 10^{-4}$	−0.16(23)

- Despite all efforts, the signal at the physical point only at 2 sigma

$$d_n = -(1.5 \pm 0.7)\ \cdot 10^{-3}\ e\,\bar{\theta}\ fm$$

- And even less for proton EDM
- We need more data and at smaller pion masses

Schiff moments

- LO ChPT: slope of form factor at small Q^2 to be pion-mass independent

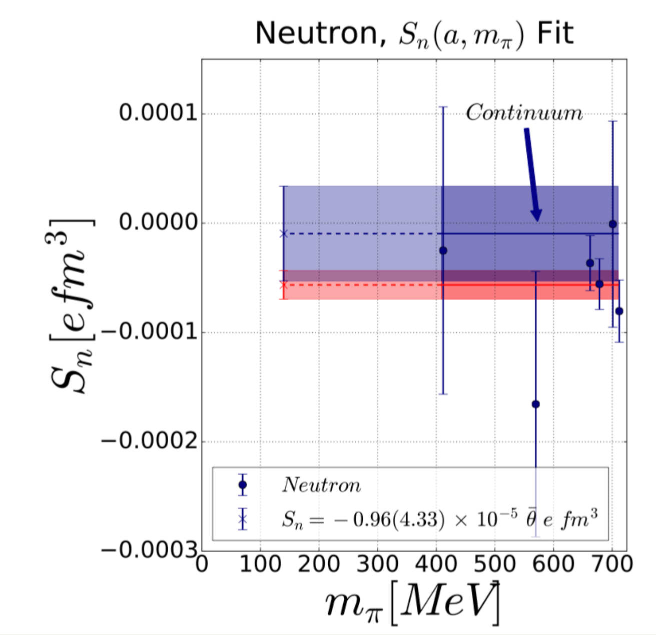
$$F(Q^2) = d + Q^2 S + Q^4 H + \ldots \qquad S_{n,p} = C_4 + C_5 a^2$$

- Size prediction $\quad S_{n,p} \sim \bar{g}_0 \,, \qquad S_{n,p} \cong \mp 7 \cdot 10^{-5}\, \bar{\theta}\, e\, fm^3$

- Attempt to extract from lattice data

$$S_n = -(1 \pm 5) \cdot 10^{-5}\, e\, \bar{\theta}\, fm$$

$$S_p = +(5 \pm 6) \cdot 10^{-5}\, e\, \bar{\theta}\, fm$$



- Numbers not crazy but clearly much more work is needed

Status

	Theta term	Quark CEDMs	Four-quark operators	Weinberg	Quark EDM
g_0	🟢	🟠	🔴	🔴	Don't matter
g_1	🟠	🟠	🟠	🔴	Don't matter
$d_{n,p}$	🟠 $\xrightarrow{?}$ 🟢	🔴 $\xrightarrow{?}$ 🟠	🔴	🔴	🟢

🟢 ~<25% uncertainty

🟠 Some estimate (~100% uncertainty) and/or lattice-QCD in progress

🔴 A long way to go ….

- Modest improvements would help a lot in interpreting EDM experiments !
- Gradient flow in progress for qCEDMs and Weinberg, but flow-time dependence must be understood.

Conclusion/Summary/Outlook

EDMs

- ✓ Very powerful search for BSM physics (probe the highest scales)
- ✓ Heroic experimental effort and great outlook
- ✓ Theory needed to interpret measurements and constraints

EFT framework

- ✓ Framework exists for CP-violation (EDMs) from 1^{st} principles
- ✓ Keep track of **symmetries** (gauge/CP/chiral) from multi-Tev to atomic scales
- ✓ Need lattice input for LECs: in particular pion-nucleon and nucleon EDMs

Nucleon EDM from strong CP violation

- ✓ Gradient flow useful tool
- ✓ Improved S/N by only summing over relevant regions
- ✓ Reasonable neutron EDM and g_0 but large uncertainties → more data needed
- ✓ **Have to go beyond theta term !!**

Backup

Trust issues

- The relations are no longer unique if we use SU(3) chPT

$$g_0 = \delta m_N \frac{m_*}{\bar{m}\varepsilon}\bar{\theta} \qquad\qquad g_0 = (m_\Xi - m_\Sigma)\frac{2m_*}{(m_s - \bar{m})}\bar{\theta}$$

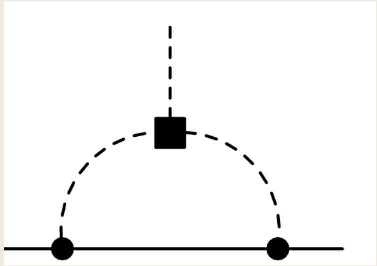
- Numerically: LO relations differ by **more than 100%** (sometimes sign...)

$$g_0 = (15.5 \pm 2.5)\cdot 10^{-3}\,\bar{\theta}$$

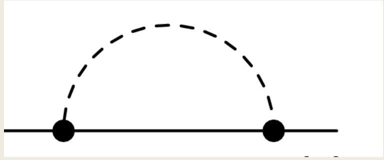
Can this be trusted ??

