Light-quark connected vacuum polarization and muon g-2

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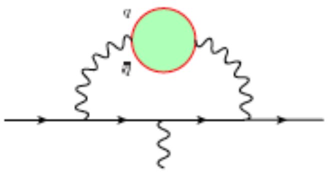
Motivation:

- Light-quark connected part is the "hardest" part: sub-percent precision required
- Improve statistics: try Low Mode Averaging and All Mode Averaging
- Improve/understand systematics: finite-volume effects to two loops in chiral perturbation theory (include taste-breaking to one loop)
  Provides comparison with GSLL (Mainz) and other (HPQCD) models
- Provide (staggered) result to compare with other (staggered) results (BMW, FNAL-HPQCD-MILC)
- Here: focus on finite-volume effects and continuum extrapolation

Definitions

Compute $a_\mu^{\rm HVP}$ in *time-momentum representation* (Bernecker & Meyer, ‘11):

$a_\mu^{\rm HVP} =$ 

$$= 4\alpha^2 \int_0^\infty dq^2 f(q^2)(\Pi(q^2) - \Pi(0))$$

$$= 4\alpha^2 \sum_t \int_0^\infty dq^2 f(q^2) \left(\frac{\cos(qt) - 1}{q^2} + \frac{t^2}{2} \right) C(t)$$

$$= \sum_t w(t) C(t)$$

with $$C(t) = \frac{1}{3} \sum_{\vec{x},i} \langle j^i(\vec{x},t) j^i(0) \rangle_{\rm conn}$$

the connected (doubly) subtracted light-quark current correlator

Assume isospin symmetry; no QED corrections

Finite volume strategy

- Finite-volume (FV) effects dominated by pions $\Rightarrow$ $SU(2)$ FV ChPT
  One loop (lowest order) (Aubin et al., '15) does not capture full effect
  (Mainz, '13; RBC/UKQCD, '18; Shintani & Kuramashi, '19)
  $\Rightarrow$ calculate $C(t)$ to two loops directly
  (seems easier than mom. space calculation: Bijnens & Relefors, '17)
  also provides estimate for convergence of ChPT

- ChPT only reliable for large t, but FV effects are a long-distance effect;
  can use ChPT to compute FV effects in a_μ^{HVP},
  we will compute

$$\Delta a_\mu^{\mathrm{HVP}} = \lim_{L\to\infty} a_\mu^{\mathrm{HVP}}(L) - a_\mu^{\mathrm{HVP}}(L)$$

  in ChPT, and use this to correct lattice results

Finite volume strategy - 2

- Two strategies:
  1) Extrapolate lattice results to continuum; use continuum FV ChPT
  2) Correct lattice results for FV effects, then extrapolate

     advantage: different ensembles have (slightly) different volumes

     disadvantage: need staggered ChPT

- Use hybrid method: correct at one loop using SChPT, extrapolate, then apply two-loop continuum FV correction

- All lattice results at physical pion mass;
  use leading-order ChPT (= scalar QED) to adjust for slight pion mass mistunings (small effect)

A peek at ChPT results

- To NNLO (*i.e.*, two loops)

$$C(t) = \frac{10}{9}\frac{1}{3}\left(\frac{1}{L^d}\sum_{\vec{p}}\frac{p^2}{E_p^2}e^{-2E_pt}\left[1 - \frac{1}{F^2L^d}\sum_{\vec{k}}\frac{1}{E_k} - \frac{8(p^2+m_\pi^2)}{F^2}\ell_6\right]\right.$$

$$\left. + \frac{1}{2dF^2L^{2d}}\sum_{\vec{p},\vec{k}}\frac{p^2k^2}{E_p^2E_k^2}\frac{E_ke^{-2E_pt} - E_pe^{-E_kt}}{k^2-p^2}\right)$$

$$= \frac{10}{9}\frac{1}{6\pi^2}\sum_{\vec{n}}\int_0^\infty dp\,\frac{p^3}{E_p^2}\,e^{-2E_pt}\frac{\sin(npL)}{nL} + \mathrm{NNLO}$$

↑ Poisson resummation

- Contains (only) two well-known low-energy constants,
$F_\pi = 92.21\ \mathrm{MeV}$, ℓ_6 from Bijnens, Colangelo & Talavera, '98

Ensembles used

- Used HISQ ensembles from the MILC collaboration with parameters

m_π (MeV)	a (fm)	size	L (fm)	$m_\pi L$
133	0.12121(64)	$48^3 \times 64$	5.82	3.91
130	0.08787(46)	$64^3 \times 96$	5.62	3.66
134	0.05684(30)	$96^3 \times 192$	5.46	3.73

- Taste breaking (courtesy Doug Toussaint)

a (fm)	mass range (MeV)
0.12121(64)	133–326
0.08787(46)	130–210
0.05684(30)	134–151

FV corrections, numerical results from ChPT

- Compare $10^{10}\Delta a_\mu^{\rm HVP}$ in NLO, NNLO in continuum ChPT, and NLO in SChPT:

