Lattice QCD calculation of the two-photon contributions to $K_L \rightarrow \mu^+ \mu^-$ and $\pi^0 \rightarrow e^+ e^-$ decays

Lattice QCD

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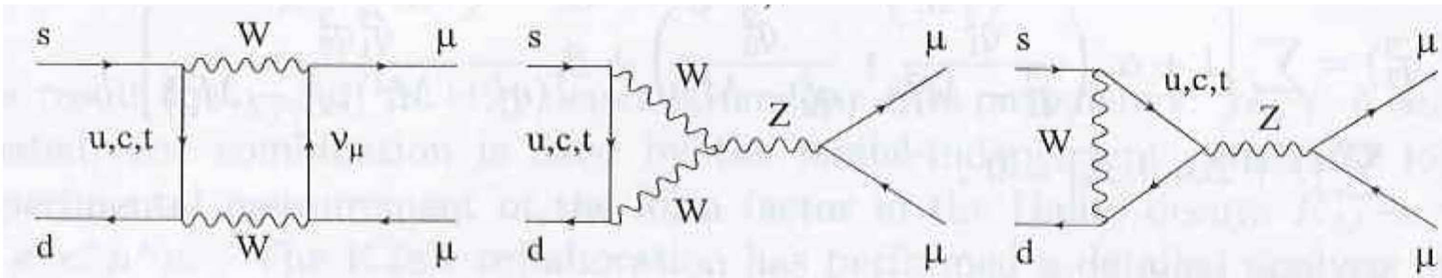
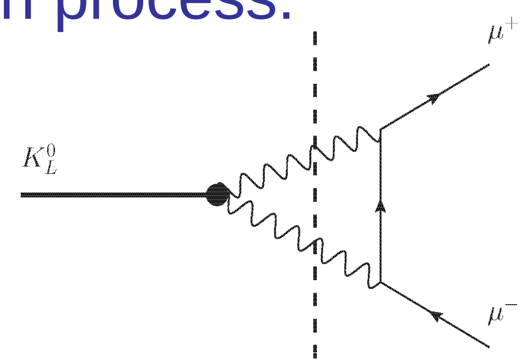
Physics of $K_L \to \mu^+ \mu^-$

- A second order weak, ``strangeness changing neutral current''



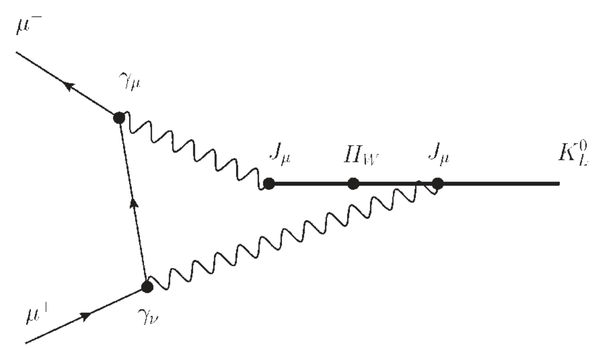
- $K_L \to \mu^+ \mu^-$ decay rate is known:
  - BR($K_L \to \mu^+ \mu^-$) = (6.84 ± 0.11) x 10^{-9}
- Large ``background'' from two-photon process:
  - Third-order electroweak amplitude
  - Optical theorem gives imaginary part.
  - $K_L \to \gamma\gamma$ decay rate is known



Physics of $K_L \rightarrow \mu^+ \mu^-$ (con't)

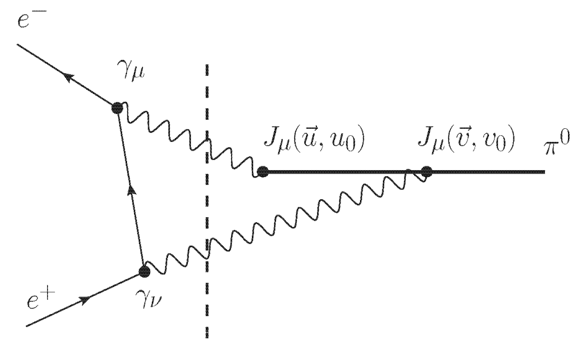
$$\boxed{-5.209 \pm 0.03} \qquad \boxed{\left|(F_{\text{real}})_{\text{E\&M}} + (F_{\text{real}})_{\text{Weak}}\right| = 1.167 \pm 0.094}$$

