Monge-Kantorovich Optimal Transport:

Theory and Applications

October 19-21, 2009

Santa Fe, New Mexico, USA

Conference Proceedings

Monge – Kantorovich Optimal Transport: Theory and Applications

October 19-21, 2009 | Eldorado Hotel, Santa Fe, New Mexico, USA

Schedule of Events:

Monday, 10/19/09

8:30 – 8:45	Welcome
8:45 – 12:10	Aspects of Optimal Transport
2:00 – 5:30	Optimal Transport and Imaging

Tuesday, 10/20/09

8:30 – 12:20	Optimal Transport and Mesh Adaption
2:15 – 3:00	Contributed Talks
3:00 – 5:00	Poster Session
6:00	Dinner Banquet

Wednesday, 10/21/09

8:45 – 12:10	Optimal Transport and Geophysical Fluid Dynamics
12:10 – 12:30	Discussion and Adjournment

Organizing Committee:
John Finn, Gian Luca Delzanno, Balu Nadiga, Rick Chartrand

Sponsors:

Center for Nonlinear Studies

IGPP Los Alamos

Los Alamos National Laboratory EST. 1943

MONDAY, OCT 19th

SESSION: ASPECTS OF OPTIMAL TRANSPORT
CHAIR: **John M. Finn**

8:00 Breakfast and Welcome

8:45 **Rick Chartrand**
'A simple algorithm for solving the Monge-Kantorovich problem'

9:25 **Robert McCann**
'When is multidimensional screening a convex program? Uniqueness and stability of optimal strategies in the principal-agent problem'

10:05 Coffee break and Poster Set-up

10:50 **Adrian Tudorascu**
'On the velocities of flows consisting of cyclically monotone maps'

11:30 **Dejan Slepcev**
'Biological aggregation, interfaces, and coarsening'

12:10-2:00 Lunch on your own

SESSION: OPTIMAL TRANSPORT AND IMAGING
CHAIR: **James Theiler**

2:00 **Gary Sandine**
'Metrics, measures and PDEs in image analysis'

2:40 **Eldad Haber**
'Numerical methods for optimal mass transport'

3:20 Coffee break with Poster Viewing

4:10 **Sameer Sheorey**
'Fast approximation of the Kantorovich-Rubinstein metric and applications to computer vision'

4:50 **Yaron Lipman**
'Volume transportation for surfaces'

A simple algorithm for solving the Monge-Kantorovich problem

Rick Chartrand
LANL

In this talk we present a simple method for solving the Monge-Kantorovich problem, at least in the case of quadratic cost. This optimization problem is nonlinear with a nonlinear constraint, but we show how to convert into an unconstrained minimization of a continuous, convex functional. Moreover, we can compute the derivative, allowing a simple gradient-descent algorithm to be used to compute the solution.

When is multidimensional screening a convex program? Uniqueness and stability of optimal strategies in the principal-agent problem

Robert McCann
University of Toronto

A principal wishes to transact business with a multidimensional distribution of agents whose preferences are known only in the aggregate. Assuming a twist (= generalized Spence-Mirrlees single-crossing) hypothesis, we identify a structural condition on the Preference b(x,y) of agent type x for product type y -- and on the principal's costs c(y) -- which is necessary and sufficient for reducing the profit maximization problem faced by the principal to be a convex program. This is a key step toward making the principal's problem theoretically and computationally tractable; in particular, it allows us to derive uniqueness and stability of the principal's optimum strategy -- and similarly of the strategy maximizing the expected welfare of the agents when the principal's profitability is constrained. We call this condition non-negative cross-curvature: it is also (i) necessary and sufficient to guarantee convexity of the set of b-convex functions, (ii) invariant under reparametrization of agent and/or product types by diffeomorphisms, and (iii) a strengthening of Ma, Trudinger and Wang's necessary and sufficient condition for continuity of the correspondence between an exogenously prescribed distribution of agents and of products. We derive the persistence of economic effects such as the desirability for a monopoly to establish prices so high they effectively exclude a positive fraction of its potential customers, in nearly the full range of non-negatively cross-curved models.