- Investigate higher-order corrections to left- right-sides of relations

g_0 @ NLO

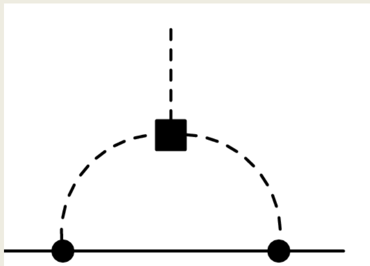


Mass terms @ NLO

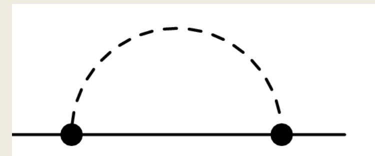


Protected relations

g_0 @ NLO



Mass terms @ NLO



$$g_0 = \delta m_N \frac{m_*}{\bar{m}\varepsilon}\bar{\theta}$$

$$g_0 = (m_\Xi - m_\Sigma)\frac{2m_*}{(m_s - \bar{m})}\bar{\theta}$$

- Relation 1: All corrections obey the relation
- Relation 2: Explicit violation already at NLO

$$\frac{g_0}{(m_\Xi - m_\Sigma)} = \left[1 + \frac{(D^2 - 6DF - 3F^2)}{6(4\pi f_\pi)^2}\frac{(m_K - m_\pi)^2(m_K + m_\pi)}{(m_\Xi - m_\Sigma)}\right]\frac{2m_*}{(m_s - \bar{m})}\bar{\theta}$$

$$\approx [1 - 0.7]\frac{2m_*}{(m_s - \bar{m})}\bar{\theta}$$

JdV, Mereghetti, Walker-Loud ‘15

Wrap-up

- Identify **protected relations** (including N2LO) for various couplings

JdV et al '15

	Values obtained here ($\times 10^{-3}\,\bar{\theta}$)
$\bar{g}_0/(2F_\pi)$	15.5 ± 2.5
$\bar{g}_{0\eta}/(2F_\eta)$	115 ± 37
$\bar{g}_{0N\Sigma K}/(2F_K)$	-36 ± 11
$\bar{g}_{0N\Lambda K}/(2F_K)$	-44 ± 13

- Values recommended for **lattice extrapolations** of neutron EDM

- Used to estimate **short-range CPV NN** forces
- Similar couplings appear in axion phenomenology   Stadnik et al '14

- Isospin-violating coupling g_1 has **no** protected relation.

$$g_1 = -(3 \pm 2) \cdot 10^{-3}\,\bar{\theta}$$

Partially based on resonance saturation
Bsaisou et al '12

$$\frac{\bar{g}_1}{\bar{g}_0} = -(0.2 \pm 0.1)$$

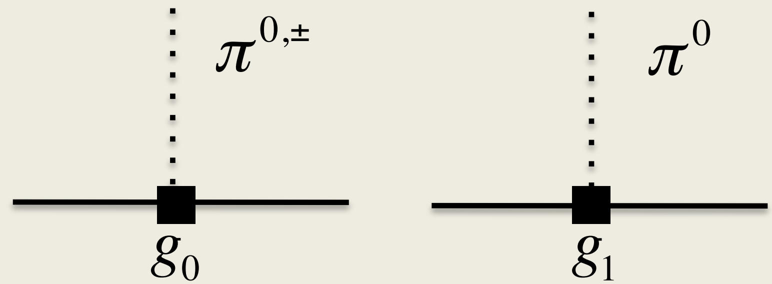
Chromo-EDM and lattice spectroscopy

- Quark chromo-EDM in many BSM scenarios (SUSY, 2HDM, leptoquarks..)

