A magnetic analysis of Casimir (Polder) forces

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“Wir konstruieren und konstruieren
und doch ist Intuition immer noch
eine gute Sache. Man kann ohne
sie Beträchtliches, aber nicht alles.
Man kann lange tun, mancherlei
und vielerlei tun, Wesentliches tun,
aber nicht alles.”

Paul Klee
“Exakte Versuche im Bereich Kunst” 1928

“…intuition is still a good thing.
…you can work long, do many things,
essential things,
but you can not do everything without it.”

Outline

history: vdWaals (London) and charge fluctuations

Fluctuations of atom and fields

electric vs magnetic

Johnson noise and magnetic fields near metals

direct measurements

Magnetic Casimir–Polder interaction

temperature dependence

metal vs superconductor

Remarks on repulsion

van der Waals → Casimir & Polder

van der Waals (1873) & London (1930)

fluctuating dipole A ⟶ polarizes dipole B

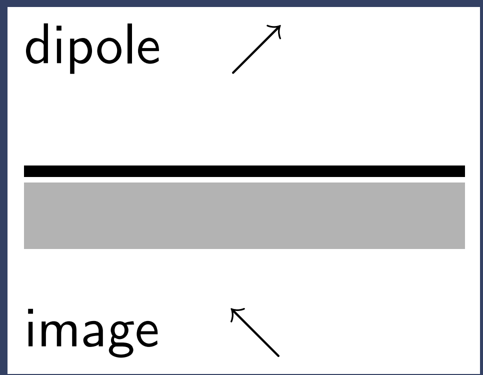
field on dipole A ⟵

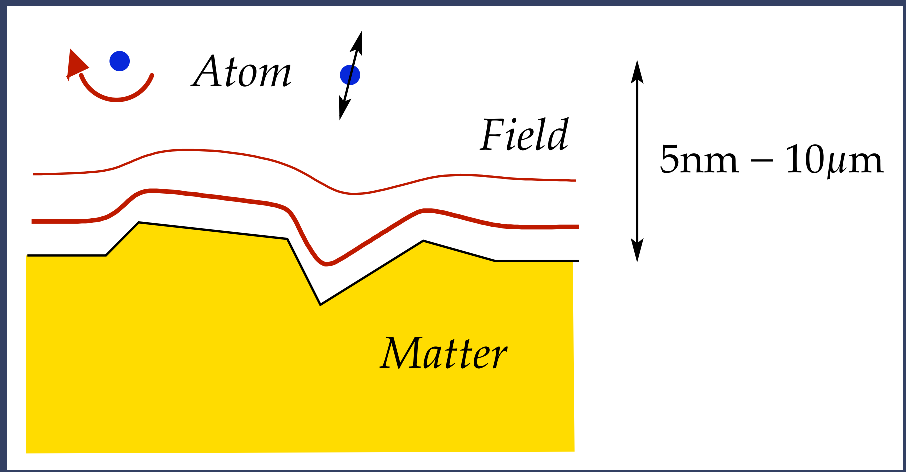
Casimir & Polder (1948)

retarded response

⟶ large-distance quenching

"intrinsic" field fluctuations





— dipole fluctuations & field response

— field fluctuations & dipole response

Dalibard & Cohen-Tannoudji 1982, 1984

Meschede, Jhe, Haroche 1990

Dipole fluctuations

dipole $\frac{1}{2}\langle\{\mu(t)\mu(t') + \mu(t')\mu(t)\}\rangle_a \quad \mapsto \quad$ spectrum $S_\mu^{(a)}(\omega)$

$$S_\mu^{(a)}(\omega) = \coth\left(\frac{\hbar\omega}{2T}\right)\mathrm{sgn}(\omega)\sum_b |\langle b|\mu|a\rangle|^2 \pi\left[\delta(\omega - \omega_{ba}) + \delta(\omega + \omega_{ba})\right]$$

electric dipoles

$\langle b|d|a\rangle \sim e a_{\mathrm{Bohr}}$

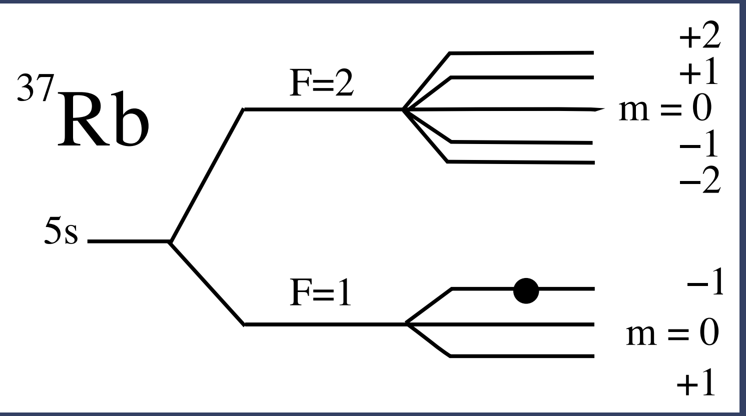
$\hbar\omega_{ba} \sim \dfrac{e^2}{\varepsilon_0 a_{\mathrm{Bohr}}} \sim$ few eV $\gg T$

magnetic dipoles

$\langle b|\mu|a\rangle \sim \mu_{\mathrm{Bohr}} \sim \alpha_{\mathrm{fs}} e a_{\mathrm{Bohr}}$

$\omega_{ba} \sim 0 \ldots 10^9\,\mathrm{Hz} \ll T/\hbar$ or $\gg T/\hbar$

Zeeman … hyperfine splitting



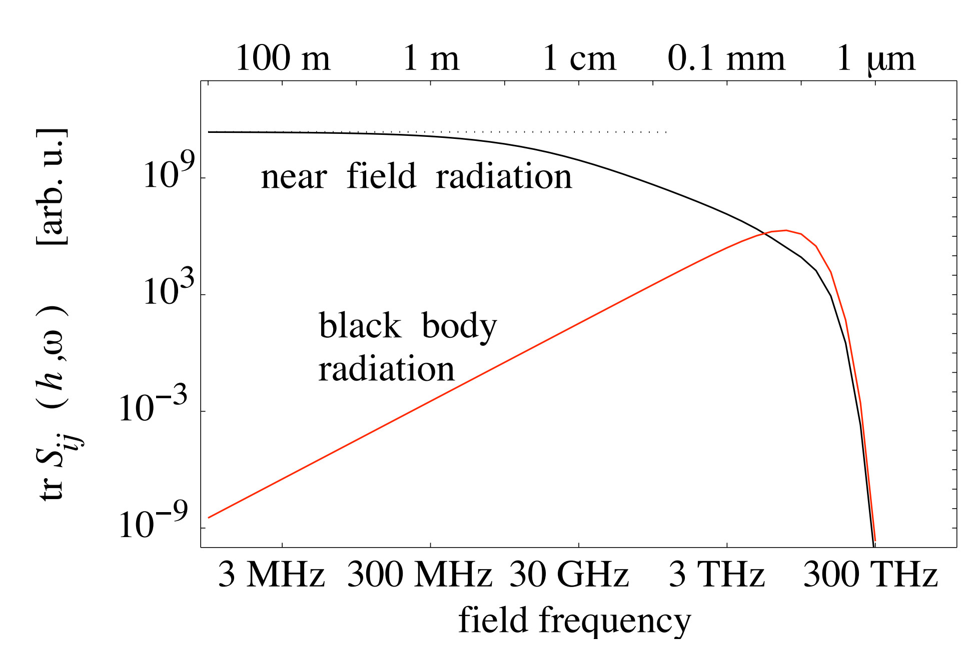
Field fluctuations

field $\frac{1}{2}\langle\{B(\mathrm{r},t)B(\mathrm{r}',t') + B(\mathrm{r}',t')B(\mathrm{r},t)\}\rangle_T \quad \mapsto \quad$ spectrum $S_B^{(T)}(\mathrm{r},\mathrm{r}',\omega)$

