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# Short-depth circuits for efficient expectation value estimation

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# Premise

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  - Can be seen as a two level system $^2$H

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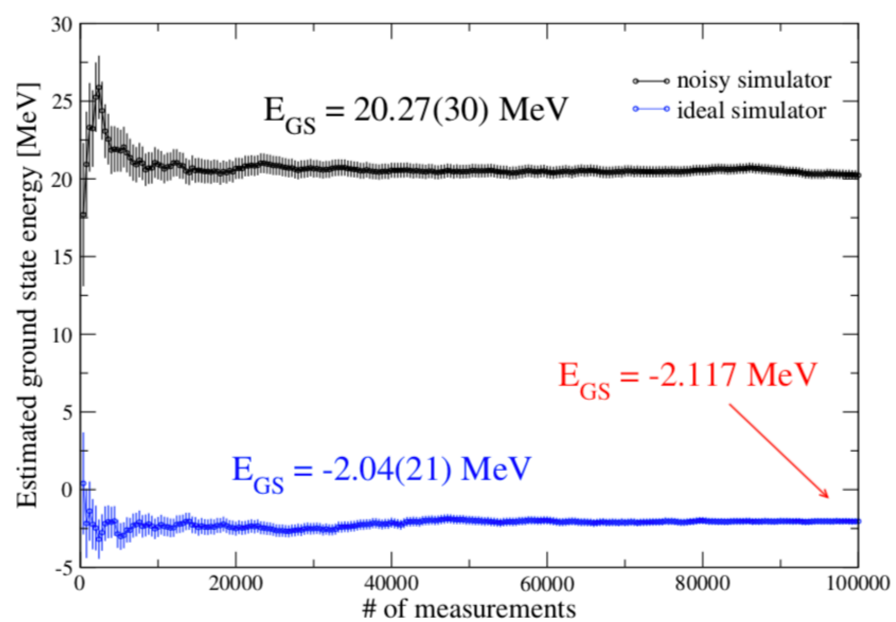

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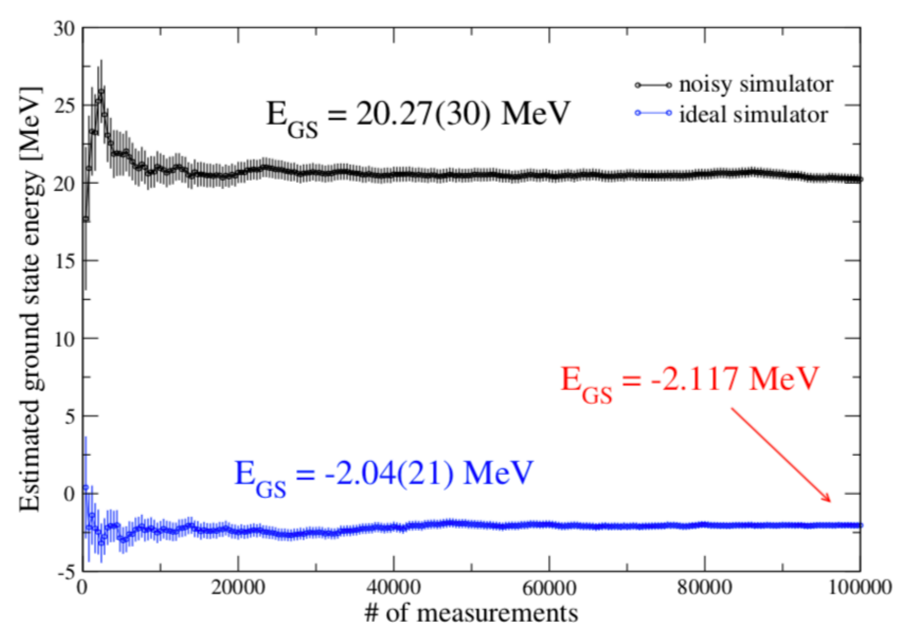
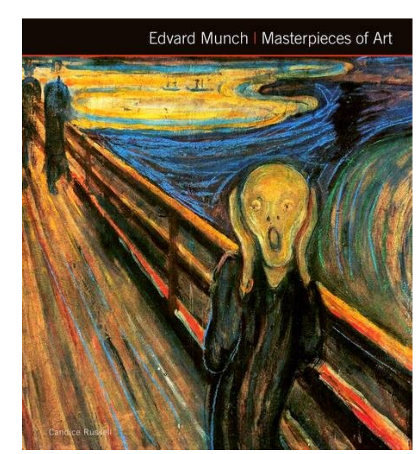
$1\% \quad accuracy$



