The Dilaton in the Composite Higgs Framework

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Introduction

Compositeness of the Higgs could provide a natural solution to the hierarchy problem of the Standard Model.

<u>Challenges</u>

- <u>Flavor Problem</u>

  It is not simple to generate masses for the fermions without large contributions to flavor violation. Conformal dynamics?

- <u>Top Yukawa</u>

  It is a challenge to generate a large mass for the top quark. Top compositeness?

- <u>Little Hierarchy Problem</u>

  Precision electroweak requires that compositeness scale $\gtrsim$ 5 TeV. Then radiative corrections to the Higgs mass from the top loop are not small, leading to "little hierarchy problem". Light top partners?

The problems are very similar to those of technicolor a decade ago. The "little hierarchy problem" is the new face of precision electroweak bound.

Strong conformal dynamics above the weak scale offers a solution to flavor problem of composite Higgs models.

The conformal symmetry is broken at the compositeness scale, with the Higgs emerging as a composite of the strong dynamics.

In theories where an exact conformal symmetry is spontaneously broken, the low energy effective theory contains a massless scalar, the dilaton.

The dilaton can be thought of as the NGB asociated with the breaking of scale invariance. (Just 1 NGB, not 5, because conformal invariance is a space-time symmetry.)

The form of the dilaton couplings is fixed by the requirement that the symmetry be realized non-linearly. Very predictive.

However, in the class of theories of interest for electroweak symmetry breaking, conformal symmetry is explicitly broken.

- Is there a light dilaton with mass below the compositeness scale, in addition to the Higgs?
- If so, how are its couplings modified by conformal symmetry violating effects?

The Mass of the Dilaton

Consider a theory where conformal symmetry is spontaneously broken. Then the low energy effective theory contains a dilaton field $\sigma(x)$.

Below the breaking scale the symmetry is realized non-linearly. Under scale transformations,

$$x^\mu \to x'^\mu = e^{-\omega} x^\mu$$

the dilaton transforms as

$$\sigma(x) \to \sigma'(x') = \sigma(x) + \omega f$$

where f is the symmetry breaking scale.

It is convenient to define the object $\chi(x)$, which transforms linearly under scale transformations.

$$\chi(x) = f e^{\sigma(x)/f}$$

Under the scale transformation

$$x^\mu \to x'^\mu = e^{-\omega} x^\mu$$

$$\chi(x) \to \chi'(x') = e^\omega \, \chi(x)$$

The low energy effective theory for the dilaton will in general contain all terms consistent with this transformation.

What terms does the Lagrangian contain?

The symmetry allows derivative terms of the form

$$\frac{1}{2}\partial_\mu \chi \partial^\mu \chi + \frac{c}{\chi^4}\left(\partial_\mu \chi \partial^\mu \chi\right)^2 + \ldots$$

However, crucially, a non-derivative term is also allowed.

$$V(\chi) = \kappa_0 \chi^4$$

The effect of this term is to drive f to zero, corresponding to unbroken conformal symmetry, if κ_0 is positive. If κ_0 is negative, f is driven to infinity, and conformal symmetry is never realized.

Only if κ_0 is identically zero is the symmetry spontaneously broken. The potential then vanishes and there is a massless dilaton. However, in general setting κ_0 to zero is associated with tuning, since there is no symmetry reason for it to vanish.

The situation changes if conformal symmetry violating effects are present. Add to the theory an operator $\boldsymbol{O}(\boldsymbol{x})$ of dimension Δ close to 4.

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + \lambda_{\mathcal{O}}\mathcal{O}(x)$$

Under scale transformations,

$$x^{\mu} \to x'^{\mu} = e^{-\omega}x^{\mu}$$
$$\mathcal{O}(x) \to \mathcal{O}'(x') = e^{\omega\Delta}\mathcal{O}(x)$$

Define a dimensionless coupling constant ,

$$\hat{\lambda}_{\mathcal{O}} = \lambda_{\mathcal{O}}\mu^{\Delta-4}$$

The operator $\boldsymbol{O}(\boldsymbol{x})$ is normalized such that $\lambda \sim 1$ corresponds to strong coupling. For small $\lambda \ll 1$, it satisfies the RG equation,

$$\frac{\mathrm{d}\,\log\hat{\lambda}_{\mathcal{O}}}{\mathrm{d}\,\log\mu} = -(4-\Delta)$$

We can determine how λ appears in the low energy theory by promoting it to a spurion. For small λ the form of the UV theory is invariant under the following transformation.

$$x^\mu \to x'^\mu = e^{-\omega} x^\mu$$
$$\lambda_{\mathcal{O}} \to \lambda'_{\mathcal{O}} = e^{(4-\Delta)\omega} \lambda_{\mathcal{O}}$$