$$S_B^{(T)}(\mathrm{r},\mathrm{r}',\omega) = \hbar \coth\left(\frac{\hbar\omega}{2T}\right) \mathrm{Im}\,\mathcal{H}(\mathrm{r},\mathrm{r}',\omega) \qquad (\mathcal{H} = \text{Green function})$$

geometry: one planar surface

magnetic fluctuation spectrum



distance $1\,\mu$m, metal surface (Cu), 300K

Field fluctuations

field $\frac{1}{2}\langle\{B(\mathrm{r},t)B(\mathrm{r}',t')+B(\mathrm{r}',t')B(\mathrm{r},t)\}\rangle_T \quad\mapsto\quad$ spectrum $S_B^{(T)}(\mathrm{r},\mathrm{r}',\omega)$

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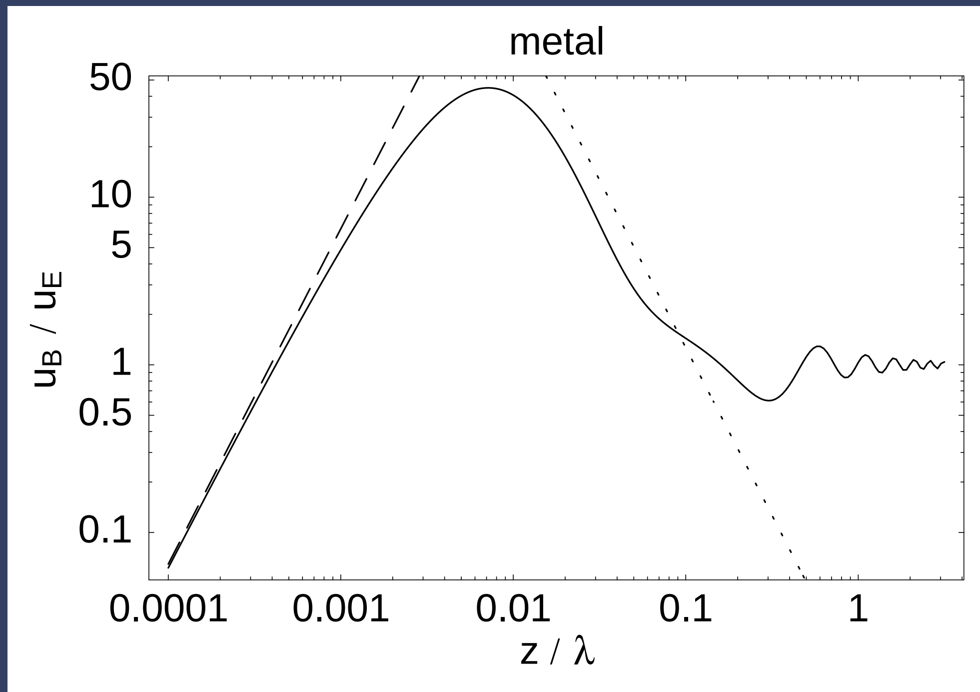
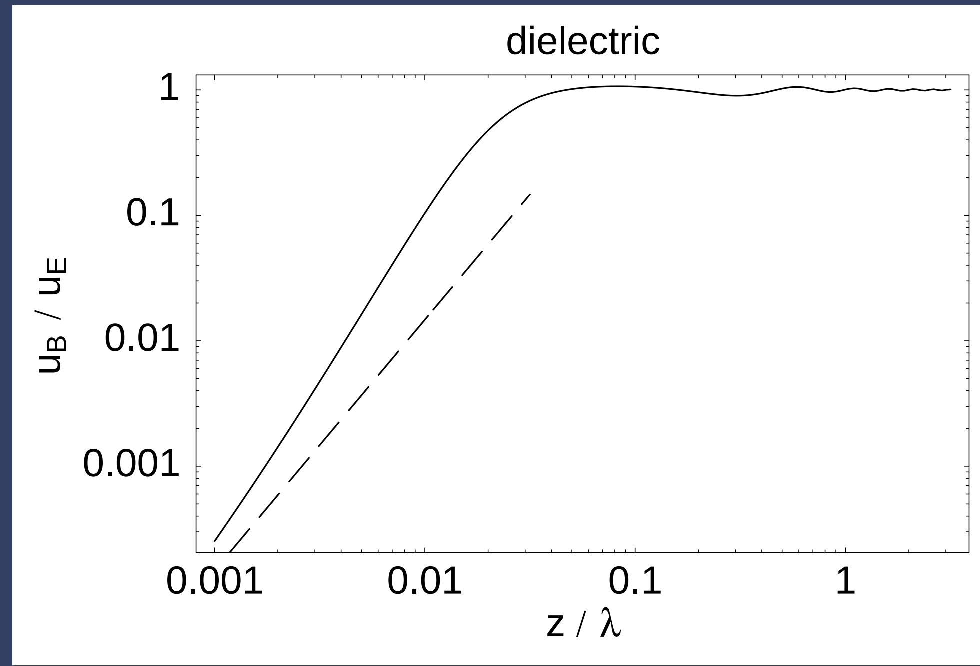
geometry: one planar surface

- Johnson noise outside a conducting surface

ratio magnetic / electric noise



metal $\varepsilon(\omega) \sim 800\,\mathrm{i} - 10$ (far IR)



dielectric $\varepsilon(\omega) \sim 2.3 + 0.1\,\mathrm{i}$

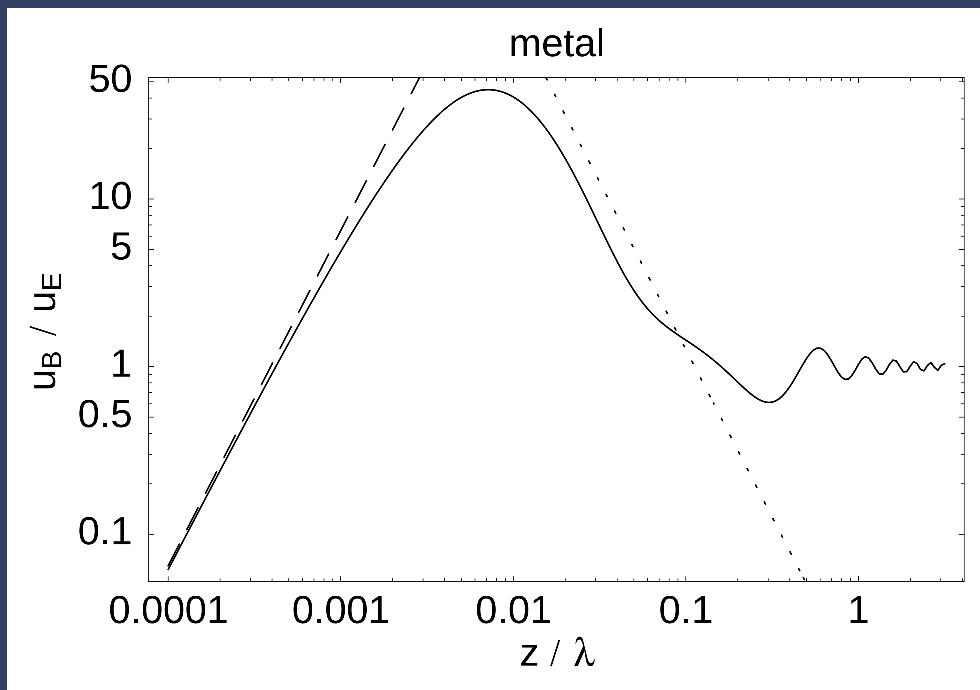
Field fluctuations

field $\frac{1}{2}\langle\{B(\mathrm{r},t)B(\mathrm{r}',t')+B(\mathrm{r}',t')B(\mathrm{r},t)\}\rangle_T \quad\mapsto\quad$ spectrum $S_B^{(T)}(\mathrm{r},\mathrm{r}',\omega)$

