# *Dipolar Superfluids*

*Introduction:* Superfluidity was discovered in 1938 as the ability of liquid Helium ($He^4$) to carry momentum without dissipation and its phenomenological theory is based on the condensate wave function as an order parameter that describes the superfluid component of the liquid. The cornerstone of the phenomenological theory is the notion that phase of the superfluid condensate $\phi$ couples to the gauge potential $\mathbf{A}$ and that the condensate current is given by $\mathbf{J} = \rho_s(\nabla\phi - e\mathbf{A})$ where $\rho_s$ is a measure of the superfluid density. $He^4$, being neutral, does not carry electrical current although it can produce nontrivial gauge potentials by rotation. Another example of a neutral superfluid is an excitonic condensate in semiconductors, where electrons and holes can form (metastable) bound states called excitons which are expected to behave as neutral bosons at low densities and therefore can undergo Bose-Einstein condensation. This condensate, being neutral, will possess properties similar to superfluidity. In recent times , attempts to experimentally realize the excitonic condensate have focused on creating electron-hole plasma by optically exciting electrons from a valence band to the conduction band and then spatially confining the resulting electrons and holes to different quantum wells using a static electric field. The investigation of their properties has been limited to photoluminescence measurements: processes which probe phase-coherence but not superfluidity, and which destroy the condensate. In particular, they have not been investigated via transport measurements which can provide a *direct* signature of their superfluid properties.

Recent developments in heterojunction fabrications open up the exciting possibility of electron-hole bilayer systems where the electrons reside in one layer and holes in the other layer separated by a distance $d \sim 200$Å, and where the density of electrons or holes in individual layers can be adjusted using independent gates. These systems have very weak but *nonzero* interlayer tunneling which allows the electrons and holes to couple with an in-plane magnetic field applied between the two layers. As a result of Coulomb attraction between electrons and holes, the excitonic condensate is expected to occur when the typical distance $r_s$ between electrons (holes) within a layer exceeds the distance $d$ between the two layers. These systems offer an alternate view of the electron-hole excitonic condensate where the condensate has a well defined dipole moment associated with each exciton.

We argue that this excitonic condensate represents a qualitatively new kind of superfluid where the condensate is neutral and carries no momentum density. We call this nominally neutral superfluid a *dipolar superfluid*. The dipole moment associated with each exciton in the condensate allows this liquid to couple to electromagnetic fields in a nontrivial fashion. We find that the phase of the dipolar superfluid couples to the *gradient of the gauge potential*. As a result, we predict that it will exhibit a neutral persistent dipolar current, consisting of equal and oppositely directed currents in the two layers, upon application of an in-plane magnetic field $\mathbf{B}_{||}$. Thus, in the present case, the composite structure of excitons is manifest in the macroscopic condensed superfluid state. In the following paragraphs, we present the hydrodynamics of such a superfluid based on the Ginzburg-Landau (GL) energy functional and discuss various predictions which follow from it [1].

*GL Energy Functional and Effective Action:* Let us consider a bilayer system with electrons in the top layer and holes in the bottom layer. We introduce a notation where $\pm$ subscript corresponds to the top and bottom layer respectively. We define the excitonic condensate order parameter as

$$\Delta(\mathbf{r}) = \langle c_+^\dagger(\mathbf{r})c_-(\mathbf{r})\rangle = |\Delta(\mathbf{r})| \exp\left[i\Phi(\mathbf{r})\right] \qquad (1)$$

where $c_\pm^\dagger(\mathbf{r})$ creates an electron in the top (bottom) layer at position $\mathbf{r}$, and we have used the fact that $c_-(\mathbf{r}) = c_h^\dagger(\mathbf{r})$ where $c_h^\dagger(\mathbf{r})$ creates a *hole* at position $\mathbf{r}$ in the bottom layer. Upon a gauge transformation $c_\pm(\mathbf{r}) \to \exp\left[ie\varphi_\pm(\mathbf{r})\right] c_\pm(\mathbf{r})$, the order parameter $\Delta(\mathbf{r})$ will transform as $\Delta(\mathbf{r}) \to \exp\left[i\Phi(\mathbf{r})\right]\Delta(\mathbf{r})$ with

$$\Phi(\mathbf{r}) \to \Phi(\mathbf{r}) - e\varphi_+(\mathbf{r}) + e\varphi_-(\mathbf{r}) \qquad (2)$$

where $-e < 0$ is the electron charge. We will call the phase of the order parameter, $\Phi(\mathbf{r})$, the *dipolar phase*. It is "approximately" charge-neutral upon gauge transformation since the condensate is an electron-hole condensate. However, the crucial observation is that the electron and the hole operators are always spatially separated: the electrons are in the top layer and the holes are in the bottom layer. Therefore as one winds the phases of $c_+(\mathbf{r})$ and $c_-(\mathbf{r})$ in the same direction, since $\varphi_\pm(\mathbf{r}) = \varphi(\mathbf{r}\pm\mathbf{d}/2)$, where $\mathbf{d} = d\hat{z}$ is a vector normal to the two layers, the phases that enter into the shift of the dipolar phase $\Phi(\mathbf{r})$ *are not fully compensated*. This phase shift that enters in the gauge transformation of a nominally charge-neutral dipolar phase, Eq.(2), is crucial for the hydrodynamics of the dipolar superfluid. Now we determine the coupling of the dipolar phase to external gauge potentials in the top and bottom layers, $\mathbf{A}_\pm(\mathbf{r}) = \mathbf{A}(\mathbf{r} \pm \mathbf{d}/2)$. To be consistent with gauge transformation $\mathbf{A}_\pm \to \mathbf{A}_\pm + \nabla\varphi_\pm$ the dipolar phase must transform as $\nabla\Phi \to \nabla\Phi - e(\mathbf{A}_+ - \mathbf{A}_-)$. The gauge potentials in top and bottom layers enter with opposite signs since they couple to oppositely charged electrons and holes respectively, and therefore, in contrast to an ordinary superfluid, *the phase of the dipolar superfluid couples to the difference of the gauge potentials between the two layers.*