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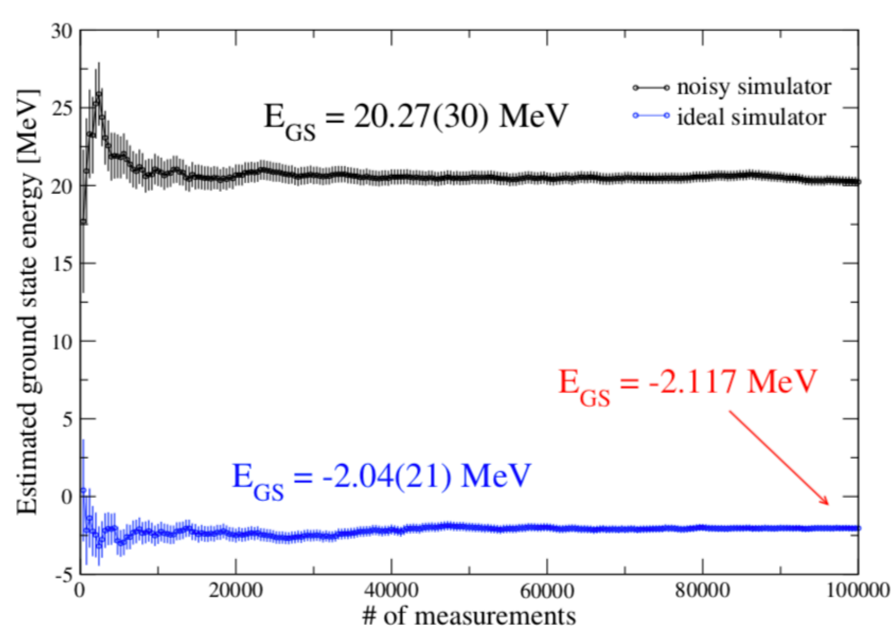
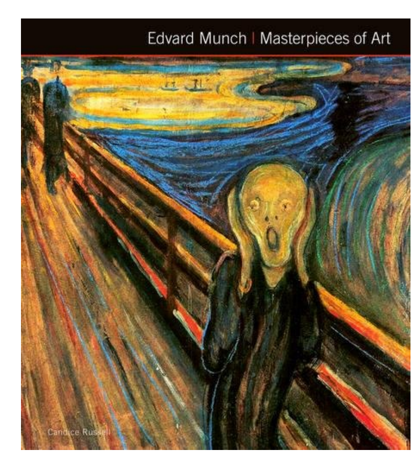
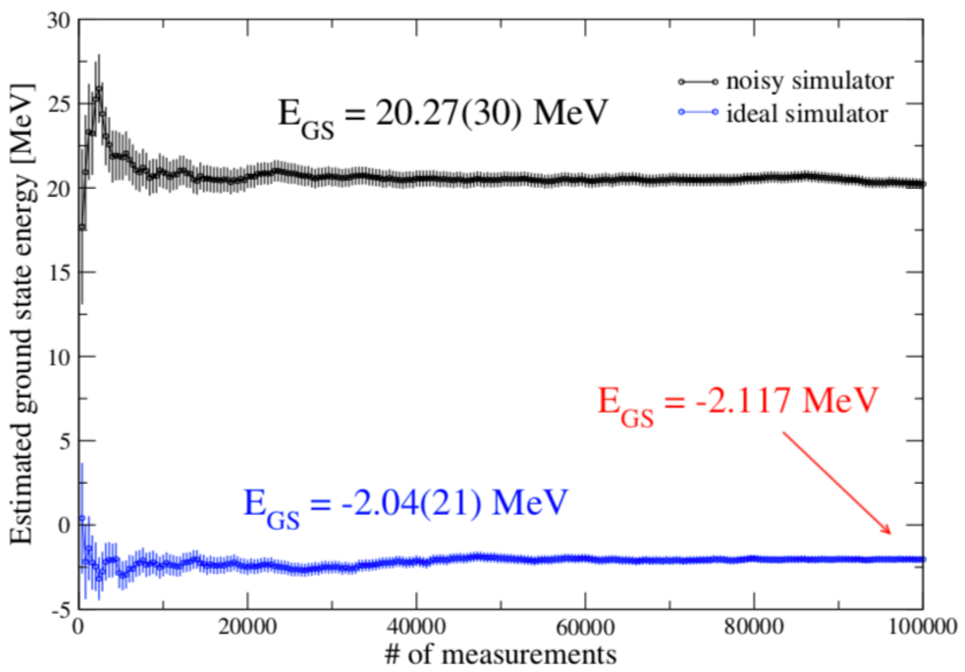


- Number of measurements large $\simeq 10^7$

$$\langle H \rangle = \alpha_0 + \alpha_x \langle X \rangle + \alpha_z \langle Z \rangle$$

$$\downarrow \quad N_X \qquad \downarrow \quad N_Z$$




$$\mathrm{var}[\langle H \rangle] \leq \frac{|\alpha_x|^2}{N_X} + \frac{|\alpha_z|^2}{N_Z}$$

$$N_X = N_Z = N \qquad\qquad N \leq (|\alpha_x|^2 + |\alpha_y|^2)/\epsilon^2$$

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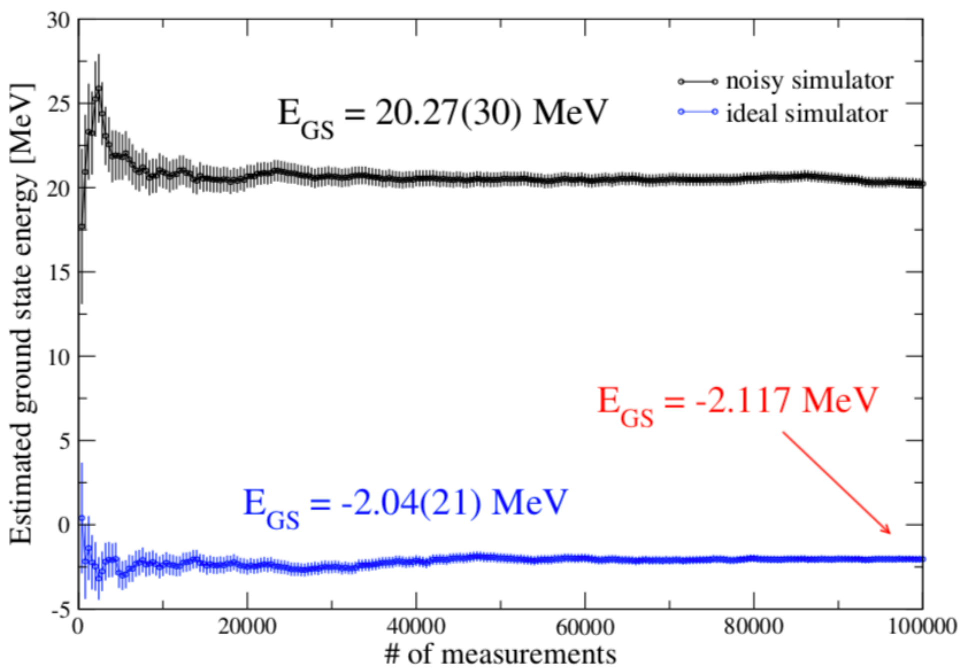