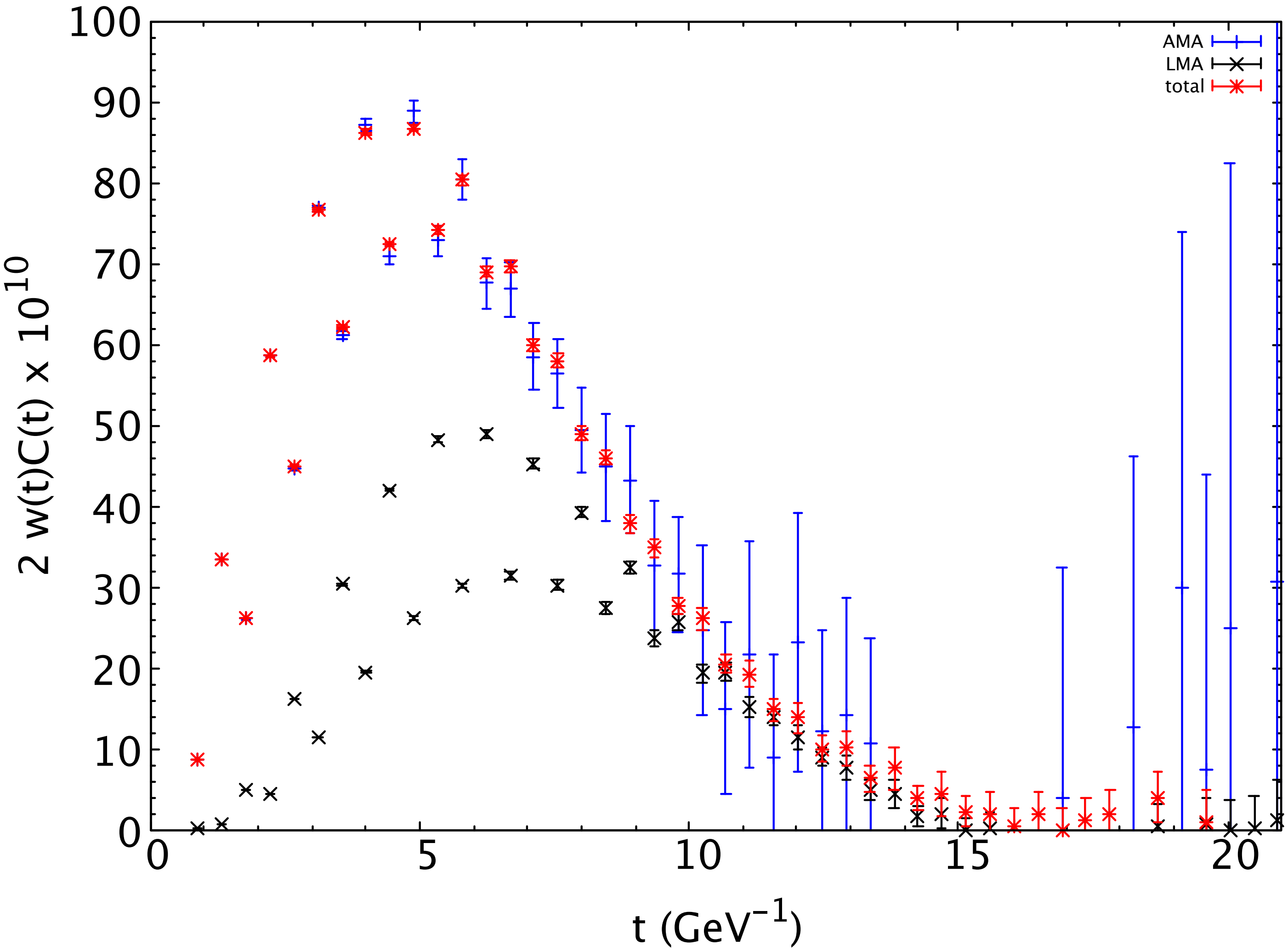
L/a	NLO (cont.)	NNLO	NLO (SChPT)
96	20.6	9.1	15.6
64	21.6	9.0	6.9
48	18.1	7.4	2.1

- Taste-breaking corrections in infinite volume to NLO (times 10^{10})

