Nonconvex Quadratic Cuts for Global Optimization of Mixed Integer Quadratic Programs

Nonconvex quadratic programs (QPs) and mixed-integer quadratic programs (MIQPs) arise in a wide variety of scientific and engineering applications including facility location, quadratic assignment, molecular conformation and max-cut problems. Given their practical importance, these classes of problems have been studied extensively in the literature and are known to be very challenging to solve to global optimality.

In this talk, we introduce a new family of quadratically constrained programming (QCP) relaxations which are derived via nonconvex quadratic cuts. However, the nonconvex cuts in conjunction with equality constraints yield convex QCP relaxations for the MIQP. In order to construct these quadratic cuts, we solve a separation problem involving a linear matrix inequality with a special structure that allows the use of specialized solution algorithms. We investigate the theoretical properties of the proposed relaxations and show that they are an outer-approximation of a semi-infinite convex program which under certain conditions is equivalent to a particular semidefinite program. We implement the new quadratic relaxations in the global optimization solver in BARON. We test our implementation by conducting an extensive computational study on a large collection of problems. Numerical results show that the new quadratic relaxations lead to a significant improvement in the performance of BARON, resulting in a new version of this solver which outperforms other state-of-the-art solvers such as CPLEX and GUROBI for many of our test problems.