$$\tilde{d}_{CE}\ \bar{q}\sigma^{\mu\nu}i\gamma^5\lambda^a q\ G^a_{\mu\nu}$$

- Induces both g_0 and g_1 at leading order. ChPT gives **no info** about sizes…

$$L = g_0\ \bar{N}\pi\cdot\boldsymbol{\tau}N + g_1\ \bar{N}\pi^0 N$$



- QCD sum rules estimate uncertain

Pospelov '02

$$\bar{g}_0 = (5 \pm 10)(\tilde{d}_u + \tilde{d}_d)\,\mathrm{fm}^{-1} \qquad \bar{g}_1 = (20^{+20}_{-10})(\tilde{d}_u - \tilde{d}_d)\,\mathrm{fm}^{-1}$$

$$|\bar{g}_1| \geq |\bar{g}_0|$$

Chromo-EDM and lattice spectroscopy

- Repeat the same trick as for theta term

$$\tilde{d}_{CE}\ \bar{q}\sigma^{\mu\nu} i\gamma^5 \lambda^a q\ G^a_{\mu\nu} \quad \overset{\longleftrightarrow}{\text{SU}_A(2)} \quad \tilde{d}_{CM}\ \bar{q}\sigma^{\mu\nu}\lambda^a \tau^3 q\ G^a_{\mu\nu}$$

- Add **CP-even** quark chromo-magnetic dipole moments
- Isospin + CP violation leads to vacuum instability (pion tadpoles)
- Align vacuum via $SU_A(2)$ rotations

Pospelov/Ritz '00, JdV et al '12, Bsaisou et al '14

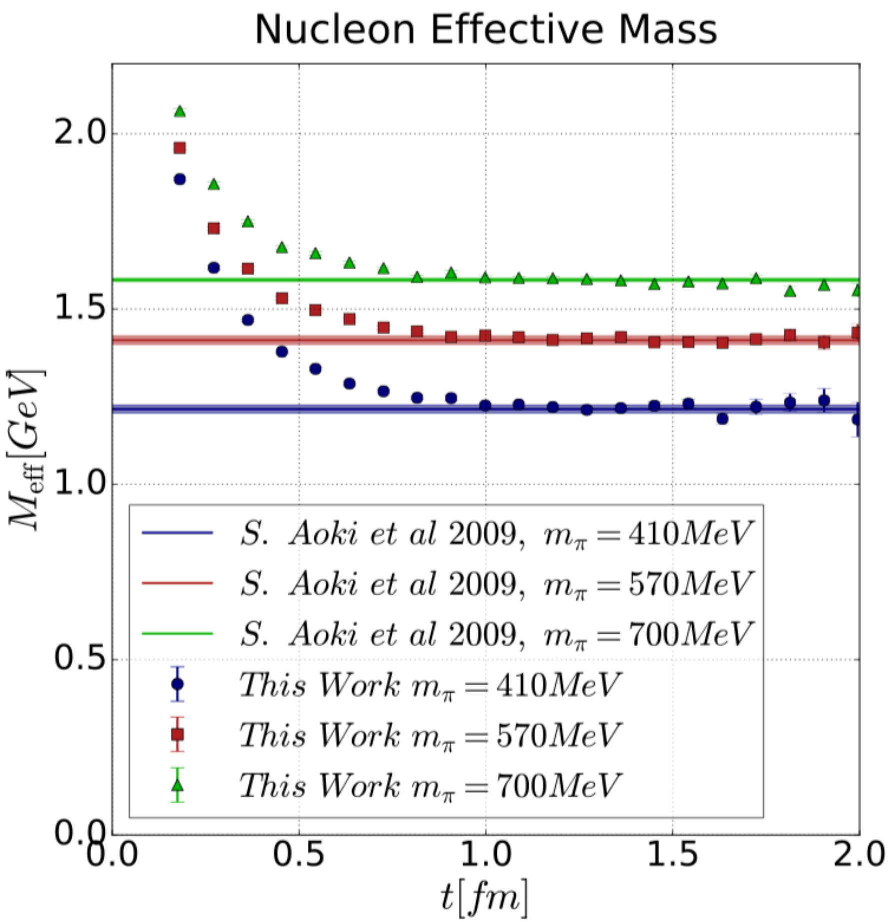
$$L_{\mathrm{dim}\,6} = r\ \bar{q}\ \tilde{d}_{CE}(i\gamma^5)q - \bar{q}\sigma^{\mu\nu}\lambda^a(\tilde{d}_{CM} + \tilde{d}_{CE} i\gamma^5)q\ G^a_{\mu\nu}$$