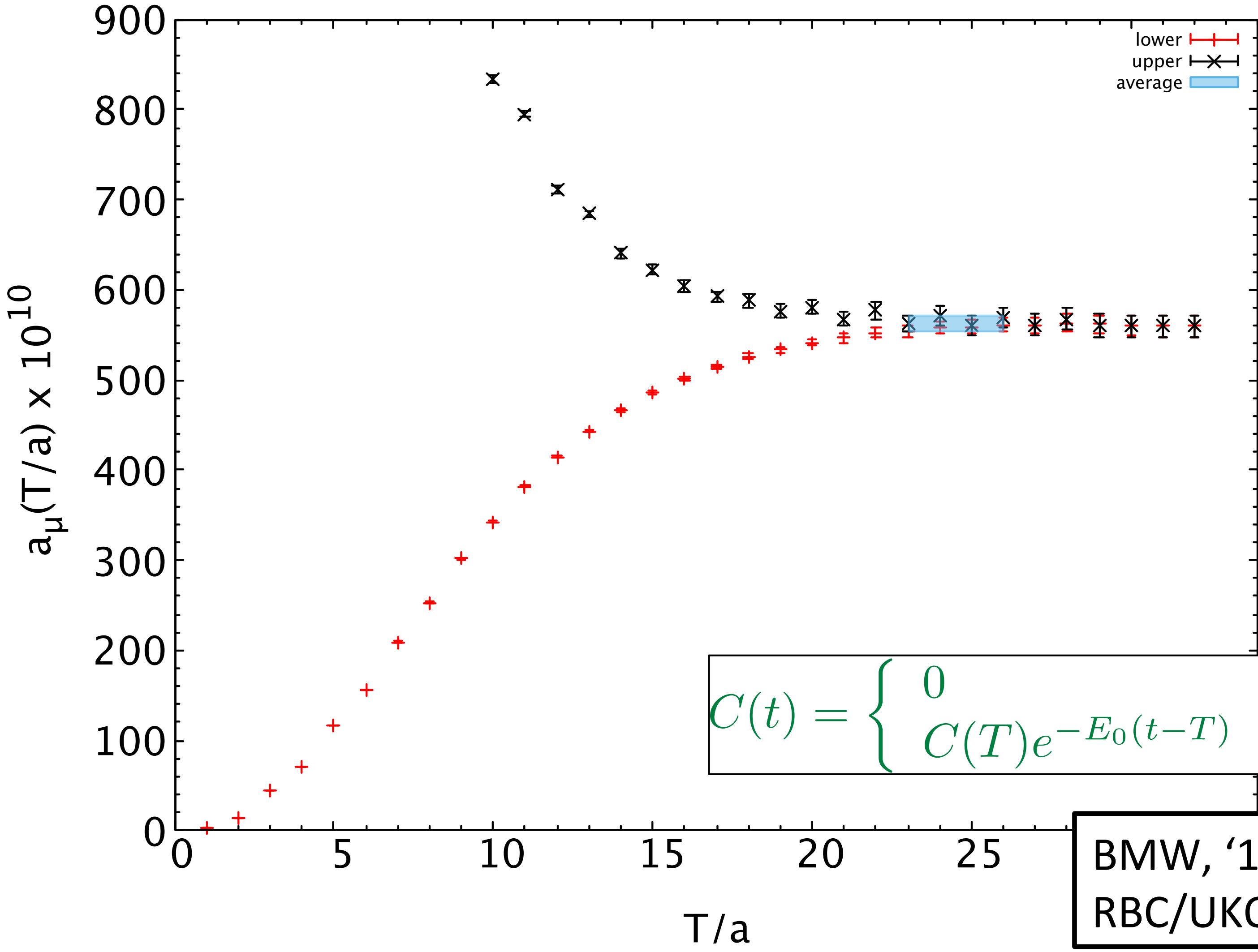
a (fm)	taste correction
0.05684	9.5
0.08787	34.2
0.12121	51.6

- Continuum extrapolation independent of this correction:
  varying method gives estimate of continuum extrapolation error

Weighted correlator results (64^3 x 96 example)



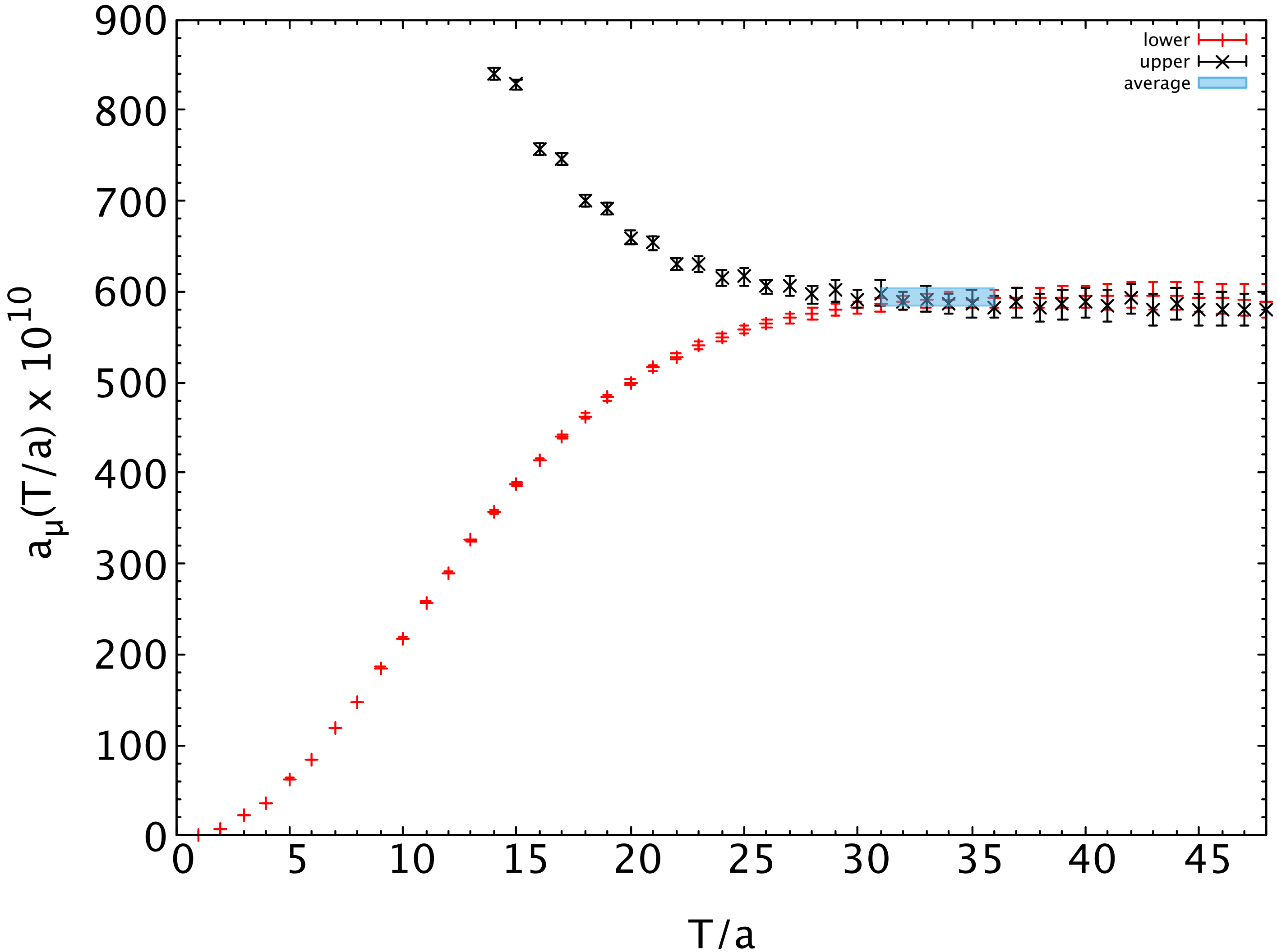
a_μ^{HVP} using bounding method, switch point T ($48^3 \times 64$)