On the velocities of flows consisting of cyclically monotone maps

Adrian Tudorascu
University of Wisconsin, Madison

Motivated by work on one-dimensional Euler-Poisson systems, Gangbo et al. proved a surprisingly general flow-map formula which unequivocally links an absolutely continuous curve in the Wasserstein space to the corresponding family of optimal maps pushing forward a given reference measure to each measure on the curve. In this talk we shall discuss a similar result for higher dimensions. We shall also provide some applications in the form of variational solutions for pressureless gas dynamics systems. They are obtained as absolutely continuous curves in a new metric space which is topologically equivalent to the Wasserstein space.

Biological aggregation, interfaces, and coarsening

Dejan Slepcev
Carnegie Mellon University

Mutual interaction between individuals lead to formation of clusters in a variety of species. We will discuss two continuum models of biological aggregation. Both have a structure of a gradient flow in Wasserstein metric. In the first one only the attraction plays a role. In some cases this leads to blowup of solutions and formation of delta masses. Nevertheless the theory of gradient flows in Wasserstein metric enables one to show global well posedness of weak measure solutions.

We will then discuss the model of Topaz, Bertozzi, and Lewis, which also takes into consideration the effects of short-range repulsion among individuals. The model describes the attraction via a nonlocal operator, while the repulsion is modeled by a differential operator. We will demonstrate that the density profile develops interfaces between a near-constant-density aggregate state and the empty space. At the interfaces surface-tension like effects appear. More precisely: The sharp interface limit for the interfacial motion is the Hele-Shaw flow.

We will also discuss the coarsening in this and related nonlocal interfacial models. In particular the ways to obtain rigorous upper bounds on the coarsening rate.

Metrics, measures and PDEs in image analysis

Gary Sandine
LANL

In this talk we introduce several threads from mathematically inclined image analysis including the need for metrics for comparisons between data and simulation, how metrics lead us into interesting investigations involving geometric measure theory and why PDE methods show up in the first place. Even though current research will be touched on, the purpose of this talk is primarily expository and as such will also be used to remind or inform the listeners of various bits of PDE, variational methods, measure theory and geometry.

Numerical methods for optimal mass transport

Eldad Haber
University of British Columbia

In this talk we discuss and compare numerical methods for the solution of optimal transport problems. In particular we concentrate on the fluid dynamic approach that embed the problem in a space-time control problem. We discuss appropriate discretization, efficient optimization techniques and preconditioning that allow us to obtain a fast and robust algorithm.

Fast approximation of the Kantorovich-Rubinstein metric and applications to computer vision

Sameer Sheorey

Toyota Technological Institute at Chicago

The Kantorovich-Rubinstein (KR) metric has been widely used in computer vision as the Earth Mover's Distance (EMD) to compare image descriptor histograms. We will first review some of its applications to colour and texture matching and image registration. The practical uses have been restricted because of the computational complexity of the KR problem. We present a fast linear time approximation algorithm for computing the KR metric between probability measures and provide error bounds. The main observation that inspires this algorithm is that the dual KR problem can be transformed into a wavelet domain problem with an explicit solution. The resulting wavelet EMD metric is equivalent to EMD, i.e. the ratio of the two is bounded. We also provide a fast approximation to Hanin's extension of the KR metric to unbalanced measures. We experimentally show that wavelet EMD is a good approximation to EMD and has similar performance, but requires much less computation

This is joint work with David Jacobs.

Volume transportation for surfaces

Yaron Lipman

Princeton University

We will present a method for automatic surface comparison and alignment based on principles from conformal geometry and optimal mass transportation. One application of the method is a novel definition of distance function between surfaces. The new distance forms a tractable alternative to the theoretically sound but hard to compute Gromov-Hausdorff distance. The second application is efficient calculation of point correspondences between surfaces.

TUESDAY, OCT 20th

SESSION: OPTIMAL TRANSPORT AND MESH ADAPTATION
CHAIR: **Markus Berndt**

7:30 Breakfast

8:30 **Patrick Knupp**
'Remarks on grid generation, equidistribution, and solution-adaptation'

9:10 **Weizhang Huang**
'A variational mesh adaptation method based on mesh equidistribution and alignment'

9:50 **Glen Hansen**
'Mesh generation and adaptation using the Laplace-Beltrami target metric'

10:30 Coffee break with Poster Viewing

11:00 **Luis Chacon**
'A robust multidimensional mesh motion approach based on Monge-Kantorovich optimization'