$$S_B^{(T)}(\mathrm{r},\mathrm{r}',\omega) = \hbar\coth\left(\frac{\hbar\omega}{2T}\right)\mathrm{Im}\,\mathcal{H}(\mathrm{r},\mathrm{r}',\omega)$$ (Green function)

geometry: one planar surface

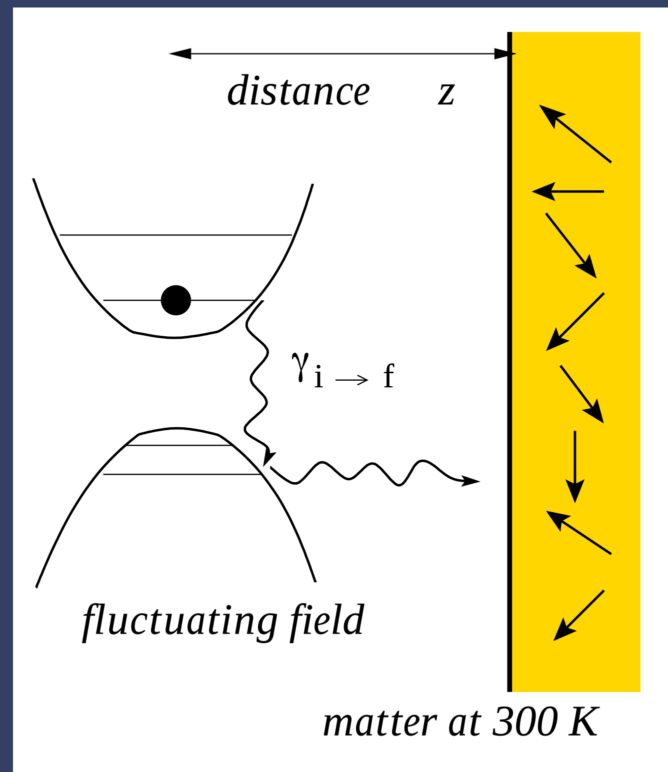
ratio magnetic / electric noise



metal $\varepsilon(\omega) \sim 800\,\mathrm{i} - 10$ (far IR)

- Johnson noise detector = spin flip

energy dumped into eddy currents



- spin *entangles* with matter

Field fluctuations from Johnson noise

Metallic layer: thickness t, conductivity σ, skin depth $\delta = (\mu_0 \omega \sigma / 2)^{-1/2}$

Varpula & Poutanen (1984), Henkel & al (1999), Sidles & al (2003)

Large skin depth, short distance

$$S_{B,ij}^{(T)}(\mathrm{r};\omega) = \frac{\mu_0 k_\mathrm{B} T}{8\pi\,\omega} s_{ij} \times \begin{cases} \dfrac{1}{z\delta^2}, & z \ll t \ll \delta \\ \dfrac{t}{z^2\delta^2}, & t \ll z \ll \delta \end{cases}$$

$s_{ij} = \mathrm{diag}(\frac{1}{2}, \frac{1}{2}, 1)$, white noise

- additive in material thickness

Small skin depth, "large" distance

$$S_{B,ij}^{(T)}(\mathrm{r};\omega) \approx \frac{\mu_0 k_\mathrm{B} T}{8\pi\,\omega} s_{ij} \times \begin{cases} \dfrac{\delta^2}{2tz^4}, & t \ll \delta \ll \sqrt{zt} \\ \dfrac{3\delta^2}{2z^4}, & \delta \ll \min(z, t) \end{cases}$$

- non-additive
- worst material: $\delta \sim \min(z, \sqrt{zt})$

Field fluctuations – experiment 1

Metallic layer: thickness t, conductivity σ, skin depth $\delta = (\mu_0 \omega \sigma / 2)^{-1/2}$

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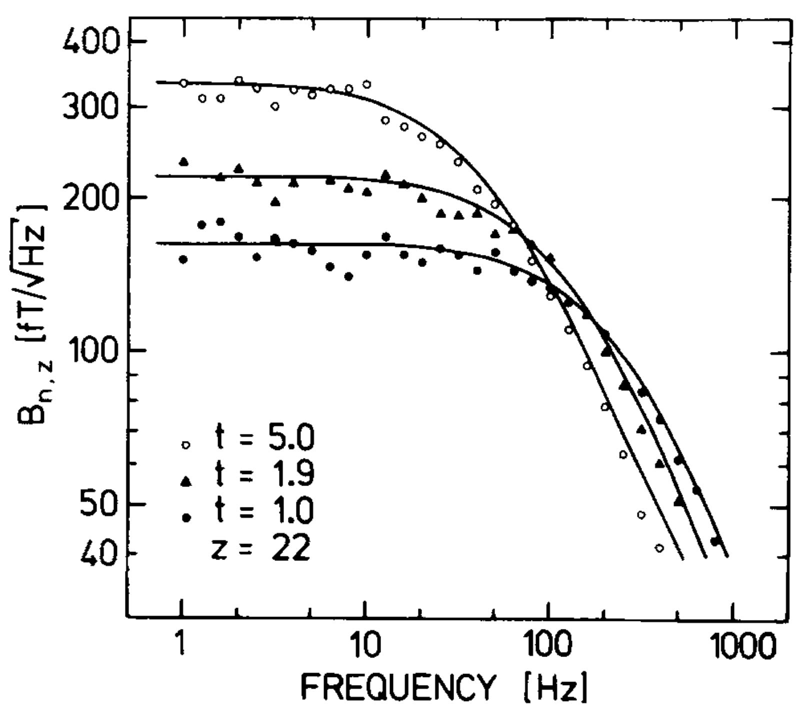


FIG. 3. Thermal magnetic noise amplitudes versus frequency measured from the copper plates at a temperature of 293 K having thicknesses of 5.0, 1.9, and 1.0 mm. The center of the gradiometer pickup coil was at a distance of 22 mm from the upper surface of the respective plate. Measured values are readings from the spectrum analyzer with magnetometer noise subtracted. The solid-line curves are calculated with Eq. (42).