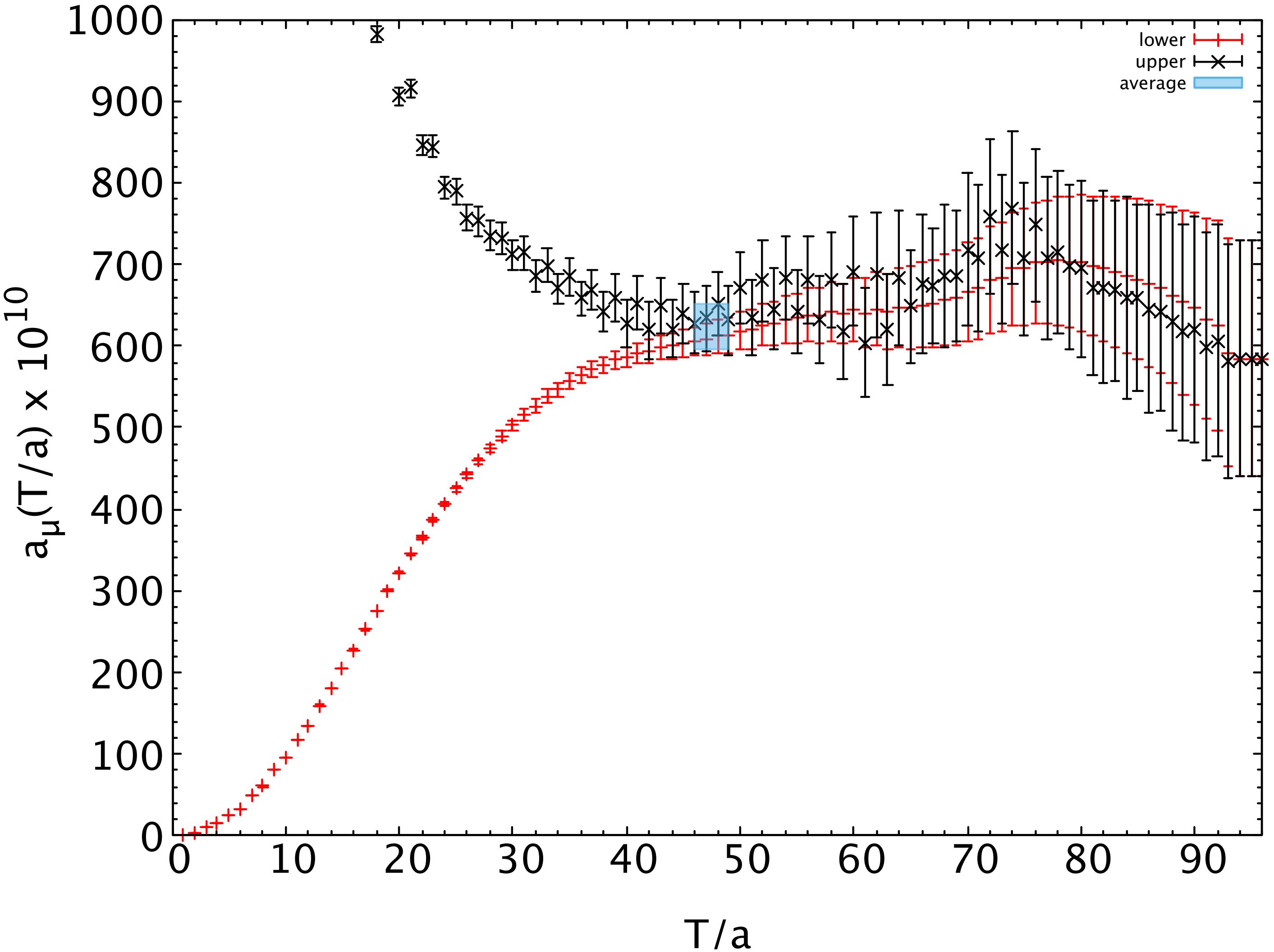
$$C(t) = \begin{cases} 0 & \text{(lower)} \\ C(T)e^{-E_0(t-T)} & \text{(upper)} \end{cases}$$