11:30 **Chris Budd**
'Optimal transport methods for grid generation, with applications to numerical weather prediction'

12:20 Lunch on your own

CONTRIBUTED TALKS AND POSTER SESSION
CHAIR: **Rick Chartrand**

2:15 **Tauseef Rehman**
'Surface warping and image registration'

2:30 **Jean-Francois Cossette**
'Monge-Ampere solvers for semi-Lagrangian trajectory schemes'

2:45 **Brittany Froese**
'Numerical methods for the elliptic Monge-Ampere equation'

3:00 Coffee Break

3:00-5:00 POSTER SESSION

6:00-9:00 BANQUET (Sunset Room)
Speaker - **Gary Geernaert**
'Climate science challenges and its systems dynamics coupling
with infrastructure design and finance initiatives'

Remarks on grid generation, equidistribution, and solution-adaptation

Patrick Knupp
SNL

The generation of grids and meshes for the solution of partial differential equations on complex domains spans a broad range of interdisciplinary methods in applied mathematics and computer science. Many methods exist for generating a grid; major requirements on such grids are that they do not cause needless discretization error (due to poor quality) and that they can be easily refined. For a fixed number of degrees of freedom, discretization error can be minimized using solution-adaptive mesh generation methods which locate mesh vertices where they are most needed. Equidistribution plays an important role in both solution-adaptive and initial mesh generation techniques. A brief survey of these topics is presented to provide context for subsequent talks in this session.

A variational mesh adaptation method based on mesh equidistribution and alignment

Weizhang Huang
University of Kansas

In this talk a derivation of the mesh equidistribution and alignment conditions from the perspective of uniform meshes in a metric space will be presented. A variational mesh adaptation method based on the conditions will be developed and its basic properties such as the existence and invertibility of the minimizer will be addressed. Numerical examples will be given to demonstrate the mesh adaptation ability of the method.

Mesh generation and adaptation using the Laplace-Beltrami target metric

Glen Hansen
INL

The Laplace-Beltrami target metric method is based on a finite element discretization of the Laplace-Beltrami equation system. In this method, the equidistribution principle driving mesh motion is derived by a combination of a monitor surface and the use of a 'coarse-grained' target element. The approach lends itself to unstructured meshes and complex geometry; results will be shown on meshes consisting of hexahedral and tetrahedral elements. Results will also be presented that show adaptation to an error indicator derived from a mesh data transfer algorithm.

A robust multidimensional mesh motion approach based on Monge-Kantorovich optimization

Luis Chacon
ORNL

We discuss a new multidimensional mesh-motion approach based on Monge-Kantorovich optimization (MK). The method employs a constrained minimization approach to formulate a rigorous multidimensional equidistribution principle. A connection between MK and the grid smoothness functional ensures grid quality even in the presence of highly structured monitor functions. Further, MK results in a single elliptic nonlinear PDE, the Monge-Ampere equation, which is solved optimally with a Newton-Krylov iterative scheme. We will demonstrate that the approach can be naturally extended to a time-stepping context without resorting to ad-hoc temporal schemes, thus providing a robust, parameter-free mesh-motion approach. In this presentation, we will derive the method, and demonstrate its efficiency, stability, and robustness with various challenging time-dependent examples.

Optimal transport methods for grid generation, with applications to numerical weather prediction

Chris Budd
University of Bath

When numerically solving partial differential equations with evolving structure, it is often important to use some form of adaptive numerical method. The class of r-adaptive methods continuously moves a grid to follow such structure. In this talk I will describe a method, the Parabolic-Monge-Ampere method, which constructs such a grid using ideas based on optimal transport. I will demonstrate that this method gives and excellent mesh which has good regularity properties and correctly follows solution features. It also naturally couples to the underlying PDE which needs to be solved by using the adaptive method. I will describe both the theory behind the optimal transport based method and show how it can be coupled to the PDE. This method will then be applied to the Eady problem which models large scale meteorological features and we will show that it is very effective at resolving localised features such as tropical storms.

Surface warping and image registration

Tauseef Rehman
Georgia Institute of Technology

In this talk, we present a novel and efficient numerical method for the computation of the L2 optimal mass transport mapping in two and three dimensions. Our method uses a direct variational approach. We have formulated a new projection to the constraint technique that can yield a good starting point for the method as well as a second order accurate discretization to the problem. The numerical experiments demonstrate that our algorithm yields accurate results in a relatively small number of iterations that are mesh independent. We show the efficacy of our method via application to various image processing tasks including image registration, morphing and visual tracking.