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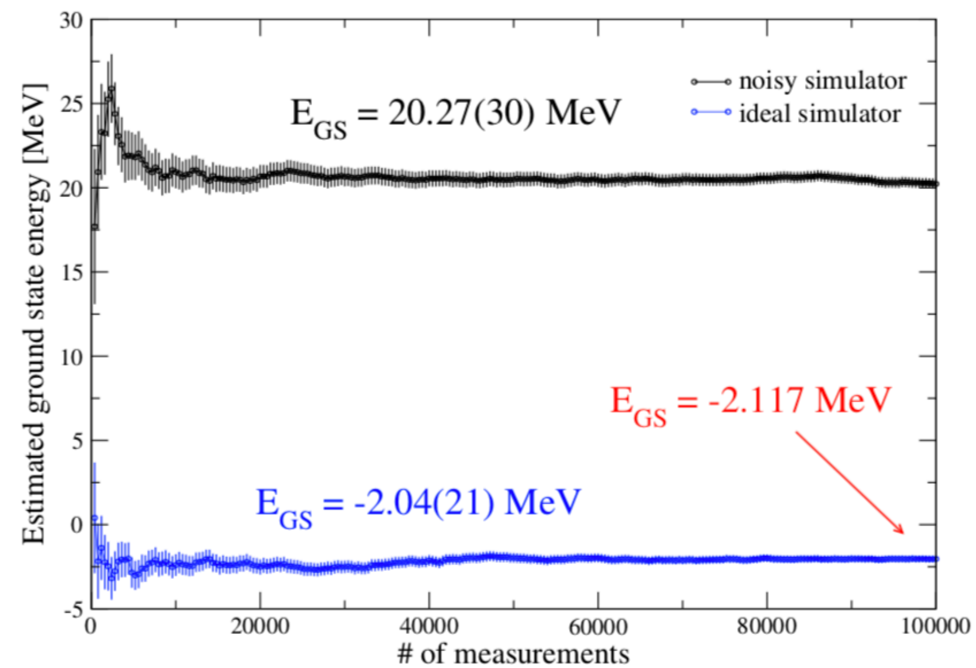


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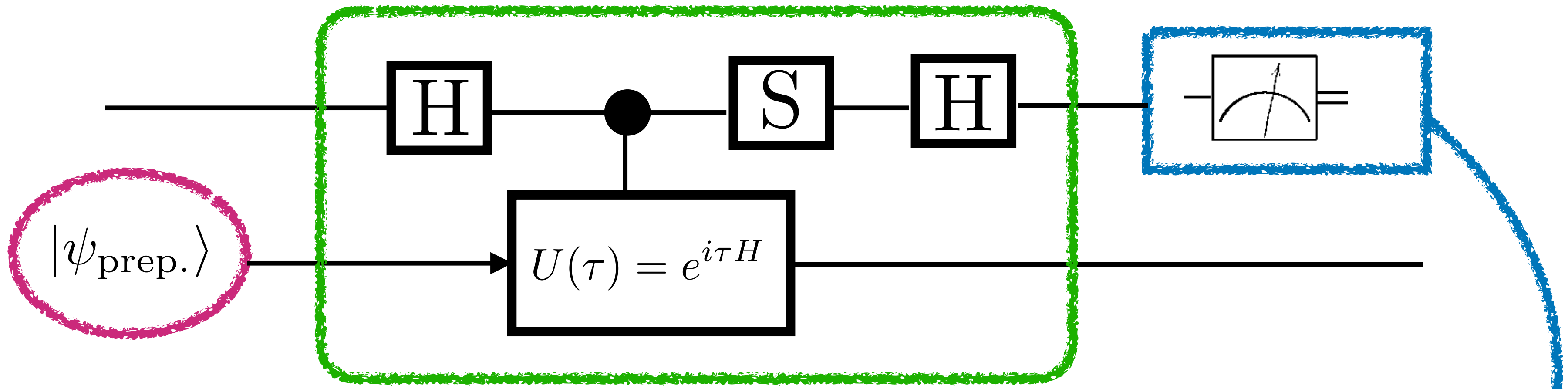
- Stumbled upon a specific case of a general problem

$$O = \alpha_0 I_d + \alpha_x X + \alpha_y Y + \alpha_z Z + \alpha_{xy} XY + \alpha_{XZ} XZ + \cdots$$

$$N \leq \left( |\alpha_x|^2 + |\alpha_y|^2 + |\alpha_{xy}|^2 + \cdots \right) / \epsilon^2$$

