Quantum Monte Carlo Approaches to Neutrino-Nucleus Scattering

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Aug, 2019

- Motivation
- Interactions/Currents
- Inclusive Electron Scattering
- Short-time Evolution and two-nucleon dynamics
- Summary / Outlook

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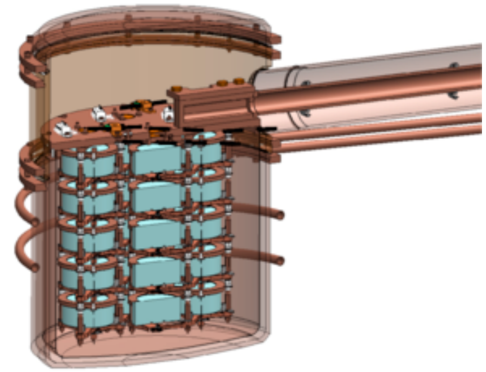
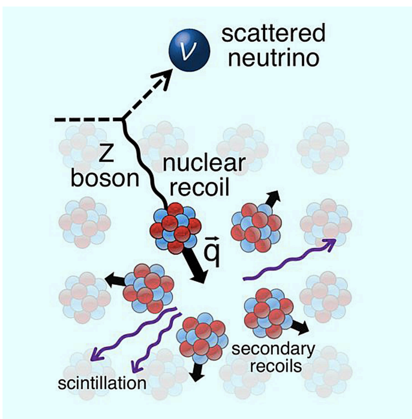
Why study Neutrinos and Nuclei

Neutrinos and nuclei are fundamental to some of the largest and most exciting experiments and observations

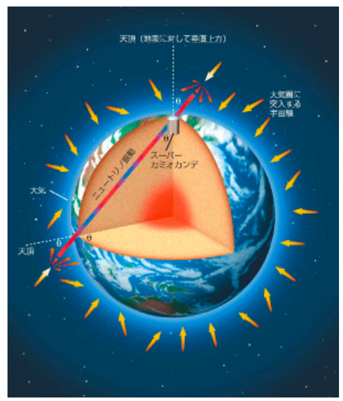
Double Beta decay
Majorana nature of the neutrino



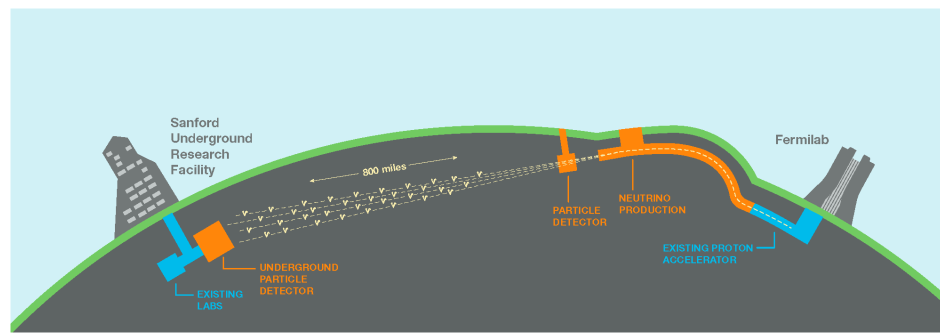
Coherent neutrino scattering at SNS



Atmospheric Neutrinos

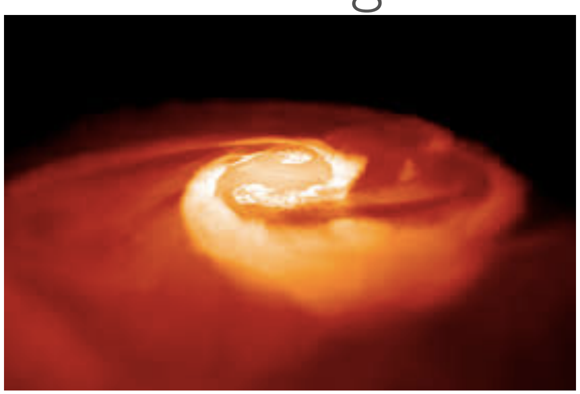


Accelerator Neutrinos



Quasi-Elastic scattering
At higher energies resonance and deep inelastic dominate

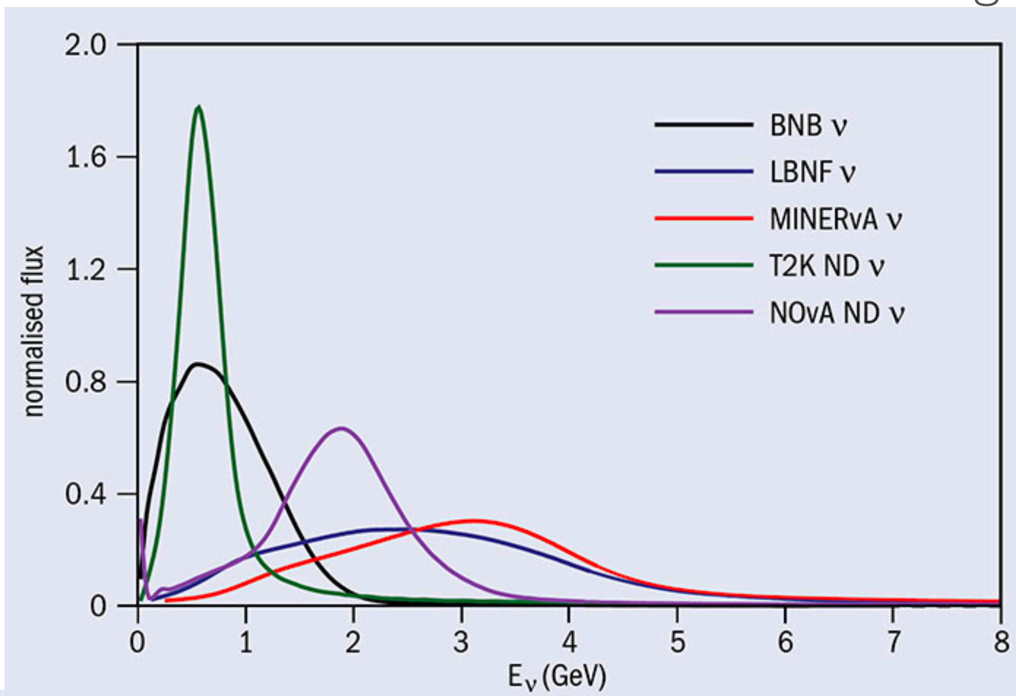
Supernovae/ Neutron star mergers

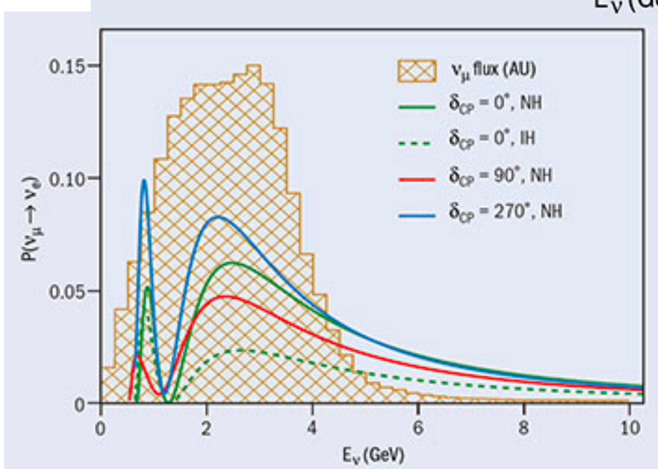


Accelerator Neutrino Experiments

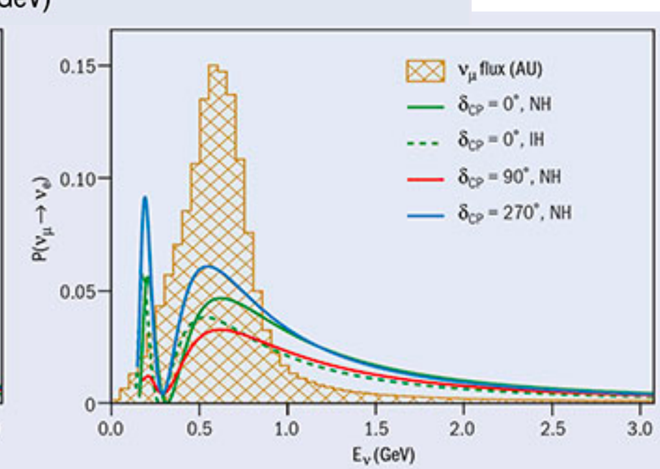
wide range of neutrino energies

importance of oscillations/cross sections for energies ~1-3 GeV





DUNE



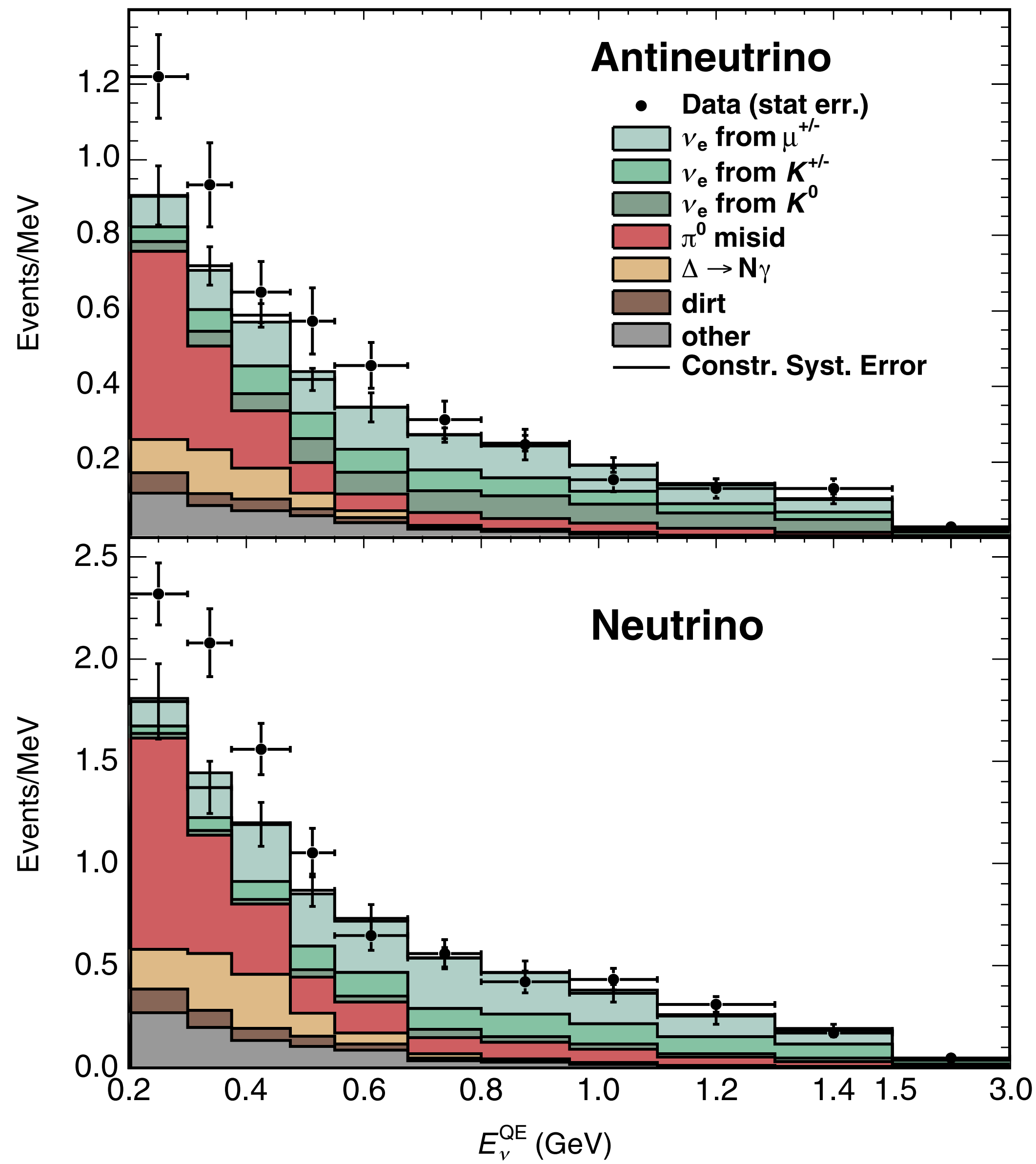
T2K

MiniBooNE experiment (2018)

$$\bar{\nu}_\mu \to \bar{\nu}_e$$

Excess of electron antineutrino events particularly at low reconstructed energy

Electron neutrino background reconstructed from muon neutrinos



Why study electron scattering?