Monge-Ampère solvers for semi-Lagrangian trajectory schemes

Jean-Francois Cossette
Université de Montréal

Demanding compatibility of semi-Lagrangian trajectory schemes with the fundamental Euler expansion formula, leads to the Monge-Ampère (MA) nonlinear second-order partial differential equation. Given standard estimates of the departure points of flow trajectories, solving the associated MA problem provides a corrected solution capable of satisfying the Lagrangian form of the mass-continuity equation to a round-off error. A hierarchy of MA solvers is considered with different degrees of complexity and numerical efficiency. The solution behavior of the MA equation arising is discussed in diverse scenarios of fluid dynamics applications, from elementary passive-scalar advection to fully developed turbulence problems. Improvements of the overall accuracy depend on the problem at hand and can be substantial.

Numerical methods for the elliptic Monge-Ampere equation

Brittany Froese
Simon Fraser University

The numerical solution of the elliptic Monge-Ampere partial differential equation, which arises in the context of optimal transport, has been a subject of increasing interest in recent years. However, while there are already some convergent methods available, many of the newly proposed methods converge only for solutions which are in H^2, which is more regularity than is generally available for solutions of the equation. We present and study the performance of two methods. The first method, which is simply the natural finite difference discretisation of the equation, is demonstrated to be the best performing method (in terms of convergence and solution time) currently available for generic (possibly singular) problems. The second method, which involves the iterative solution of a Poisson equation involving the Hessian of the solution, is demonstrated to be the best performing (in terms of solution time) when the solution is regular.

WEDNESDAY, OCT 21st

SESSION: OPTIMAL TRANSPORT AND GEOPHYSICAL FLUID DYNAMICS
CHAIR: **Balu Nadiga**

8:00 Breakfast

8:45 **Rick Salmon**
'Constructing numerical algorithms that maintain conservation laws'

9:25 **David Dritschel**
'Balance in non-hydrostatic rotating stratified turbulence'

10:05 Coffee break and Poster Take-down

10:50 **Andrei Sobolevskii**
"Transport in shock manifolds in Hamilton-Jacobi equations'

11:30 **Qinglan Xia**
'Diffusion-limited aggregation driven by optimal transportation'

12:10-12:30 Discussion and adjournment

Constructing numerical algorithms that maintain conservation laws

Rick Salmon
University of California, San Diego

Even sophisticated numerical methods that fail to maintain fundamental conservation laws can exhibit a kind of structural instability, in which the solution slowly drifts into very unrealistic states. However, designing numerical algorithms that maintain fundamental conservation laws can be very difficult, and no general method exists. In this talk I survey some methods of algorithm construction that utilize the Hamiltonian formulation of fluid mechanics, and I apply the methods to the rotating shallow-water equations, a set of fluid equations that are particularly important in meteorology and oceanography. The algorithms conserve mass, circulation, energy, and potential enstrophy in domains of arbitrary shape. In meteorology and oceanography, the conservation of energy and potential enstrophy are thought to be very important.

Balance in non-hydrostatic rotating stratified turbulence

David Dritschel
University of St Andrews

Two distinct types of fluid motion occur in rotating, stably-stratified flows, (1) slow, vortical motion associated with the material transport of a dynamical scalar called 'Potential Vorticity' (PV), and (2) relatively fast motion associated with 'gravity waves', or undulations of nearly horizontal density surfaces. In many circumstances, particularly in the atmosphere and oceans, the slow vortical motion greatly dominates over gravity waves. This state of affairs is referred to as 'balance'. Balance means that the flow evolution is entirely governed by the distribution of PV. Although balance is not strictly observed, it is astonishing just how accurate it can be in highly-nonlinear turbulent flows, examined here, even when the stabilizing effects of rotation and stratification are only marginal.

Mathematically, we show that re-casting the equations describing incompressible fluid motion under the Boussinesq approximation in an invariant vector form naturally leads to a double Monge-Ampere equation. Moreover, the solvability condition of this equation embeds the conditions for both static and inertial stability.