# Circuit

- A way to circumvent this is the use of an ***ancillary*** qubit




$$\langle Z\rangle(\tau) = -\langle\psi_{\text{prep}}|\sin(\tau H)|\psi_{\text{prep}}\rangle = -\tau\langle\psi_{\text{prep}}|H|\psi_{\text{prep}}\rangle + \frac{\tau^3}{6}\langle\psi_{\text{prep}}|H^3|\psi_{\text{prep}}\rangle + \mathcal{O}(\tau^5)$$

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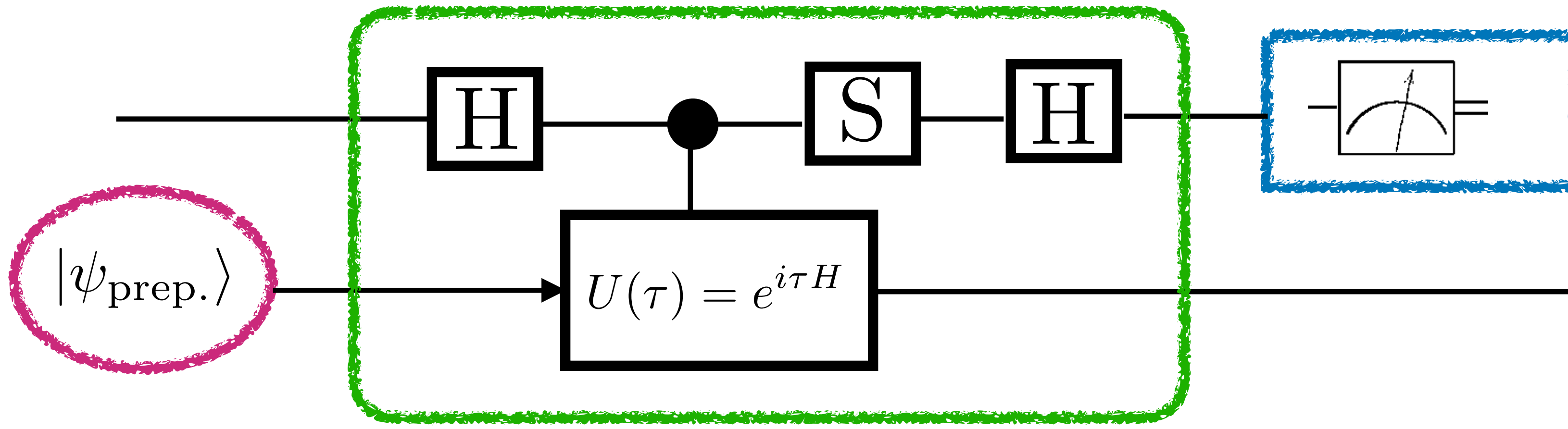


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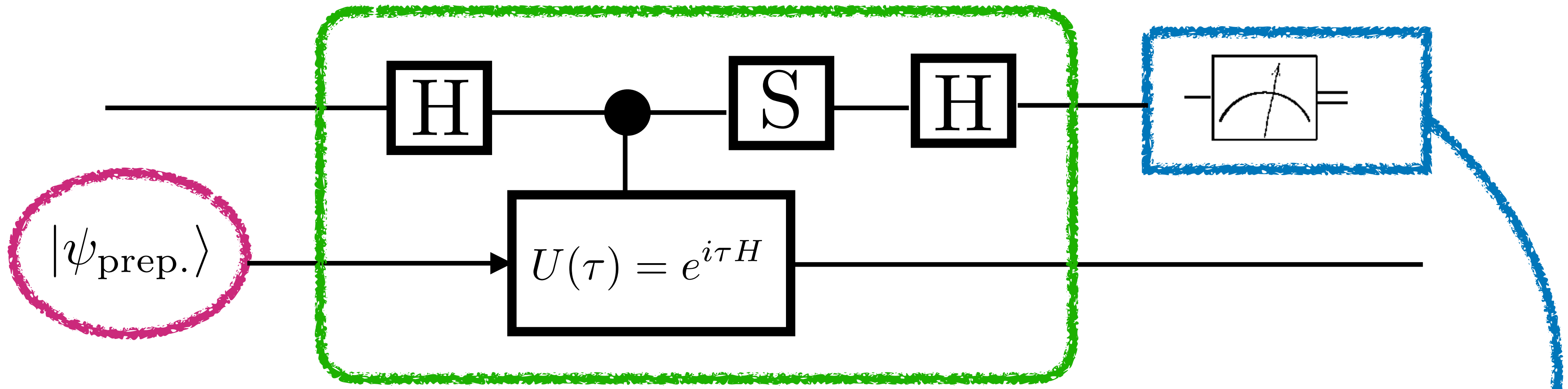