To leading order in λ the form of the potential now becomes

$$V(\chi) = \kappa_0 \chi^4 - \kappa_1 \lambda_{\mathcal{O}} \chi^\Delta$$ (Rattazzi & Zaffaroni)

The dilaton potential admits a minimum at

$$f^{(\Delta-4)} = \frac{4\kappa_0}{\kappa_1 \lambda_{\mathcal{O}} \Delta}$$

From this, we find the dilaton mass at the minimum,

$$m_\sigma{}^2 = \kappa_1 \lambda_{\mathcal{O}} \Delta (4-\Delta) f^{\Delta-2} = 4\kappa_0 (4-\Delta) f^2$$

The dilaton mass is suppressed if the operator $O(x)$ that breaks conformal symmetry is marginal! (Goldberger, Grinstein & Skiba)

Need a large hierarchy between the flavor and electroweak scales. Then either the deformation λ must be hierarchically small in the ultraviolet, or the operator O must be marginal.

If O is indeed marginal, potentially a very important result! A new light scalar state below the compositeness scale!

Unfortunately, the analysis that led up to this conclusion is only valid at small λ, corresponding to weak coupling. To validate this result, must establish it at strong coupling.

Our approach will be to assume small (perturbative) λ, but work to all orders in this parameter. Check if the result survives when $\lambda \rightarrow 1$, its strong coupling value.

Working to all orders in λ involves incorporating 4 distinct effects.

- In writing down the Lagrangian, did not take into account the breaking of scale invariance by the regulator. Must include this.

- In determining the vacuum structure used the potential, not the effective potential. This needs to be accounted for.

- Need to include terms with all powers of λ in the Lagrangian. Setting $\epsilon = 4 - \Delta$, the potential becomes

$$V(\chi) = \kappa_0 \chi^4 - \sum_{n=1}^{\infty} \kappa_n \lambda_{\mathcal{O}}^n \, \chi^{(4-n\epsilon)}$$

- As λ approaches strong coupling, its RG evolution is affected. The RG for λ now takes the more general form

$$\frac{\mathrm{d}\ \log \hat{\lambda}_{\mathcal{O}}}{\mathrm{d}\ \log \mu} = -g(\hat{\lambda}_{\mathcal{O}})$$

  where $g(\lambda)$ is a polynomial in λ. The constant term in this polynomial is $\epsilon = (4 - \Delta)$.

Of these 4 effects, the first 3 do not alter the conclusions of the naive small λ analysis. The underlying reason is that in each of these 3 cases, the corrections are of order λ times the leading order effect and are therefore at most of the same size.

The 4th effect is qualitatively different. Consider again the RGE

$$\frac{\mathrm{d}\,\log \hat{\lambda}_{\mathcal{O}}}{\mathrm{d}\,\log \mu} = -g(\hat{\lambda}_{\mathcal{O}}) \qquad g(\hat{\lambda}_{\mathcal{O}}) = \sum_{n=0}^{\infty} c_n \hat{\lambda}_{\mathcal{O}}^n$$

The leading order term in the polynomial g(λ) is $(4 - \Delta) = \epsilon \ll 1$, while the corrections begin at order λ. Even before strong coupling is reached the higher order terms dominate, and their effects can alter the conclusions of the naïve small λ analysis.

The dilaton mass is given by

$$m_\sigma^2 = 4\frac{\kappa_0}{4!}g(\hat{\lambda}_{\mathcal{O}})f^2$$

It is the scaling behavior of the deformation $O(x)$ at the breaking scale that determines the dilaton mass, not scaling dimension at fixed point!

$$g(\hat{\lambda}_{\mathcal{O}}) = \sum_{n=0}^{\infty} c_n \hat{\lambda}_{\mathcal{O}}^n$$

To obtain a light dilaton, it is not sufficient that $c_0 = \epsilon \ll 1$. Require $g(\lambda) \ll 1$ at the breaking scale.

This is equivalent to requiring that not just c_0 but all the $c_n \ll 1$.

Although this can happen naturally in some special cases, for example in theories with fixed lines, this criterion is not expected to be satisfied in most theories of interest for EWSB.

Light dilatons are not a generic feature of composite Higgs models that address the flavor problem through strong conformal dynamics.

For a light dilaton to be present, the explicit violation of scale invariance at the breaking scale must be small. Either

- the deformation λ must be small at the breaking scale, or
- the scaling behavior of the operator O near the breaking scale must be close to marginal.

If a light dilaton is present, we can construct a consistent effective theory, treating the violation of scale invariance at the breaking scale as the small expansion parameter.

The Dilaton Couplings

If the UV theory was conformally invariant, the dilaton would couple so as to formally restore this symmetry to the interactions in the low energy effective field theory.

The form of the dilaton interactions with the SM fields would then be completely predicted. (Goldberger, Grinstein & Skiba)

However, in realistic composite Higgs models, the conformal symmetry is explicitly violated. There are 2 sources of conformal symmetry violation.

