Casimir-like drag forces in superfluids

Andrew Sykes, 1 Matthew Davis 1 and David Roberts 2

1 The University of Queensland, School of Mathematics and Physics, ARC Centre of Excellence for Quantum-Atom Optics, Old 4072, Australia
2 Theoretical Division and Center for Nonlinear Studies, Los Alamos national Laboratory, Los Alamos, New Mexico 87545, USA

Summary. We calculate the drag force acting on an impurity moving through a quasi-1D BEC using analytic solutions for the quantum fluctuations in the system [4]. We find a non-zero force arises for zero and finite temperature at all velocities due to an imbalance in the Doppler shift in the scattering of quantum and thermal fluctuations. The force is proportional to $\sqrt{m g/(k_B T)}$ and a cross-over from a thermal to a quantum regime occurs near $T \sim 10^{-3} \text{g/m}_c$.

Introduction

One of the most remarkable properties of superfluidity is the emergence of a critical velocity, below which, an obstacle can move through a superfluid without experiencing any friction.

Landau’s phenomenological explanation [1]: The excitation spectrum of the fluid may be such, that it only allows for the creation of quasiparticles above a critical velocity. It is commonly believed that BEC lies at the heart of superfluidity. Experiments looking at BEC in dilute gases provide exciting opportunities to further our understanding of superfluidity.

Formalism

Hamiltonian in second-quantisation:

$\hat{H} = \int dx \left( -\frac{\hbar^2}{2m} \Delta^2 + i\hbar \delta \dot{\psi} + \eta \delta(x) + \frac{g}{2} \Delta^2 \psi \right) \psi.$

where $n$ is the mass of the atoms, $\Delta$ is the velocity of the impurity, $\gamma$ is the strength of the impurity, and $g$ is the interaction strength between atoms.

Bogoliubov’s perturbation expansion for a weakly interacting BEC:

$\psi = \psi_0 + \delta \psi$.

where $\psi_0$ is the condensate wavefunction and $\delta \psi$ is the quantum operator for the fluctuations.

We wish to calculate the force acting on the impurity which is given by

$F = -\left( \psi_0^* \nabla [\eta \delta(x)] \psi_0 + \psi_0 \nabla [\eta \delta(x)] \psi_0^* \right)$

Generalised Gross-Pitaevskii equation

The fluctuations of the field gives the operator its quantum properties.

The equation governing these fluctuations is called the Gross-Pitaevskii-de Gennes equation:

$i \hbar \delta \psi_0 = \left[ -\frac{\hbar^2}{2m} \Delta^2 + i\hbar \delta \dot{x} + \eta \delta(x) + 2g |\psi_0|^2 \delta \psi_0 \right] \psi_0 + 2g \delta \psi_0 \psi_0^* \psi_0 + \psi_0^* \Delta^2 \delta \psi_0 \psi_0$

The above equation defines a scattering problem [3], and it involves finding all four zero eigenvalue solutions [4]. Difficult… but it looks quite pretty once you're done. Hopefully it will impress the girls.

References:

Once the generalised GPE has been solved one can be quantitative about the size of the force, also, one can look at finite temperature fluctuations.