- Define: $$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} = 2\beta_\mu \left(\frac{\alpha}{\pi}\frac{m_\mu}{M_K}\right)^2 \left(|F_{\text{imag}}|^2 + |F_{\text{real}}|^2\right)$$
- $(F_{\text{real}})_{\text{Weak}} = -1.82 \pm 0.04$ (*)
- A 10% lattice calculation of $(F_{\text{real}})_{\text{E\&M}}$ would allow a test of (*) to 13%.
- Lattice calculation more difficult than ΔM_K
  - 5 vertices, 60 time orders
  - many states |n> with $E_n < M_K$
- First try simpler $\pi^0 \rightarrow e^+ e^-$

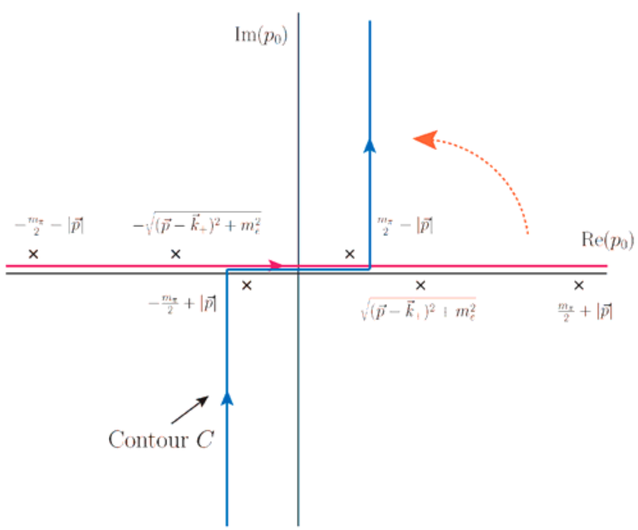


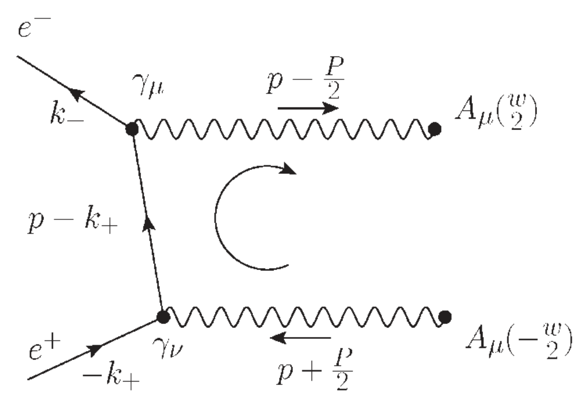
Consider simpler $\pi^0 \rightarrow e^+ e^-$

- Euclidean non-covariant P.T. difficult:
  - 12 time orders,
  - $E_{\gamma\gamma} < M_{\pi^0}$
- Try something different:
  - Evaluate in Minkowski space
  - Wick rotate integral over time argument:



$$\mathcal{A}_{\pi^0\to e^+e^-} \to \int d^4w\, \widetilde{L}(k_-, k_+, w)_{\mu\nu} \langle 0| T\left\{ J_\mu(\frac{w}{2}) J_\nu(-\frac{w}{2}) \right\} |\pi^0(\vec{P}=0)\rangle$$