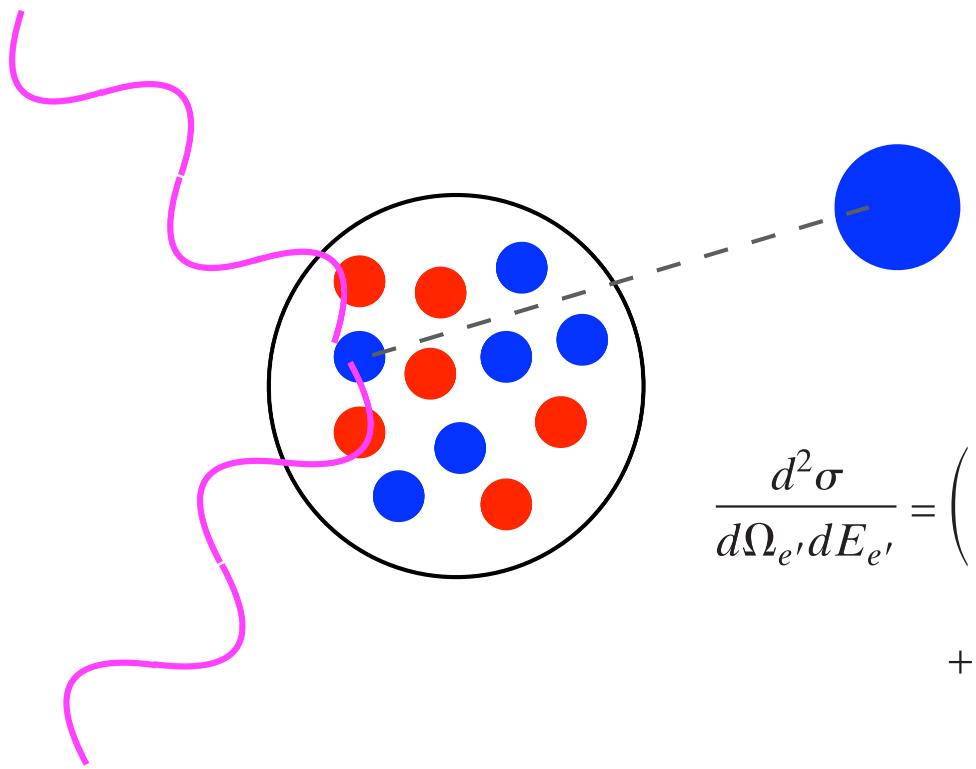
not to determine properties of electron or photon

Quasi-elastic scattering: higher q, E

Scaling with momentum transfer: ‘y’-scaling

incoherent sum over scattering from single nucleons

- scaling of 1st kind-



$$\frac{d^2\sigma}{d\Omega_{e'}dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)_M \left[\frac{Q^4}{|\mathbf{q}|^4} R_L(|\mathbf{q}|,\omega) + \left(\frac{1}{2}\frac{Q^2}{|\mathbf{q}|^2} + \tan^2\frac{\theta}{2}\right) R_T(|\mathbf{q}|,\omega)\right]$$

Electron Scattering:
Longitudinal and Transverse Response

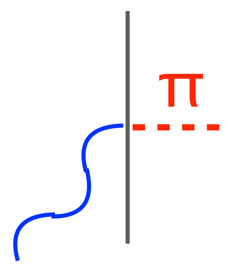
Transverse (current) response:

$$R_T(q,\omega) \; = \; \sum_f \langle 0| \, \mathbf{j}^\dagger(q) \, |f\rangle\langle f| \, \mathbf{j}(q) \, |0\rangle \; \delta(w - (E_f - E_0))$$

Longitudinal (charge) response:

$$R_L(q,\omega) \; = \; \sum_f \langle 0| \, \rho^\dagger(q) \, |f\rangle\langle f| \, \rho(q) \, |0\rangle \; \delta(w - (E_f - E_0))$$

$$\mathbf{j} \, = \sum_i \, \mathbf{j}_i \, + \, \sum_{i<j} \, \mathbf{j}_{ij} + ...$$



Two-nucleon currents required by current conservation
Response depends upon all the excited states of the nucleus

Connections to Lattice QCD: one- and two-N matrix elements

- Elastic Nucleon form factors (particularly axial)
- Inelastic form factors:
  - Inclusive (sum over all all hadronic final states): constrains hadronic input
  - Exclusive (e.g. specific pi-N final state)
- Two-Nucleon matrix elements w/ current insertions (particularly for NN final state)

Solutions or advances on dealing with

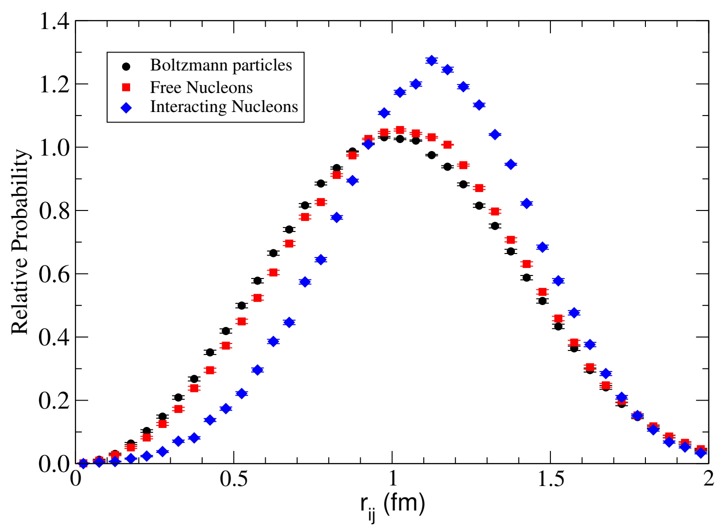
sign problem

imaginary to real time response

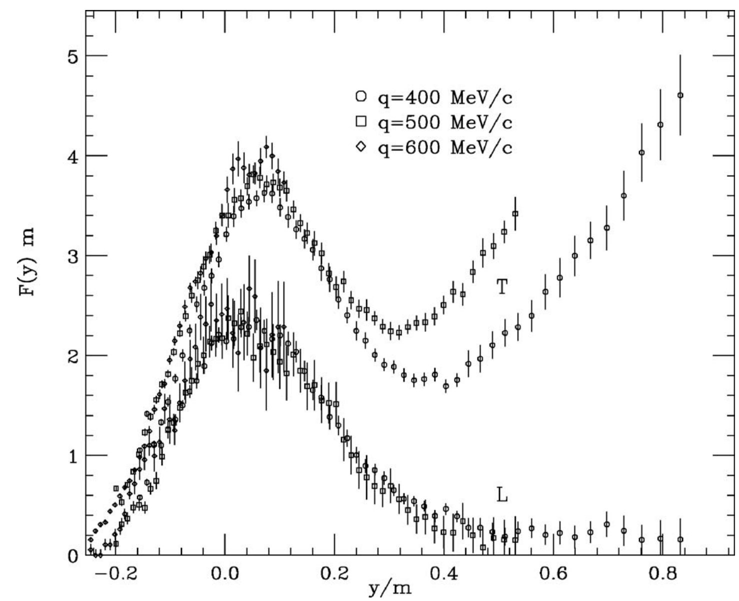
dynamics

....

Electron Scattering: Longitudinal vs. Transverse
Single Nucleon form factors (squared) divided out