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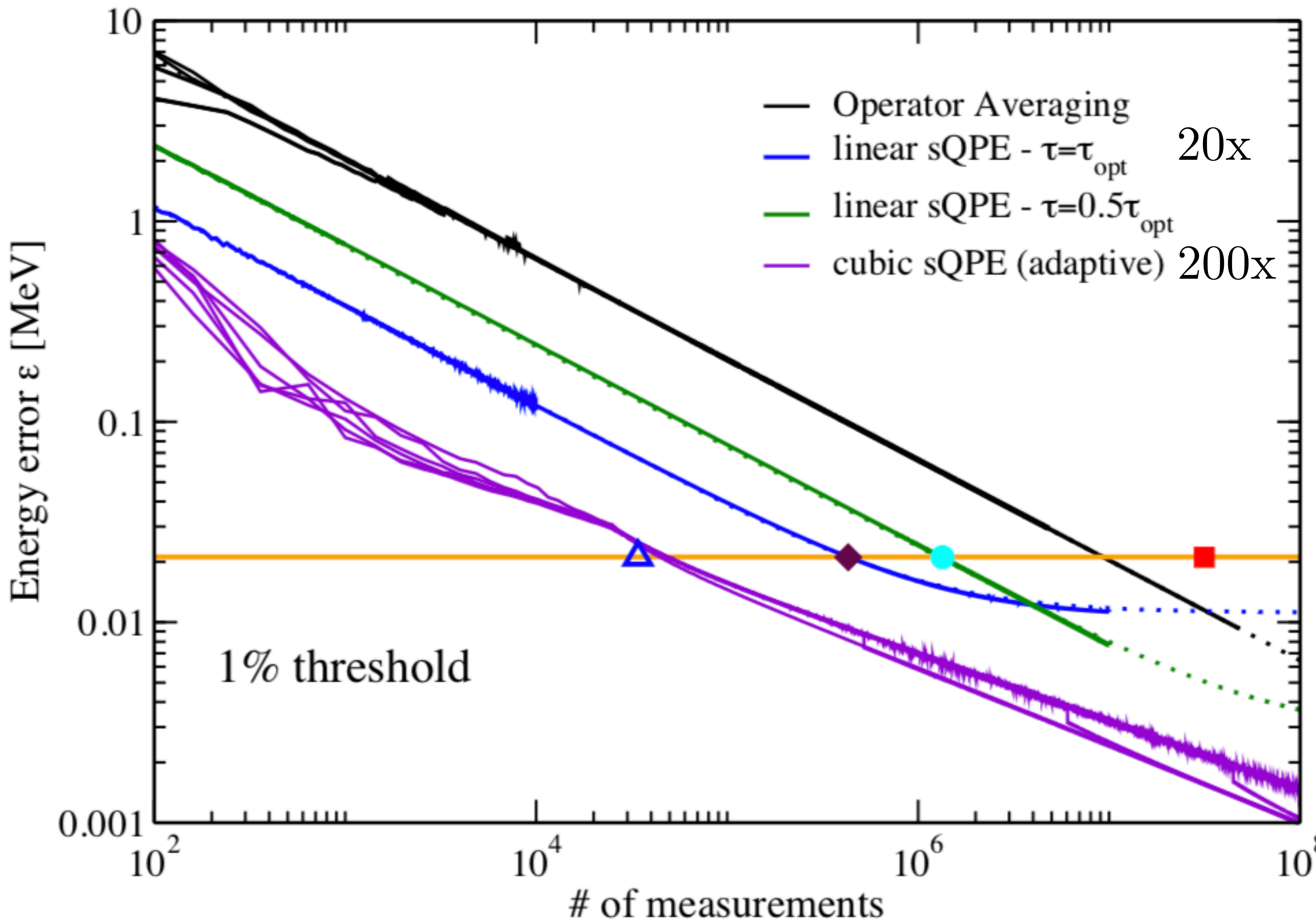
- Find optimal time step minimizing the total number of measurements for fixed precision
  - Using proper error metric (MSE) is possible to find a relation

$$N_{tot} = f(\tau, \epsilon) \longrightarrow \text{minimizing} \longrightarrow \tau_{opt}$$

- Linear and cubic implementation reduced significantly the number of measurements

