The Hadronic Vacuum Polarization Contribution to the Muon g-2 from Four-Flavor Lattice QCD

Aida X. El-Khadra
University of Illinois

Fermilab Lattice-HPQCD-MILC Collaboration

Santa Fe Workshop on Lattice QCD and
[Beyond] the Standard Model
26-30 August 2019

Fermilab/MILC/HPQCD g-2 group

Subgroup of members of the three collaborations actively engaged in the g-2 project:

Fermilab Lattice Collaboration

- Aida El Khadra (Illinois)
- Andreas Kronfeld (Fermilab)
- Ethan Neil (Colorado)
- Ruth Van de Water (Fermilab)

HPQCD

- Daniel Hatton (Glasgow)
- Christine Davies (Glasgow)
- Peter Lepage (Cornell)
- Craig McNeile (Plymouth)

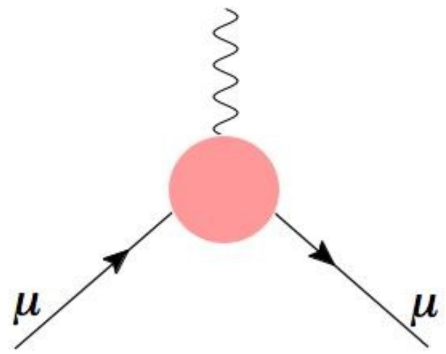
MILC Collaboration

- Carleton DeTar (Utah)
- Steve Gottlieb (Indiana)
- Jack Laiho (Syracuse)
- Yuzhi Liu (Indiana)
- Doug Toussaint (Arizona)
- Alejandro Vaquero (Utah)
- Shuhei Yamamoto (Utah)

Outline

- Introduction
- Set-Up
- Correlator analysis
- FV and taste breaking corrections
- light quark connected a_μ^{HVP}
- disconnected a_μ^{HVP}
- Corrections and comparisons
- Summary and outlook

Muon anomalous magnetic moment



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

- muon anomalous magnetic moment: $a_\mu = F_2(0)$
  - is generated by quantum effects (loops).
  - receives contributions from QED, EW, and QCD effects in the SM.
  - is a sensitive probe of new physics.
- QED + EW correction are known precisely:

$$\delta a_\mu^{\rm QED} \times 10^{11} = 0.08 \qquad \delta a_\mu^{\rm EW} \times 10^{11} = 1$$

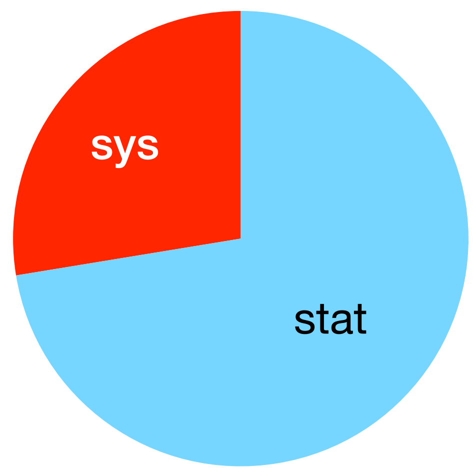
- QCD corrections are the dominant source of error in the SM prediction:

$$\delta a_\mu^{\rm had} \times 10^{11} \sim 50$$

(Keshawarzi et al arXiv:1802.02995; Davier at al arXiv:1706.09436, Jegerlehner arXiv:1704.07409, Davier et al. 2011, Hagiwara et al 2011, Kurz et al 2014, Prades et al 2009, Colangelo et al 2014, Jegerlehner 2015, Benayoun et al 2015,…)