The GL energy functional for the dipolar superfluid will depend on the order parameter $\Delta(\mathbf{r})$, and for inhomogeneous state should depend only on the gauge-invariant combinations involving the gradient of the dipolar phase $\Phi(\mathbf{r})$. In particular, the gradient part of free energy will be given by

$$F = \frac{1}{2}\rho_d \int_{\mathbf{r}} \left[\nabla\Phi(\mathbf{r}) - e\mathbf{A}_+(\mathbf{r}) + e\mathbf{A}_-(\mathbf{r})\right]^2 \qquad (3)$$

where $\rho_d$ is the dipolar superfluid density. The currents in the top and bottom layers are given by $\mathbf{J}_\pm(\mathbf{r}) = \pm e\rho_d\left[\nabla\Phi(\mathbf{r}) - e\mathbf{a}(\mathbf{r})\right]$ where $\mathbf{a} \equiv (\mathbf{A}_+ - \mathbf{A}_-)$ is the antisymmetric combination of gauge potentials. The excitonic condensate carries a net dipolar current

$$\mathbf{J}_d(\mathbf{r}) = 2e\rho_d\left[\nabla\Phi(\mathbf{r}) - e\mathbf{a}(\mathbf{r})\right]. \qquad (4)$$

Eq.(4) has exactly the same form as a supercurrent in a superconductor. Therefore, in analogy with a superconductor, we expect persistent dipolar currents produced by the external antisymmetric gauge potential. For a smoothly varying gauge potential, the antisymmetric combination $\mathbf{a}(\mathbf{r}) = \mathbf{A}(\mathbf{r} + \mathbf{d}/2) - \mathbf{A}(\mathbf{r} - \mathbf{d}/2) \approx d\partial_z\mathbf{A}(\mathbf{r})$ can be tuned by varying a uniform in-plane magnetic field. For example, let us consider a uniform magnetic field $\mathbf{B}_{||} = -B_{||}\hat{y}$ between the two layers, generated by gauge potential $\mathbf{A}(\mathbf{r}, z) = (-B_{||}z, 0, 0)$. Such uniform field leads to a dipolar supercurrent given by

$$\mathbf{J}_d = 2e^2\rho_d d B_{||}\hat{x}. \qquad (5)$$

Thus, we predict that *a uniform in-plane magnetic field will induce persistent and opposite currents in the top and the bottom layers in the direction perpendicular to magnetic field.* One can view the dipolar persistent current as arising from "perfect diamagnetism". Indeed, turning on the in-plane magnetic field produces electric fields which are equal and opposite in the two layers. These electric fields accelerate electrons and holes *in the same direction* in respective layers. The phase stiffness of the condensate does not allow the resulting current to decay and gives rise to dipolar supercurrent.

This supercurrent can be detected via separate contacts to the two layers and will provide a *direct signature* of the superfluid properties of excitonic condensates. As a possible realization of this experiment in available samples, we propose the study of induced charges in each layer in response to an *ac* in-plane

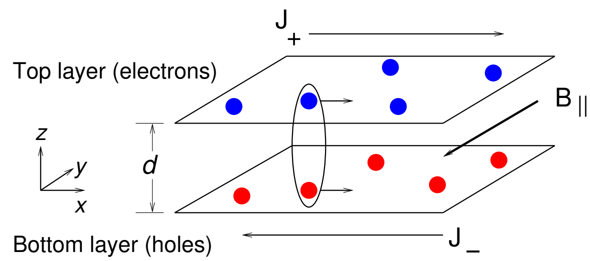


Figure 1: Schematic bilayer electron-hole system. The electrons in the top layer and holes in the bottom layer form excitons which condense at low densities. We predict that an in-plane field $B_{||}$ will produce equal but opposite currents in the two layers, corresponding to a dipolar supercurrent $J_d \propto B_{||}$

magnetic field with frequency $\omega$. In the absence of the excitonic condensate, Faraday induction will lead to a dipolar current $J_d \sim \omega B_{||}$ which in turn will induce equal and opposite charges $Q_\pm \sim \pm B_{||}$ in the two layers. In contrast, in the presence of condensate, the dipolar current will be $J_d \sim B_{||}$ and therefore the induced charges will be given by $Q_\pm \sim \pm B/\omega$, leading to a very different frequency dependence.

Since the dipolar phase couples to the in-plane magnetic field, introducing vortices in the dipolar phase requires gradients in this field over the length-scale $d$ and is necessarily small for externally applied fields. In addition, the zero divergence of the in-plane field necessitates other gradients compensate for the gradients which induce vorticity. Thus creation of vortices in the dipolar phase requires non-trivial texture in the external magnetic field over very short length-scales. In this sense, the dipolar superfluid is robust against creation of vortices (and subsequent destruction of superfluidity) by external magnetic fields. The superfluidity can also be destroyed when the dipolar supercurrent velocity (proportional to the applied field) exceeds the velocity of the collective mode associated with the dipolar phase fluctuations. This criterion gives an estimate for the critical field $B^c$ above which the excitonic condensate is destroyed. For typical system parameters, the critical field is given by $B^c \sim 100$T, much larger than typical values of applied in-plane field, and suggests that dipolar condensates are indeed robust.

# References


[1] A.V. Balatsky, Y.N. Joglekar and P.B. Littlewood, Phys. Rev. Lett. **93**, 266801 (2004) and references therein.



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