Spectral Line Shapes with Strong Collisions By Plasma Electrons

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Abstract: Spectral line broadening is important for modeling and understanding white dwarf atmospheres. Modeling the broadening of spectral lines is complex and requires approximations to keep the problem tractable. The Coulomb interaction between charged particles is usually approximated with a long-range dipole interaction (dot product of dipole moment with plasma electric field). This approximation is valid for low-density plasmas where the average distance between particles is greater than a few atomic radii. But for high densities, close-range interactions become important giving rise to several non-dipole effects: plasma polarization, quantum-particle exchange, atomic reaction to the collision, and line merging. We have so far studied only hydrogen-like radiators due to the extra complexities with multi-electron radiators. The perturbing electrons are treated quantum mechanically, which can accurately account for energy exchange as well as charge exchange (which neglected by classical calculations). Including charge exchange reduces line widths nearly 50% for neutral hydrogen, but is less for more highly charged ions. Only when penetration and exchange are included do quantum-mechanical calculations agree with semi-classical calculations—which are known to be accurate—for neutral hydrogen. However, this correspondence does not happen for charged radiators. Plasma polarization, which is a measure of the average charge inside the radiator, is unimportant for neutral hydrogen, but is significant for charged radiators. Strong presence of plasma charge inside the radiator wavefunction induces a back reaction from the atom, which can significantly broaden spectral lines.

Interaction Potential

The interaction between radiators and perturbers is through the Coulomb interaction, which is commonly approximated with a Taylor expansion with only the dipole term is retained (dipole approximation),

$$\frac{1}{|r_e - r_p|} = \sum_{j} \frac{e^2}{r_{e,j}^{2j+1}} P_{j}(\cos \theta) \approx r_e \cdot r_p \frac{1}{r_e^2} \approx \vec{D} \cdot \vec{E}$$

where $r_e$ is the coordinate of the radiator electron, $r_p$ is coordinate of plasma electron. At low densities, the dipole approximation is valid, though at high densities, this approximation breaks down and quadrupole terms become important (Kidcrease et al. 1993; Gomez et al. 2016) and at even higher densities, penetration becomes important (Woltz & Hooper 1984). When penetration occurs, then it is necessary to model the plasma electron with quantum motion and treat various effects, such as exchange.

The total wavefunction of two identical particles needs to reflect the physical indistinguishability of the two particles (Bethe & Salpeter 1957),

$$\Psi(r_e, r_p) = \frac{1}{\sqrt{2}} \left[ \psi(r_e) \chi(r_p) - \chi(r_e) \psi(r_p) \right],$$

and when the size of the radiator is small, then this can be approximated by retaining only the first term and ignoring exchange (O’Brien & Hooper 1974; Woltz & Hooper 1984; Junkel et al. 2000). However, for hydrogen, exchange is necessary to achieve correspondence between quantum and semi-classical calculations (see figure 1).

However, for helium, our new calculations—with the details of exchange—were narrower than the semi-classical results of Kepple (1972); though the Coulomb-wave without exchange calculation agreed well. This result is contrary to the neutral hydrogen case. Figure 2 shows a comparison of Kepple’s calculation and our new detailed calculation with experiment.

Atomic Back Reaction

The electron broadening operator, $\phi$, is defined by the radiation theory (Fano 1963)

$$\phi = \frac{1}{1 + (M/\Delta \omega)^2} < M >$$

where $L_1$ is the Coulomb interaction and $\Delta k l$ is the change in energy of the plasma electron. This is a complicated form and is not easily solved. Therefore a Taylor expansion is used for practical evaluation of $\phi$:

$$\phi \approx \left( L_1^2 + \left< L_1 \Delta \omega - \Delta k l \right> \right) - \left< L_1 \right> \left( \Delta \omega \right)^2 - \left< L_1 \right>.$$

The third term of $\phi$ can be evaluated with the use of a projection operator (Fano 1963; Smith & Hooper 1967; Smith et al. 1969), but has been ignored due to the dipole approximation and is ignored even when $\lambda < l$ is not assumed to be zero (Junkel et al. 2000). The back reaction does not exist in the impact theories of Baranger (1958) or Kolb & Griem (1958).

For neutral hydrogen and ionized helium, the back-reaction term is fairly insignificant, but can contribute substantially to broadening of more highly-charged radiators and multi-electron systems, such as helium-like systems and carbon, sometimes resulting in factors of two increase.

Figure 3 demonstrates the effect of the atomic back reaction term. In this case we studied Li-like carbon, which can occur in hot white dwarfs. Even though this calculation ignores some exchange collision processes, we can get a sense for the importance of this term in the broadening. In this case of C IV, the widths increase substantially. This effect becomes more dramatic with increasing atomic number.

Future Work: We want to extend these calculations to more complicated atoms like He-like or Li-like ions. This requires much greater effort since we cannot accurately treat line broadening by only the valence electron (Glener et al. 1992; 1996).

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