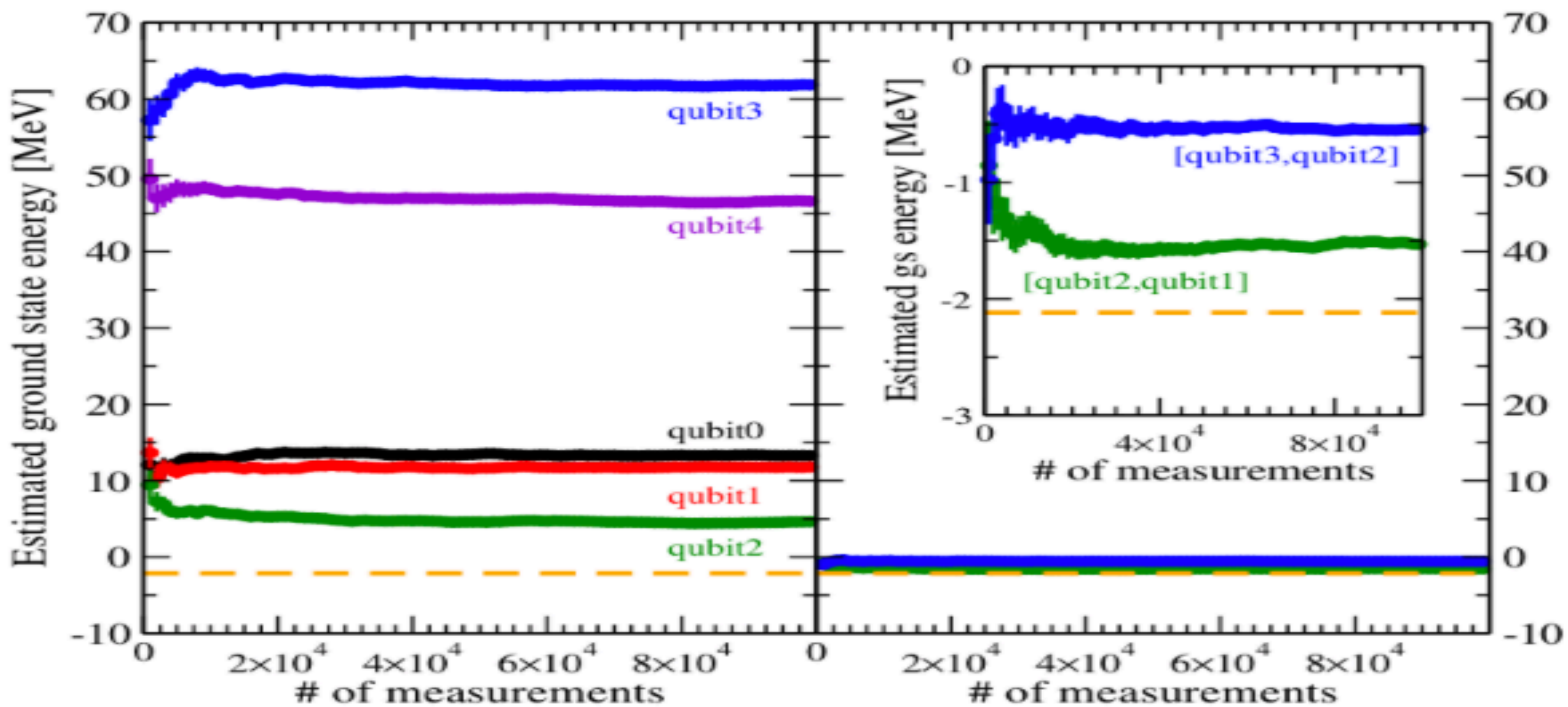
Issues: longer circuit (controlled t ev) ( increases error with gates)

# Results (no noise)



# Results (noise)

- Just blindingly applying OAM and the linear algorithm




Ibmqx4

- Major source of error is readout error, how do we correct for it?

$$O = \alpha_0 + \sum_{i=1}^{3} \beta_i P_i \quad \vec{P} = (X, Y, Z)$$

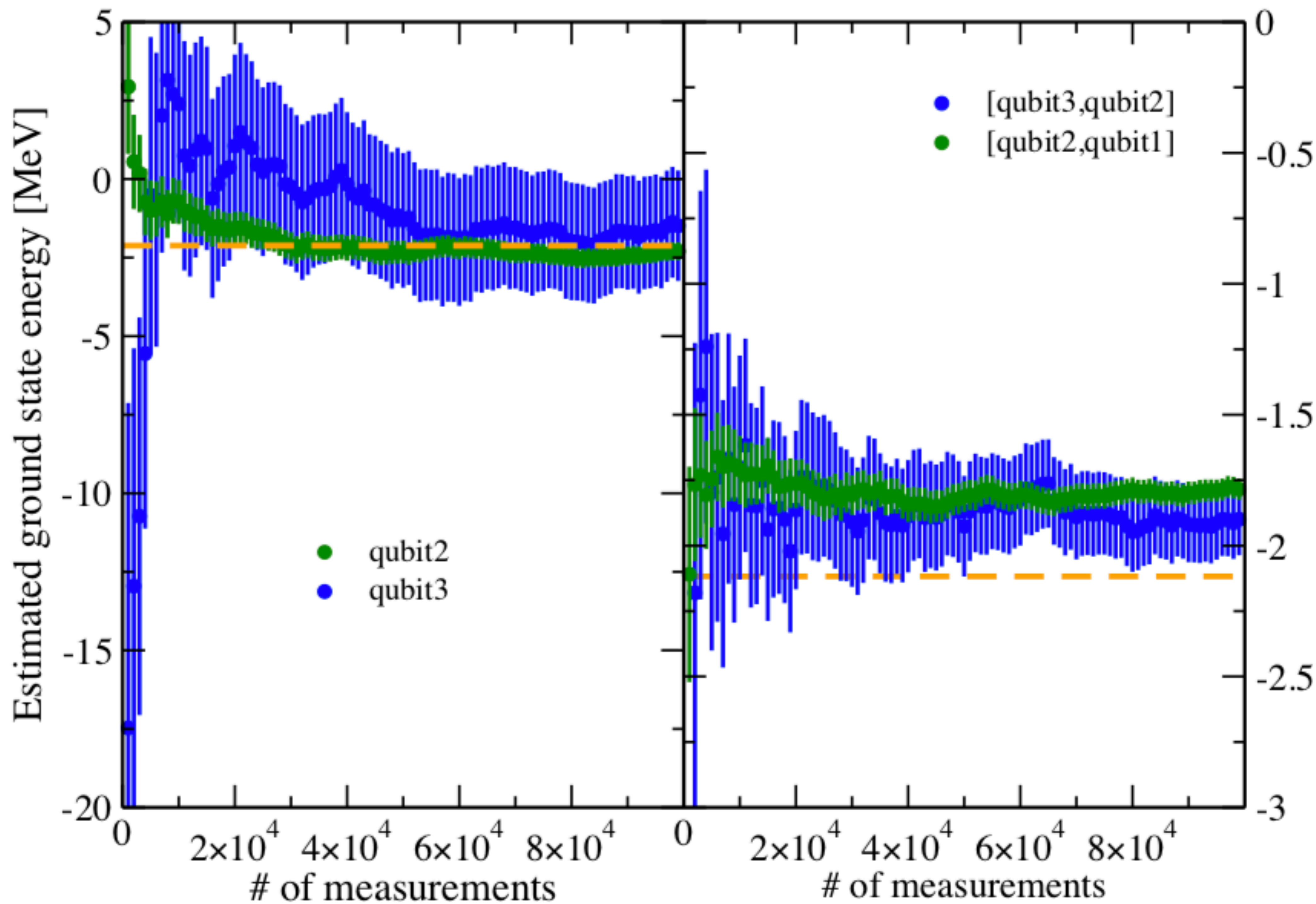
$$\langle \widehat{O} \rangle = \alpha_0 + \sum_{i=1}^{3} \frac{\beta_i}{1 - 2\widehat{p}} \langle \widetilde{P}_i \rangle$$