Muon g-2: experiment vs theory

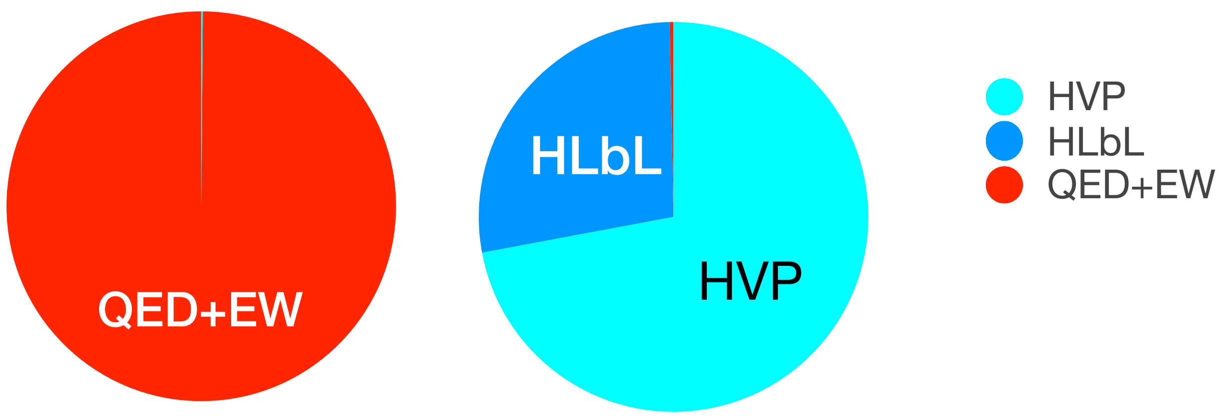
BNL E821



SM theory

contribution

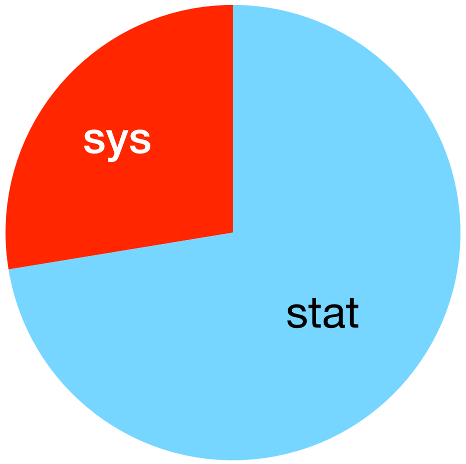
error



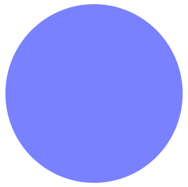
FNAL E989 + J-PARC E34

Muon g-2: experiment vs theory

BNL E821



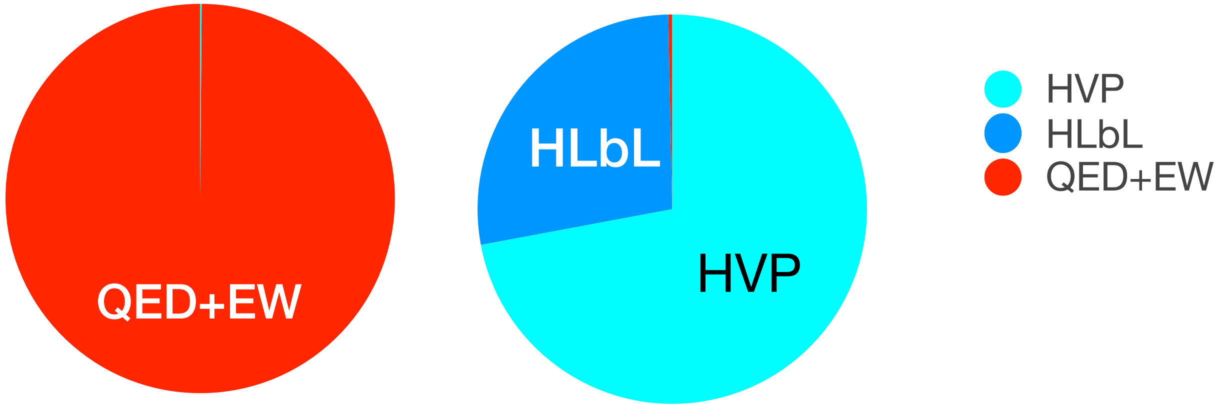
FNAL E989 + J-PARC E34

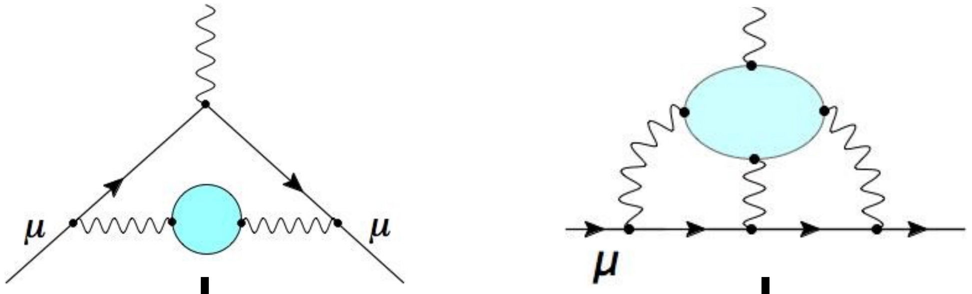


SM theory

contribution

error

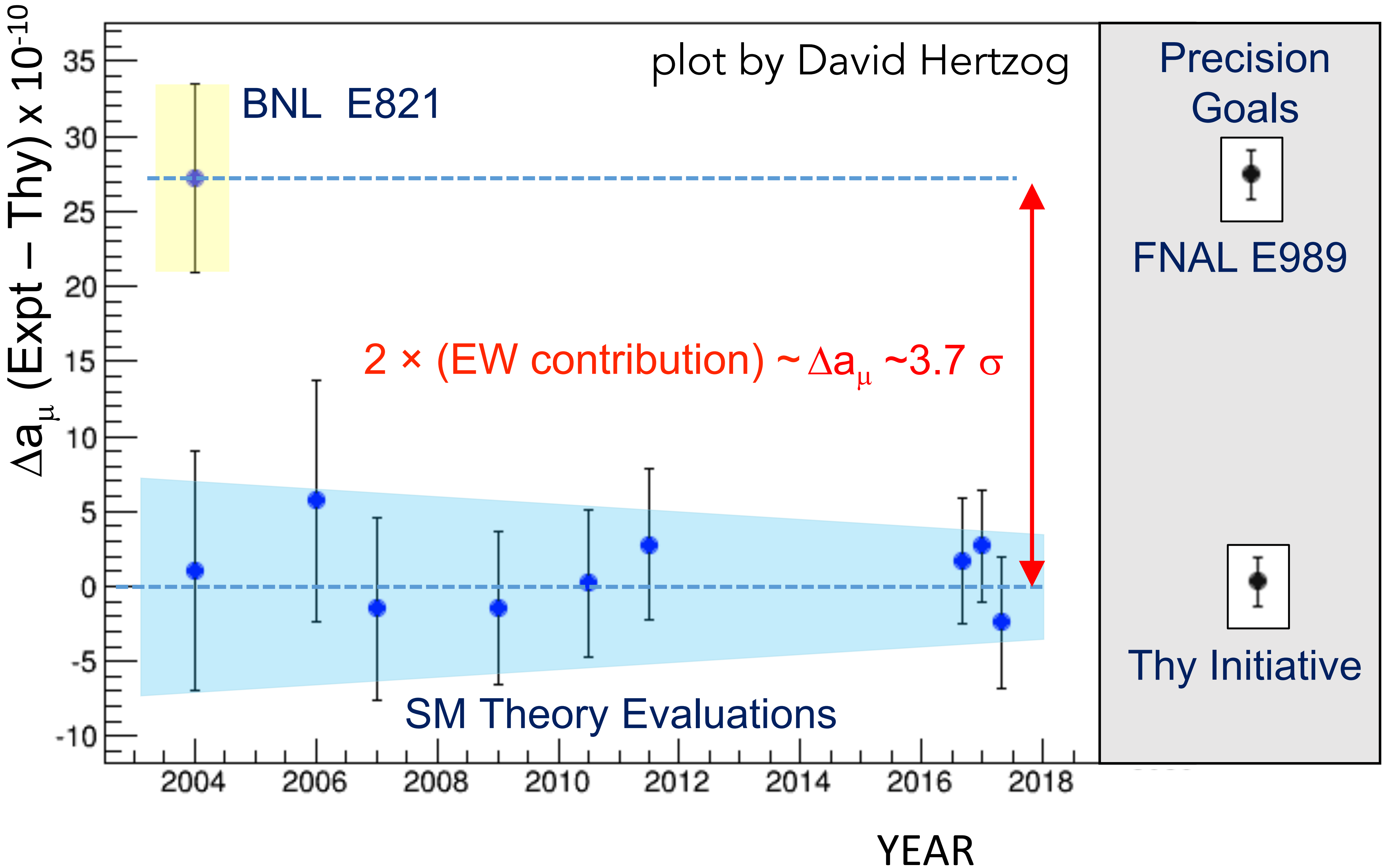


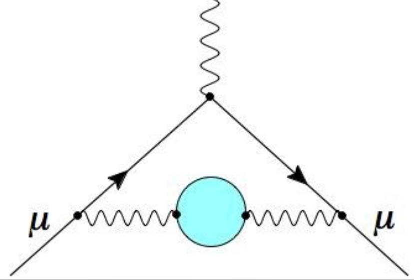


~0.2 %

10 %

Muon g-2: experiment vs theory

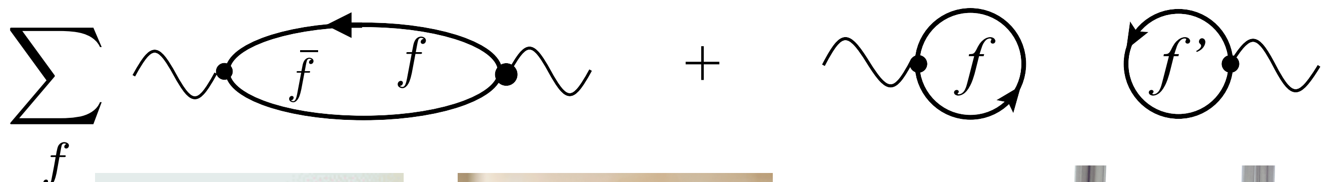




Hadronic vacuum polarization

Calculate a_μ^{HVP} in Lattice QCD: $$a_\mu^{\text{HVP,LO}} = \sum_f a_{\mu,f}^{\text{HVP,LO}} + a_{\mu,\text{disc}}^{\text{HVP,LO}}$$

- Separate into connected and disconnected contributions
  + calculate connected contribution for each quark flavor separately
  (gluon and sea-quark background not shown in diagrams)

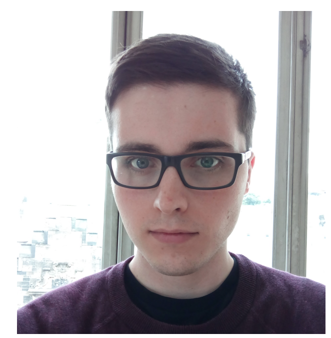


f= *ud, s, c, b*

Ruth Van de Water

Peter Lepage

+ SIB

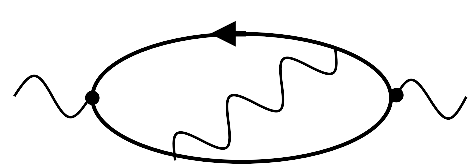


Dan Hatton
(Glasgow student)

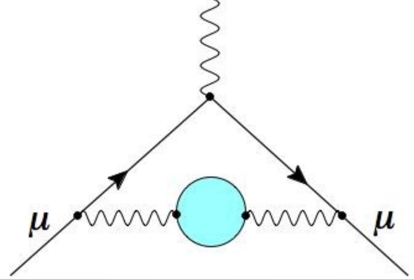
+ 2-pion contributions

Shaun Lahert
(UIUC student)

- add QED corrections
  (in progress/planned)

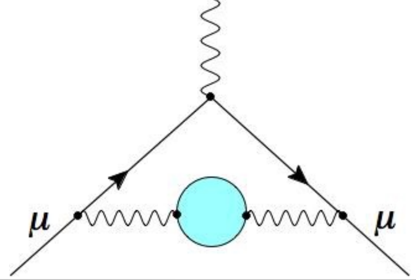


+ …



Hadronic vacuum polarization

- Target: < 0.5% total error
- current uncertainties at 1-2% level
- Challenges:
  - ✓needs ensembles with (light sea) quark masses at their physical values
  - include disconnected contributions
  - include QED and strong isospin breaking corrections ($m_u \neq m_d$)
  - finite volume corrections, continuum extrapolation: guided by EFT
  - growth of statistical errors at large Euclidean times
    - ➠ noise reduction methods
      include guidance from EFT
    - ➠ include two-pion channels into analysis



Hadronic vacuum polarization

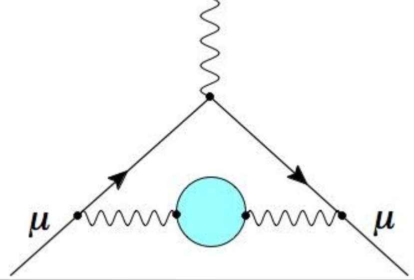
Leading order HVP correction: $\boxed{a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)}$

- Calculate $a_\mu^{\rm HVP}$ in Lattice QCD:
  - ✦ Time-momentum representation:
    reorder the integrations and compute $G(t) = \frac{1}{3}\sum_{i,x} \langle j_i(x,t)\, j_i(0,0)\rangle$

$$\boxed{a_\mu^{\rm HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int dt\, \tilde{\omega}(t)\, G(t)}$$

[Bernecker & Meyer, EPJ 12]

Need $G(t)$ over the entire range of t.



Hadronic vacuum polarization

Leading order HVP correction: $\boxed{a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)}$

- Calculate a_μ^{HVP} in Lattice QCD:
  - ✦ Time-moments: Taylor expand $\hat{\Pi}(q^2) = \sum_k q^{2k} \Pi_k$

    and compute the Taylor coefficients from time moments

    $G_{2n} = a \sum_t t^{2n} G(t)$: $\quad \Pi_k = (-1)^{k+1} \dfrac{G_{2k+2}}{(2k+2)!}$

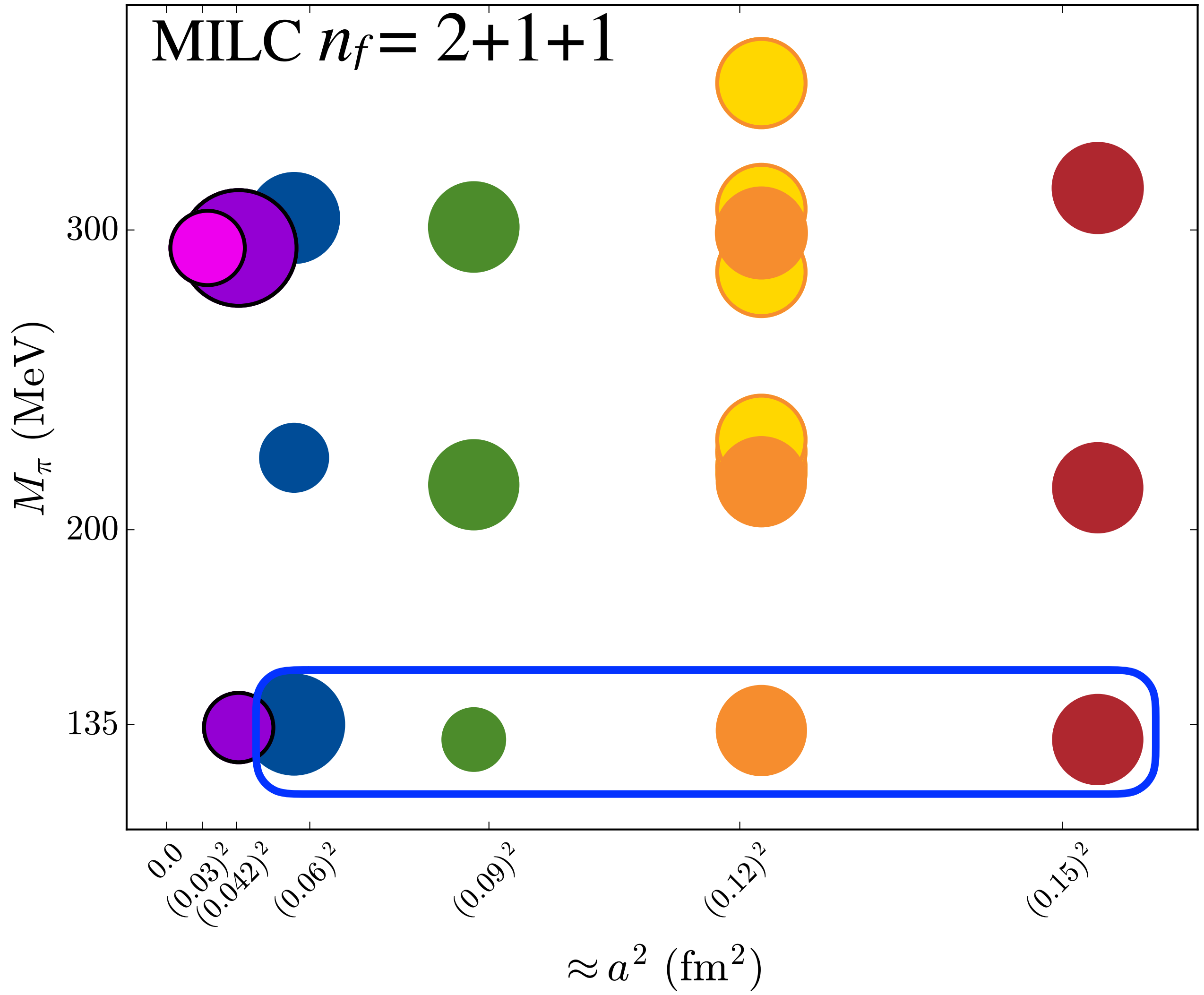
    and replace $\hat{\Pi}(q^2)$ by its $[n,n]$ and $[n,n\text{-}1]$ Padé approximant
    [HPQCD (Chakraborty et al), PRD 14]

    Can apply corrections (finite volume, discretization) to the Taylor coefficients **before** constructing a_μ

  - ✦ Note: The time-moments method yields results that are numerically equivalent to the time-momentum representation.

Fermilab/MILC/HPQCD project set-up

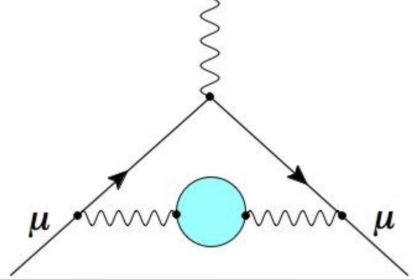
- Use MILC ensembles with physical sea pion masses at 4 lattice spacings



- connected light quark contribution
  [Davies et al, arXiv:1902.04223]
- disconnected contribution
  [Dan Hatton (Glasgow student) + Craig McNeile]

- also working on QED corrections using QCD+QED ensemble

- new project on a direct calculation of the two-pion contributions [Shaun Lahert, UIUC student]