Small skin depth, "large" distance

$$S^{(T)}_{B,ij}(\mathrm{r};\omega) \approx \frac{\mu_0 k_\mathrm{B} T}{8\pi\,\omega} s_{ij} \times \begin{cases} \dfrac{\delta^2}{2tz^4}, & t \ll \delta \ll \sqrt{zt} \\[2ex] \dfrac{3\delta^2}{2z^4}, & \delta \ll \min\left(z, t\right) \end{cases}$$

- non-additive
- worst material: $\delta \sim \min(z, \sqrt{zt})$

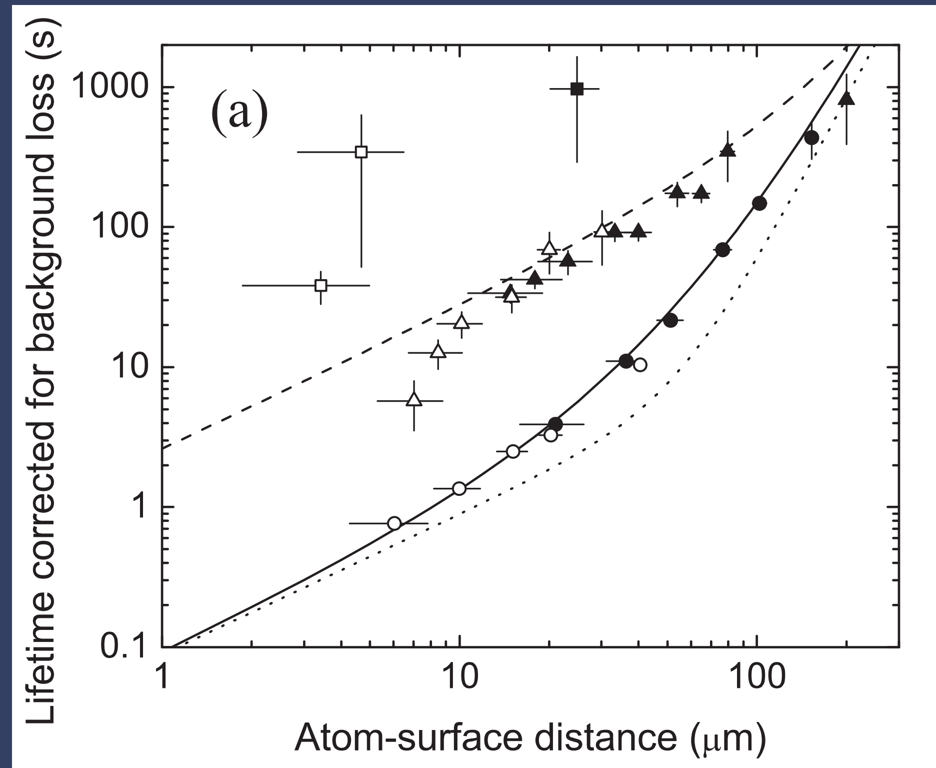
th/expt: V & P, J Appl Phys 1984
review: Nenonen & al, Rev Sci Instr 1996

Field fluctuations – experiment 2

Magnetic trap above half-space:
different heights/materials

Signal: spin flip $\longrightarrow$ trap loss rate

• 2008/09:
cooled metal, superconductor
J Fortágh group, G Noguès group



Data points: • non-condensed, ∘ BEC
Lines: *ab initio* theory (Cu, Ti surface)
E. Cornell group,
J. Low Temp. Phys. 133 (2003) 229

Henkel & al, *Appl. Phys. B* **69** (1999) 379
current noise: Chr. Bruder group,
Phys. Rev. A 68 (2003) 043618

wire experiment: E. Hinds group,
Phys. Rev. Lett. 91 (2003) 080401,
Rekdal & al, *Phys. Rev. A* 70 ('04) 013811

Magnetic Casimir–Polder interaction

free energy shift at $z = L$

Wylie & Sipe 1984/85; Henkel & al 2005;
Klimchitskaya & al 2009; Skagerstam & al 2009

$$\mathcal{F} = -\frac{\hbar}{2\pi}\int_0^\infty \mathrm{d}\omega \coth\left(\frac{\hbar\omega}{2k_B T}\right) \mathrm{Im}[\beta^T_{ij}(\omega)\mathcal{H}_{ji}(L,\omega)]$$

β^T: magnetic polarizability (equilibrium)

features at $T = 0$ (——)

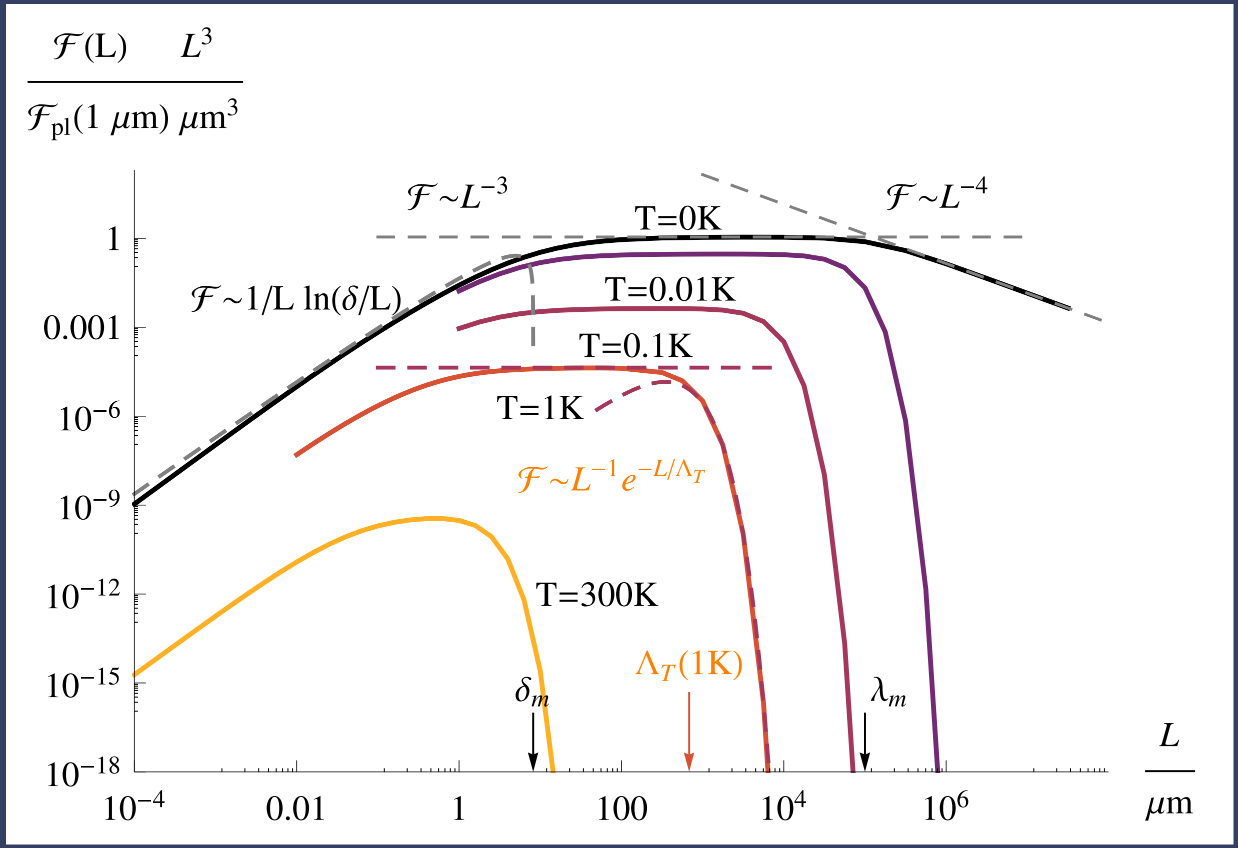
van der Waals regime

$$\mathcal{F} \approx \frac{|\langle a|\mu_x|b\rangle|^2\mu_0}{32\pi L^3}$$

C–P quenching $\mathcal{F} \approx \dfrac{\beta(0)\,\hbar c\mu_0}{16\pi^2 L^4}$

short-distance quenching

$$\mathcal{F} \approx \frac{|\langle a|\mu_x|b\rangle|^2\mu_0}{8\pi^2\delta_m^2 L}\log\frac{\delta_m}{L}$$