Nearest-neighbor distances in nuclear matter



Scaled longitudinal vs. transverse scattering from ^{12}C

from Benhar, Day, Sick, RMP 2008
data Finn, et al 1984

Distances probed at various q

q	$r \sim \pi/q$
0.3 GeV/c	2.1 fm
0.5 GeV/c	1.2 fm
1 GeV/c	0.6 fm

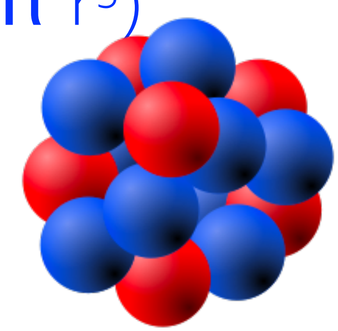
Nearest neighbor nucleons at

$\rho = 0.16\ fm^{-1} = 1 / (4/3\ \pi\ r^3)$

$r = 1.14$ fm

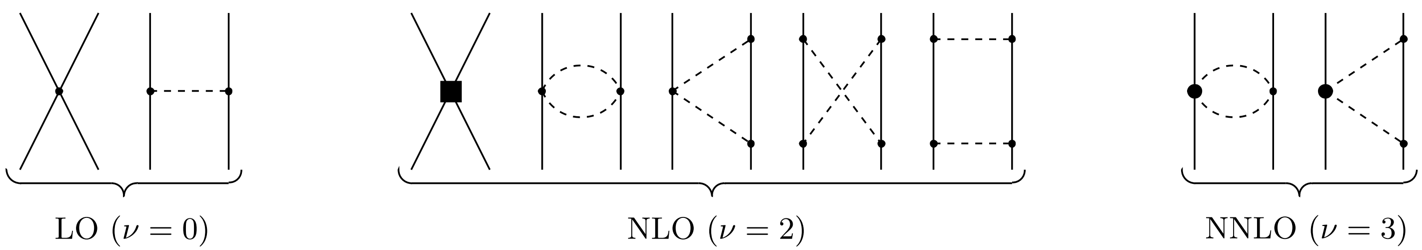
$d = 2.28$ fm

$1/m_\pi \sim$ **1.5 fm**

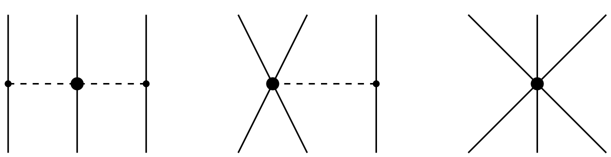


Basic building blocks: Nuclear interactions and currents

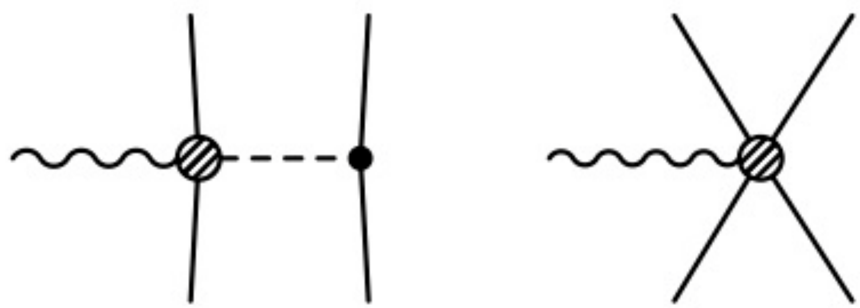
NN interactions



3N interactions

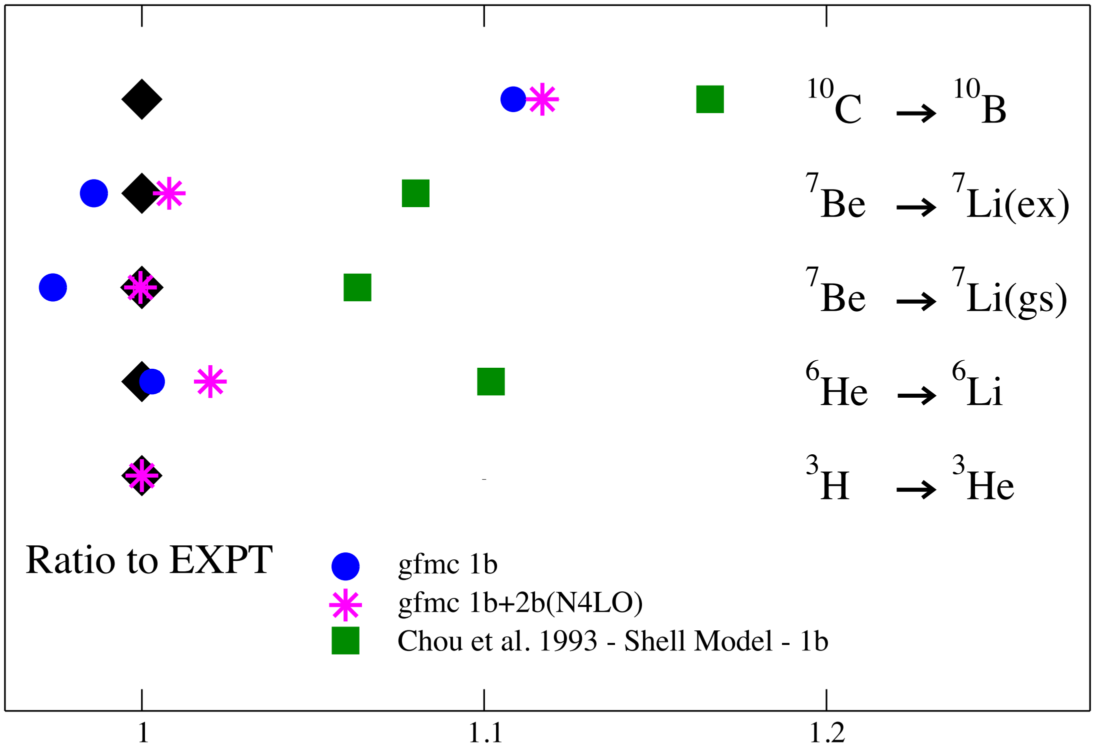


NN currents



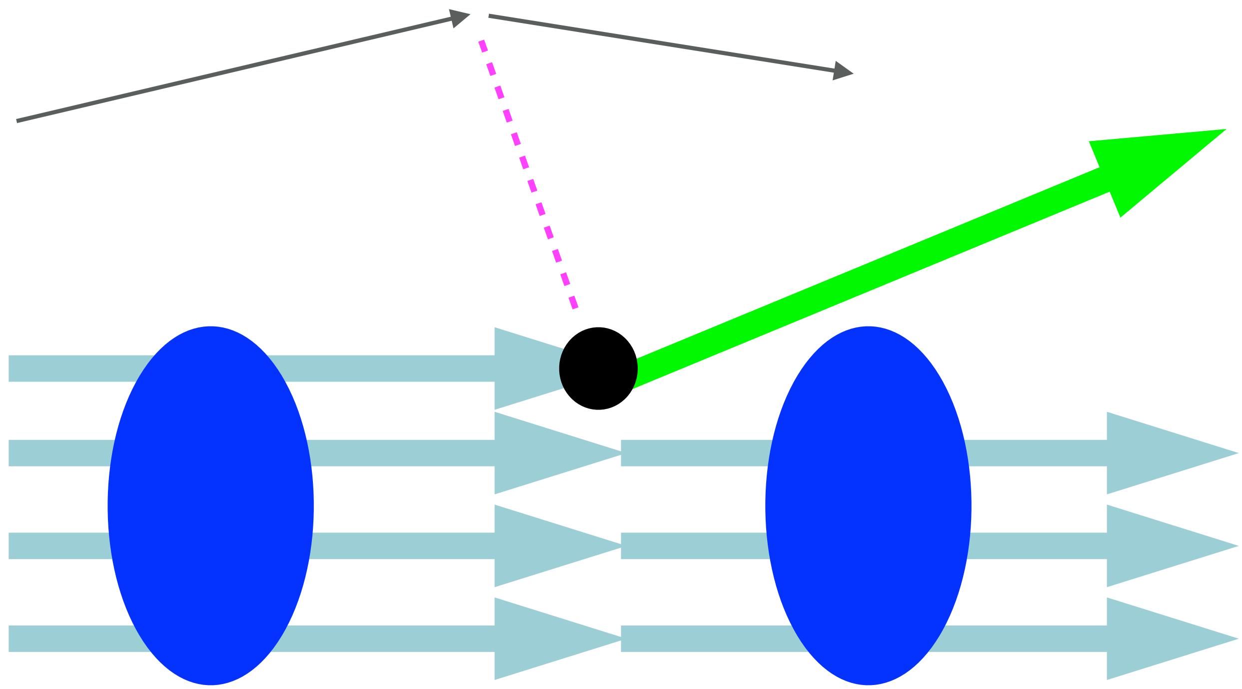
Low Momenta - Beta Decay in Light Nuclei

Pastore, et al, 2017



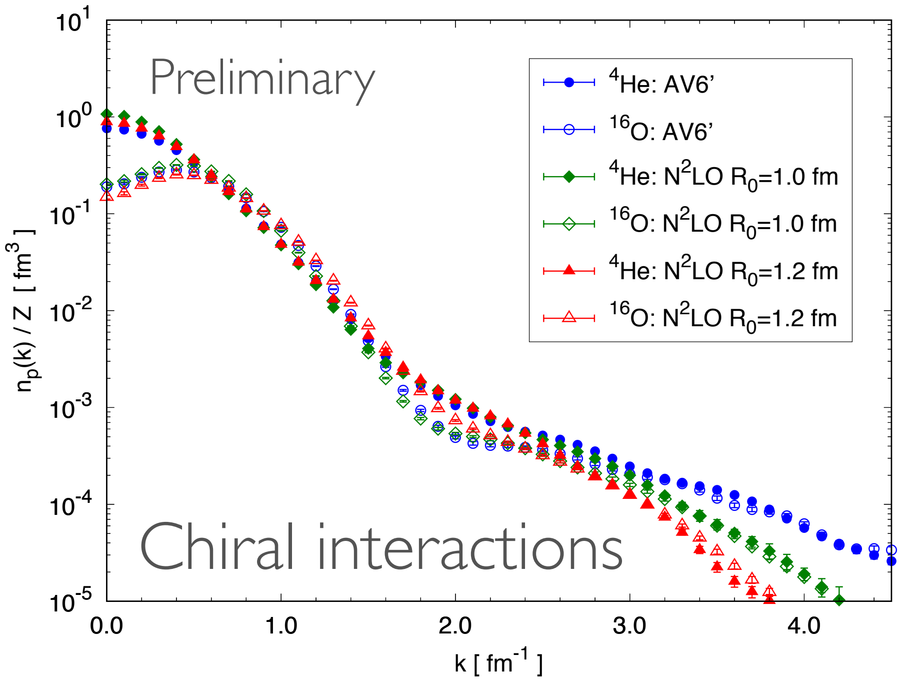
- Contact fit to Tritium beta decay
- Substantial reduction due to two-body correlations
- Modest 2N current contribution
- Good description of experimental data, explains 'quenching'
- Many calculations with larger nuclei underway

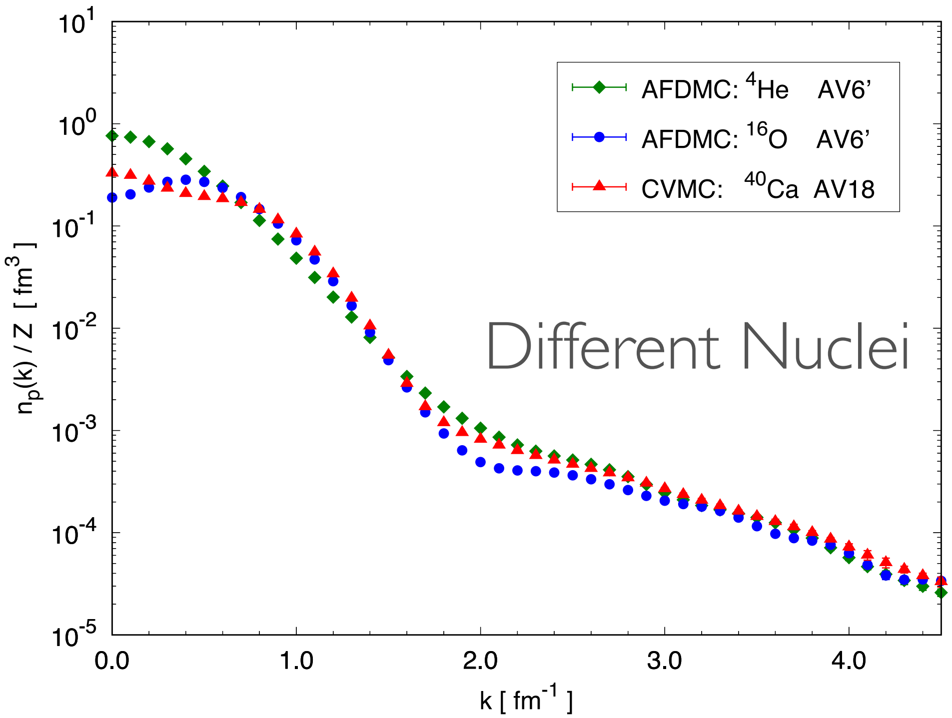
Quasi-Elastic Scattering and Plane Wave Impulse Approximation



Incorporates incoherent scattering of single nucleons:
n(k) or spectral function S(k,w)
and single-nucleon form factors

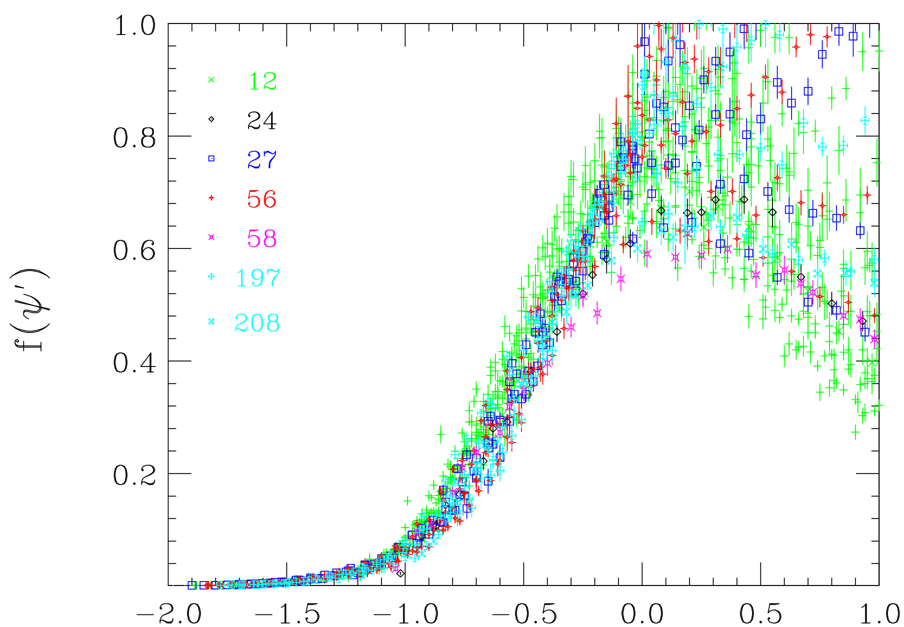
Single-Nucleon Momentum Distributions





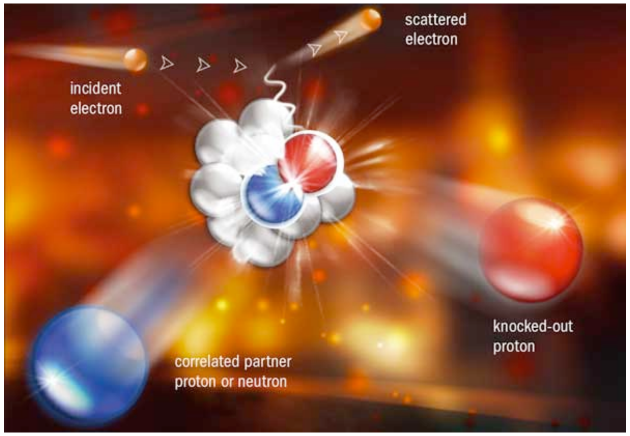
Lonardoni, Gandolfi, Wiringa, Pieper, et al