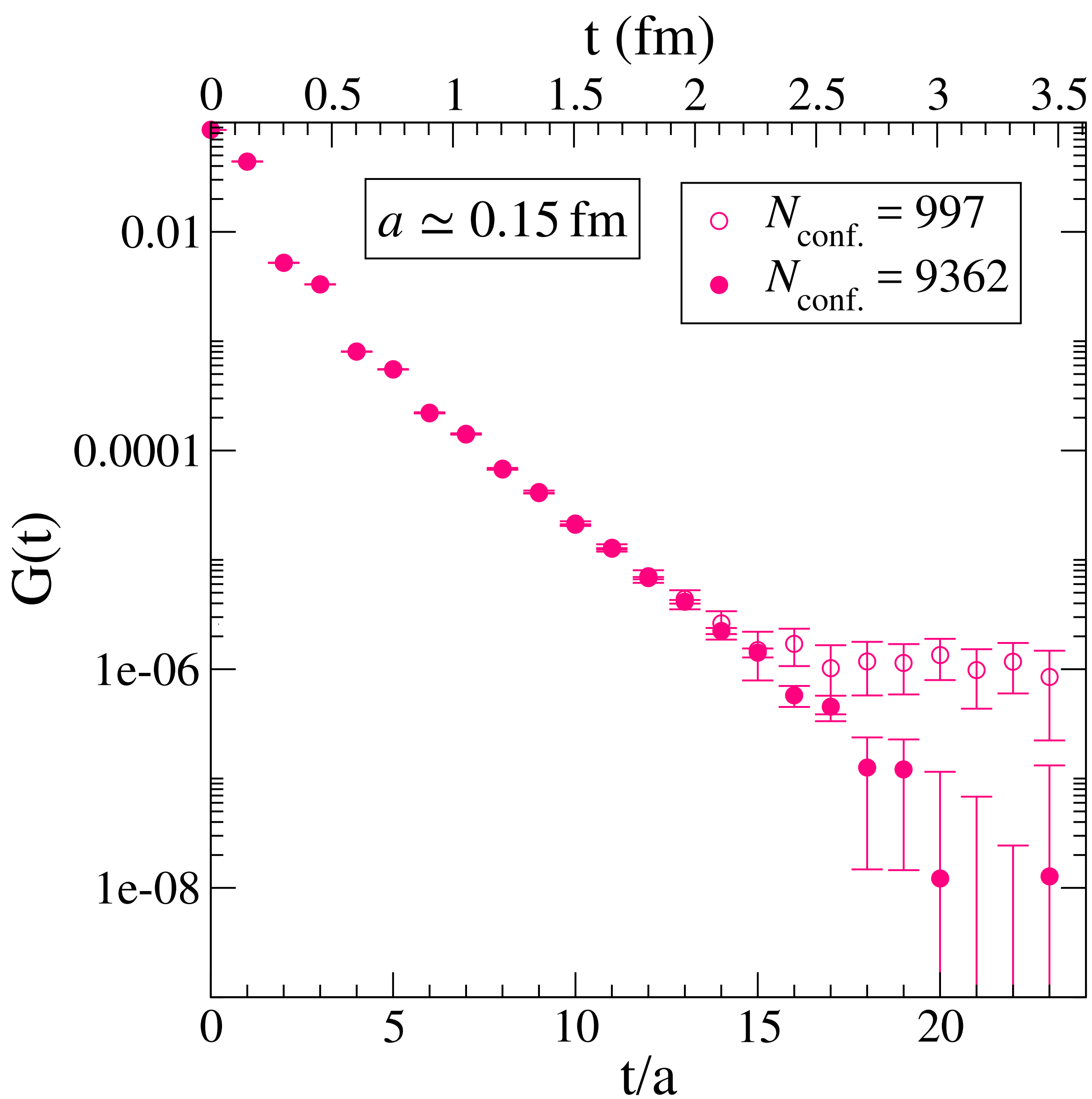
Hadronic vacuum polarization

Calculate a_μ^{HVP} in Lattice QCD:

$$a_\mu^{\mathrm{HVP,LO}} = \sum_f a_{\mu,f}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

light quark connected contribution:

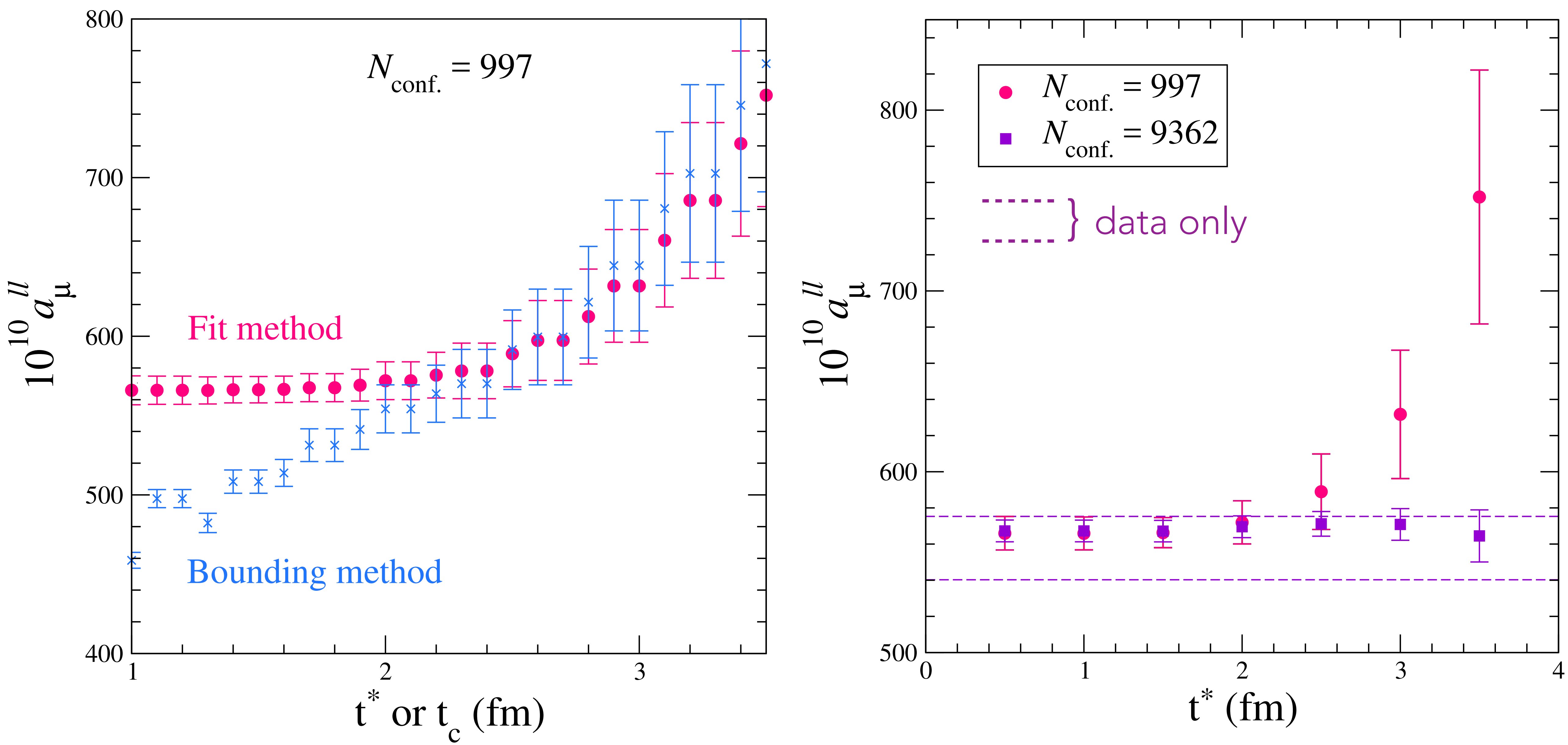
$$G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$$



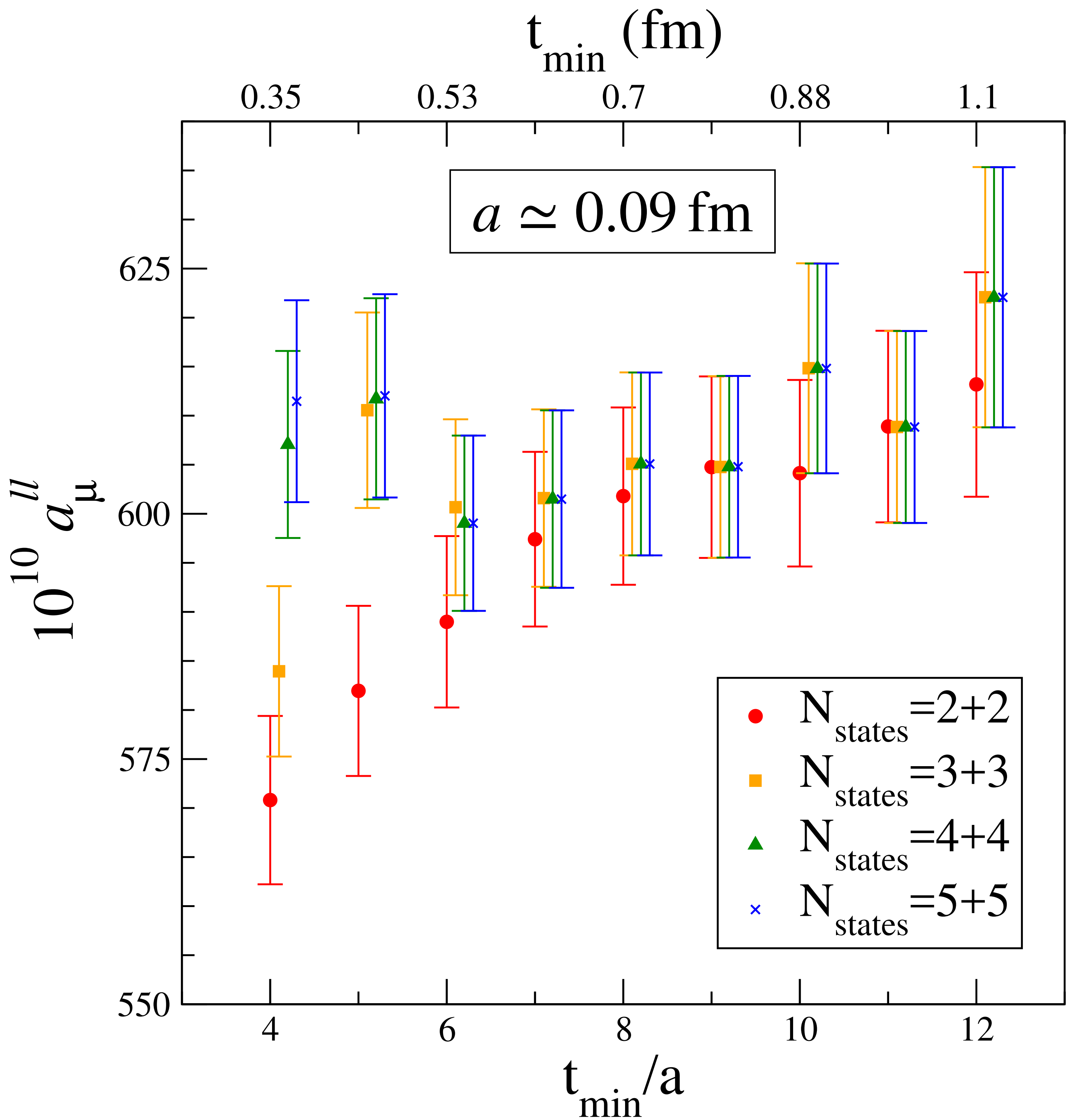
- Noise at large t
  ➠ bounding method
- spectral decomposition of G(t)
  ➠ perform multi-exponential fits to G(t) in range $t_{min} < t < t_{max}$
  + replace G(t) with fit for t > t* ~2-2.5 fm
  ➠ tests of fit method using high statistics data and EFT guidance
  ➠ add contributions from two-pion states to reconstruct G(t) at large t (in progress)