BMW, '17
RBC/UKQCD, '18

a_μ^{HVP} using bounding method, switch point T (64^3 x 96)



a_μ^{HVP} using bounding method, switch point T (96^3 x 192)



fewer low modes, could improve

Results in units of 10^{-10}

a (fm)	lattice value	FV corr.	FV + taste corr.	FV+taste+m_π
0.12121(64)	562.1(8.4)	564.2(8.4)	615.8(8.4)	613.6(8.4)
0.08787(46)	594.8(10.4)	601.7(10.4)	635.9(10.4)	630.2(10.4)
0.05684(30)	623.1(27.5)	638.7(27.5)	648.2(27.5)	647.1(27.5)
0		648.3(20.0)	657.9(20.0)	651.1(20.1)

All corrections in table are NLO ChPT only

Continuum values in columns 3 and 4 should agree ⇒ 4.8 cont. extr. error

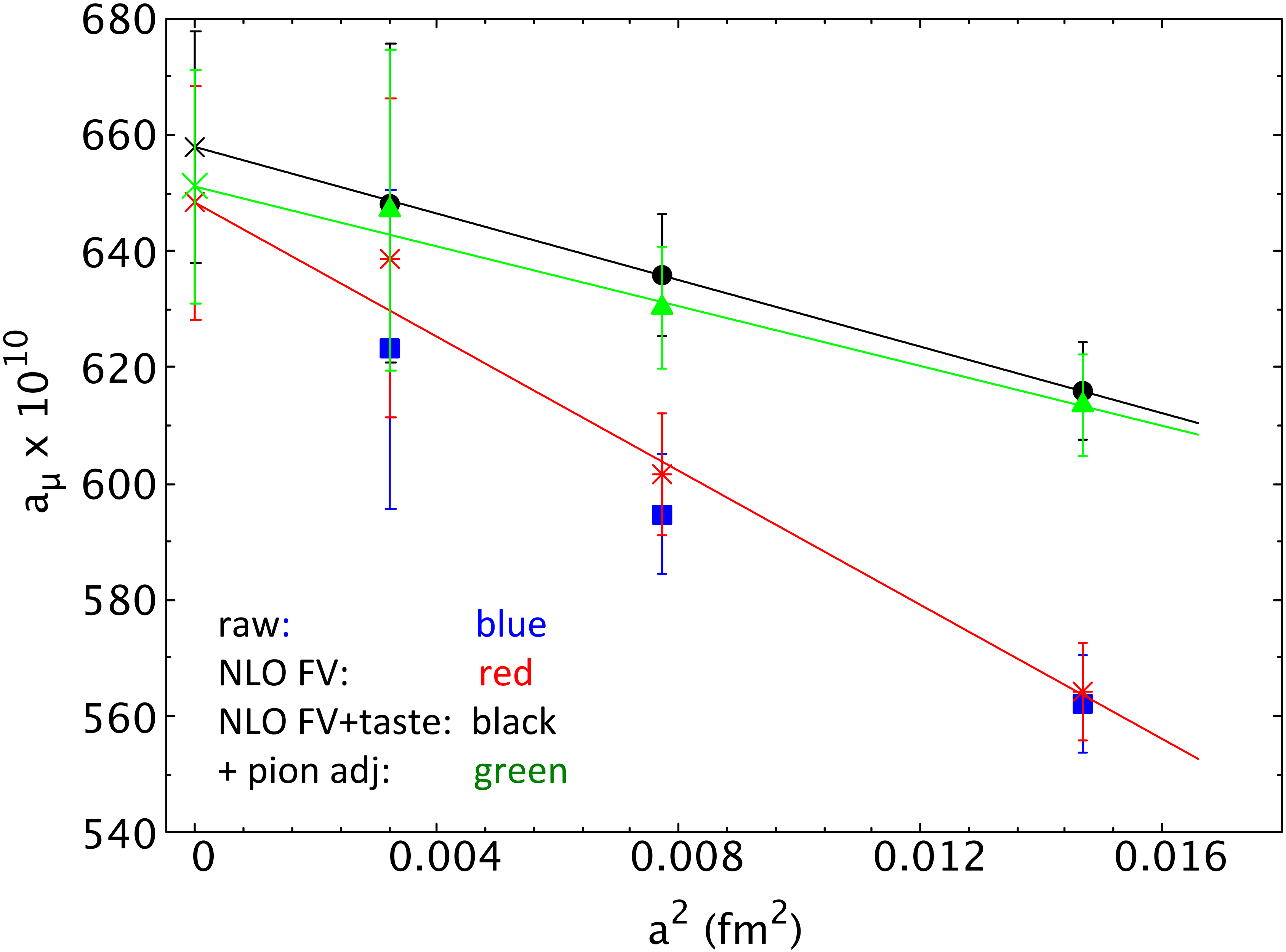
Last column: also correct for pion mass mistuning; take as central value