- r is ratio of condensates

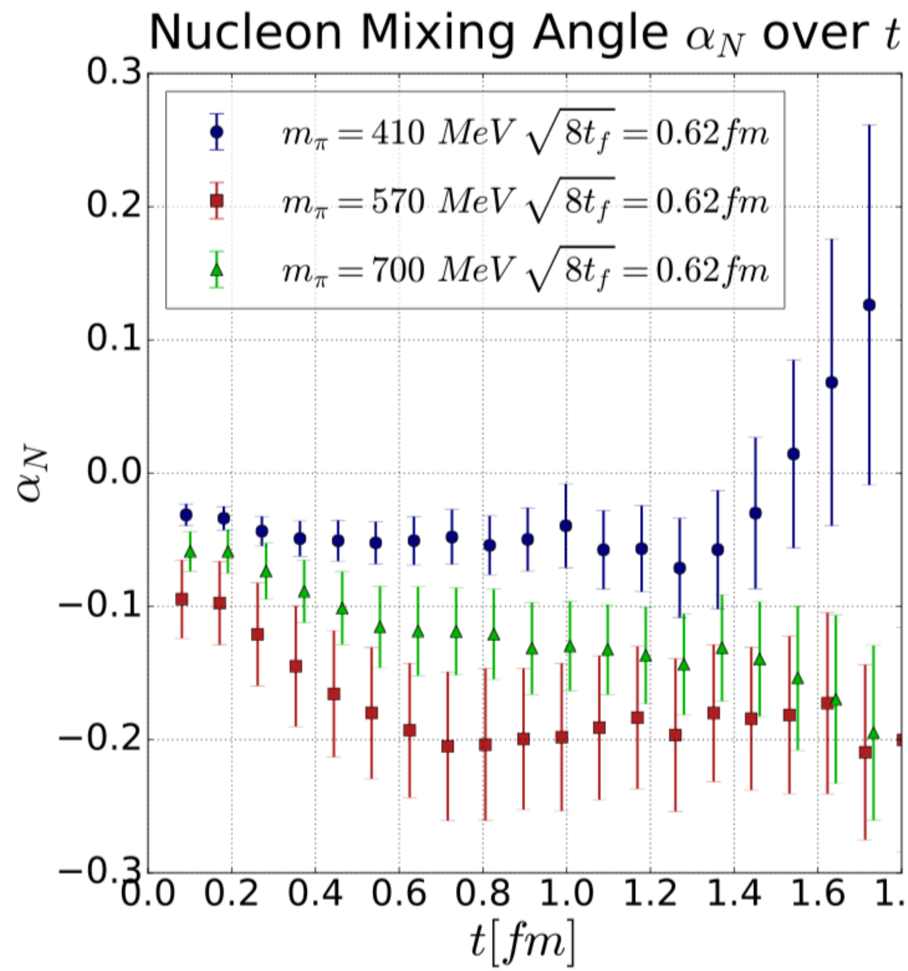
$$r \propto \frac{\langle 0 | \bar{q}\sigma^{\mu\nu}\lambda^a q G^a_{\mu\nu} | 0 \rangle}{\langle 0 | \bar{q}q | 0 \rangle} \propto \frac{\tilde{m}_\pi^2}{m_\pi^2}$$

- Now build chiral Lagrangian in usual way but with 2 chiral spurion fields

$$\chi = 2BM \rightarrow 2B(M + ir\tilde{d}_{CE}) \qquad \tilde{\chi} = 2B(\tilde{d}_{CM} + i\tilde{d}_{CE})$$


Nucleon Effective Mass
$M_{\text{eff}}[GeV]$
$t[fm]$
0.0
0.5
1.0
1.5
2.0
$S.\ Aoki\ et\ al\ 2009,\ m_\pi = 410MeV$
$S.\ Aoki\ et\ al\ 2009,\ m_\pi = 570MeV$
$S.\ Aoki\ et\ al\ 2009,\ m_\pi = 700MeV$
$This\ Work\ m_\pi = 410MeV$
$This\ Work\ m_\pi = 570MeV$
$This\ Work\ m_\pi = 700MeV$

(a)


Nucleon Mixing Angle α_N over t
α_N
$t[fm]$
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
$m_\pi = 410\ MeV\ \sqrt{8t_f} = 0.62fm$
$m_\pi = 570\ MeV\ \sqrt{8t_f} = 0.62fm$
$m_\pi = 700\ MeV\ \sqrt{8t_f} = 0.62fm$