Integrated Strength:
15-20 % above k_F,
Amplitude ~ 0.3-0.4



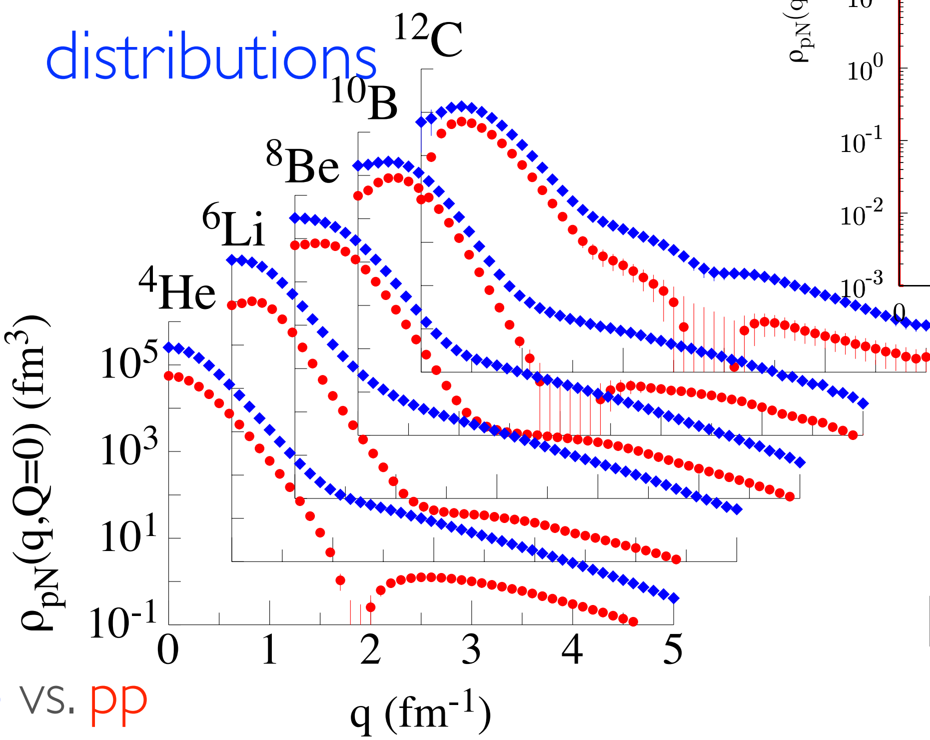
Scaling of the 1st kind (w/ p)
Donnelly & Sick (1999)

Back to Back Nucleons (total Q~0)
np pairs dominate over nn and pp



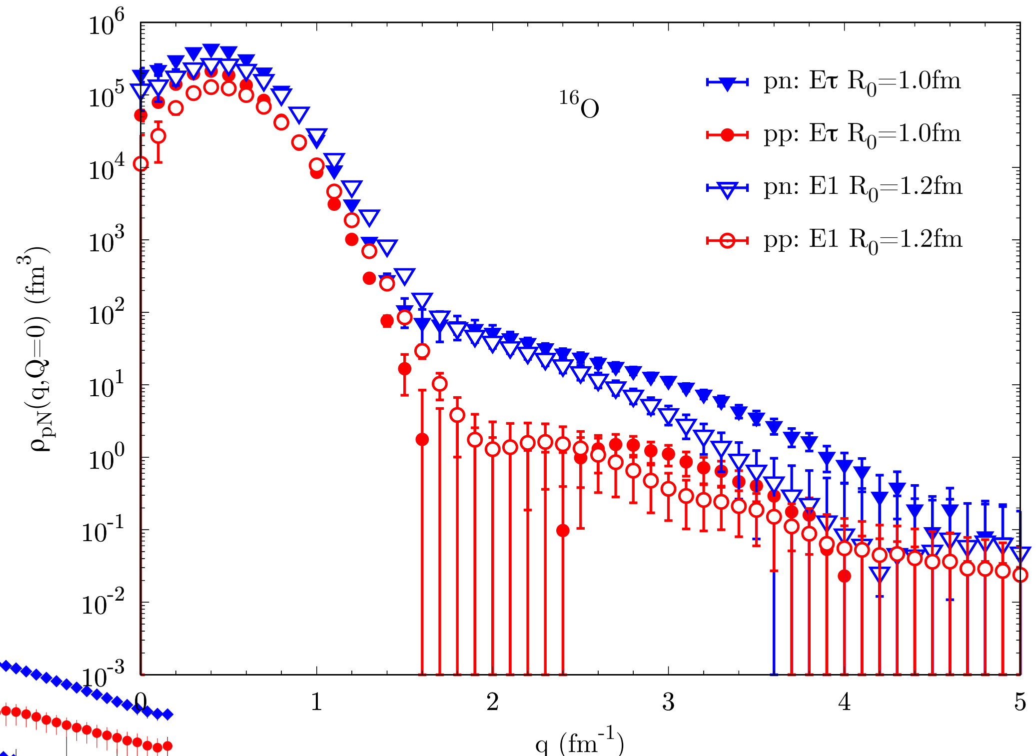
E Piasetzky *et al.* 2006 **Phys. Rev. Lett. 97** 162504.
M Sargsian *et al.* 2005 **Phys. Rev. C 71** 044615.
R Schiavilla *et al.* 2007 **Phys. Rev. Lett. 98** 132501.
R Subedi *et al.* 2008 **Science 320** 1475.

2-nucleon momentum distributions



np vs. pp

Wiringa et al.; Carlson, et al, RMP 2015

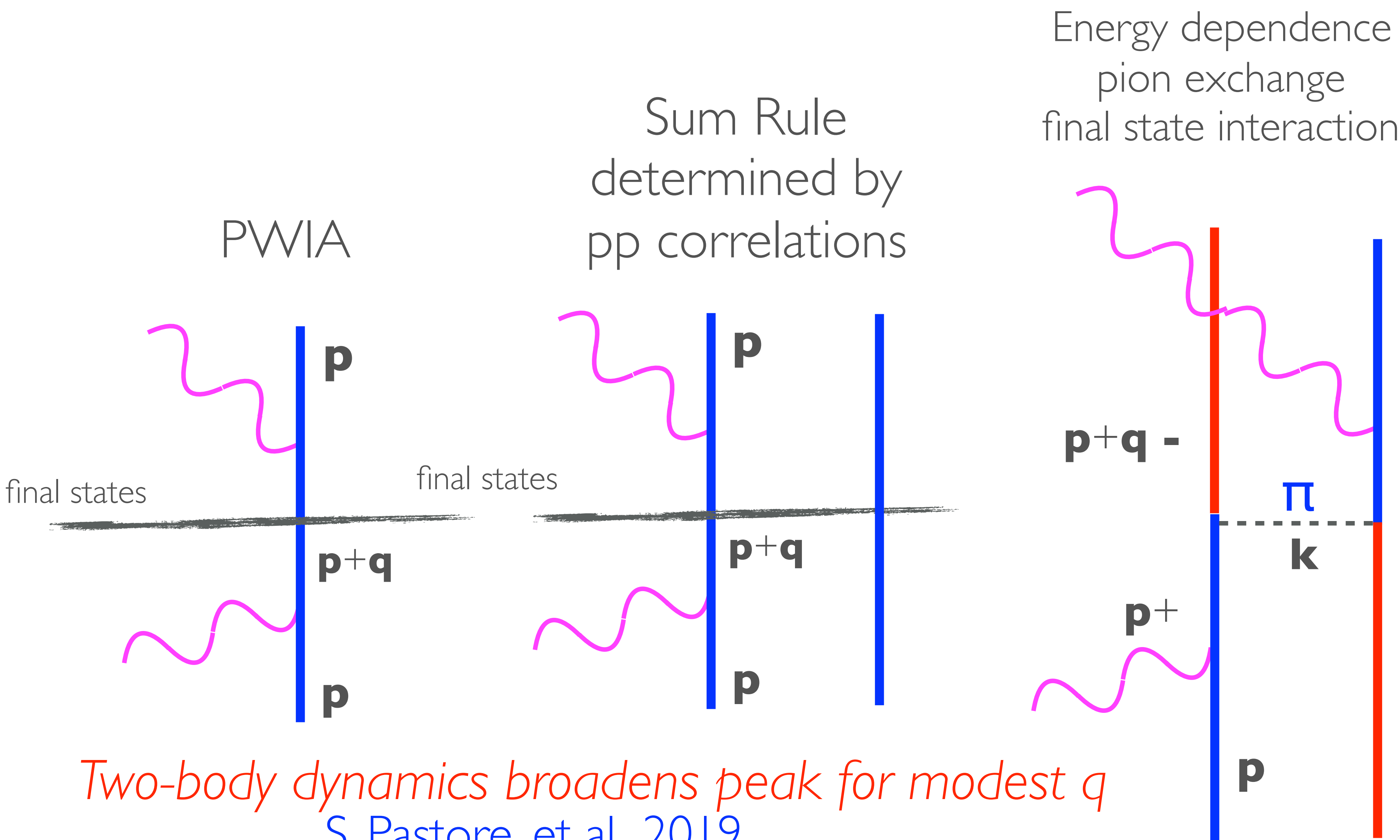


Bob Wiringa, Diego Lonardoni

Sum Rule: Longitudinal Response

$$S(q) = \langle\, 0 \mid \mathbf{j}^{\dagger}(q)\, \mathbf{j}(q)\; 0 \,\rangle$$

Gives an indication of total strength,
but not energy dependence



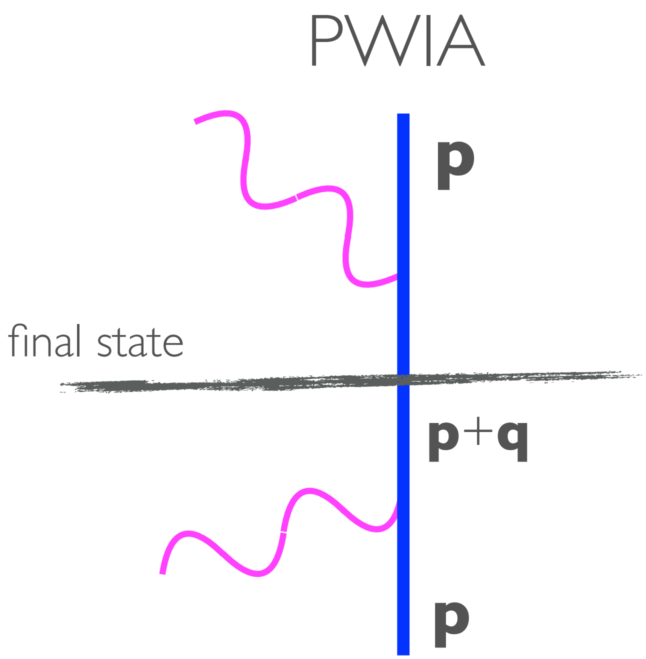
Two-body dynamics broadens peak for modest q