Add (continuum) NNLO correction (take average): add 8

Estimate higher orders in ChPT geometrically ⇒ 4 error for NNNLO ChPT

Scale setting error of 5 (from HPQCD)

TOTAL: 659 ± 20 ± 5 ± 5 ± 4 = 659 ± 22



680
660
640
620
600
580
560
540
$a_\mu \times 10^{10}$
0
0.004
0.008
0.012
0.016
a^2 (fm^2)
raw: blue
NLO FV: red
NLO FV+taste: black
+ pion adj: green

Comparison with other work (light-quark connected)

Units: 10^{-10}

Our value: 659 ± 20 ± 5 ± 5 ± 4 = 659 ± 22

FNAL/HPQCD/MILC: 630.1 ± 8.3

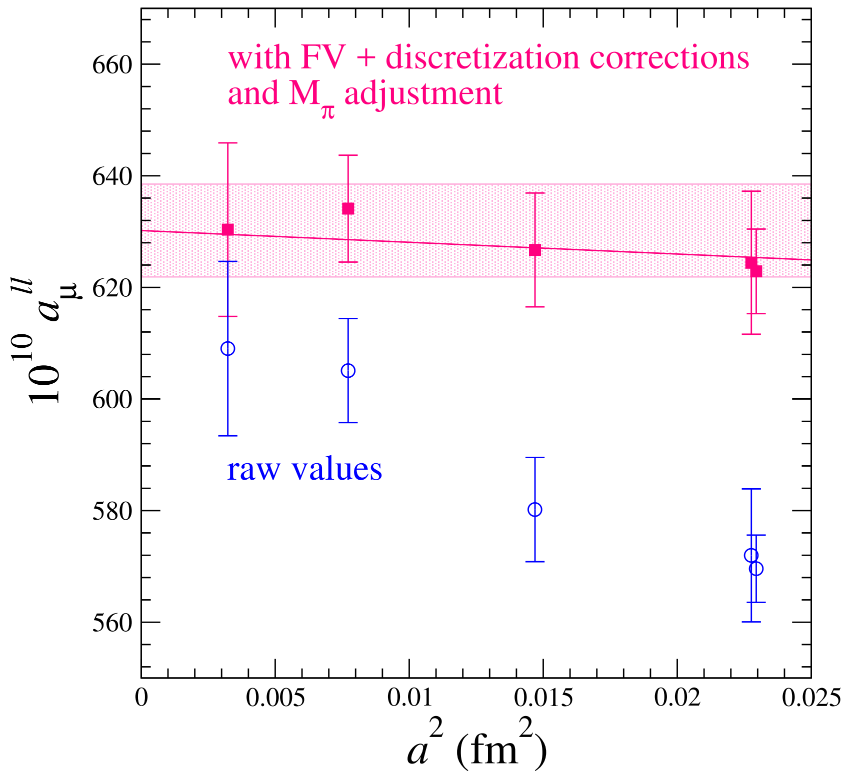
(using priors on the fit)

staggered, appears to be

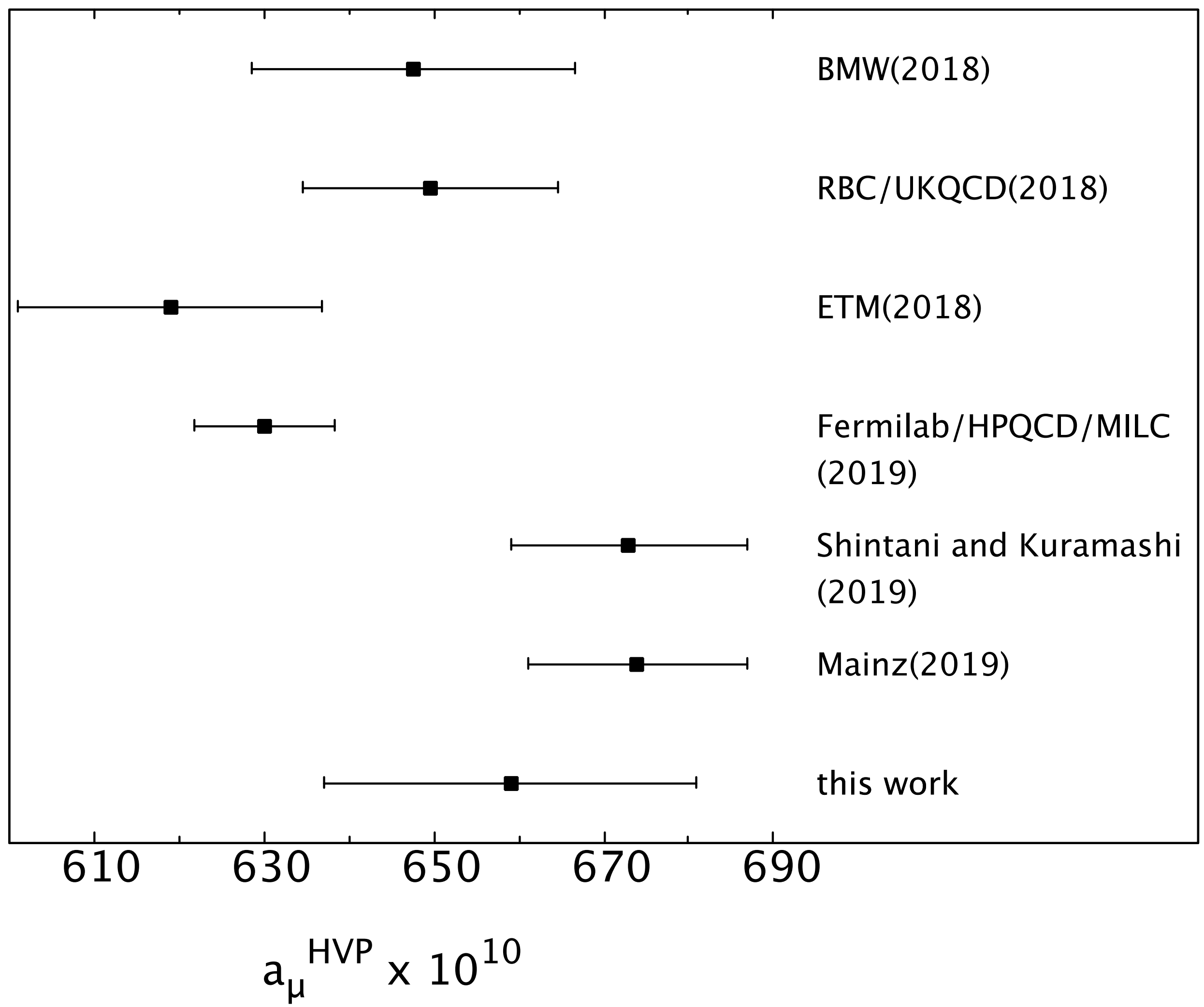
“moved down” by $\mathrm{a=0.15\ fm}$ points