$$\langle \widetilde{P_\sigma} \rangle = (1 - 2p) \langle P_\sigma \rangle$$ Noisy expectation values

Finite sample estimator of readout error rate

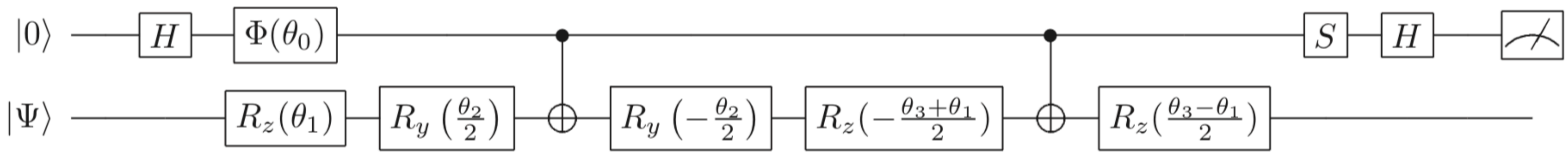
*Kandala et al. Nature (2017)*

# Results (noise)



# THANK YOU!

# Exact time evolution




Last rotation can be avoided

# Scalings

$\epsilon$ Target accuracy

| | sQPE | QPE |
|---|---|---|
| • Number of measurements | $\mathcal{O}(1/\epsilon^{2+\nu})\,, \qquad 0 < \nu \leq 1$ | $\mathcal{O}(1/\epsilon)$ |
| • Circuit depth | $\mathcal{O}(\epsilon^{\mu})\,, \qquad \mu \geq 0$ | $\mathcal{O}(1/\epsilon)$ |

# Circuit depth

$\mathcal{O}(\epsilon^{\mu})\,,\qquad \mu \geq 0 \qquad \|\widetilde{U_\tau} - U_\tau\| \leq \delta_\tau \qquad Childs\ et.\,al\,(2018)$

First order Trotter

$$r_1 = \mathcal{O}\left(\alpha \frac{1}{\sqrt{\epsilon}}\right)$$

2j-th order Trotter

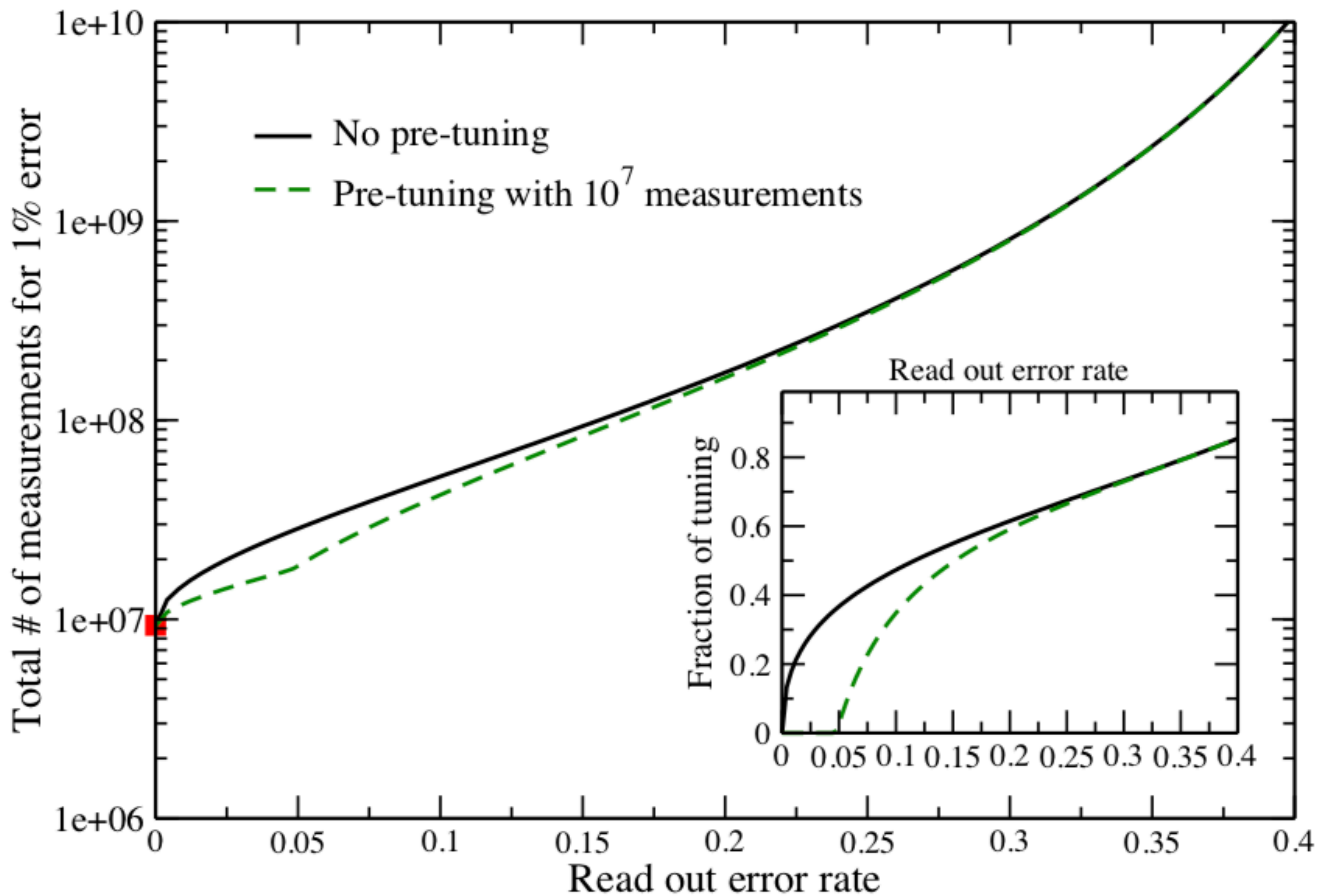
$$\|S_{2j}(\tau, r) - U_\tau\| \leq \frac{\left(2\tau 5^{j-1}\|\overline{O}\|_1\right)^{2j+1}}{3r^{2j}} e^{2\frac{\tau}{r}5^{j-1}\|\overline{O}\|_1}$$