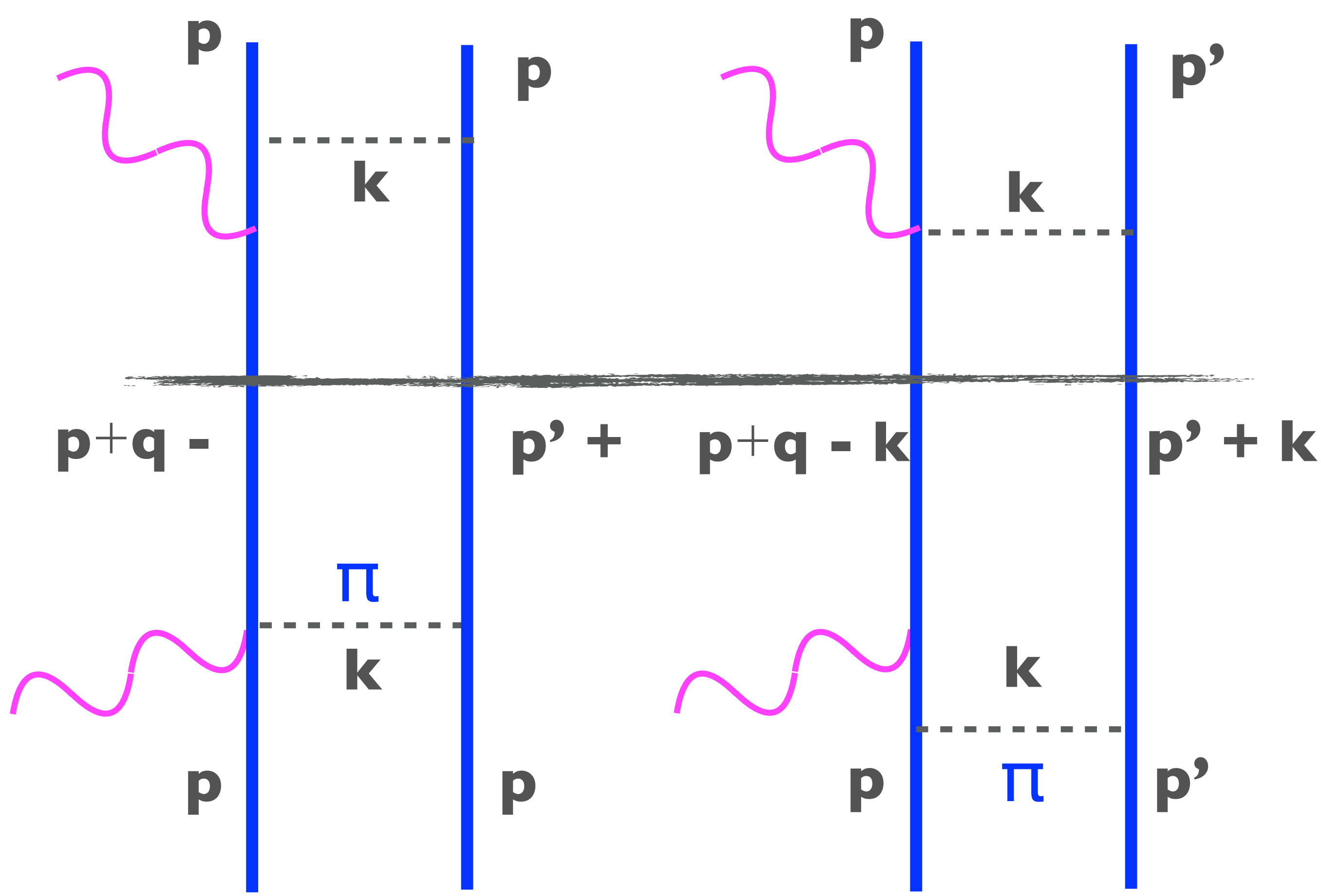
S. Pastore, et al., 2019

Vector Response

Sum Rule: Constructive Interference
between 1- and 2-body currents
w/ tensor correlations

S. Pastore, et al., 2019

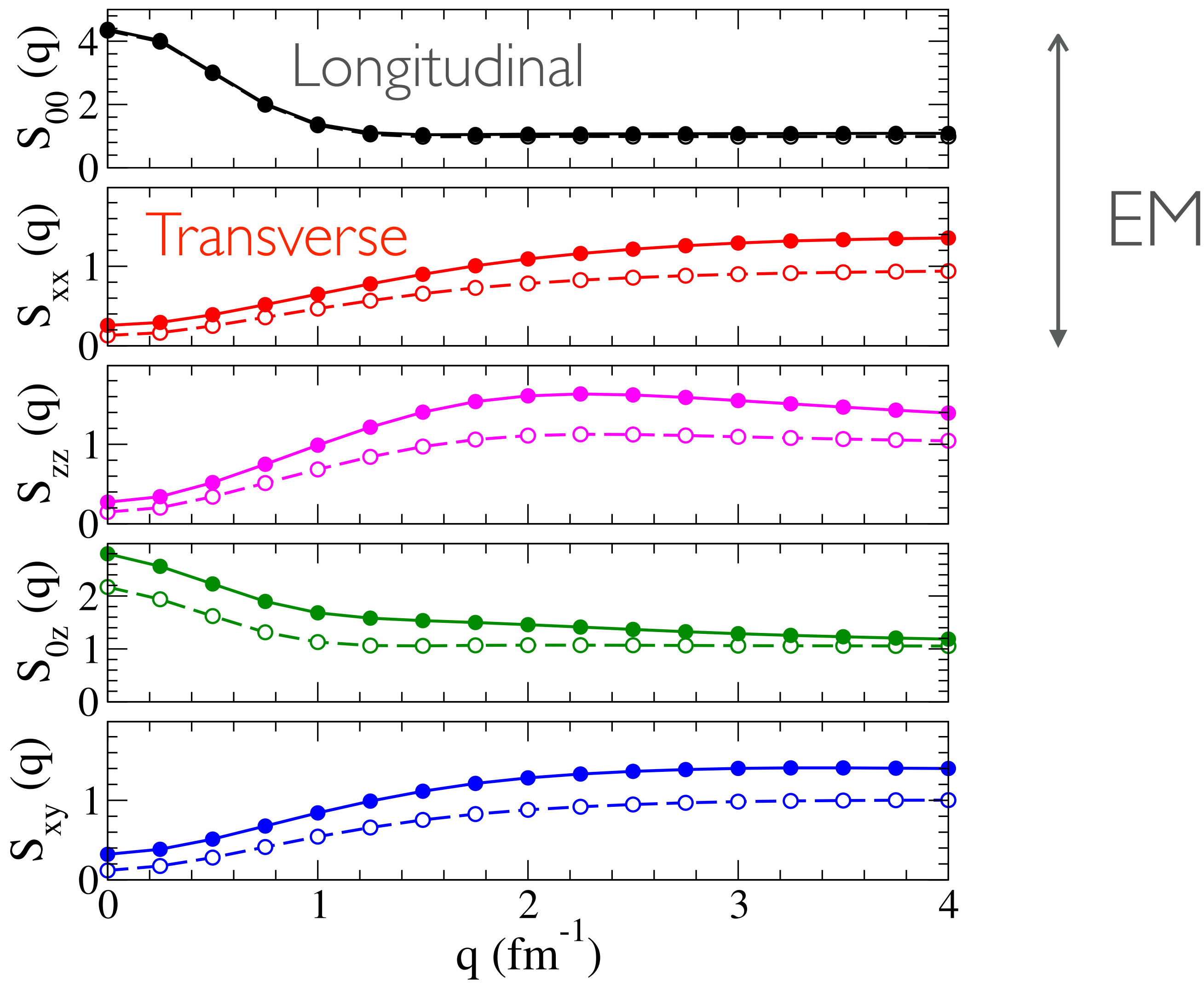




Large enhancement from
initial state correlations
and two-nucleon currents
similar in axial response
Note enhancement from
final states have larger momenta

$$\propto \; \sigma_i \cdot \mathbf{k} \; \sigma_i \cdot \mathbf{q} \; (\sigma_j \cdot \mathbf{k})^2 \; (\tau_i \cdot \tau_j)^2 \; v_\pi^2(k)$$

Sum rules in ^{12}C: neutral current scattering



Lovato, et. al PRL 2014

Single Nucleon currents (open symbols) versus
Full currents (filled symbols)

Euclidean Response

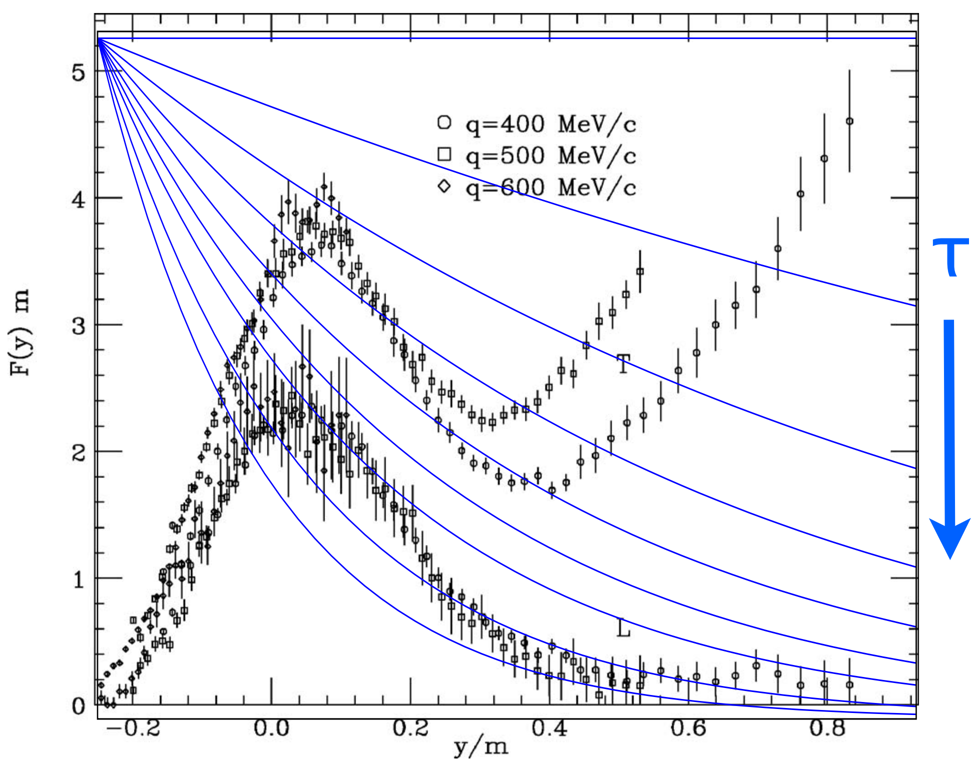
Want to calculate

$$R(q,\omega) \;=\; \int dt\ \langle 0|\ \mathbf{j}^\dagger \exp[i(H-\omega)t]\ \mathbf{j}\ |0\rangle$$

Can calculate

$$\tilde{R}(q,\tau) \;=\; \langle 0|\ \mathbf{j}^\dagger\ \exp[-(\mathbf{H}-\mathbf{E_0}-\mathbf{q^2}/(\mathbf{2m}))\tau]\ \ \mathbf{j}\ |\mathbf{0}\rangle >$$

- Exact given a model of interactions, currents
- `Thermal' statistical average
- *Full final-state interactions*
- *All contributions included - elastic, low-lying states, quasi elastic, …*