Transport in shock manifolds in Hamilton-Jacobi equations

Andrei Sobolevski
Institute for Information Transmission Problems

Characteristics of a solution to a Hamilton-Jacobi equation can be seen as trajectories of fluid particles, but usually this analogy is only pursued until trajectories hit a shock and cease to minimize the Lagrangian action. We show that any weak solution to a Hamilton-Jacobi equation with a general convex Hamiltonian gives rise to a unique, global in time, adhesive flow that can be recovered in the limit of vanishing viscosity and that in turn defines a mass transport process. In contrast, the limit of vanishing diffusivity for the same weak solution generally gives rise to a stochastic flow with non-unique trajectories. However the two limits happen to coincide for Hamiltonians quadratic in the momentum variable.

This is a joint work with K. Khanin (Toronto).

Diffusion-limited aggregation driven by optimal transportation

Qinglan Xia
University of California, Davis

The optimal transport problem aims at finding an optimal way to transport a given probability measure into another. In contrast to the well-known Monge-Kantorovich problem, the ramified optimal transportation problem aims at modeling a tree-typed branching transport network by an optimal transport path between two given probability measures. An essential feature of such a transport path is to favor group transportation in a large amount. Branching transport networks are observable not only in nature as in trees, blood vessels, etc. but also in efficiently designed transport systems such as used in railway configurations and postage delivery networks.

In this talk, I will talk about a joint work with an REU student Douglas Unger. We combine the DLA model of Witten and Sander with ideas from ramified optimal transportation. We propose a modification of the DLA model in which the probability of sticking is inversely proportional to the additional transport cost from the point to the root. We used a family of cost functions parameterized by a parameter α as studied in ramified optimal transportation. $\alpha<0$ promotes growth near the root whereas $\alpha>0$ promotes growth at the tips of the cluster. $\alpha=0$ is a phase transition point and corresponds to standard DLA. What makes this model interesting is that when α is negative enough (e.g. $\alpha<-2$) the final cluster is a one dimensional curve. On the other hand, when α is positive enough (e.g. $\alpha>2$) we get a nearly two dimensional disk. Thus our model encompasses the full range of fractal dimension from 1 to 2.

Poster Abstracts

*(*Partial List)*

Gian Luca Delzanno, Los Alamos National Laboratory

Generalization of optimal transport for grid adaptation and imaging

We will review some of our recent work on optimal transport applied to grid adaptation and imaging. For grid adaptation, we will present two methods [1] based on a generalization of the optimal transport cost function and compare them with the results obtained by the L2 theory [2] (i.e. the Monge-Ampere equation). The first method uses a cost function proportional to the Lp norm of the distance between the new and old grid, while the second uses a cost function proportional to the trace of the covariant metric tensor (which measures the smoothness of the grid). We will present detailed comparisons of these methods in terms of performance and grid quality. For imaging, we will present a fluid dynamic approach to optimal transport based on the Lp cost function and apply it to obtain the warping transformation between two images [3]. We will show that, while L2 theory performs better than Lp for $p<1$, in general optimal transport applied to imaging seems to have some problems associated with double exposure.

[1] G.L. Delzanno, J.M. Finn, Generalized Monge-Kantorovich optimization for grid generation and adaptation in Lp, submitted to SISC (2009). [2] G.L. Delzanno, L. Chacon, J.M. Finn, et al., J. Comput. Phys. 227, 9841 (2008). [3] G.L. Delzanno, J.M. Finn, The fluid dynamic approach to equidistribution methods for grid generation and adaptation, submitted to SISC (2009).

John Finn, LANL

Semi-geostrophic Equations of Geophysical Fluid Dynamics

We present a tutorial derivation of the semi-geostrophic equations. We take the 2D Euler equations in a rotating frame and expand in powers of the Coriolis parameter 1/f, where f is two times the rotation frequency. At lowest order we have the quasi-geostrophic equations, in which the potential vorticity (PV) is advected by the geostrophic flow, and the equations are closed by solving a Poisson equation for geostrophic streamfunction. At the next order the PV is advected by a velocity including the geostrophic and the ageostrophic flow. The Poisson equation is replaced by a higher order (and nonlinear) equation for the geostrophic streamfunction. This equation is the Monge-Ampere equation. The equations are simplified greatly by the use of the geostrophic coordinates of Hoskins. For these coordinates dx\'/dt is the geostrophic flow. These equations, involving advection by the ageostrophic flow, can represent frontogenesis. We will discuss the usefulness of these equations, in light of the fact that the Monge-Ampere equation is very difficult to solve, in spite of recent advances.