Magnetic dipole energy shift

free energy shift at $z = L$

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β^T: magnetic polarizability (equilibrium)

features at $T = 0$ (——)

- why repulsion? Boyer's $\varepsilon| \quad |\mu$

magnetic image dipole NS | SN

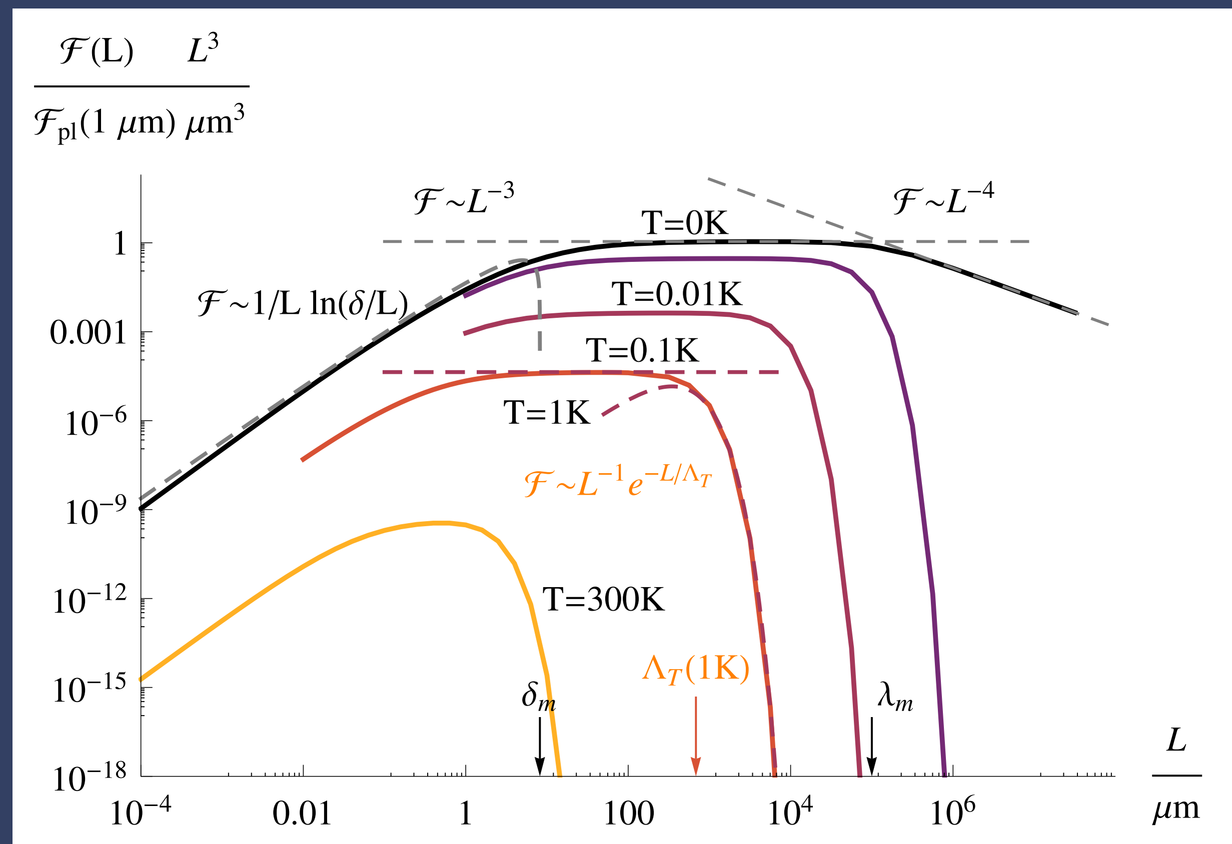
- why long-distance quenching?

delay as usual

- why short-distance quenching?

deep penetration into metal

= "softened mirror"



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β^T: magnetic polarizability (equilibrium)

$T > 0$: thermal quenching

polarizability $\beta^T \propto \tanh\dfrac{\hbar\omega_{ab}}{2T}$

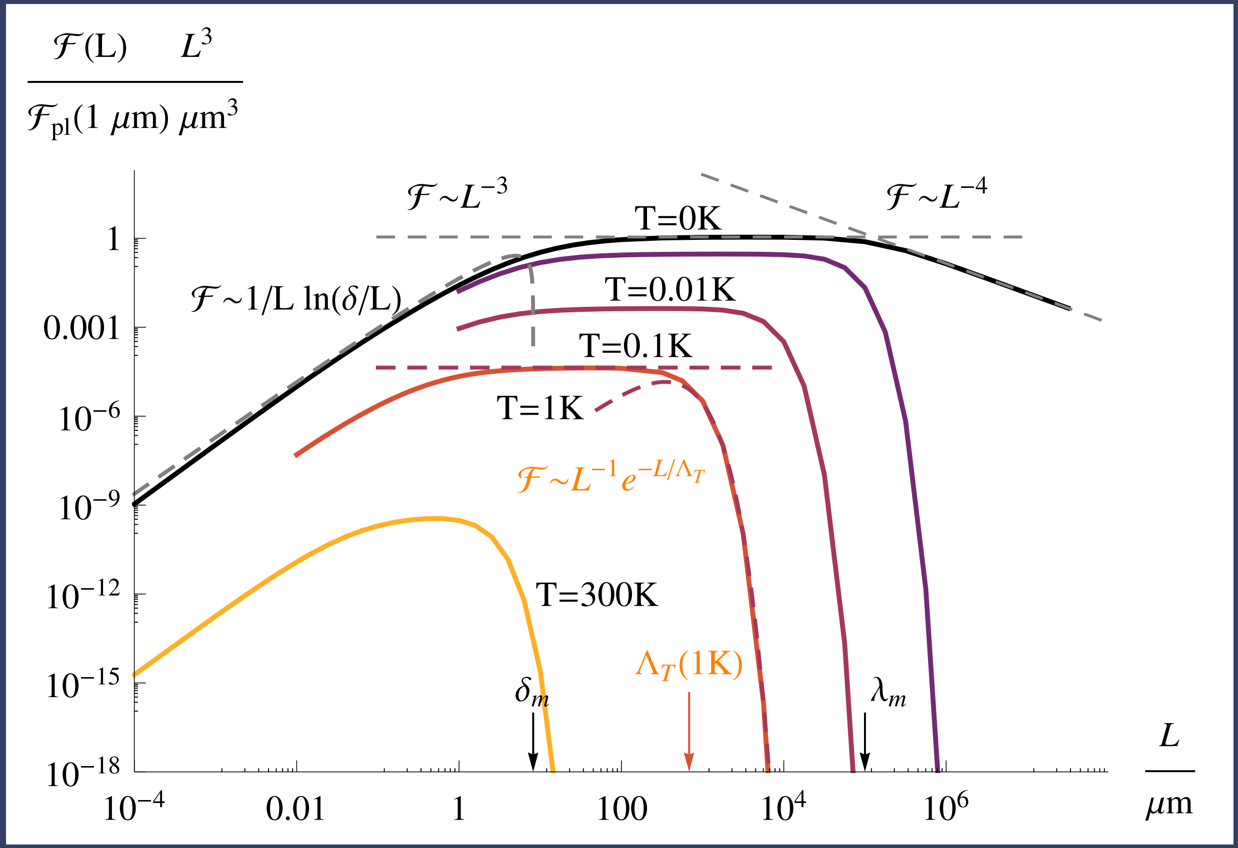
$\longrightarrow$ reduces image dipole

(para)magnetic attraction remains

$-\frac{1}{2}\beta^T(0)\langle B^2(z)\rangle_T < 0$

large distances $\mathcal{F} \sim \dfrac{\exp(-L/\Lambda_T)}{L}$

"artefacts" of thermal β^T



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β^T: magnetic polarizability (equilibrium)

superconductor (two fluid or dirty BCS)