Better scaling

$$\mathcal{O}\left(\beta \epsilon_r^{\frac{j-K}{4jK}}\right) \qquad \epsilon_r = \epsilon/|\langle H \rangle|$$

When $j = K$ Gate count constant

When $j = K + 1$ Gate count decreases as a function of precision!



No pre-tuning
Pre-tuning with 10^7 measurements
Total # of measurements for 1% error
Read out error rate
1e+06
1e+07
1e+08
1e+09
1e+10
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Fraction of tuning
0
0.2
0.4
0.6
0.8

# Cubic algorithm

Optimal design

$$P_{a/b} = \frac{1 - \langle \Psi | sin(\tau_{a/b} O) | \Psi \rangle}{2}$$

Choose two time-steps:

$$= \frac{1 - \tau_{a/b} \langle \Psi | O | \Psi \rangle + \frac{\tau_{a/b}^3}{6} \langle \Psi | O^3 | \Psi \rangle}{2} + \mathcal{O}\left(\tau_{a/b}^5\right)$$

Get maximum likelihood estimates for means $\mu_{mle}$ $\eta_{mle}$

Use inverse of Fisher information matrix to get the corresponding variances

$$Var\left[\mu_{mle}\right] = \frac{4}{M} \frac{\tau_a^6 P_b \left(1 - P_b\right) + \tau_b^6 P_a \left(1 - P_a\right)}{\tau_a^2 \tau_b^2 \left(\tau_a^2 - \tau_b^2\right)^2}$$

$$Var\left[\eta_{mle}\right] = \frac{144}{M} \frac{\tau_a^2 P_b \left(1 - P_b\right) + \tau_b^2 P_a \left(1 - P_a\right)}{\tau_a^2 \tau_b^2 \left(\tau_a^2 - \tau_b^2\right)^2}$$

**Bayesian estimators for P..**

# Cubic algorithm

Define MSE at definite time steps $\epsilon_M(\mu|\tau_a, \tau_b) = Var[\mu_{mle}] + B^2_\mu(\tau_a, \tau_b)$

Choose next two time steps minimizing quantity above

Start with random time-steps

Choose next ones to minimize the upper bound $Var\left[\mu_{mle}\right] \leq \frac{1}{M}\frac{\tau_a^6 + \tau_b^6}{\tau_a^2\tau_b^2\left(\tau_a^2 - \tau_b^2\right)^2}$

Choose next time-steps such that $\epsilon^i_M(\mu|\tau_a, \tau_b) = Var[\mu_{mle}] + (i+1)B^2_\mu(\tau_a, \tau_b)$

$Var[\mu_{mle}]$ → Minimum

$\mathrm{Var}[\mu_{mle}]$ Cannot be calculated exactly

$$\widetilde{Var}[\mu_{mle}] = \frac{4}{M}\frac{\tau_a^6\widetilde{P}_b\left(1-\widetilde{P}_b\right) + \tau_b^6\widetilde{P}_a\left(1-\widetilde{P}_a\right)}{\tau_a^2\tau_b^2\left(\tau_a^2-\tau_b^2\right)^2}$$

# Cubic algorithm

Upper bound bias $$|B_\mu(\tau_a,\tau_b)| \leq \frac{|\langle\Psi|O^5|\Psi\rangle|}{120}\tau_a^2\tau_b^2\frac{\tau_a^2+\tau_b^2}{|\tau_a^2-\tau_b^2|}$$

Quantity minimized $$\Delta_i = \widetilde{Var}[\mu_{mle}] + (i+1)B_\mu^{u\,2}(\tau_a,\tau_b)$$

Works pretty well for the deuteron

How does it go in general for randomly generated matrices?