**In collaboration with B. Nadiga, LANL.*

Nicola Gigli, University of Bordeaux

On the Heat flow on metric measure spaces.

I will first recall the definition of metric measure spaces with Ricci curvature bounded from below – due Lott/Villani & Sturm -together with some related recent developments. Then I will define the Heat flow on such spaces and prove some basic results: existence, uniqueness and stability. Specifically, for what concerns existence and uniqueness, the results follows from interesting connections between displacement convexity of the Entropy functional and the linear (i.e. usual) convexity of the squared slope. Regarding the stability property, this is a consequence of a much more general result on the stability of gradient flows of geodesically convex functionals.

Konstantin Lipnikov, Los Alamos National Laboratory

We present a new technology for generating meshes minimizing the interpolation and discretization errors or their gradients. The key element of this technology is the construction of a tensor metric from edge-based error estimates. The equidistribution principle suggests to build meshes that are quasi-uniform in this tensor metric. For such meshes, we prove optimal estimates for the interpolation error. In particular, we prove that for a mesh with N triangles, the error is proportional to N^{-1} and the gradient of error is proportional to $N^{-1/2}$. For many problems, the interpolationerror controls the discretization error. Optimal reduction of the discretization error on a sequence of adaptive meshes will be illustrated for elliptic problems with strong point and anisotropic singularities.

Chris Newman, Idaho National Laboratory

Finite element solutions of the Monge-Kantorovich problem

We examine solution strategies for finite element discretizations of the Monge-Kantorovich problem. Our methods are based on conforming finite element spaces and preconditioned Jacobian-free Newton-Krylov solution methods. This work focuses on effectiveness of solvers and preconditioners applied to this problem, and presents numerical examples to support this study. Results will be demonstrated in 2 and 3-D using INL\'s parallel MOOSE framework.

Annalisa Quaini, University of Houston

Numerical simulation of an ultrasound imaging model of mitral valve regurgitation

We developed and validated a computational model to simulate the fluid dynamics conditions in an imaging heart chamber. The model was then extended to include a simulation of the fluid-structure interaction between fluid flow and an elastic orifice modeling a ``leaky\'\' valve.

The imaging heart chamber, connected to a pulsatile flow loop, was previously designed to model in vitro the hemodynamics conditions encountered in patients with mitral regurgitation \\cite{littlei1}. The chamber consists of two (acrylic) cylinders partitioned by a divider plate containing a geometric orifice. Hemodynamics conditions in the chamber were simulated using a 3D computational fluid dynamics model based on the Finite Element Method approximation of the Navier-Stokes equations for an incompressible, viscous fluid. The flow across two rigid orifices differing in size and shape (0.4 cm^2 circle and 0.35 cm^2 slot) has been simulated. Moreover, two different regurgitant volumes have been taken into account. The comparison between the numerical solution and the experimental data shows very good agreement. Thus, the computational model has been used for the depiction of the clinically recognized flow events: proximal isovelocity surface area (PISA) zone, vena contracta, expanding distal regurgitant jet.

The replacement of the fixed divider plate by an elastic one represents a step forward in the simulation of a leaky mitral valve. To model a deformable orifice we couple the incompressible Navier-Stokes equations to the elastodynamics equations for an elastic solid. Besides being of clinical interest, this fluid-structure interaction problem is challenging from the numerical and computational point of view. For the solution of the coupled problem we adopt a non-modular method called ILUT-GMRES \\cite{badiaq2}, which is known to be robust and to possess good convergence properties. The orifice in the deformable divider plate is arc-shaped (0.4 cm^2), in order to better represent not tightly closing mitral valve leaflets. Also for this problem, clinically recognized flow events have been depicted.

This is a joint work with S. Canic, R. Glowinski, G. Guidoboni, S. R. Igo, S. H. Little, and W. A. Zoghbi

Gideon Simpson, University of Toronto

Magma Transport: Coherent Structures & Constitutive Relations

Magma, molten rock in the Earth\'s interior, surfaces at many sites of geologic interest, particularly volcanoes. Magmatism is also essential to global mantle convection and plate tectonics; it is universally present at plate boundaries, including mid-ocean ridges and subduction zones.