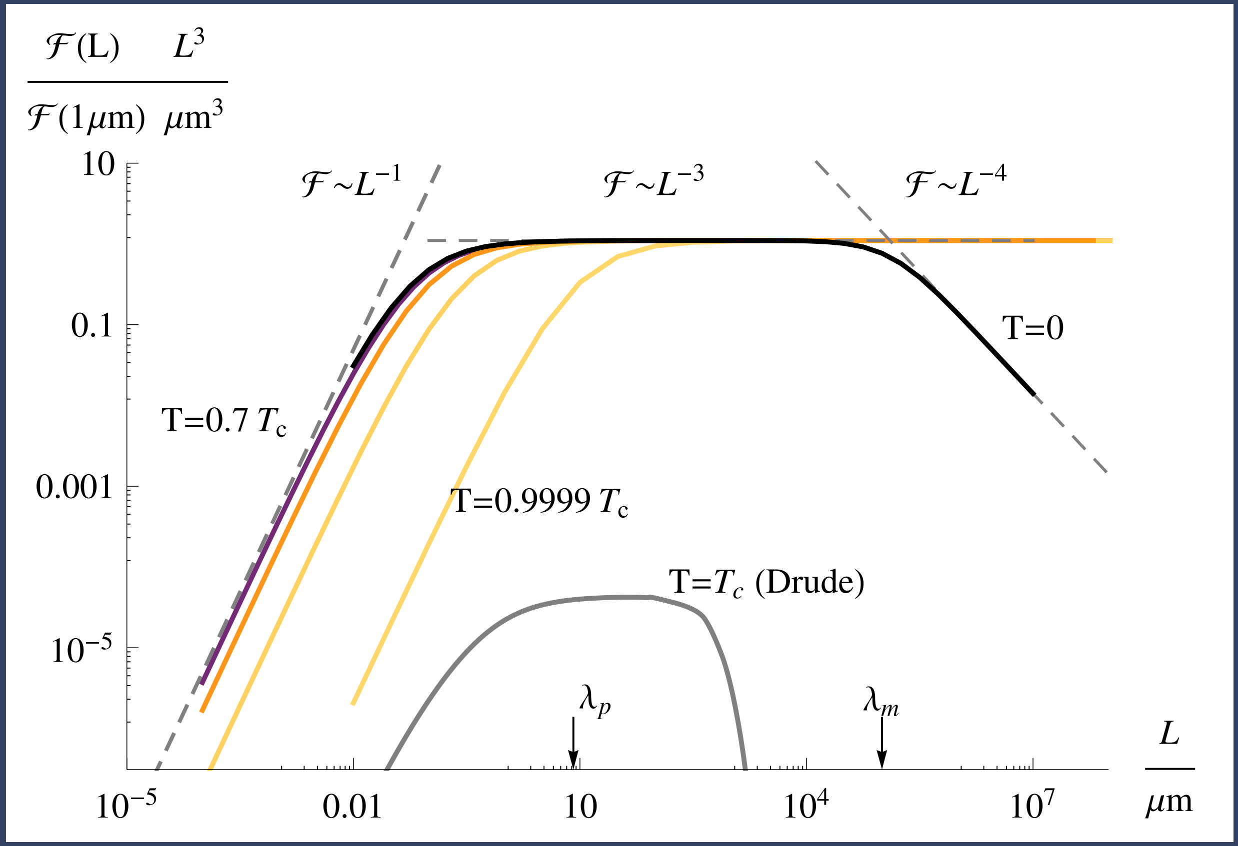
- why small distance quenching?

London length $\mathcal{F} \sim \dfrac{1}{[\lambda_\mathrm{p}(T)]^2 L}$

- why long-distance repulsion?

$\omega = \mathrm{i}\xi \to 0$ Matsubara term $\neq 0$
(Meissner effect)

$\longrightarrow$ poster F Intravaia
(non-equilibrium)



Casimir–Polder entropy

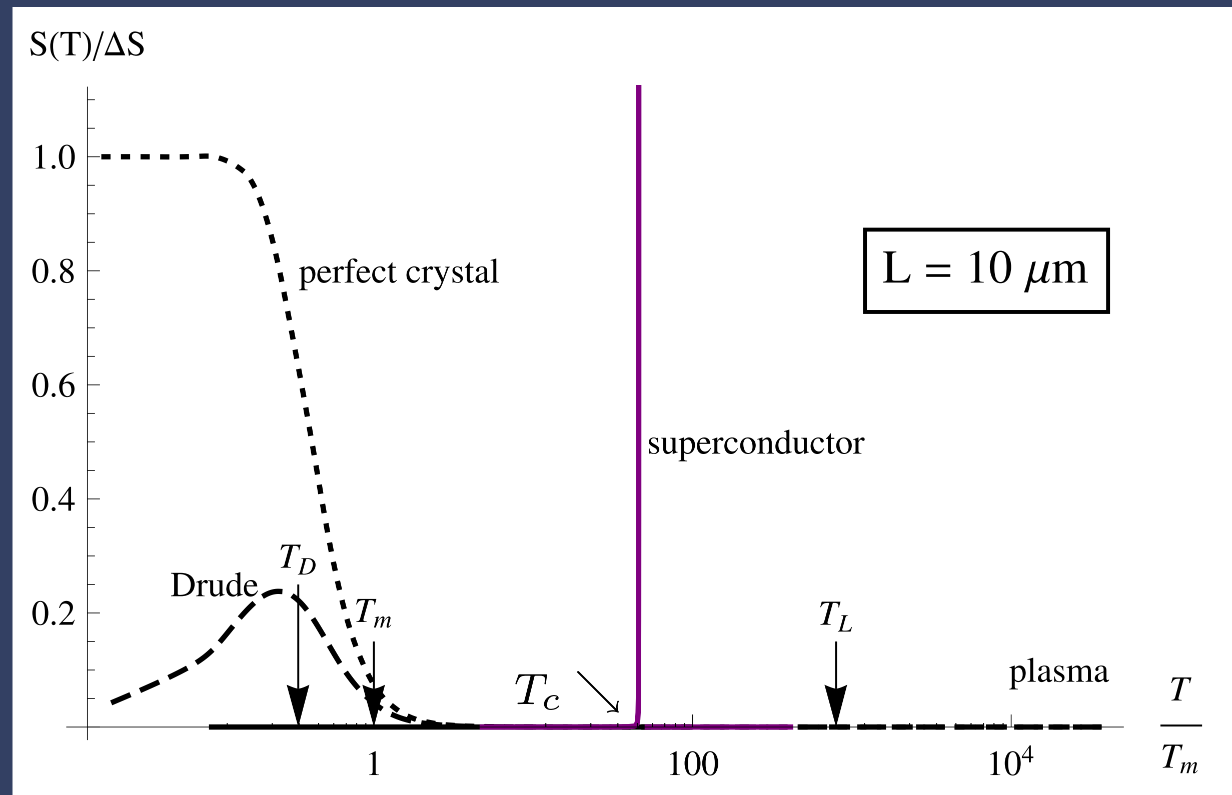
entropy shift $S = -\dfrac{\partial \mathcal{F}}{\partial T}$