Correlator analysis

$a \simeq 0.15\,\mathrm{fm}$



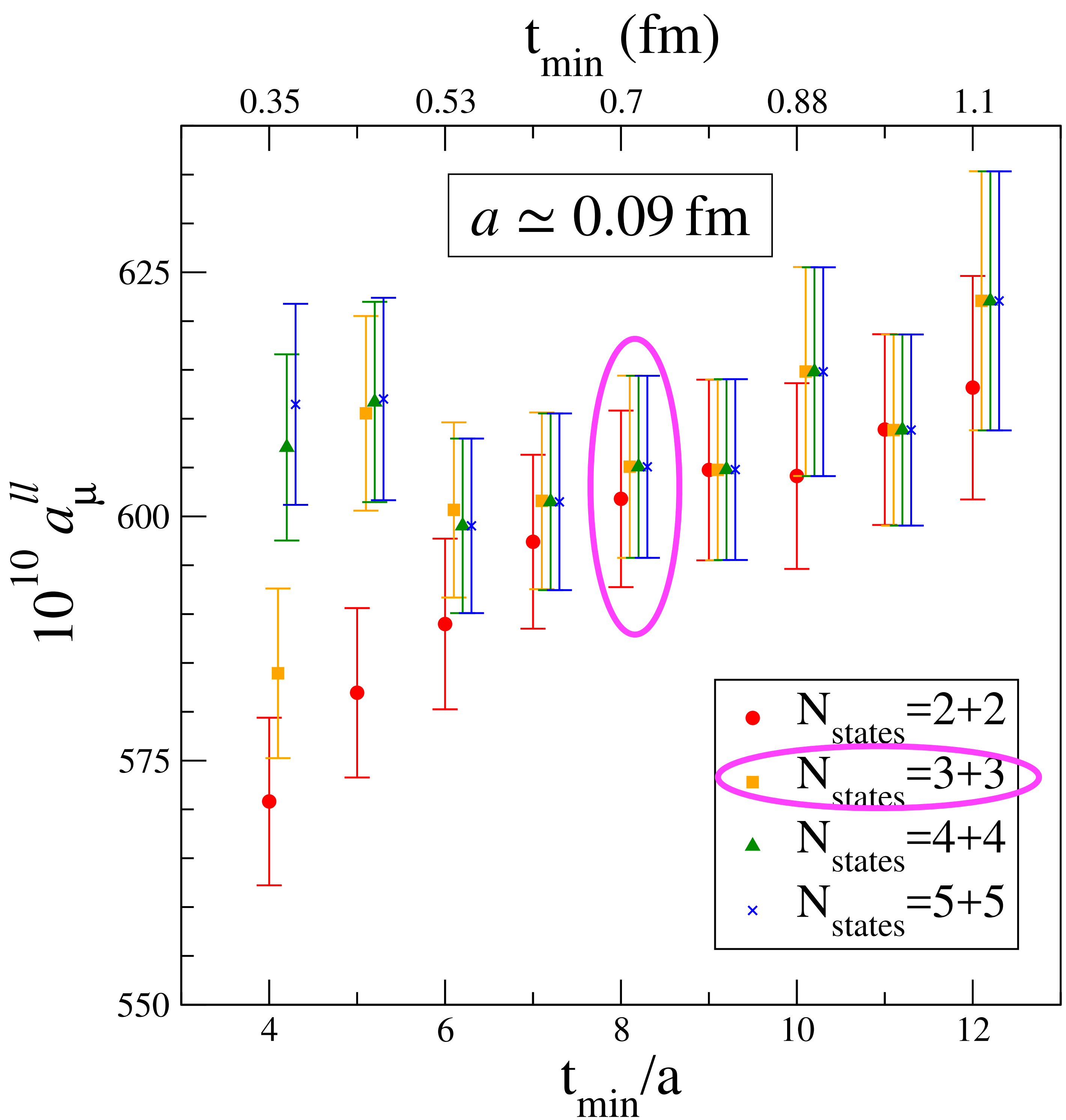
Correlator analysis



check stability under variations of:

- t_{min}
- number of states, N_{states}

Correlator analysis

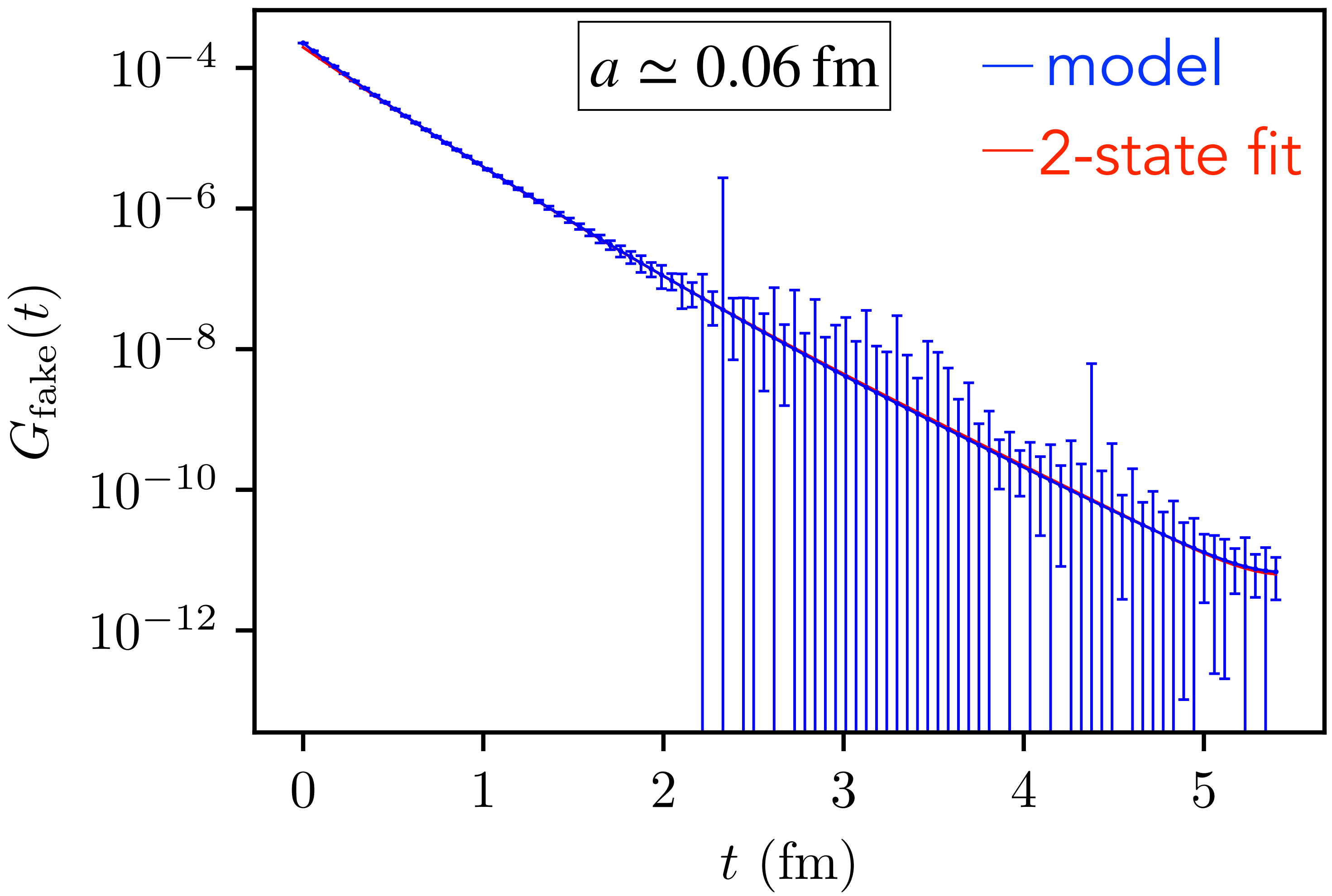


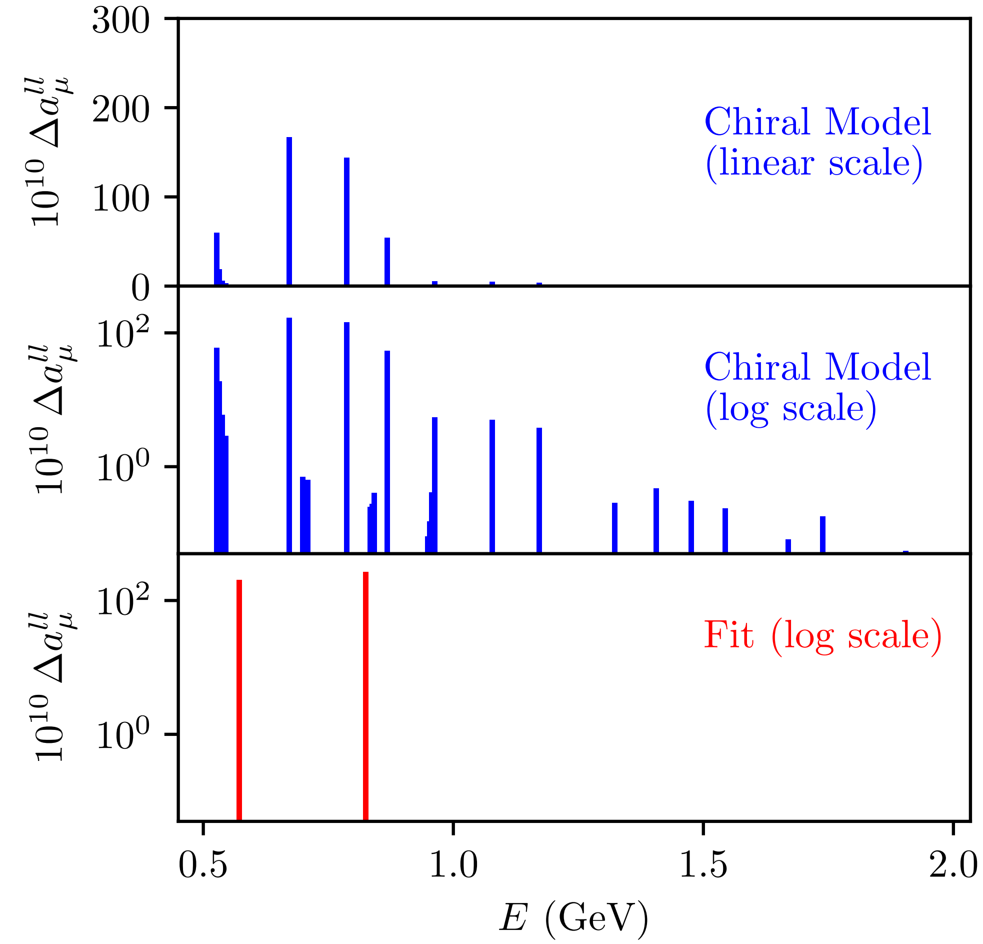
check stability under variations of:

- t_{min}
- number of states, N_{states}

Correlator analysis: two-pion contributions

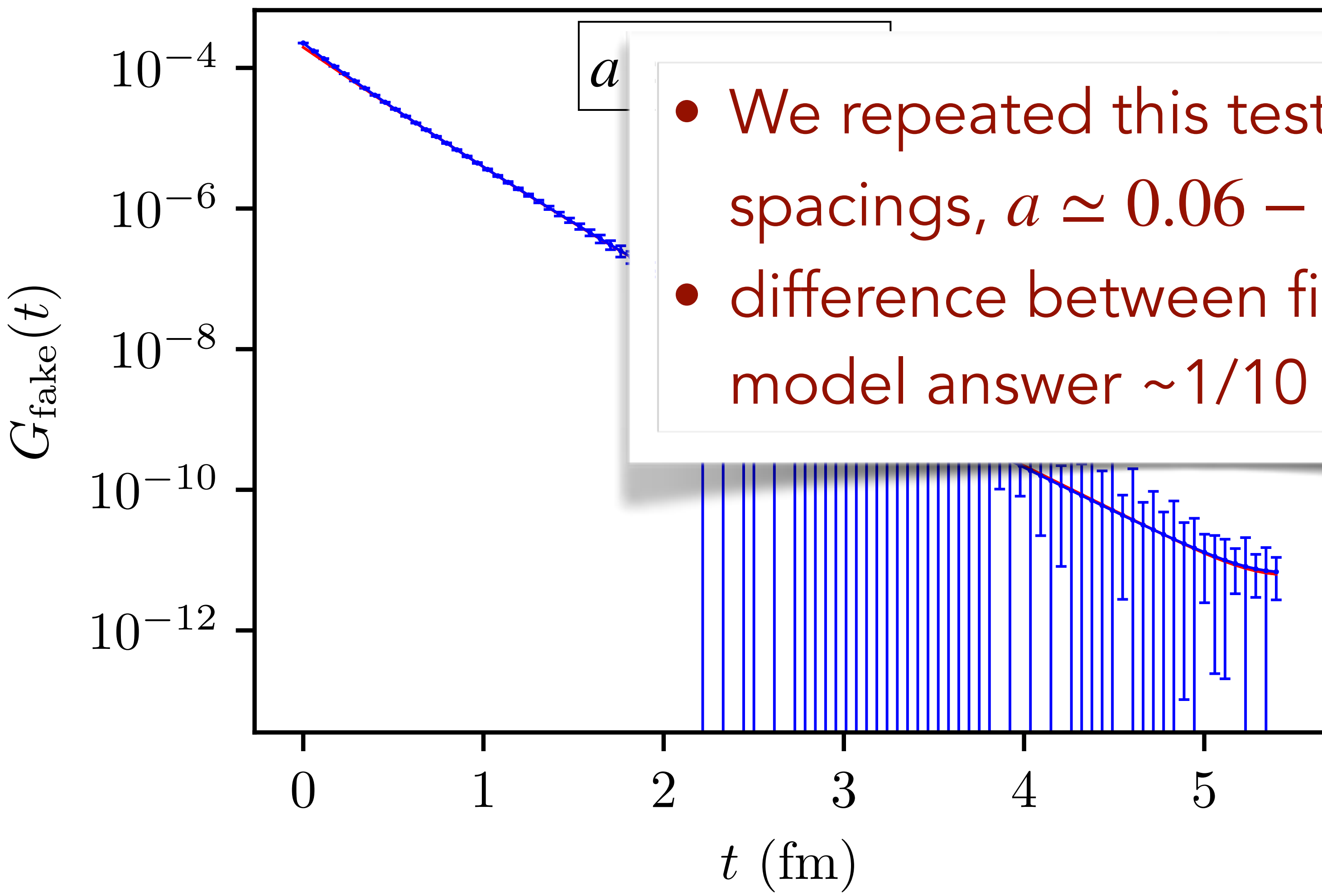
- Generate fake data with the chiral model used for FV and taste corrections (includes $\rho - \pi\pi$ interactions)
- Add covariance matrix from real data
- Fit fake data using the same procedure (fit ranges etc…) as used for fitting the ``real data" correlators