statistical error ± 5

(plot from FHM, arXiv:1902.04223)



Comparison with other work (light-quark connected)

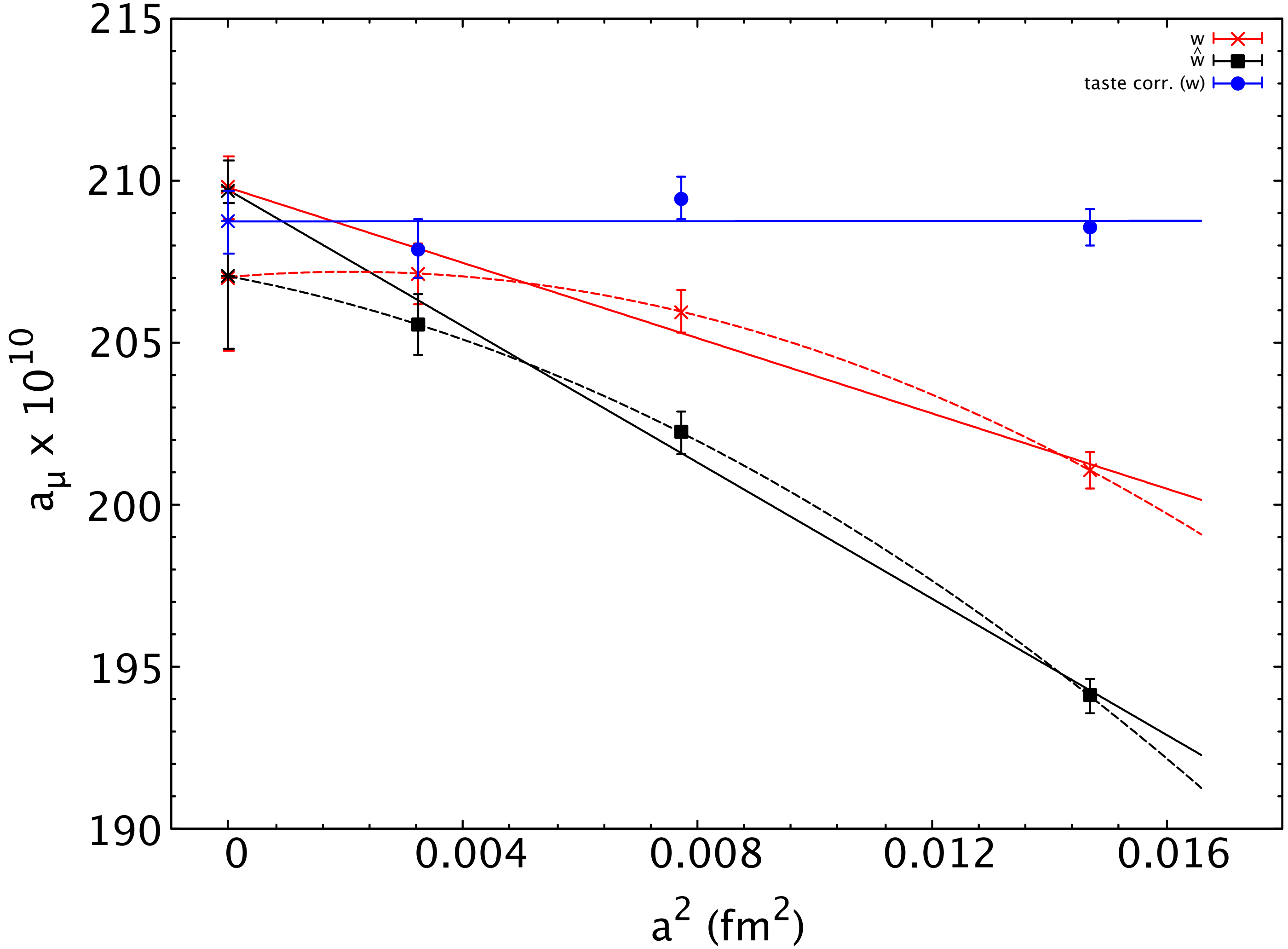


staggered

staggered

staggered

Window method



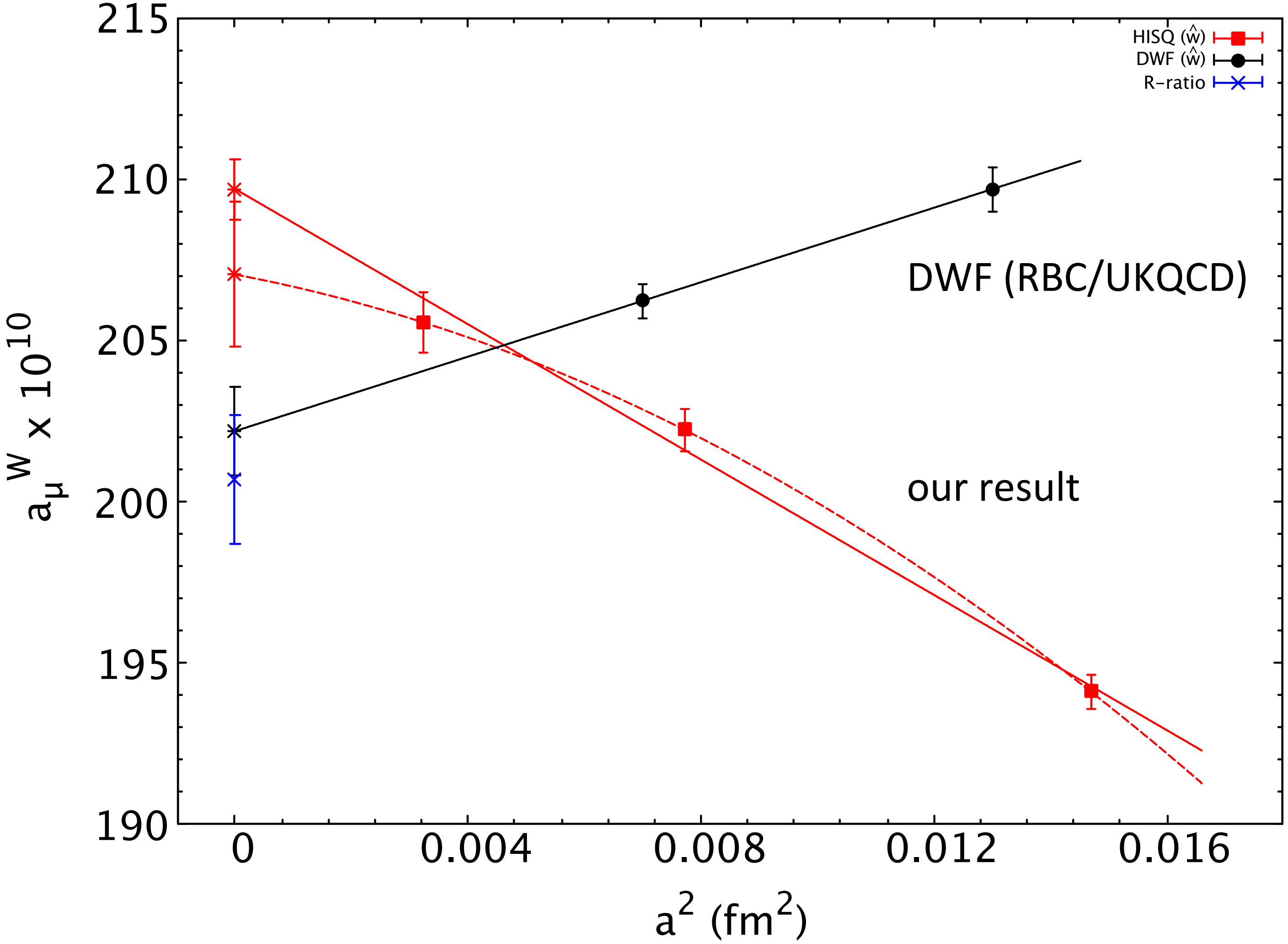
$t_0 = 0.4$ fm

$t_1 = 1.0$ fm

$\Delta = 0.15$ fm

(tiny FV effects)

Window method



$t_0 = 0.4$ fm

$t_1 = 1.0$ fm

$\Delta = 0.15$ fm

Conclusions (homework)

- Our error is dominated by statistics – room for improvement!
  Improve 96^3 x 192 ensemble (more low modes?), add 0.15 fm points

- Finite volume effects: correct with two-loop ChPT
  appears consistent with models – ChPT FV error estimated at 0.6%
  (depends only on two well-known Low Energy Constants)

- Add SChPT at two loops(?)

- Understand comparison of results with window method better
  (more statistics, 0.15 fm points?)