Klimchitskaya & al 2009; Bimonte & al 2009

understand as "interaction entropy"

- superconductor:

field "communicates" matter phase transition to atom



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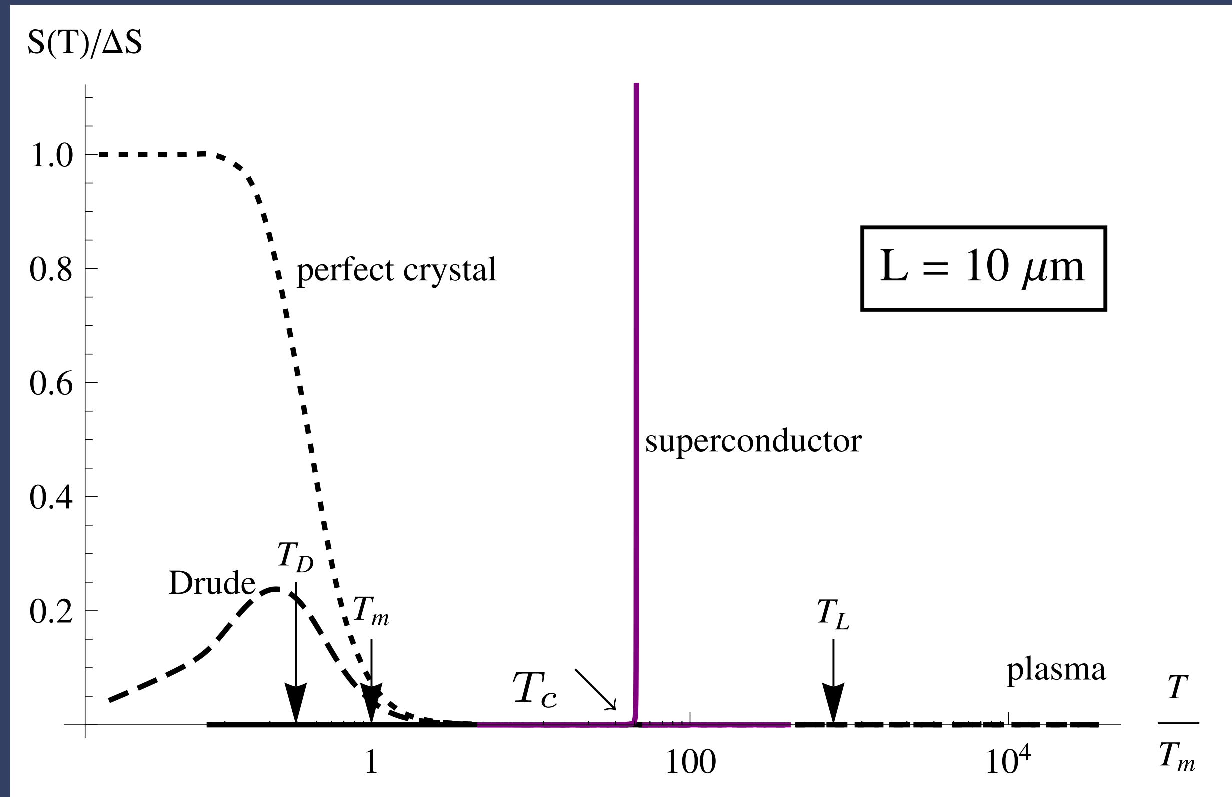


- normal conductor:

entropy of Johnson currents
"tapped" by atom (via the field)

energy of diffusive motion

$$T_D = \frac{\hbar D}{L^2} = \frac{\hbar}{\mu_0 \sigma L^2}$$

Casimir–Polder entropy

entropy shift $S = -\dfrac{\partial \mathcal{F}}{\partial T}$

Klimchitskaya & al 2009; Bimonte & al 2009

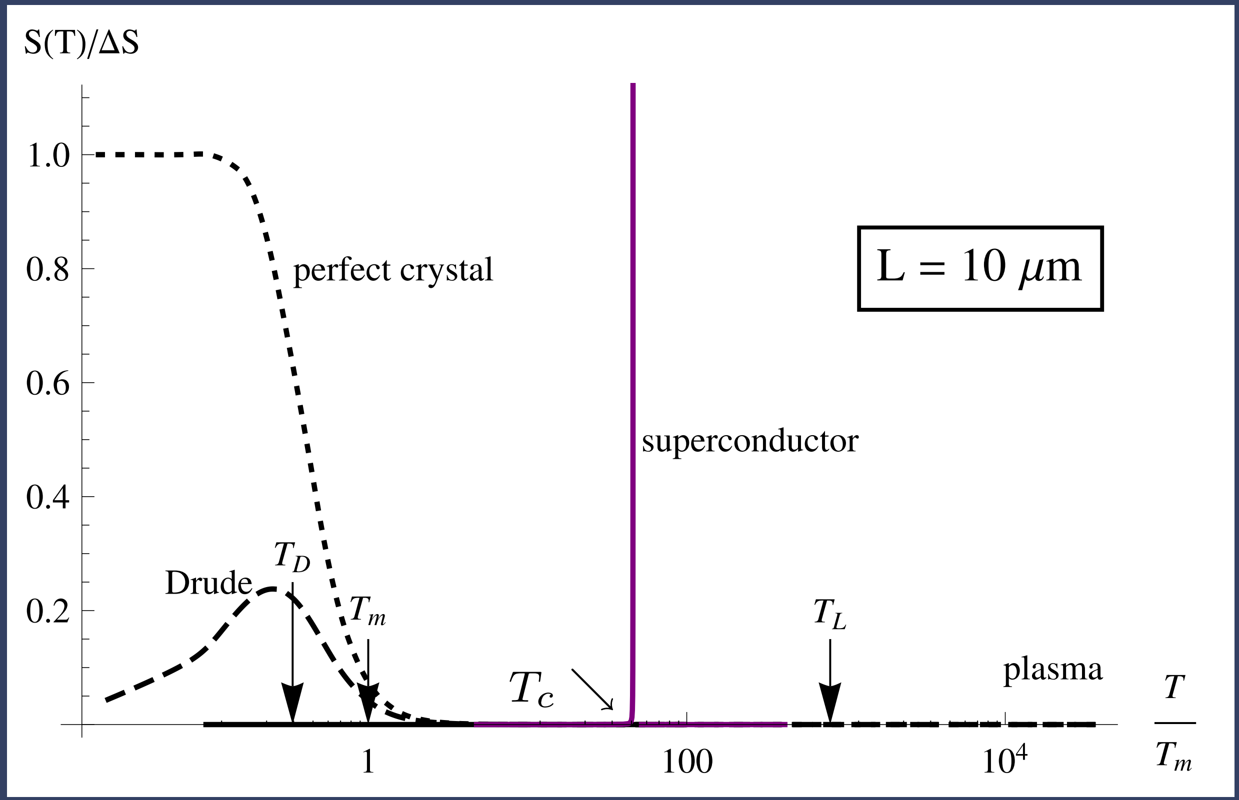
understand as "interaction entropy"

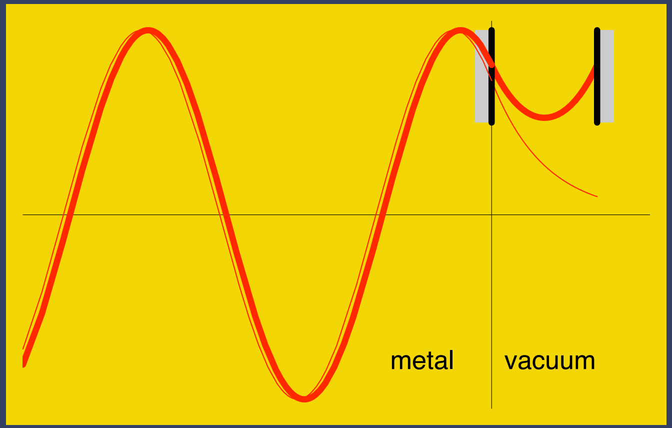
- "perfect crystal" = ideal gas

conductivity $\sigma(\omega) = \dfrac{\omega_{\rm p}^2 \varepsilon_0}{\mathrm{i}\omega + \mathcal{O}(T^2)}$

entropy of frozen currents

paramagnetic atom $\longrightarrow$ phase shift





entropy defect $\Delta S = \dfrac{|\langle a|\mu_x|b\rangle|^2 \mu_0}{16\pi\, \hbar\omega_{ba}\, L^3} > 0$ ($\rightarrow$ poster F Intravaia)

highly idealized: no anomalous skin effect, Landau damping ...