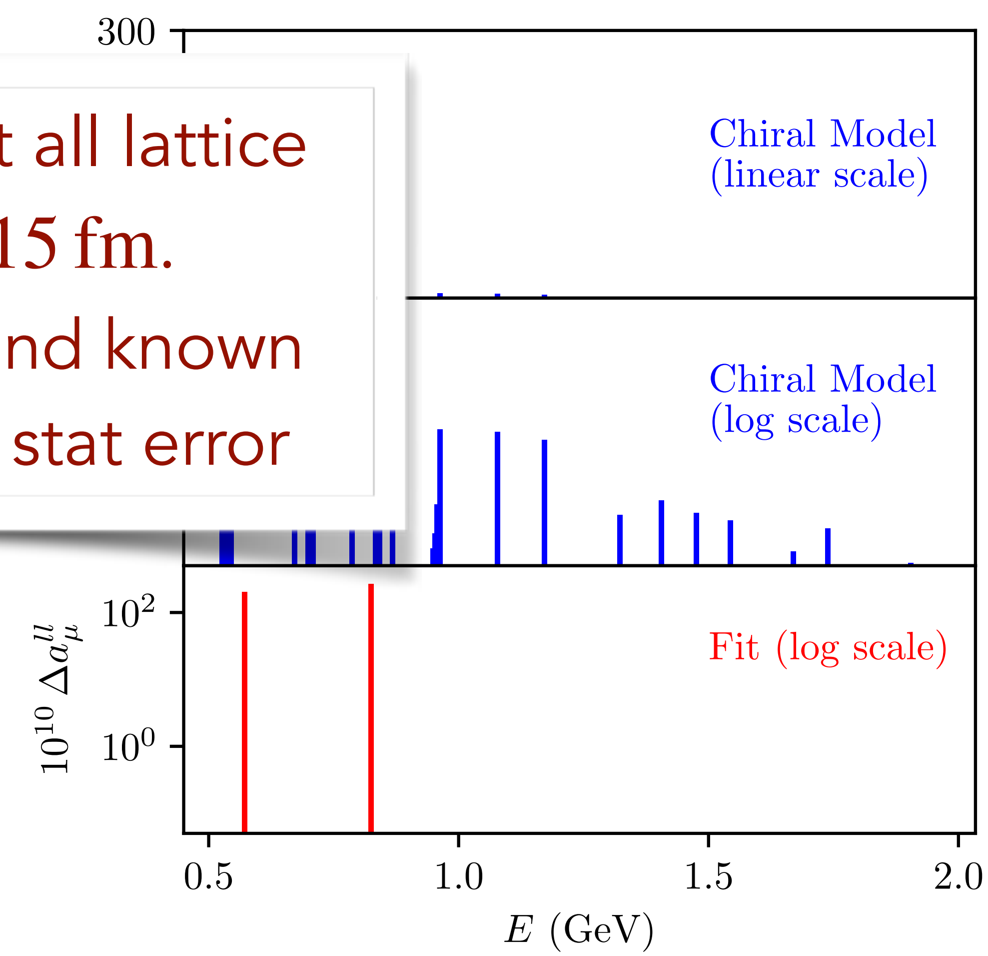


Correlator analysis: two-pion contributions

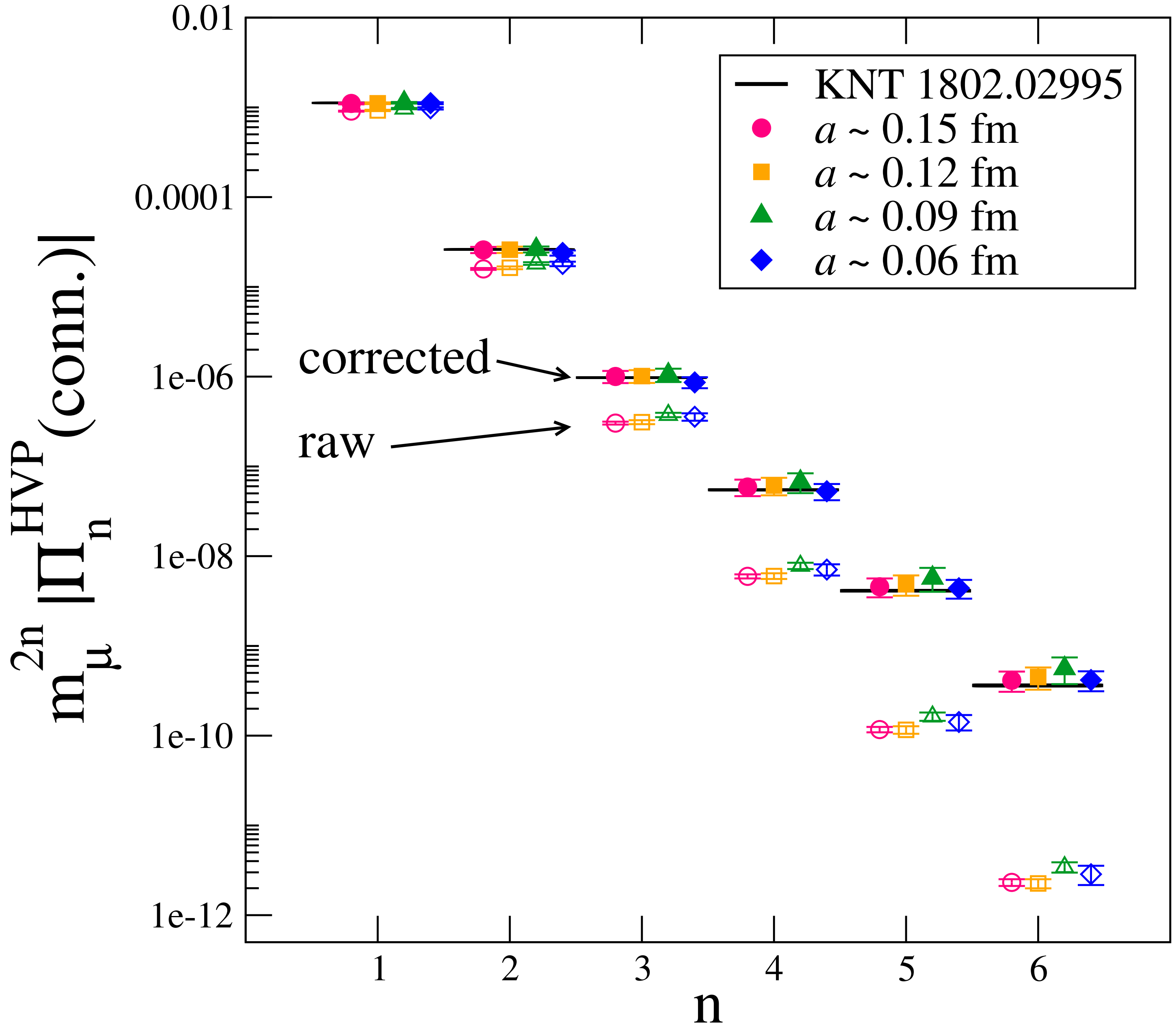
- Generate fake data with the chiral model used for FV and taste corrections (includes $\rho - \pi\pi$ interactions)
- Add covariance matrix from real data
- Fit fake data using the same procedure (fit ranges etc…) as used for fitting the ``real data" correlators



- We repeated this test at all lattice spacings, $a \simeq 0.06 - 0.15$ fm.
- difference between fit and known model answer ~1/10 of stat error



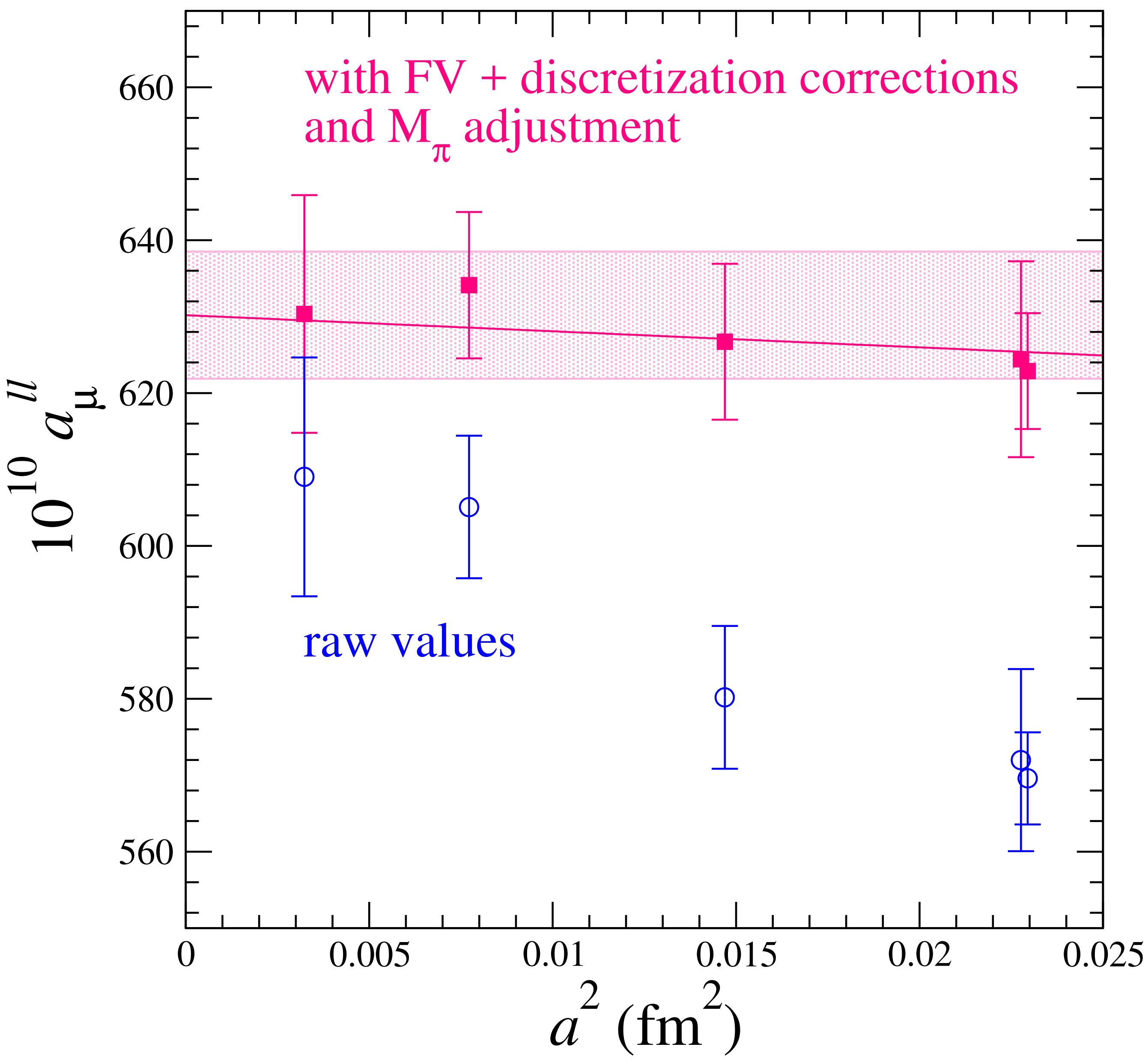
Corrections: FV and leading discretization



- Calculate $\pi\pi$ contributions to Π_n in modified chiral theory (which includes $\rho - \gamma - \pi\pi$ interactions)
  [Chakraborty et al, 1601.03071]
- Corrections obtained from difference between contributions in infinite volume & continuum and FV & taste-breaking effects
- compare with Π_n obtained from fit to R-ratio data (KNT 1802.02995)
- agreement at large n good test of corrections