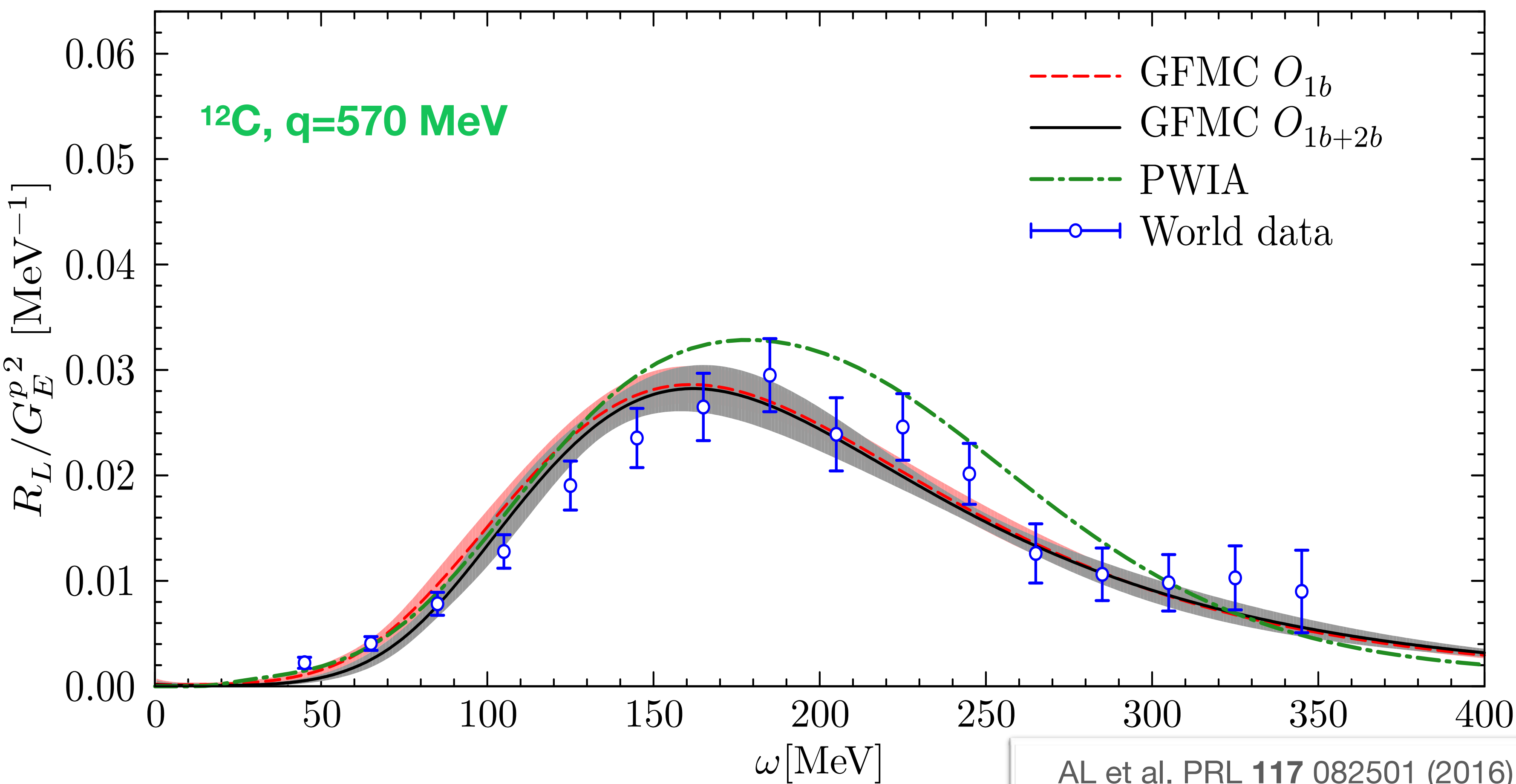


Excellent agreement
w/ EM (L & T)
response in A=4,12
Lovato, 2015, PRL 2016

Sum rule → elastic FF² w/ increasing

Electron Scattering from ^{12}C: Longitudinal Response

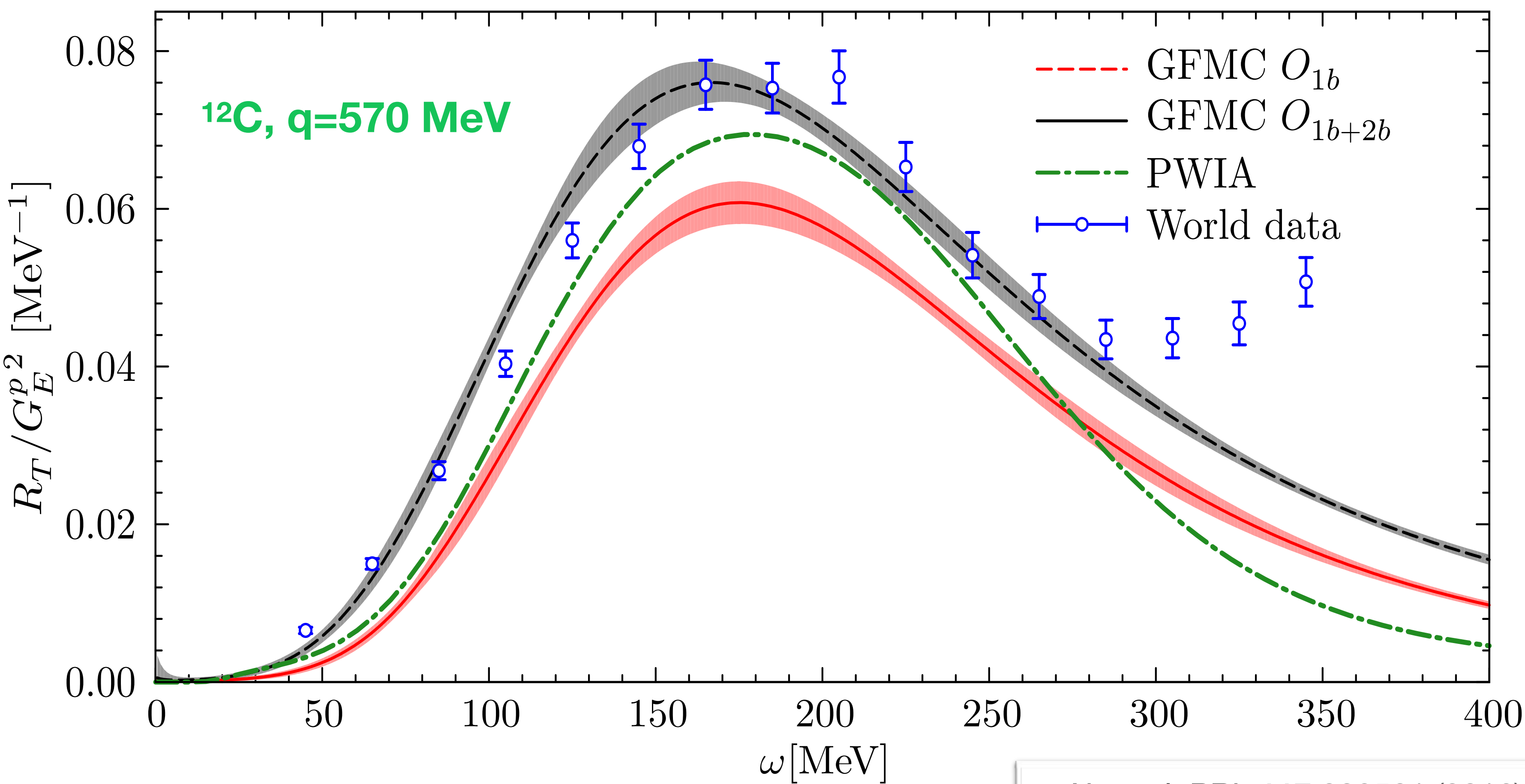
- We inverted the electromagnetic Euclidean response of ^{12}C
- Good agreement with data without in-medium modifications of the nucleon form factors
- Small contribution from two-body currents.



AL et al. PRL **117** 082501 (2016)

Electron Scattering from ^{12}C: Transverse Response

- We inverted the electromagnetic Euclidean response of ^{12}C
- Good agreement with the experimental data once two-body currents are accounted for
- Need to include relativistic corrections in the kinematics

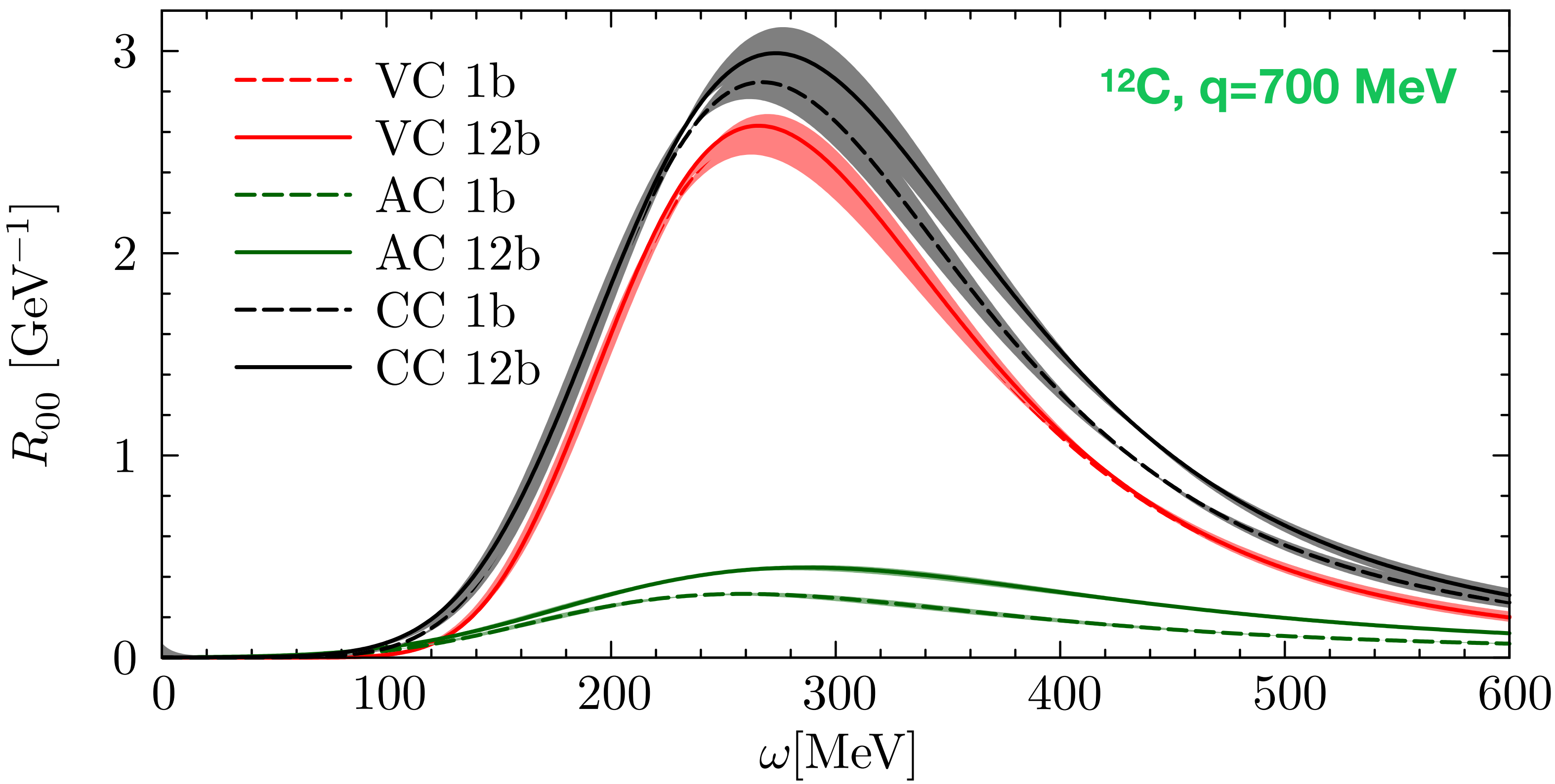


AL et al. PRL **117** 082501 (2016)