Remarks on Casimir–Polder control

Basic repulsive mechanism: $\longrightarrow$ talk J Munday

'mixed' geometry $\varepsilon|\quad|\mu$ Boyer 1974; Kenneth & al 2002

repulsion between magnetic images

attraction wins as $T > \hbar\omega_{\mathrm{p,\,mag}}$ CH & Joulain 2005 [quant-ph/0407153]

- hard to 'beat' electric attraction
  that has much larger bandwidth $\omega_{\mathrm{p,\,el}} \sim (n_{\mathrm{el}})^{1/2} \sim$ UV

Evidence with (ultracold) atoms

van der Waals regime ($< 100\,\mathrm{nm}$) well-known Beeby correction 1971

electric dipole damping well-known Drexhage 1974 ...

retarded regime ($> 1\,\mu\mathrm{m}$): ever smaller effects JILA & Trento $\geq$ 2005

- electric energies & magnetic transition rates

Conclusions

attractive magnetism …

— usually a weak effect — hard to detect

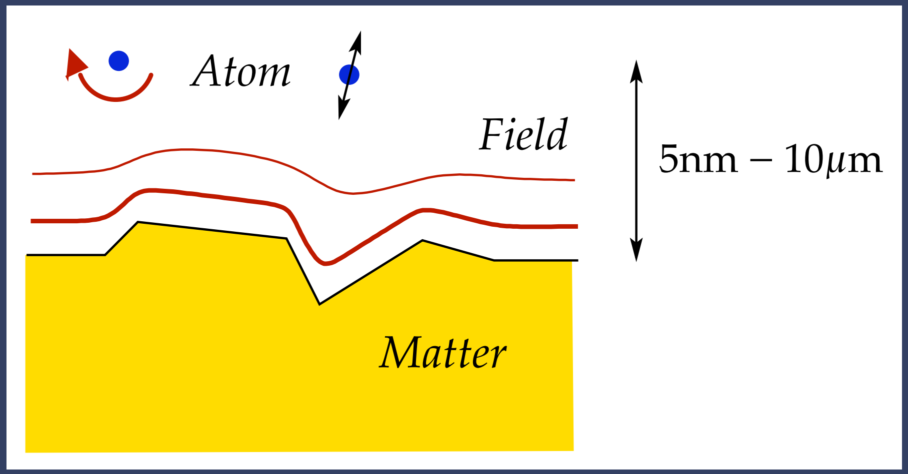
— repulsive via instantaneous images (Meissner effect)

— dissipative via induction images (eddy currents) — easy to detect

a diagnostic tool for theory …

— μ sensitive to low frequencies ($\sim \omega_{ba} \ll T$)

— μ reveals state of matter (temperature, topography, entropy)

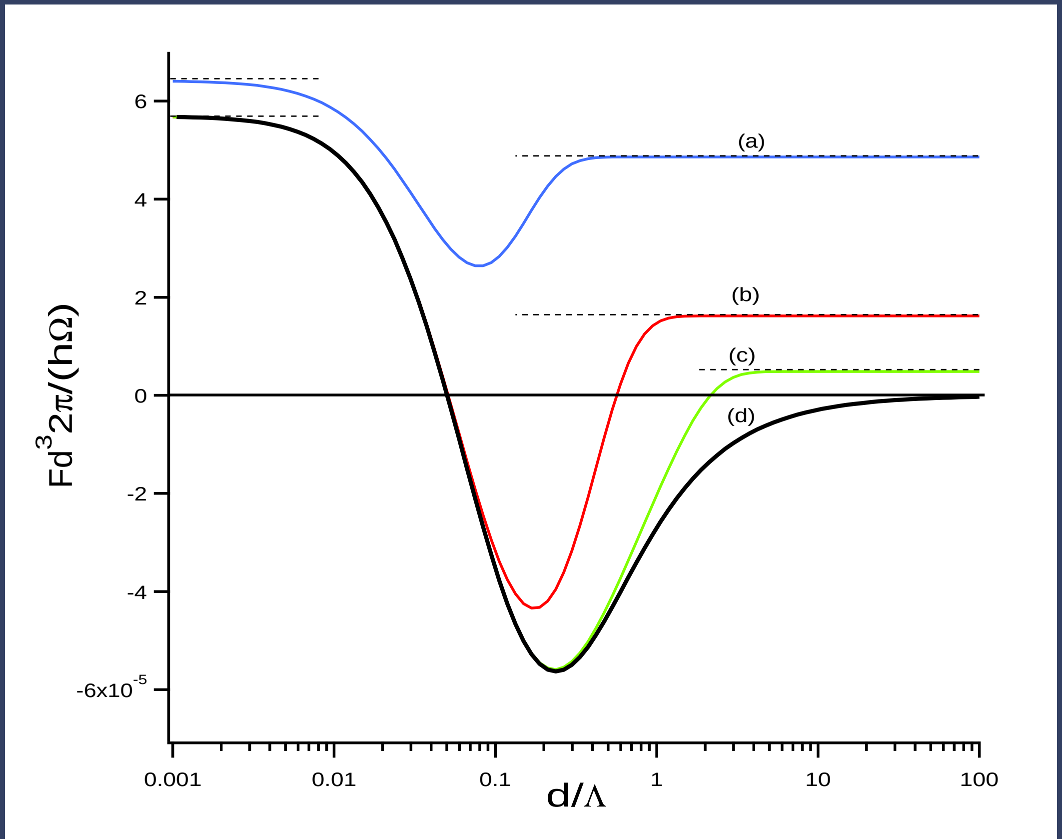


Metamaterial – repulsive Casimir effect

very attractive subject

Tomaš 2005; CH & Joulain 2005 = quant-ph/0407153; Raabe & Welsch 2005; Leonhardt & Philbin 2007; Pirozhenko & Lambrecht 2008; Rosa & al 2008; Rosa 2009; Yannopapas & Vitanov 2009 . . .

Casimir force between metamaterials



pure dielectric facing mainly diamagnet

$k_B T / \hbar\omega_\mathrm{p} =$
- 0.3 (a),
- 0.1 (b),
- 0.01 (c),
- 0 (d)

ω_p: typical plasma / resonance frequency, $\Lambda = 2\pi c/\omega_\mathrm{p}$

. . . but attraction wins as $T > \hbar\omega_{\mathrm{p,\,mag}}$

CH & Joulain 2005

van der Waals $\rightarrow$ Casimir & Polder

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