- a_μ dominated by lowest moments

Light-quark connected a_μ: Continuum limit



- adjust for small differences between simulation M_π and M_{π^0}:
  - remove cont. $\pi\pi$ contribution with pion mass set to simulation values
  - add cont. $\pi\pi$ contributions back in with M_{π^0}
- fit a_μ^{ll} to:

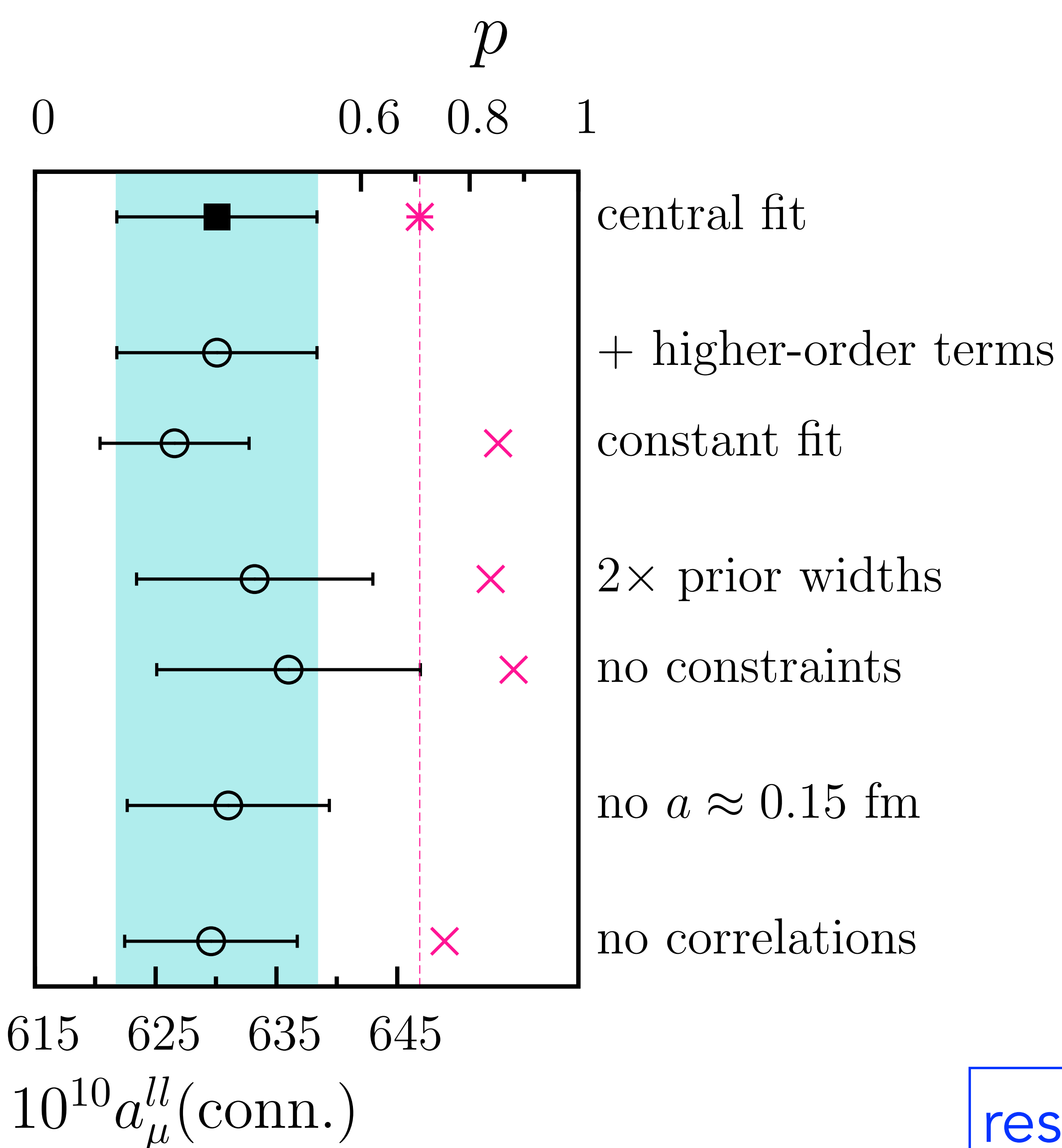
$$a_\mu^{ll}(\text{latt.}) = a_\mu^{ll}(\text{conn.})\left(1 + c_s \sum_{f=l,l,s,c} \frac{\delta m_f}{\Lambda} + c_{a^2}\frac{(a\Lambda)^2}{\pi^2}\right)$$

$$\Lambda = 500\,\text{MeV}$$

$$c_{a^2} = 0\,(1), \quad c_s = 0.0\,(3)$$

$$\boxed{10^{10}\, a_\mu^{ll}(\text{conn.}) = 630.1 \pm 8.3}$$

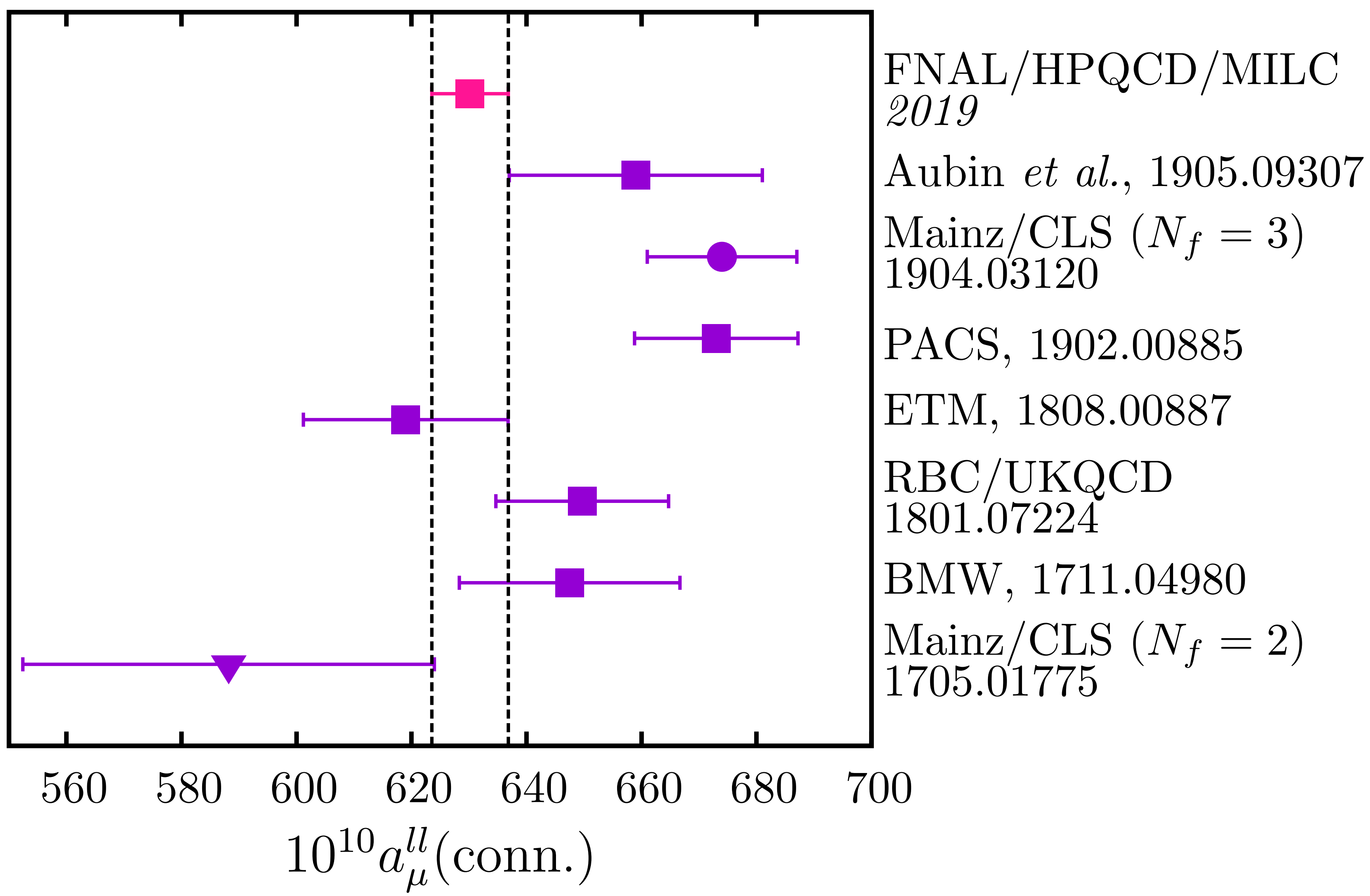
Continuum limit: stability



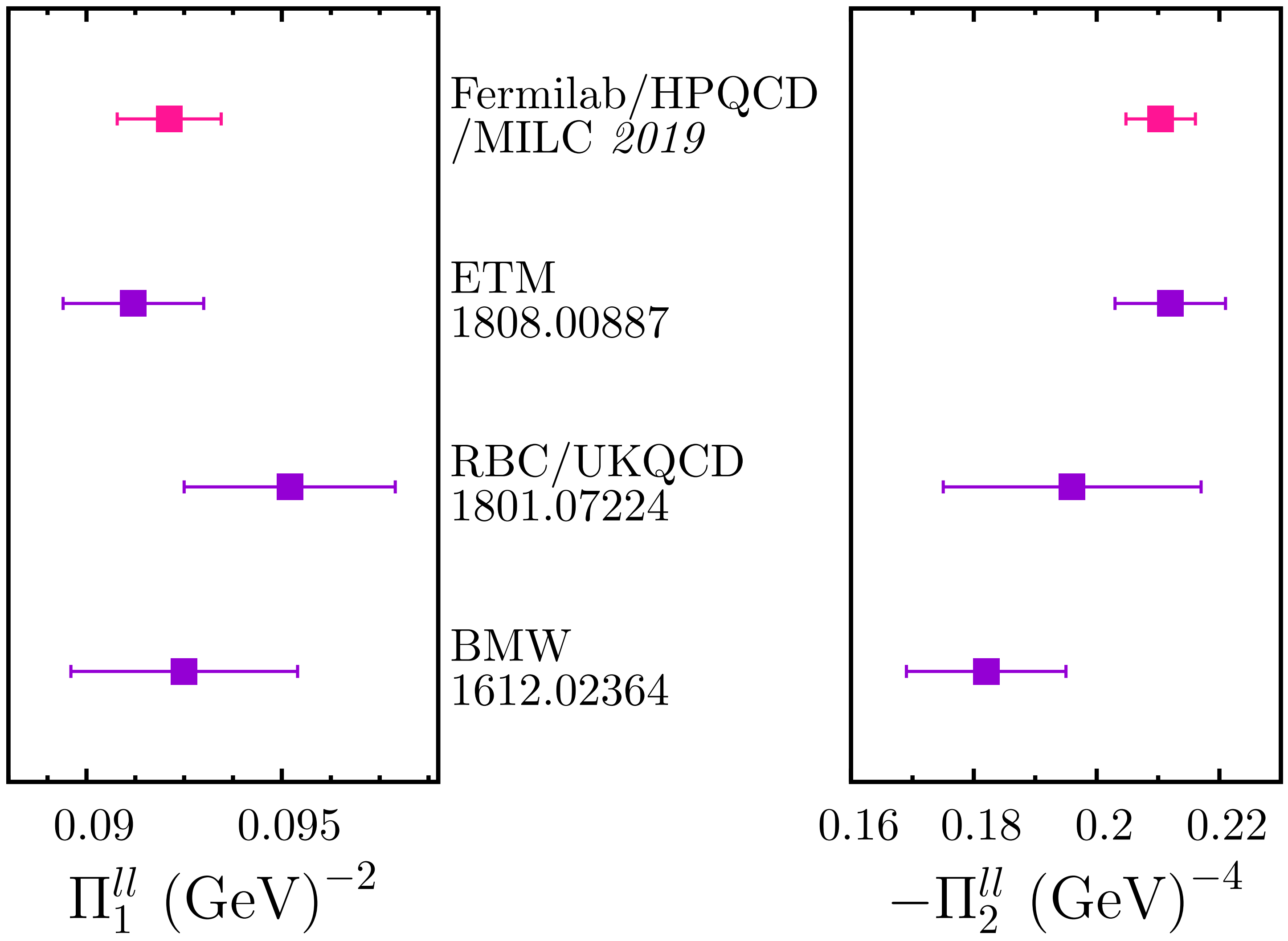
- higher order terms:
  $\sim (\delta m_f)^2, \sim (a\Lambda)^4, \sim \delta m_f (a\Lambda)^2$
- constant fit:
  no a^2, m_f dependence