However, the macroscopic mechanisms for melt generation and transport are still not well understood. Fully self-consistent models remain elusive, and our understanding of existing models is poor. We present our efforts to construct self-consistent models by homogenization. These are in close agreement with earlier models. Additionally we present some analysis of the models under simplification. This includes theorems on the existence and uniqueness of solutions and the stability of solitary waves.

Joint work with Michael I. Weinstein and Marc Spiegelman, both of Columbia University

Contact List

Aluie	Hussein	LANL	hussein@lanl.gov
Berndt	Markus	LANL	berndt@lanl.gov
Boella	Elisabetta	Politecnico di Torino	elisabetta.boella@gmail.com
Brune	Christoph	University of Muenster, Germany	christoph.brune@uni-muenster.de
Budd	Chris	University of Bath	mascjb@bath.ac.uk
Carrasco	Rafael	SMU CAAM Department	rcarrasco@smu.edu
Chacon	Luis	Oak Ridge National Lab	chaconl@ornl.gov
Chartrand	Rick	LANL	rickc@lanl.gov
Chung	Yeojin	Southern Methodist University	ychung@smu.edu
Cossette	Jean-Francois	Université de Montréal	cossette@astro.umontreal.ca
Delzanno	Gian Luca	Los Alamos National Laboratory	delzanno@lanl.gov
Dritschel	David	University of St Andrews	dgd@mcs.st-and.ac.uk
Fichtl	Chris	LANL	cfichtl@lanl.gov
Finn	John	LANL	finn@lanl.gov
Froese	Brittany	Simon Fraser University	bdf1@sfu.ca
Gigli	Nicola	University of Bordeaux	nicolagigli@googlemail.com
Haber	Eldad	UBC	haber@math.ubc.ca
Hansen	Glen	Idaho National Laboratory	Glen.Hansen@inl.gov
Horesh	Raya	Emory University	rshindm@emory.edu
Huang	Weizhang	University of Kansas	huang@math.ku.edu
Ketelaar	Christiaan	Southern Methodist University	cketelaarb@smu.edu
Knupp	Patrick	Sandia National Laboratories	pknupp@sandia.gov
Lipman	Yaron	Princeton University	ylipman@math.princeton.edu
Lipnikov	Konstantin	Los Alamos National Laboratory	lipnikov@lanl.gov
McCann	Robert	University of Toronto	mccann@math.toronto.edu
Nadiga	Balu	LANL	balu@lanl.gov
Newman	Chris	Idaho National Laboratory	christopher.newman@inl.gov
Oberman	Adam	Simon Fraser University	aoberman@sfu.ca
Quaini	Annalisa	University of Houston	quaini@math.uh.edu
Rehman	Tauseef	Georgia Tech	tauseef@ece.gatech.edu
Repetti	Cora	Politecnico di Torino	cora.r@inwind.it
Roytershteyn	Vadim	LANL	roytersh@lanl.gov
Salmon	Rick	University of California, San Diego	rsalmon@ucsd.edu
Sandine	Gary	LANL	gars@sandine.net
Shashkov	Mikhail	LANL	shashkov@lanl.gov
Sheorey	Sameer	Toyota Technological Institute at Chicag	ssameer@tti-c.org
Simpson	Gideon	University of Toronto	simpson@math.toronto.edu
Slepcev	Dejan	Carnegie Mellon University	slepcev@math.cmu.edu
Smart	Charles	University of California Berkeley	smart@math.berkeley.edu
Sobolevskiy	Andrey	A.A. Kharkevich Institute	sobolevski@iitp.ru
Tannenbaum	Allen	Georgia Tech	tannenba@ece.gatech.edu
Theiler	James	LANL	jt@lanl.gov
Tudorascu	Adrian	UW-Madison	tudorasc@math.wisc.edu
Wohlberg	Brendt	LANL	brendt@tmail.lanl.gov
Xia	Qinglan	UC Davis	qlxia@math.ucdavis.edu

Conference Information	conferences@cnls.lanl.gov
Expense Reimbursements	ellie@lanl.gov

Monge-Kantorovich Optimal Transport:

Theory and Applications

Conference Notations

Center for
Nonlinear Studies