^{12}C charged-current responses

- We recently computed the charged-current response function of ^{12}C

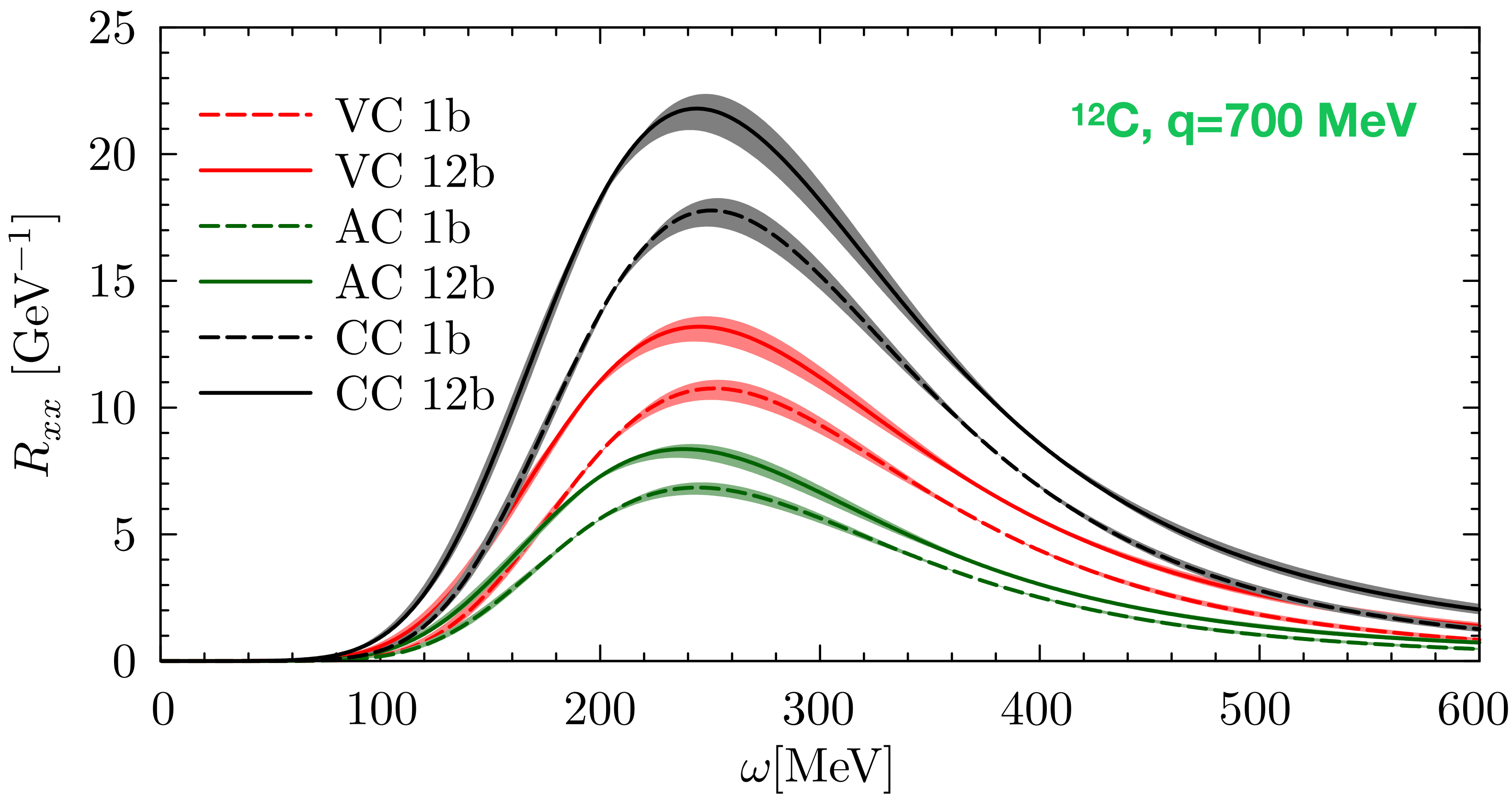
- Two-body currents have little effect in the vector term, but enhance the axial contribution at energy larger than quasi-elastic kinematics



Alessandro Lovato, 2019

^{12}C charged-current responses

- We recently computed the charged-current response function of ^{12}C
- Two-body currents have a sizable effect in the transverse response, both in the vector and in the axial contributions



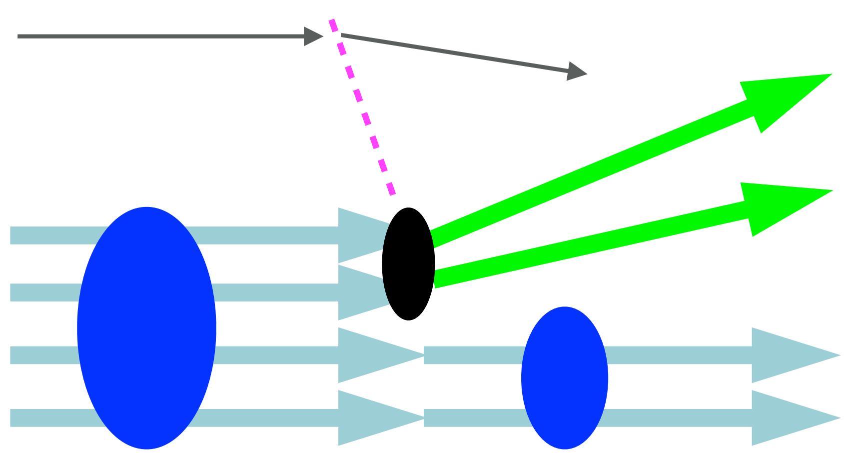
Alessandro Lovato, 2019

Desired/ Required: information on Ar and exclusive channels

Ground-state: doable with some (variational) approximations
Propagation: 12C GFMC calculations to ~ 0.1 MeV
Each particle propagates ~ 3 fm
Sign problem much worse in Ar than Carbon
Any fermion interchange in the system
contributes to the noise

Exclusive channels? Pion- Delta Production? …

How much information can we get from very short (real) times?



Short Time Approximation: Towards real-time dynamics

Saori Pastore, et al, 2019

$$R^O(q,\omega) \;=\; \frac{\int d\Omega_q}{4\pi}\sum_f \langle\Psi_0|\mathcal{O}^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|\mathcal{O}(\mathbf{q})|\Psi_0\rangle\delta(E_f - E_0 - \omega),$$

$$R^O(q,\omega) \;=\; \frac{\int d\Omega_q}{4\pi}\int \frac{dt}{2\pi}\exp[i\omega t]\langle\Psi_0|\mathcal{O}^\dagger(\mathbf{q},t')\exp[-iHt]\mathcal{O}(\mathbf{q},t=0)\Psi_0\rangle,$$

At short time evolution can be described as a product of NN propagators

$$\langle\mathbf{R}',\sigma',\tau'|\exp[-iHt]|\mathbf{R},\sigma,\tau\rangle \approx \langle\mathbf{R}',\sigma',\tau'|\prod_i \exp[-iH_i^0 t]\frac{\mathcal{S}\prod_{i<j}\exp[-iH_{ij}t]}{\prod_{i<j}\exp[-iH_{ij}^0 t]}|\mathbf{R},\sigma,\tau\rangle$$

Evaluate as a sum of matrix elements of NN states embedded in the Nucleus

Incoherent sum of single nucleon currents

$$\sum_{q,Q,J,L,S,T} \langle\Psi_0|\,\mathbf{j_i}^\dagger\,|\psi_{NN}(q,Q)\rangle\,\langle\psi_{NN}(q,Q)|\,\mathbf{j_i}\,|\Psi_0\rangle\,\delta(E_f - E_i - \omega)$$

Interference of 1- and 2-nucleon currents

$$\sum_{q,Q,J,L,S,T} \langle\Psi_0|\,\mathbf{j_{ij}}^\dagger\,|\psi_{NN}(q,Q)\rangle\,\langle\psi_{NN}(q,Q)|\,\mathbf{j_i}\,|\Psi_0\rangle\,\delta(E_f - E_i - \omega)$$

Diagonal 2-nucleon currents

$$\sum_{q,Q,J,L,S,T} \langle\Psi_0|\,\mathbf{j_{ij}}^\dagger\,|\psi_{NN}(q,Q)\rangle\,\langle\psi_{NN}(q,Q)|\,\mathbf{j_{ij}}\,|\Psi_0\rangle\,\delta(E_f - E_i - \omega)$$

Short Time Approximation: Towards real-time dynamics

Saori Pastore, et al, 2019

$$R^O(q,\omega) \;=\; \frac{\int d\Omega_q}{4\pi} \sum_f \langle \Psi_0 | \mathcal{O}^\dagger(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | \mathcal{O}(\mathbf{q}) | \Psi_0 \rangle \delta(E_f - E_0 - \omega),$$

$$R^O(q,\omega) \;=\; \frac{\int d\Omega_q}{4\pi} \int \frac{dt}{2\pi} \exp[i\omega t] \langle \Psi_0 | \mathcal{O}^\dagger(\mathbf{q}, t') \exp[-iHt] \mathcal{O}(\mathbf{q}, t=0) \Psi_0 \rangle,$$

At short time evolution can be described as a product of NN propagators

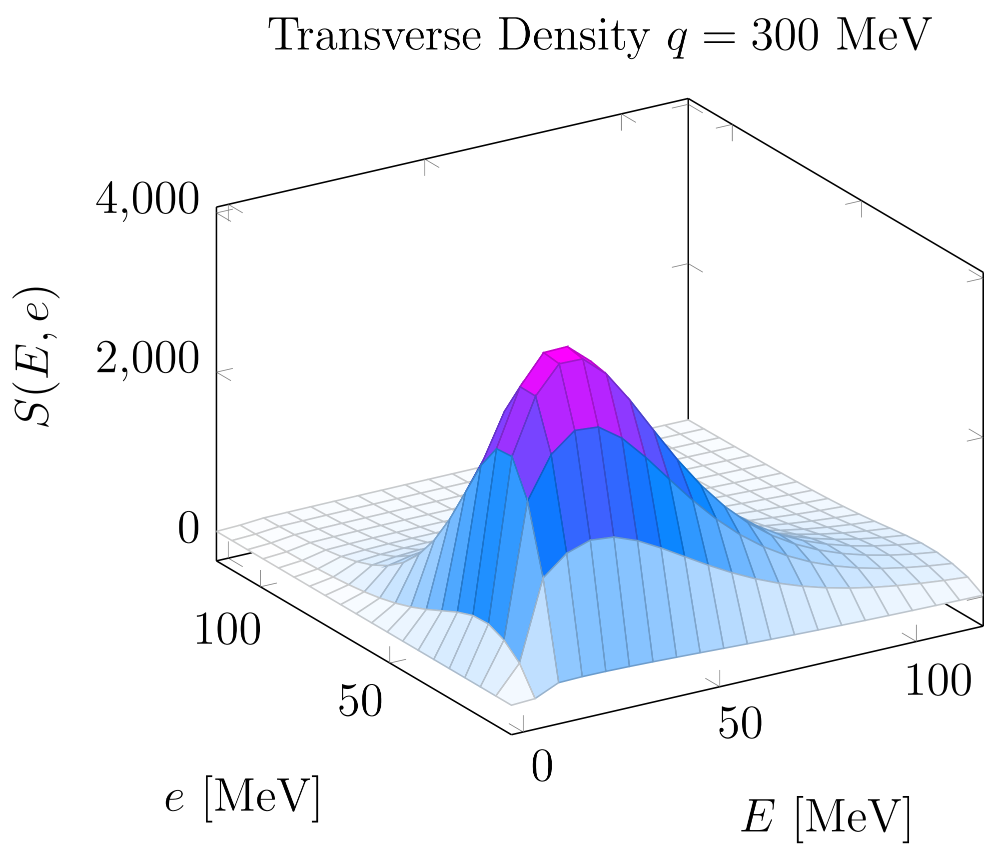
$$\langle \mathbf{R}', \sigma', \tau' | \exp[-iHt] | \mathbf{R}, \sigma, \tau \rangle \approx \langle \mathbf{R}', \sigma', \tau' | \prod_i \exp[-iH_i^0 t] \frac{\mathcal{S} \prod_{i<j} \exp[-iH_{ij} t]}{\prod_{i<j} \exp[-iH_{ij}^0 t]} | \mathbf{R}, \sigma, \tau \rangle$$