- The light fermions of the SM, and also the $SU(3) \times SU(2) \times U(1)$ gauge interactions, are generally not part of the strong conformal dynamics.
- The explicit violation from the operator $\boldsymbol{O}$ that breaks conformal symmetry and generates the weak scale must also be accounted for.

Consider the dilaton coupling to the W gauge bosons. To begin, neglect the conformal symmetry violating effects arising from the operator $\boldsymbol{O}$.

That leaves only one explicit source of conformal symmetry violation, a quantum effect from the running of the SU(2) gauge coupling. This is small and can be neglected.

Then the mass of the W boson arises as a consequence of the spontaneous breaking of conformal symmetry.

In this limit the dilaton couples as a "conformal compensator", to formally restore the conformal symmetry.

$$\left(\frac{\chi}{f}\right)^2 m_W^2 W_\mu^+ W^{\mu -} \longrightarrow 2\sigma \frac{m_W^2}{f} W_\mu^+ W^{\mu -}$$

Once conformal symmetry violating effects are incorporated, there are additional dilaton couplings that respect the spurious conformal symmetry.

$$\left(\frac{\chi}{f}\right)^2 \left[1 + \sum_{n=1}^{\infty} \alpha_{W,n} \lambda_{\mathcal{O}}^n \, \chi^{(-n\epsilon)}\right] \hat{m}_W^2 W_\mu^+ W^{\mu -}$$

Expanding this out we get

$$2\sigma \frac{m_W^2}{f} \left[1 + c_W \epsilon + \quad . . \right] W_\mu^+ W^{\mu -}$$

where c_W is of order λ.

Then correction to the dilaton coupling is of order $\epsilon\lambda$, which is the square of the dilaton mass over the strong coupling scale.

Corrections to the dilaton couplings from conformal symmetry violating effects arising from the operator O are suppressed by the square of the dilaton mass over the strong coupling scale.

Let us see how this arises in the composite Higgs framework.

For illustrative purposes, consider a theory in which the fields in the SM Higgs doublet emerge as pseudo-Nambu-Goldstone bosons from the breaking of SU(3) × U(1) to SU(2) × U(1).

Normally we would parametrize the Goldstones as

$$\phi = \hat{f} \exp\left(i h^a t^a\right) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

But now the non-linear sigma model condition changes from,

$$|\phi|^2 \;=\; \hat{f}^2 \quad \longrightarrow \quad |\phi|^2 \;=\; \hat{f}^2 \hat{\chi}^2$$

where $\hat{\chi} = \chi/f$. This corresponds to the replacement,

$$\hat{f} \to \hat{f}\hat{\chi}$$

In this parametrization Goldstones *h* are invariant under scale transforms.

Then the kinetic term for the Higgs doublet arises as

$$(D_\mu \phi)^\dagger D^\mu \phi \longrightarrow \hat{\chi}^2 \hat{f}^2 (D_\mu h)^\dagger D^\mu h$$

The potential for the Higgs can only arise from effects that violate the global symmetry, such as the gauge and Yukawa couplings.

In realistic models, the gauge interactions and top Yukawa coupling do violate the conformal symmetry, but not by very much.

The potential for the Higgs is then of the very restrictive form,

$$V = \chi^4 V_0(h)$$

This potential does not lead to Higgs-dilaton mixing. After setting the Higgs to its VEV, we obtain the dilaton coupling to the W exactly as before,

$$\left(\frac{\chi}{f}\right)^2 m_W^2 W_\mu^+ W^{\mu-} \longrightarrow 2\sigma \frac{m_W^2}{f} W_\mu^+ W^{\mu-}$$

When conformal symmetry violating effects from the operator ***O*** are included, the non-linear sigma model condition generalizes to,

$$|\phi|^2 = \hat{f}^2\hat{\chi}^2 \left[1 + \sum_{n=1}^{\infty} \alpha_{\phi,n}\overline{\lambda}_{\mathcal{O}}^{n}\, \chi^{(-n\epsilon)}\right]$$

Repeating the same steps as before, we again find for the dilaton coupling to the W boson,

$$2\sigma\frac{m_W^2}{f}\left[1 + c_W\epsilon + \quad . . \right] W_\mu^+ W^{\mu -}$$

The dilaton couplings to the other SM fields can be determined in a similar fashion.

Conclusions

In theories where the operator that breaks conformal symmetry remains close to marginal until the breaking scale, the dilaton mass can naturally lie below the scale of strong dynamics.

However, in general, this condition need not be satisfied in realistic composite Higgs models.

In this framework, if the dilaton is light, corrections to the form of dilaton couplings to SM fields from conformal symmetry violating effects are suppressed by the square of the dilaton mass over the strong coupling scale, and are under good theoretical control.