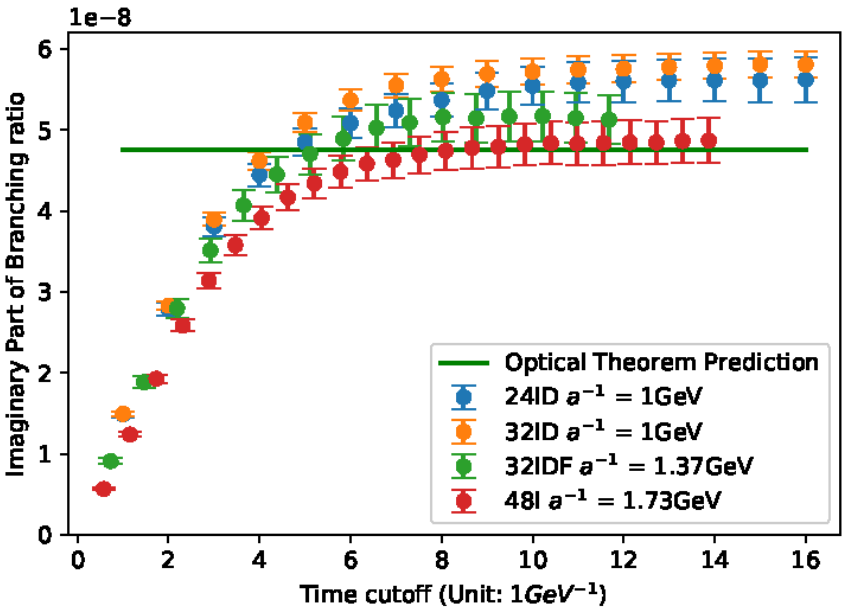


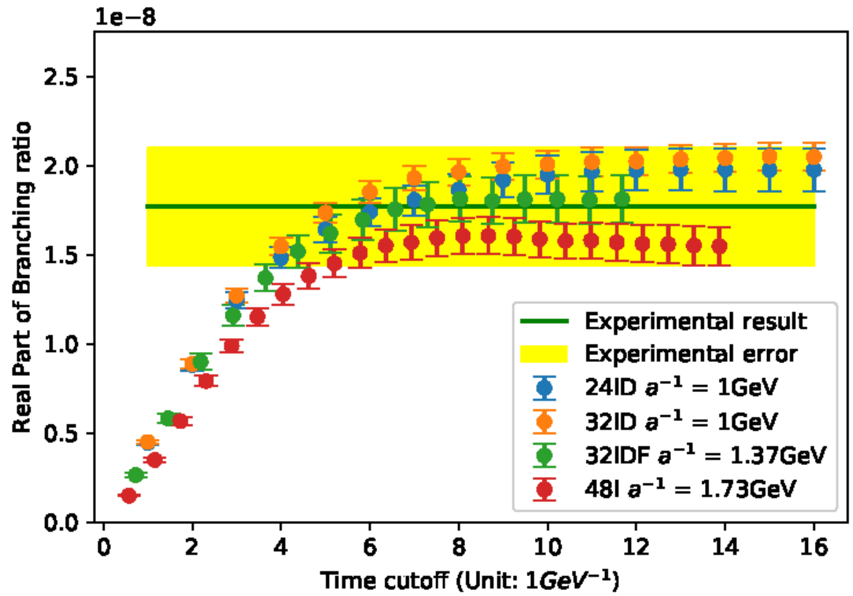


Lattice Results
(Yidi Zhao)

$$\mathcal{A}_{\pi^0\to e^+e^-} \to \int d^4w\, \widetilde{L}(k_-,k_+,w)_{\mu\nu}\langle 0|T\left\{J_\mu(\frac{w}{2})J_\nu(-\frac{w}{2})\right\}|\pi^0(\vec{P}=0)\rangle$$

- Tabulate $L(k_-,k_+,w)_{\mu\nu}$ and sum over 4-d lattice variable w : exponentially small FV corrections
- Physical kinematics, $1/a \le 1.73$ GeV.





Conclusion

- QED portion of combined QCD + QED amplitudes can be evaluated in infinite-volume Minkowski space.
- Compute complex $\pi^0 \to e^+ e^-$ amplitude decay using this method. (New results from disconnected piece could be presented in a longer talk.)
- Position-space formulation can lead to exponentially suppressed finite-volume errors ($\pi^0 \to e^+ e^-$ case).
- A 10% calculation of E&M contribution to $K_L \to \mu^+ \mu^-$ would provide a new test of the Standard Model
  - Much simplified by the proposed method.
  - Still difficult with many graphs and subtractions.
  - Need theory of finite-volume effects (generalize Christ, Feng, Martinelli & Sachrajda, arXiv:1504.01).

Backup

Disconnected part

- Smaller than connected by ~ 40x