results from fit variations are consistent with central fit result

Light-quark connected a_μ: Comparison



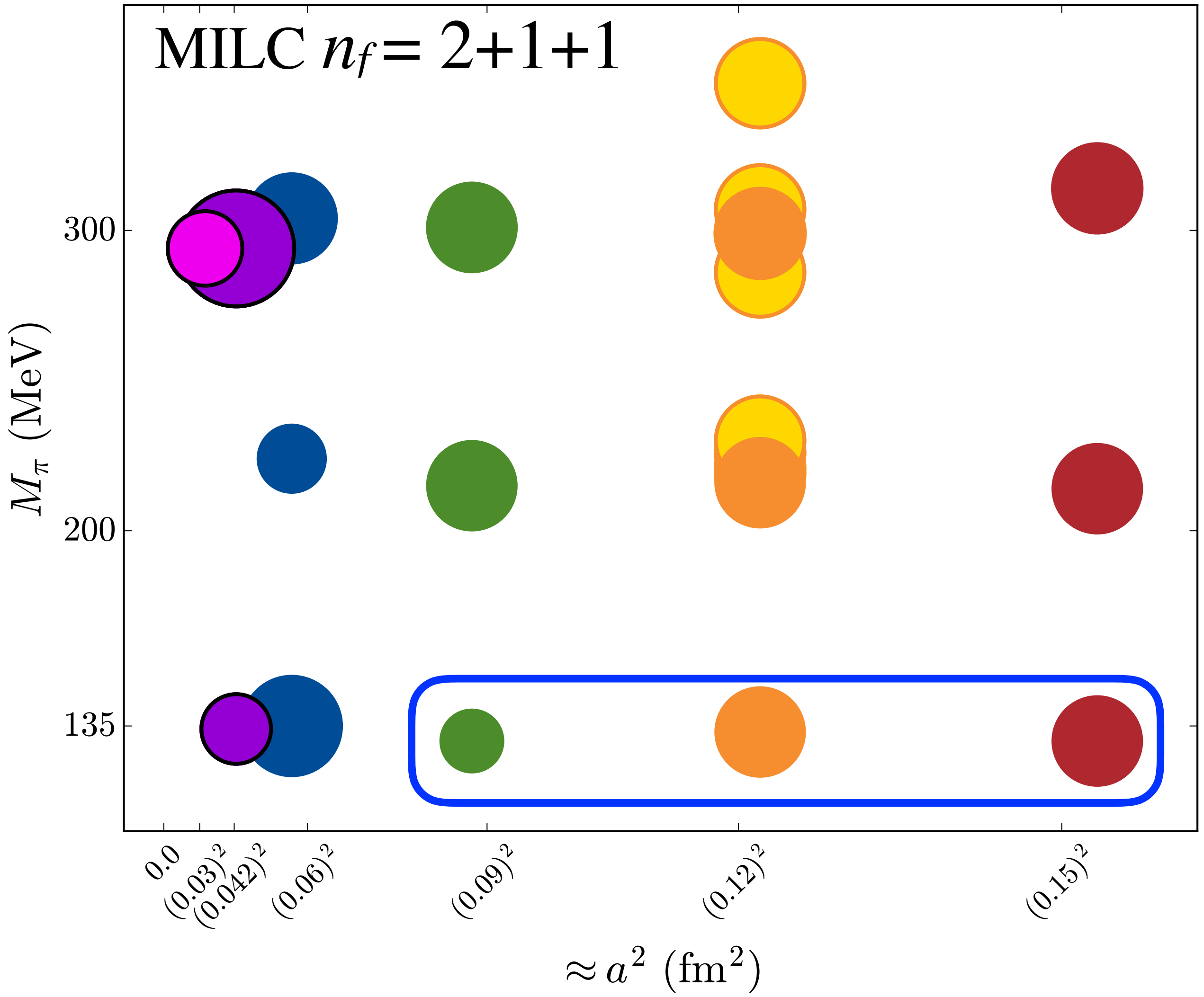
Light-quark connected Π_1, Π_2: Comparison



Light-quark connected: error budget

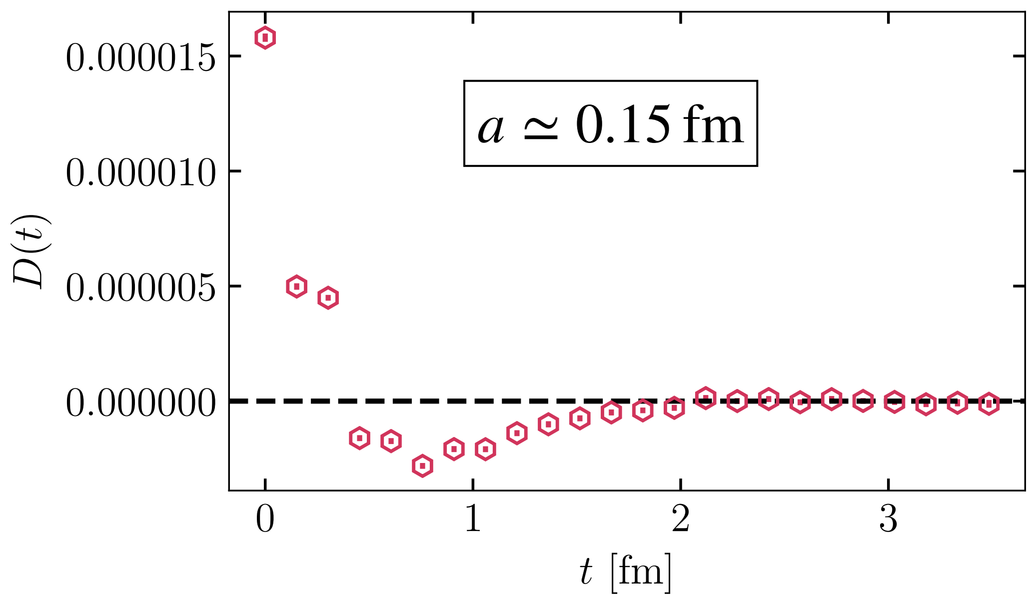
Source	a_μ^{ll}(conn.) (%)	Π_1^{ll}(conn.) (%)	Π_2^{ll}(conn.) (%)
Lattice-spacing (a^{-1}) uncertainty	0.8	0.9	1.6
Monte Carlo statistics	0.7	0.7	1.1
Continuum ($a \to 0$) extrapolation	0.7	0.7	0.8
Finite-volume & discretization corrections	0.3	0.4	1.7
Current renormalization (Z_V)	0.1	0.1	0.1
Chiral (m_l) interpolation	0.1	0.1	0.0
Sea (m_s) adjustment	0.1	0.1	0.1
Pion mass ($M_{\pi,5}$) uncertainty	0.0	0.0	0.1
Total	1.3%	1.4%	2.7%

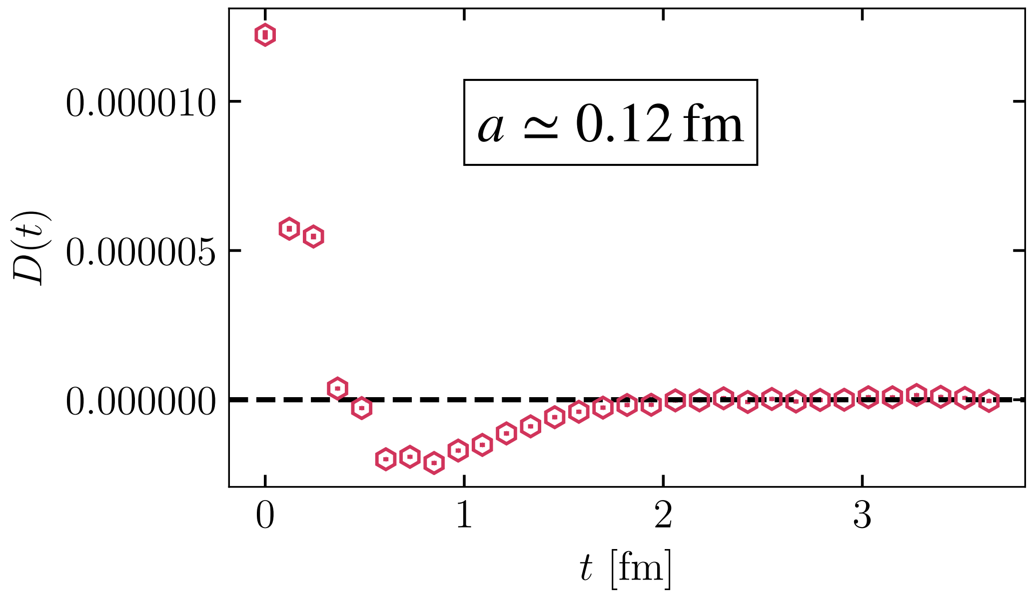
disconnected a_μ^{HVP}



- Analysis still preliminary
  [Dan Hatton (Glasgow student) + Craig McNeile]
- Two ensembles so far:
  0.15 fm, 0.12 fm
- Correlator generation for 0.09 fm ensemble is in progress

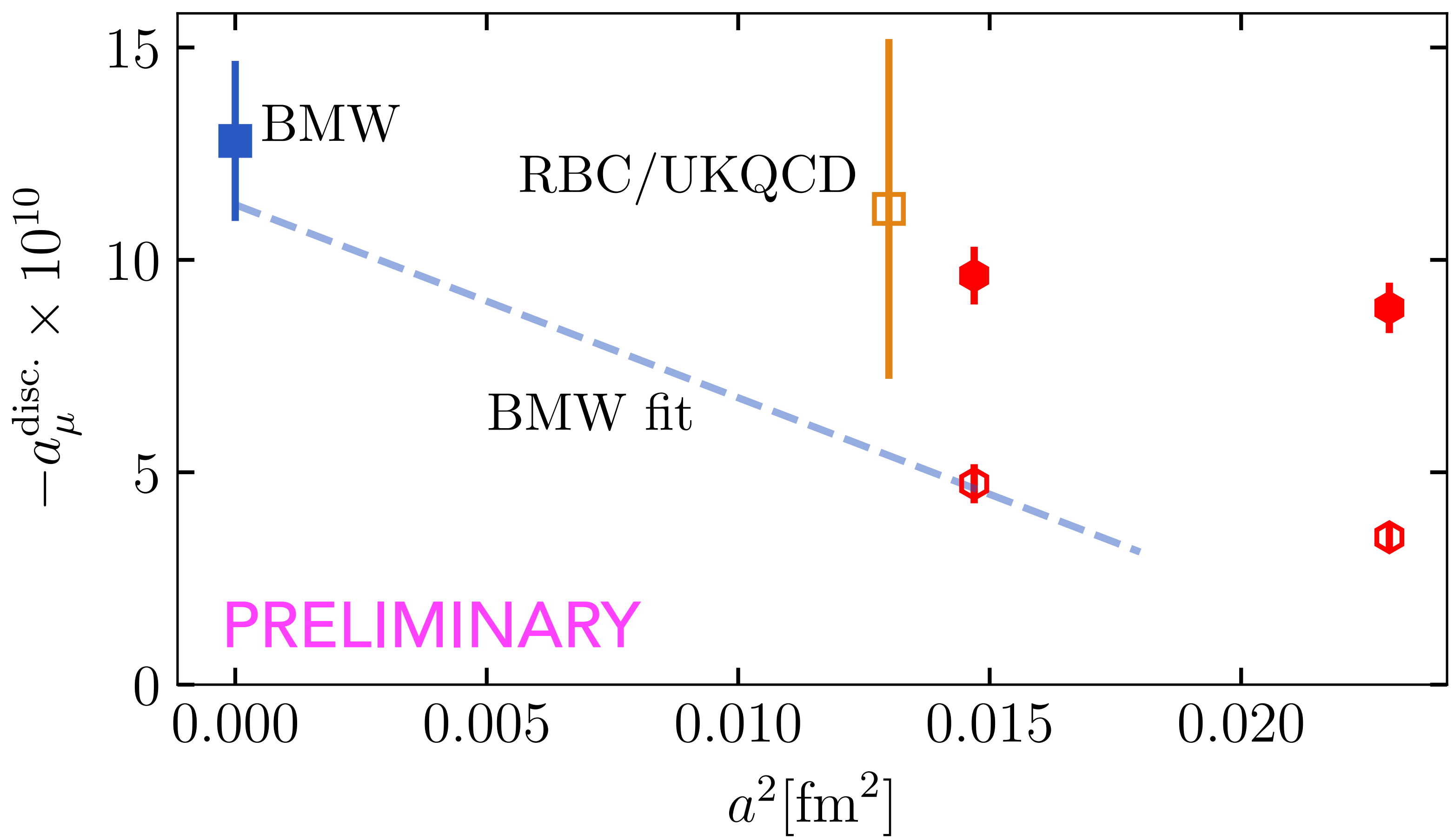
disconnected $a_\mu^{\rm HVP}$





- Similar analysis strategy as for connected $a_\mu^{\rm HVP}$
- fit to exponentials including ground state $\omega - \rho$ + excited states for ρ, ω
- replace correlator data with fit for t > 2 fm

disconnected $a_\mu^{\rm HVP}$: comparison



with FV + discretization corrections

raw values

Plan:

-add ensemble at $a \simeq 0.09\,\rm fm$

-calculate SIB correction to disconnected $a_\mu^{\rm HVP}$

Complete light-quark $a_\mu^{\mathrm{HVP,LO}}$

Our calculation of the disconnected and IB contributions is not yet complete. ➠ estimate them using chiral model, phenomenology

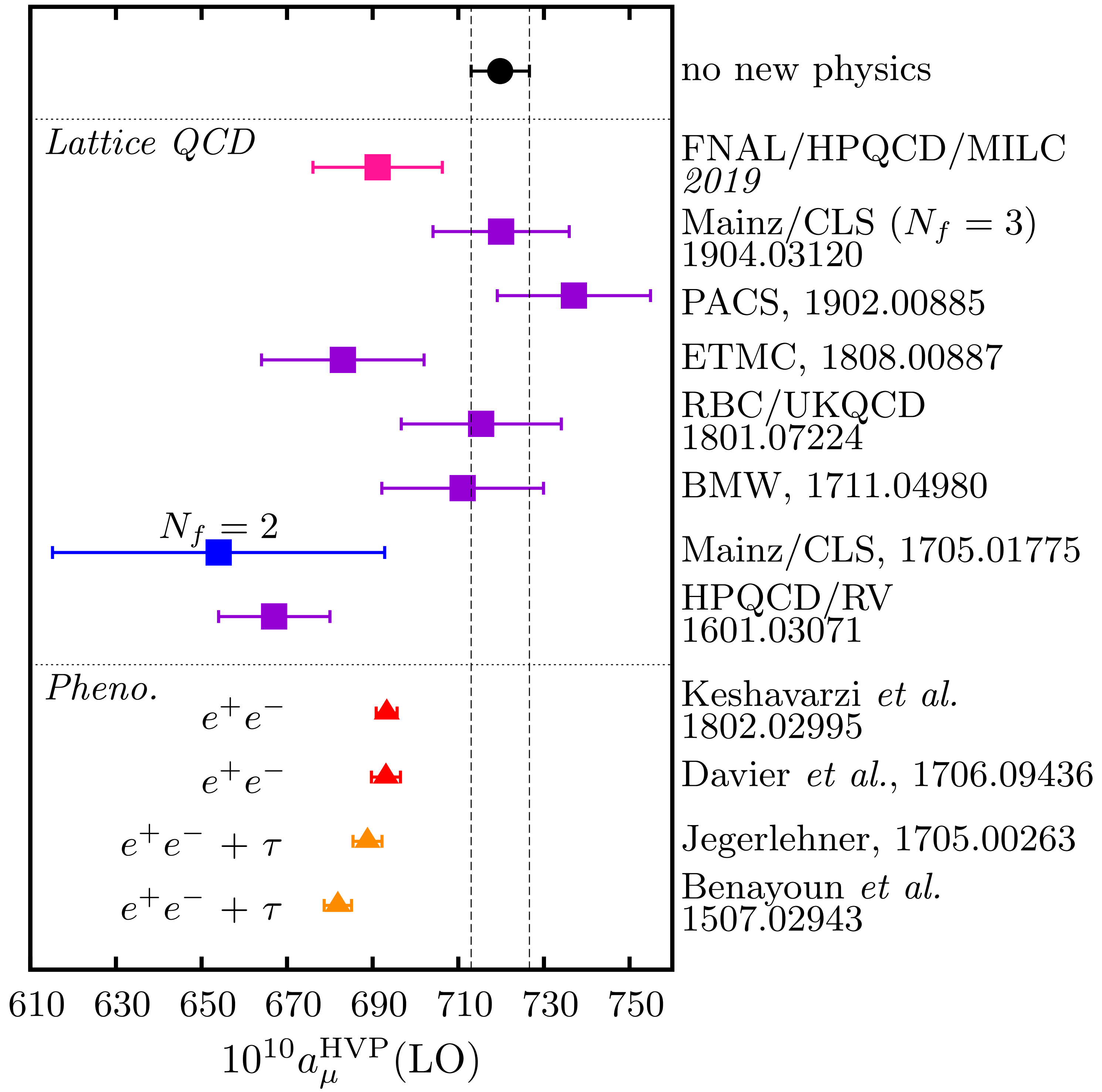
Contribution	$10^{10} a_\mu^{ll}(\text{conn.})$	$\Pi_1^{ll}(\text{conn.})$ (GeV^{-2})	$\Pi_2^{ll}(\text{conn.})$ (GeV^{-4})
$M_{\pi^0} \to M_{\pi^+}$	-4.3	-0.00075	0.0057
$\pi\pi$ disconnected	-7.9	-0.00120	0.0044
Total $\pi\pi$	$-12(3)$	$-0.0020(5)$	$0.010(3)$
ρ, ω disconnected	$-5(5)$	$-0.0008(8)$	$0.002(1)$
Strong-isospin breaking	$10(10)$	$0.0015(15)$	$-0.006(6)$
Electromagnetism	$0(5)$	$0.0000(6)$	$0.000(2)$
Total correction	$-7(13)$	$-0.0013(19)$	$0.006(7)$

Complete $a_\mu^{\mathrm{HVP,LO}}$: adding all flavors

For the contributions from the heavier flavors (s,c,b), use results from previous calculations [Chakraborty, et al arXiv:1403.1778; Donald et al arXiv:1208.2885; Colquhoun et al, 1408.5768]

Contribution	$10^{10} a_\mu^{\mathrm{HVP,LO}}$	$\Pi_1^{\mathrm{HVP,LO}}(\mathrm{GeV}^{-2})$	$\Pi_2^{\mathrm{HVP,LO}}(\mathrm{GeV}^{-4})$
light	623.1(8.3)(13)	0.0921(13)(19)	−0.2104(56)(71)
strange	53.40(60)	0.007291(78)	−0.00587(12)
charm	14.40(40)	0.001840(49)	−0.0001240(43)
bottom	0.270(40)	0.0000342(48)	$-2.28(37)e-07$
Total	691(15)	0.1000(23)	−0.2104(90)

Complete $a_\mu^{\mathrm{HVP,LO}}$: Comparison



Summary and Outlook

- We have calculated the light-quark connected contribution to $a_\mu^{\mathrm{HVP,LO}}$ with a precision of 1.3%.
- Using phenomenological estimates for missing IB corrections and disconnected contributions we obtain the light-quark $a_\mu^{\mathrm{HVP,LO}}$ with 2.2% precision.
- We presented preliminary results for the disconnected contributions at two lattice spacings (Dan Hatton). Analysis on 0.09 fm ensemble is in progress.
- We will continue to improve our calculation of the light-quark connected contribution by
  - adding more statistics (more measurements, more configs)
  - adding 2-pion analysis (Shaun Lahert)
  - improving the scale setting
- Working on including QED effects

A cute story

Anomalous magnetic moment: $\boxed{a \equiv \frac{g-2}{2} = F_2(0)}$

- electron: $10^{14} a_e = \begin{array}{l} 115965218073\,(28) \\ 115965218161\,(23) \end{array}$ exp. SM
- muon: $10^{11}\, a_\mu = \begin{array}{l} 116592089\,(63) \\ 116591803\,(49) \end{array}$ exp. SM
- tau lepton: $-0.052 < a_\tau < 0.013$ exp.
  $a_\tau = 1.17721(5)\,10^{-3}$ SM

Sensitivity to heavy new physics: $a_\ell^{\mathrm{NP}} \sim \frac{m_\ell^2}{\Lambda^2}$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

A cute story

Anomalous magnetic moment: $\boxed{a \equiv \frac{g-2}{2} = F_2(0)}$

- electron: $10^{14} a_e = \begin{array}{l} 115965218073\,\boxed{(28)} \\ 115965218161\,\boxed{(23)} \end{array}$ exp. 2.4σ tension SM
- muon: $10^{11}\, a_\mu = \begin{array}{l} 116592089\,(63) \\ 116591803\,(49) \end{array}$ exp. SM
- tau lepton: $-0.052 < a_\tau < 0.013$ exp. $a_\tau = 1.17721(5)\, 10^{-3}$ SM

Sensitivity to heavy new physics: $a_\ell^{\mathrm{NP}} \sim \frac{m_\ell^2}{\Lambda^2}$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

A cute story

Anomalous magnetic moment: $\boxed{a \equiv \frac{g-2}{2} = F_2(0)}$

- electron: $10^{14} a_e = \begin{array}{l} 115965218073\,\boxed{(28)} \\ 115965218161\,\boxed{(23)} \end{array}$ exp. SM 2.4σ tension
- muon: $10^{11}\, a_\mu = \begin{array}{l} 116592089\,(63) \\ 116591803\,(49) \end{array}$ exp. SM $3.6\ \sigma$ difference
- tau lepton: $-0.052 < a_\tau < 0.013$ exp.
  $a_\tau = 1.17721(5)\,10^{-3}$ SM

Sensitivity to heavy new physics: $a_\ell^{\mathrm{NP}} \sim \frac{m_\ell^2}{\Lambda^2}$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

A cute story

Anomalous magnetic moment: $\boxed{a \equiv \frac{g-2}{2} = F_2(0)}$

- electron: $10^{14} a_e = \begin{array}{l} 115965218073\boxed{(28)} \\ 115965218161\boxed{(23)} \end{array}$ exp. SM 2.4σ tension
- muon: $10^{11}\, a_\mu = \begin{array}{l} 116592089\,(63) \\ 116591803\,(49) \end{array}$ exp. SM $3.6\,\sigma$ difference

•

Se

$(m_\mu/m_e)^2 \sim 4 \times 10^4$

- The student project at the SLAC Summer Institute (SSI 2019) on "The leptonic g-2 puzzle" won the first prize.
- The team constructed a model (Z' with chiral couplings) which gives rise to contributions to the muon and electron (g-2) with opposite signs.

Thank you!

Farah Willenbrock

Appendix

Muon g-2 Theory Initiative

Steering Committee:

- Gilberto Colangelo (Bern) gilberto@itp.unibe.ch
- Michel Davier (Orsay) davier@lal.in2p3.fr
- Simon Eidelman (Novosibirsk) eidelman@cern.ch
- Aida El-Khadra (UIUC & Fermilab) axk@illinois.edu
- Christoph Lehner (BNL) clehner@bnl.gov
- Tsutomu Mibe (KEK) mibe@post.kek.jp
  J-PARC E34 experiment
- Andreas Nyffeler (Mainz) nyffeler@uni-mainz.de
- Lee Roberts (Boston) roberts@bu.edu
  Fermilab E989 experiment
- Thomas Teubner (Liverpool) thomas.teubner@liverpool.ac.uk

Muon g-2 Theory Initiative

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
- two working groups, one for HVP and one for HLbL
  community participation: 53 people in HVP WG, 33 in HLbL WG
- White Paper: in progress, plan to post it **before** the Fermilab experiment announces its first measurement with ``Brookhaven-level'' statistics
  target date: November 2019

First Workshop of the Muon g-2 Theory Initiative

3-6 June 2017 *Q Center*
US/Central timezone

Search

66 registered participants, 40 talks, 15 discussion sessions (525 minutes)