Evaluate as a sum of matrix elements of NN states embedded in the nucleus

A set of two-nucleon off-diagonal density matrix elements:

Calculate for each operator and each q
Incorporates: Exact sum rule
Full Pauli Principle (A-nucleon ME)
Information on the 2-nucleon quantum state right after the vertex
- couple with semi-classical event generators

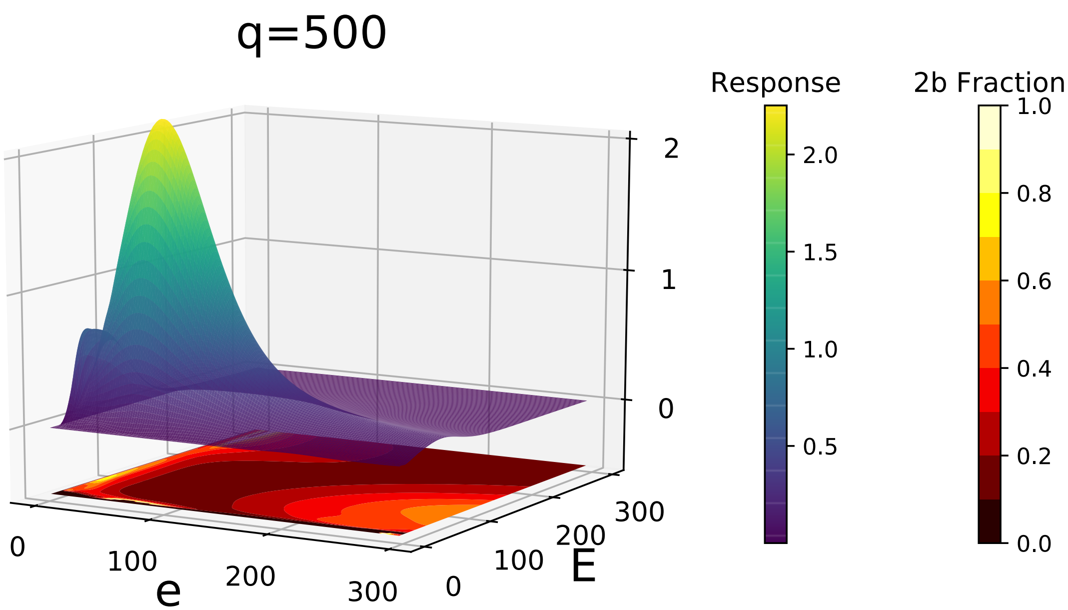
Response Densities



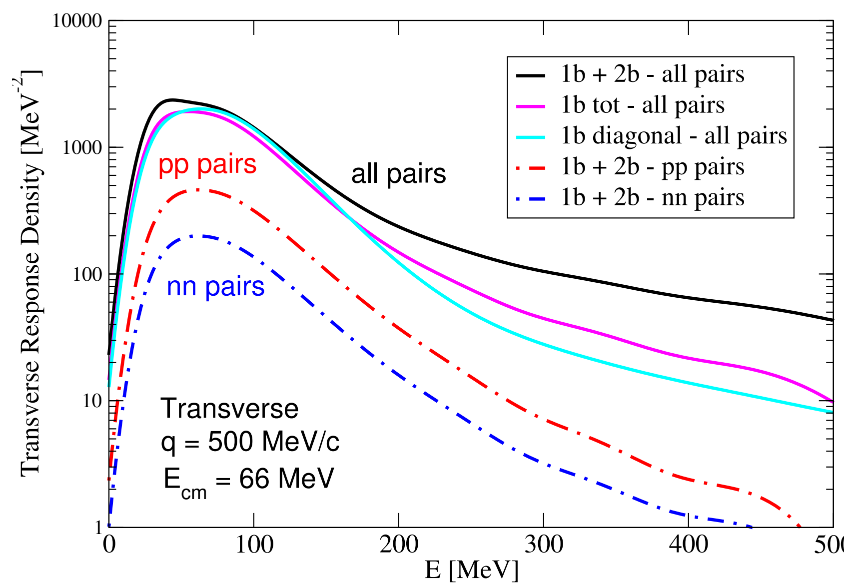
- Integral over surfaces w/ constant e+E gives full response

Response Densities

Fraction of Transverse response that include a 2N current

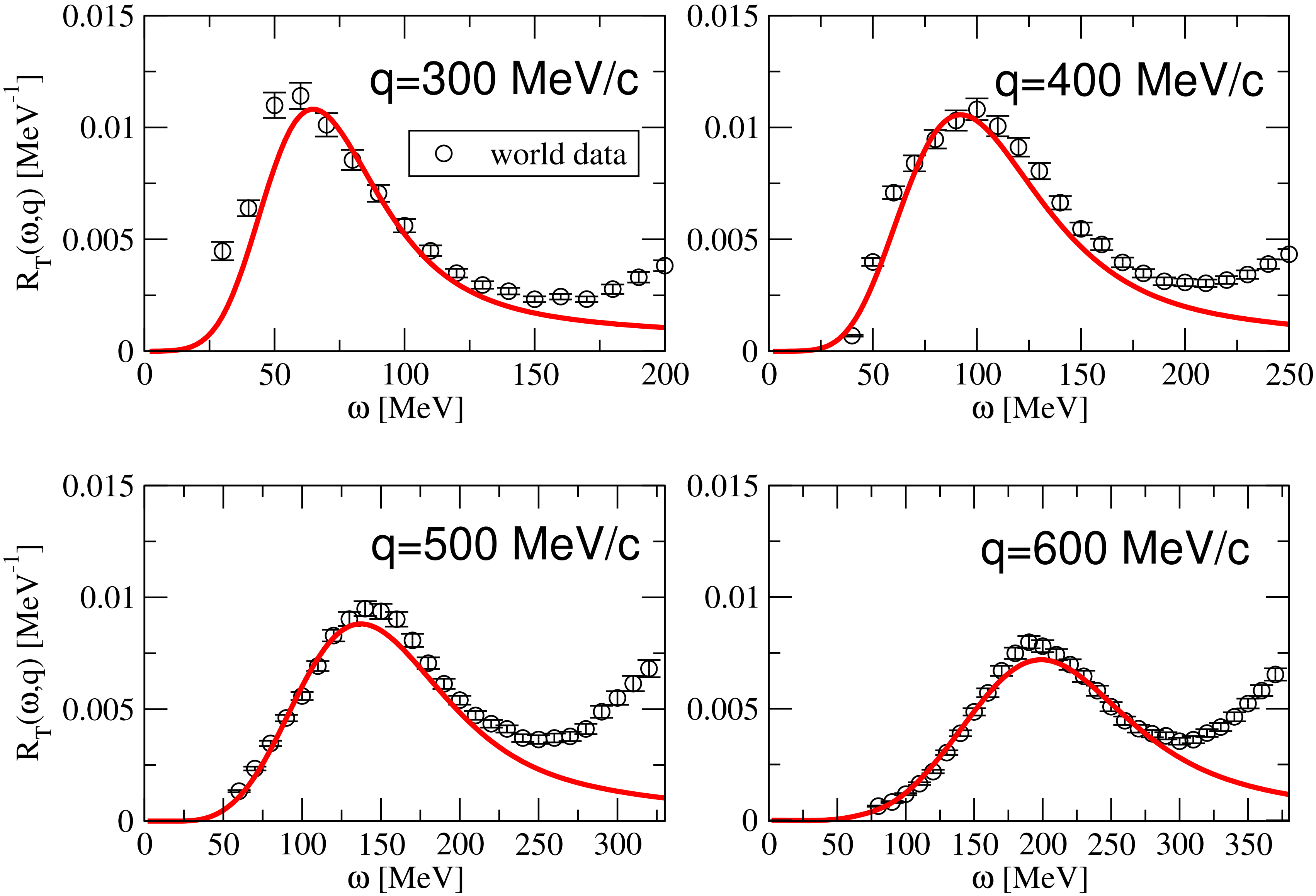


Large impact of 2-body currents at high relative energy
np vs. pp, etc.



np vs pp in back-to-back kinematics

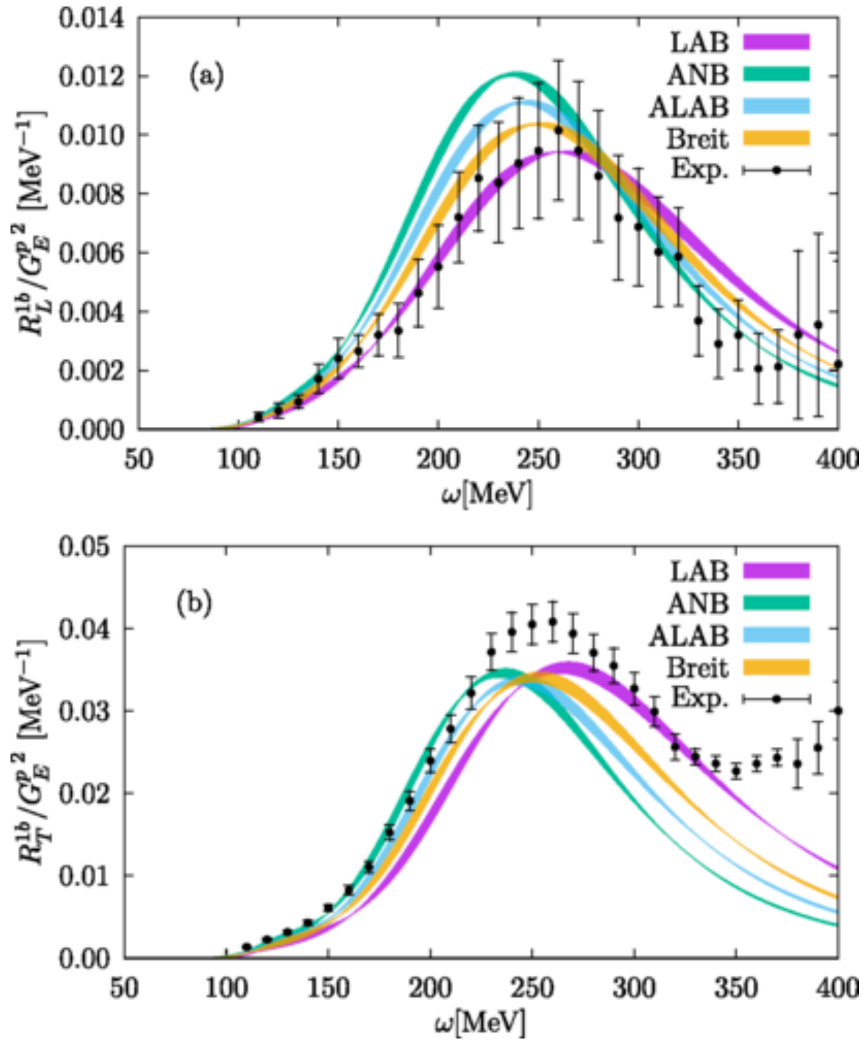
Comparison to Data



Future directions

- Couple to Generators
- A=12, 40
- Relativistic Dynamics
- Pion Production and Deltas
  from two hadrons

  requires model of NN inelastic processes
  can we match to lattice calculations?
- Quantum Computing: even a short coherence time may be valuable.



Noemi Rocco, et al (2018)